A Software of Generating a Symbolic Circuit Model with Computers for Wireless Power Transmission System

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Abstract: Conventional researches on Wireless Power Transmission systems use experimental or numerical method to analyze the systems. However, the situation will be better if symbolic method is available. This paper develops a software which generates a state equation directly from schematics. The software helps engineers that design circuits and obtain characteristics of circuits to analyze the behavior of circuits with circuit parameters by symbolic computing. Our software also provides various perspectives of the system with circuit parameters.

Key–Words: symbolic circuit analysis, modelling, power transfer

1 Introduction

There are numerous number of products powered by electric power in the world, such as mobile phones, cars, computers and televisions — the heart of those products is electric circuits which are proposed, tested, and fabricated by circuit engineers. Nowadays, the design process is dominated by circuit numerical simulators such as SPICE, a de facto standard tool in academia as well as industry. Using SPICE, engineers and designers can analyze the circuits which may contain nonlinear elements such as diodes and transistors, and any complex circuits supposing real electric parts in the time and frequency domains. Circuit engineers design their circuits with deep consideration of how the circuit parameters will affect the outputs of the circuits. Since the consideration sums up the results from each value of parameters, they need to repeat many sets of numerical simulation on SPICE. On the other hand, it is obvious that if their circuit is simple enough to write equations they will be able to understand the whole behavior of the circuit. In this situation, they have symbolic equations which govern the behavior of the circuits in terms of voltages and currents. However, it is also true that such a symbolic analysis is not popular in practical use. One reason for it is that computer aided tools available now with symbolic expressions are less sophisticated than SPICE. A symbolic solution to practical circuits are very difficult to find due to an explosion of computation steps as the number of elements increase. Even if the symbolic solution is found, it tends to be too complicated to be interpret by engineers. Some of these problems have been handled by recent studies[1]-[5], but other problems have been still open, and engineers continue to use SPICE. However, there are some researches on circuits with symbolic analysis[7][8].

The heart of a wireless power transfer system with magnetic resonance is to transmit much electric power with high efficiency to the loads in the secondary from a voltage source in the primary. The system is realized by the circuits which is composed of resistors, inductors, and capacitors, thus, linear circuits. The efficiency and quantity of the transfer power are different among different circuits. Additionally, the way to design the circuits which can transmit the power maximally is not established. Thus we have to design many circuits and compare the circuits from the point of view of efficiency and the quantity of the power.

In this paper, we propose the method to automatically make circuit models which express the wireless power transmission behavior from a primary circuit to a secondary circuit. The model is expressed in so-called state equation fully embedded with symbols of the value of electrical elements as well as voltages and currents in the circuit. Some of conventional software[4][6] does not handle wireless situation. The software which can compute wireless circuit equations are based on transfer functions. Since our algorithm outputs a state equation instead of transfer functions, one has a benefit: we can utilize other sophisticated computation tools seamlessly with our algorithm, and apply theoretical idea arranged and developed on a state equation in other academic field.

2 Proposed Method

In general circuit equations which represent the behavior of provided circuits are composed of KVL (Kirchhoff's Voltage Law), KCL (Kirchhoff's Current Law) and Laws which are voltage and current law of the elements. We consider here that a circuit is composed of resistors, inductors, capacitors and a voltage source. Then circuit equations are written by linear differential equations and linear algebraic equations. The circuit equations contain variables (function of time) – each of voltages and currents –representing circuit states. In these equations, there are *redundant* quantities, voltage and current of resistors, current of capacitors and voltage sources, voltage of inductors and nodes.

We call circuit equations which contain the redundant quantities and redundant circuit equations. Additionally, we call circuit equations which are expressed as a linear combination of only independent quantities *non-redundant* circuit equations. In this paper, we propose a method which is suitable for a computer algorithm and is to transform redundant circuit equations into non-redundant circuit equations systematically.

Linear differential equations in redundant circuit equations are written as

$$z = F\dot{x} \tag{1}$$

where z is a vector which arranges vertically voltage of inductors and current of capacitors. x is a vector which arranges vertically current of inductors and voltage of capacitors. A matrix F is determined by x consistency.

Algebraic equations in redundant circuit equations can be written as (2) after eliminating voltage and current of resistors, current of a voltage source, voltage of nodes.

$$K_1 z + K_2 x + K_3 u = 0 \tag{2}$$

where u is the voltage of a voltage source. The matrices K_1, K_2, K_3 are determined consistently by z, x, u respectively.

