

Ant Colony Optimization for Optimal Low-Pass Butterworth Filter Design

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Abstract: - Performance analog filters depend strongly values of discrete components (Resistors and capacitors) which are selected from the series following constant values chosen. Exhaustive search on all possible combinations of value choices for an optimized design is not feasible. The metaheuristics have proved a capacity to treat such problem effectively. In this paper, we present an optimization algorithm based on the Ant Colony Optimization (ACO) technique for dealing with the active filter design. The ACO algorithm is applied to the Low-Pass Butterworth Filter Design realized with components selected from different manufactured series. SPICE simulations are used to validate the obtained result/performances. A comparison with published works is highlighted.

Key-Words: - Metaheuristic, Ant Colony Optimization, Low-pass Butterworth Filter.

1 Introduction

Despite the wide use of integrated circuits, discrete components are always preferred in analog design, particularly in the active filters where passive components used are chosen with equal values. This approach simplifies the design process, but also limits the freedom of design.

Otherwise, the components are supposed to be having the exact values for the analog design process. However, discrete components such as resistors and capacitors are produced according to a series of values constants such as the series: E12, E24, E48, E96 or E192. To reduce costs and make the design faster, discrete components are selected from the industrial series. An exhaustive search of all possible combinations of values for selection of an optimized design is not always possible. Consequently, an intelligent search method requires short computation time with high precision, must be used.

Some (meta-)heuristics and algorithms are also proposed in the literature and are used by the designers, such as Particle Swarm Optimization (PSO) and finite element method [1,2], Genetic Algorithm (GA) [3], artificial bee colony (ABC) [4], Genetic Programming (GP) [5], Tabu search TS [6], artificial immune algorithm (AIA) [7].etc

Recently, an important interest has been paid to the Ant Colony Optimization which is inventive, resourceful and efficient algorithm. The ACO

technique's basic idea is to imitate the cooperative behavior of ant colonies in order to solve combinatorial optimization problems within an acceptable amount of time. Ant Colonies (AC) is a general purpose heuristic (meta-heuristic) that has been proposed by Dorigo *et al.* in [8,9].

In our previous works, we have presented, successfully, the ACO technique to deal with analog circuits design and sizing [10-14], it consist of finding the more convenient transistors sizes, including the channel widths and lengths, in order to meet or reach the specified requirements.

In this work, we focus on the use of the ACO technique to solve a typical analog circuit sizing problem: a Low-Pass Butterworth Filter Design mainly composed of discrete elements (capacitors and resistors) which are to choose among industrial series to comply with clearly defined specifications. SPICE software was used for performing simulations in order to check reached performances. The comparison with other techniques shows the ability of the ACO to achieve better results.

The remainder of the paper is structured as follows: The second section presents an overview of the used algorithms. The third section illustrates the application example and the fourth section, deals with the simulations and comparison results. The final section is devoted to some concluding remarks.

2 Ant colony optimization ACO

ACO technique is inspired by the collective behavior of deposit and monitoring of slopes that is observed in insects colonies [15], such as ants. Figure 1 shows an illustration of the ability of ants to find the shortest path between food and their nest.

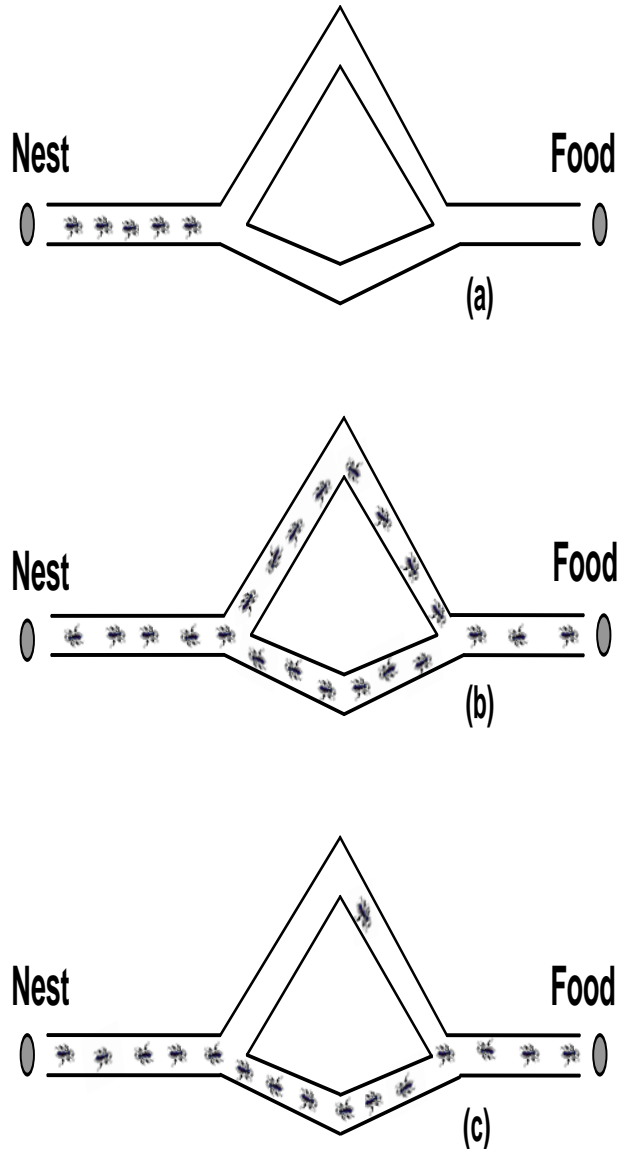


Fig. 1 Self-adaptive behavior of a real ant colony

- (a) Ants go in search of food;
- (b) Ants follow a path between nest and food source. They; choose, with equal probability, whether to shortest or longest path;
- (c) The majority of ants have chosen the shortest path.

It is illustrated through the example of the appearance of an obstacle on their path. Ants communicate indirectly through dynamic changes in their environment (pheromone trails). Pheromones are chemical substances that are laid down by ants. Thus, when other ants find the path taken by the

former ant, they are no more likely to ‘walk randomly’, but instead they follow the trail and reinforce it if they eventually find food [16].

ACO was initially used to solve graph related problems, such as the traveling salesman problem (TSP) [15], vehicle routing problem [16]... For solving such problems, ants randomly select the vertex to be visited. When ant k is in vertex i , the probability of going to vertex j is given by expression (