We define a matrix Q as the following.

$$Q = \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix} \tag{3}$$

If $\operatorname{rank} Q = r$, then we can transform (2) into the following.

$$z = -(K_1^T K_1)^{-1} K_1^T (K_2 x + K_3 u) \tag{4}$$

Even if $\operatorname{rank} Q \neq r$, we can transform Q into the form $\operatorname{rank} Q = r$ after we eliminate linearly dependent rows of Q.

By using (1), if $\det F \neq 0$, we can rewrite (4) as the following.

$$\begin{aligned} \dot{x} &= -F^{-1}(K_1^T K_1)^{-1} K_1^T (K_2) x \\ &\quad -F^{-1}(K_1^T K_1)^{-1} K_1^T (K_2) u \end{aligned} \tag{5}$$

If we put

$$A = -F^{-1}(K_1^T K_1)^{-1} K_1^T (K_2) \tag{6}$$

$$B = -F^{-1}(K_1^T K_1)^{-1} K_1^T (K_3), \tag{7}$$

then (5) can be rewritten as

$$\dot{x} = Ax + Bu \tag{8}$$

and we obtain non-redundant circuit equations. The matrices A and B contain symbolic inversion of matrices. Generally, larger size matrices require longer time of computation. Thus, we found a way of being illustrated Figure 1 in a process of eliminating voltage and current of resistors, current of a voltage source, voltage of nodes in redundant circuit equations. Hereby, we can perform the computing with a little computational effort.

2.1 Implementation

Our proposed method is described in a flowchart of Figure 1.

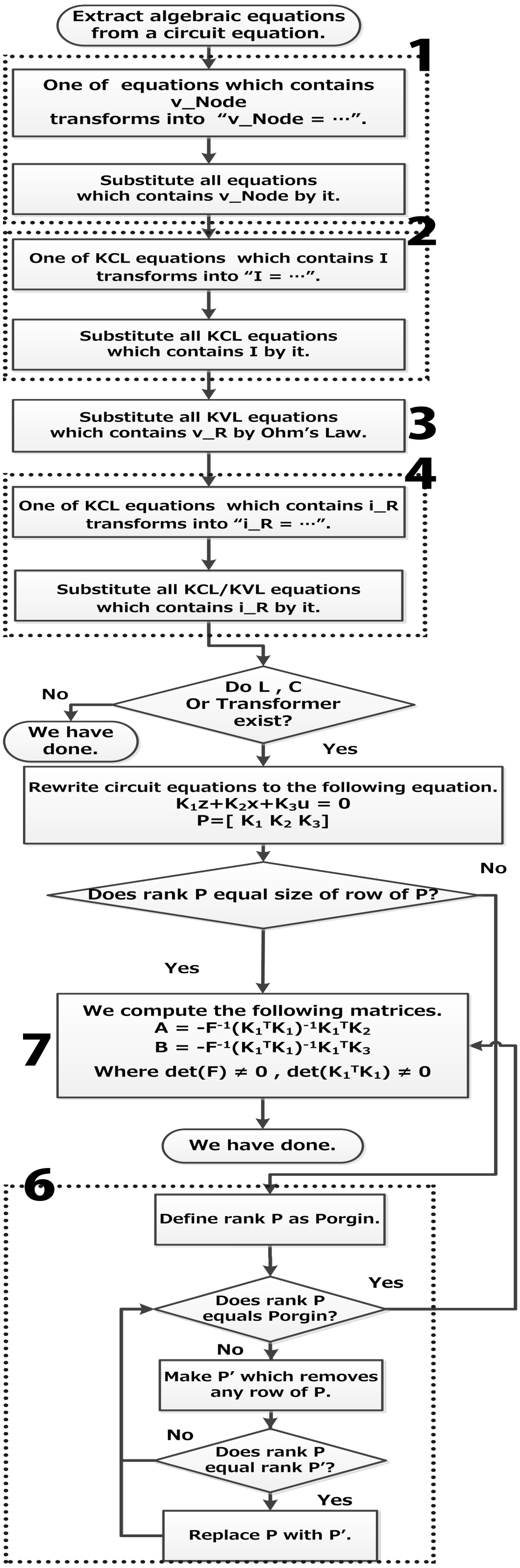

Figure 1: A flowchart of our method

Note v_{Node}, I, v_R, i_R represent voltage of nodes, current of a voltage source, and voltage of resistors and current of resistors respectively.

2.2 An Illustrative Example

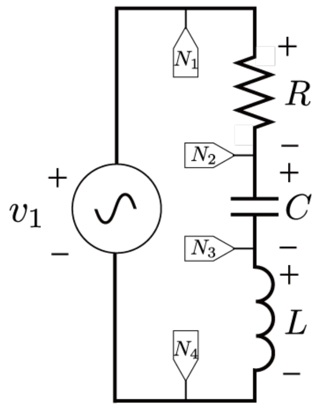

Figure 2: LCR circuit

Table 1: Notation(Figure. 2)

Symbol	Representation
v_*	Voltage of *.
i_*	Current of *. The current direction of * is opposite to its voltage respectively.
N_*	Node name
v_1	Voltage of a voltage source

We apply the proposed method to the circuit shown on Figure 2. The circuit is expressed in equations (9) - (18) with circuit theory.

KVL:

$$v_R = v_{N1} - v_{N2} \tag{9}$$
$$v_L = v_{N2} - v_{N3} \tag{10}$$
$$v_C = v_{N3} - v_{N4} \tag{11}$$
$$v_1 = v_{N1} - v_{N4} \tag{12}$$

KCL:

$$i_{v_1} - i_R = 0 \tag{13}$$
$$i_R - i_L = 0 \tag{14}$$
$$i_L - i_C = 0 \tag{15}$$
$$i_C - i_{v_1} = 0 \tag{16}$$

The others(e.g. Ohm's law):

$$v_R = Ri_R \tag{17}$$
$$z = F\dot{x} \tag{18}$$

where

$$z = \begin{bmatrix} v_L & i_C \end{bmatrix}^T, \ F = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix},$$
$$x = \begin{bmatrix} i_L & v_C \end{bmatrix}^T \tag{19}$$

First, we apply the step 1 in Figure 1 to (9) - (12).
KVL:

$$v_1 = v_R + v_L + v_C \tag{20}$$

Second, we apply the step 2 in Figure 1 to (13) - (16).
KCL:

$$\begin{aligned} i_C - i_R &= 0 &(21)\\ i_R - i_L &= 0 &(22)\\ i_L - i_C &= 0 &(23)\end{aligned}$$

Third, we apply the step 3 in Figure 1 to (20).
KVL:

$$v_1 = Ri_R + v_L + v_C \tag{24}$$

Next, we apply the step 4 in Figure 1 to (20).
KCL:

$$\begin{aligned} i_C - i_L &= 0 &(25)\\ i_L - i_C &= 0 &(26)\end{aligned}$$

KVL:

$$v_1 = Ri_C + v_L + v_C \tag{27}$$

Next, We apply the step 5 in Figure 1 to (25)-(27).

$$P = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & -R & 0 & -1 & 0 \end{bmatrix} \tag{28}$$

Since $\mathrm{rank}\, P = 3 \neq 2$, next we apply the step 6 in Figure 1 to P.

$$P = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ -1 & -R & 0 & -1 & 0 \end{bmatrix} \tag{29}$$

Finally, we can find A, B by calculating the equations shown on the step 7 in Figure 1.

$$A = \begin{bmatrix} -\frac{R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{bmatrix},\ B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \tag{30}$$

where

$$K_1 = \begin{bmatrix} 0 & -1 \\ -1 & -R \end{bmatrix},\ K_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{31}$$

3 A Transformer Model for Wireless Transfer

As a model for receiving and transmitting coils of a wireless power transfer, we consider that there are $(n-1)$ loads in the secondary windings in the model on Figure 3.

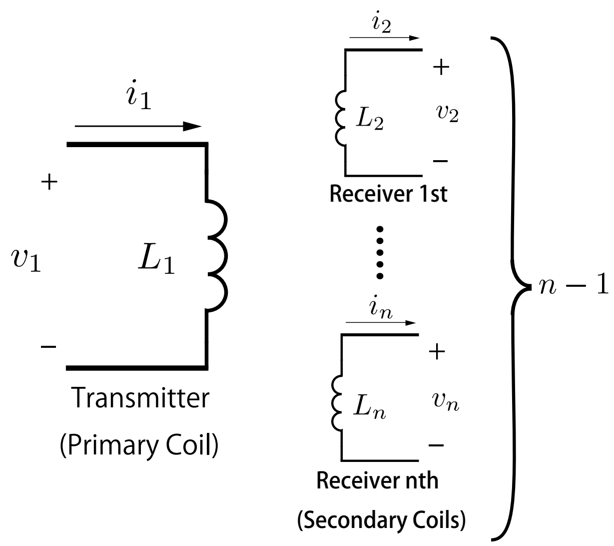

Figure 3: A transformer model for a wireless power transfer system

By Neumann's law, we can write the mutual inductor M in the following equation[10]-[12].

$$M_{ij} = \frac{\mu_0}{4\pi} \oint_{C_i} \oint_{C_j} \frac{dl_i \cdot dl_j}{r_{ij}} \tag{32}$$

where $1 \leq i \leq n, 1 \leq j \leq n$, as $i = j$, M_{ij} represents L_i. C_i and C_j represent the path around Coil i and Coil j respectively. dl_i and dl_j show vectors of Coil i and Coil j respectively. r_{ij} represents the distance between Coil i and Coil j. A mutual inductance M_{ij} is determined by the distance between coils and a shape of a coil(the number of turns) and so on.

We can consider that a model of a transmission coil and a receiver coil is in the case of a variable mutual inductance M_{ij} by a distance between coils and a relative position of a coil in a general electric circuit. Hence we adopt the relation expression between voltage and current in an ideal transformer in the case of the variable mutual inductance M_{ij} as the model for a wireless power transfer.

$$v = H\dot{i} \tag{33}$$

where v, i are $v = \begin{bmatrix} v_1 & v_2 & \ldots & v_n \end{bmatrix}^T$, $i = \begin{bmatrix} i_1 & i_2 & \ldots & v_n \end{bmatrix}^T$, respectively and

$$H =$$

$$\begin{bmatrix} L_1 & \pm M_{12} & \pm M_{13} & \dots & \pm M_{1n} \\ \pm M_{21} & L_2 & \pm M_{23} & \dots & \pm M_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \pm M_{n1} & \pm M_{n2} & \dots & \pm M_{nn-1} & L_n \end{bmatrix}. \quad (34)$$

4 An Example of Analyzing a WPT Circuit with Proposed Algorithm

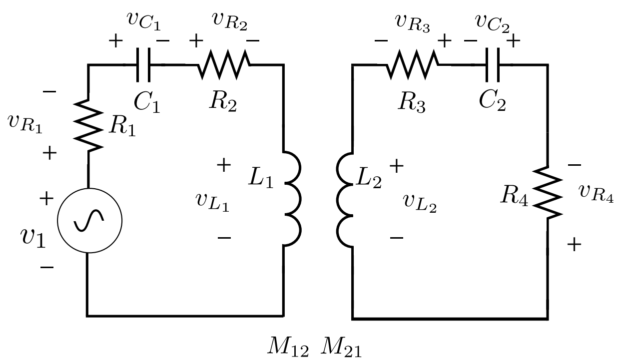

Figure 4: Wireless power transfer circuit

Table 2: Notation(Figure. 4)

Symbol	Representation
v_1	Voltage of a voltage source
$R_1, R_2,$ R_3, R_4	Resistor
L_1, L_2	Coil(Inductor) The winding direction of L_1 is the same as the direction of L_2.
C_1, C_2	Capacitor
v_*	Voltage of *
i_*	Current of * The current direction of * is opposite to its voltage respectively.

We consider a wireless power transfer system shown on Figure 4 which consists of a circuit that is composed of resistors, inductors(including coils), capacitors and a voltage source. Thus, we can make a model by proposed method and by use the model, we can analyze an effect of power transfer by variation of circuit parameters. This modelling is performed by Wasabi that we implement our proposed method as a computer program.

$$\dot{x} = Ax + Bu \quad (35)$$

$$A = \begin{bmatrix} 0 & 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & 0 & \frac{1}{C_2} \\ \frac{-L_2}{\Delta} & \frac{M_{12}}{\Delta} & \frac{(-R_1-R_2)L_2}{\Delta} & \frac{(R_4+R_3)M_{12}}{\Delta} \\ \frac{M_{21}}{\Delta} & \frac{-L_1}{\Delta} & \frac{(R_1+R_2)M_{21}}{\Delta} & \frac{(-R_4-R_3)L_1}{\Delta} \end{bmatrix} \quad (36)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{L_2}{\Delta} \\ \frac{-M_{21}}{\Delta} \end{bmatrix} \quad (37)$$

$$x = \begin{bmatrix} v_{C_1} & v_{C_2} & i_{L_1} & i_{L_2} \end{bmatrix}^T \quad (38)$$

$$u = v_1 \quad (39)$$

where $\Delta = L_2L_1 - M_{12}M_{21}$. The other symbols are represented in Table 2.

In this system, by using symbols that are an average power $\bar{P}_4$ of a load resistor R_4 and an average power $\bar{P}_1$ of a voltage source, including an internal resistance, we can write an efficiency η as the following.

$$\eta = \left| \frac{\bar{P}_4}{\bar{P}_1} \right| \quad (40)$$

If we choose v_1 which is the voltage of a voltage source for $v_1(t) = \sin(\omega t)$, then we can write the efficiency η as

$$\eta = \frac{C_2^2 M_{12}^2 R_4 \omega^4}{\alpha} \quad (41)$$

where

$$\begin{aligned} \alpha \;=\; & |C_2^2 M_{12} M_{21}(R_3 + R_4)\omega^4 \\ & +R_2(1 - 2C_2L_2\omega^2 + \\ & C_2^2\omega^2(R_3^2 + 2R_3R_4 + R_4^2 + L_2^2\omega^2))| \end{aligned} \quad (42)$$

and ω, t are angular frequency, time respectively. Note the obtained equation is complicated, but we can analyze the efficiency by leaving symbolic parameters that we are only interested in. This is possible since the model as expressed in symbolic equations. Next we introduce concretely similar situation through some examples.

4.1 A Case of Only a Variation of M_{12} and M_{21}

By (32), mutual inductances M_{12} and M_{21} vary with a distance between the primary coil L_1 and the secondary coil L_2. Thus, if we would like to analyze that how the distance effects the efficiency, then we regard (40) as a function about the mutual inductances. For example, let $M_{12} = M_{21} = M$ and let M be a variable. We consider the case of varying with $0.01L_1 \leq M \leq 0.1L_1$ by changing the distance.

Table 3: Parameter Value

Element	Value
R_1, R_4	50Ω
R_2	0.03Ω
R_3	0.012Ω
L_1	5μH
L_2	5μH
C_1	286pF
C_2	1.43nF
ω	13Mrad/s

Then by (41), the efficiency $\eta(M)$ is expressed as

$$\eta(M) = \frac{2.92022 \times 10^{12} M^2}{0.0272338 + 2.92092 \times 10^{12} M^2}. \quad (43)$$

A change of the efficiency $\eta(M)$ is shown in Figure 5.

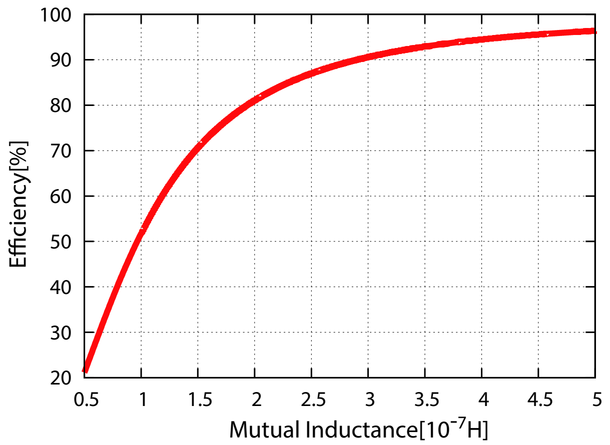

Figure 5: Efficiency $\eta(M)$ $[0.01L_1 \leq M \leq 0.1L_1]$

4.2 A Case of Only a Variation of Angular Frequency ω

Let us be interested in angular frequency ω that maximizes the efficiency η. Then let ω be a variable and $M_{12} = M_{21} = 0.05L_1$, and the other parameters are shown on Table 3. Then the efficiency $\eta(\omega)$ is expressed as

$$\eta(\omega) = \frac{6.39031 \times 10^{-30}\omega^4}{\beta} \quad (44)$$

where

$$\beta = 0.03 - 2.75559 \times 10^{-16}\omega^2 + 7.92552 \times 10^{-30}\omega^4. \quad (45)$$

We plot the efficiency $\eta(\omega)$, $|\bar{P}_4|$, $|\bar{P}_1|$ in $10^5 \leq \omega \leq 10^8$ on Figure 6.

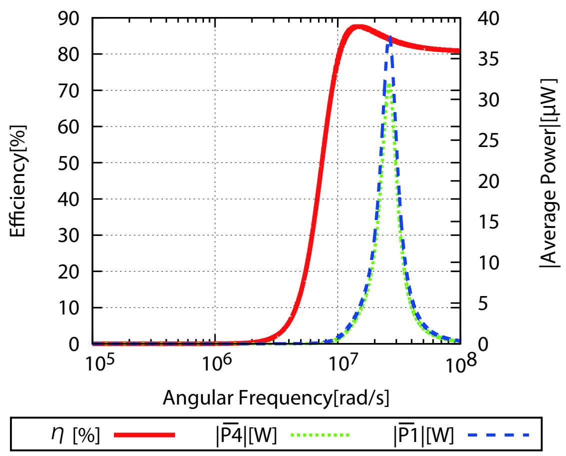

Figure 6: Efficiency and average power $[10^5 \leq \omega \leq 10^8]$

This result shows that the peak frequency of the efficiency, $\omega_1 = 14.8$Mrad/s is different from the peak frequency of the average power, $\omega_2 = 26.3$Mrad/s. Furthermore, the efficiency at ω_1 is lower than one at ω_2, but the average power is greater at ω_2 than one at ω_1.

Generally, the frequency of the power supply on a wireless power transmission system at maximum efficiency point is different from the frequency at the maximum power point[14]. Furthermore, we can find the optimized input as the power supply to improve efficiency and power transmission of the system[15].

4.3 A Variation of the Power Transfer due to Varying the Load

The solution to a state space equation in the form (8) is (46).

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau \quad (46)$$

where e^{At} is the state transition matrix. The first term of (46) represents an initial state response and the second term of it represents a transient response. In general, the more eigenvalues of the matrix A are near the imaginary axis, the more slowly the time of the initial state response converges, and it takes longer to become a steady state. Most studies[9][10][12] view the consumption power of the load if it is fixed, but it is time varying generally, and we choose the frequency of a voltage source which maximize the quantity of transfer power. Thus the system is largely affected by the initial state response. We consider this situation next.

The parameters of the circuit are represented in Table 4. In the circuit, we consider the situation where

Table 4: Parameter Value

Element	Value
L_1, L_2	50mH
R_1	50Ω
R_2	0.03Ω
R_3	0.012Ω
M_{12}, M_{21}	0.05 L_1
C_1	2.86μF
C_2	14.3μF
ω	2.5krad/s

the resistance R_4 changes from $R_4 = 50[\Omega]$ to $R_4 = 49[\Omega]$ at $t = 0.3$[s] where the initial state $x(0) = 0$.

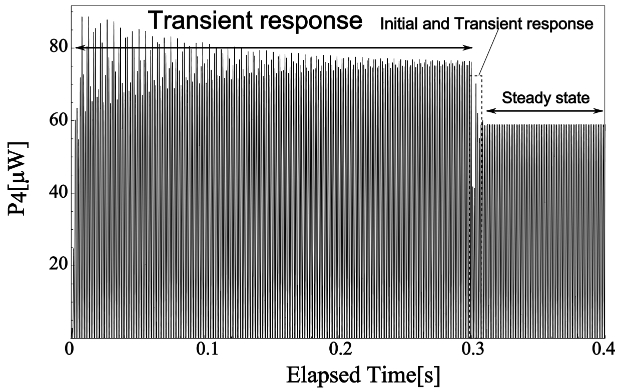

Figure 7: Response of P_4

It takes 0.3s to get the steady state from 0s. At 0.3s, the load resistor R_4 varies to 49Ω, and it takes 0.01s to get the steady state again. After all, in 0.4s, it takes 0.31s and the average power only expresses in the power if a steady state has remained since 0s, thus, there is an extra estimation about the load power.

5 Conclusion

In wireless power transfer, we must search a suitable frequency of a voltage source for high efficiency and high power supplying capability. We have proposed a method of circuit modelling automatically for computing the suitable frequency and the efficiency. We define a model a transfer model between the coils for wireless power transfer and we have been able to analyze more circuit through simplifying the process which makes models for searching high efficiency circuits.

We have shown that we can analyze the behavior of circuits with leaving the symbolic parameters that we interested in since models are expressed in symbolic equations. We have confirmed the achievement of these objects by our proposed method.

In this paper, we have been able to reduce the computing time of analysis of circuits, and analyze circuits if the circuit topology is large and the circuit is composed of many elements, and as a consequence, we have been able to search suitable circuits for wireless power transfer systems more widely.

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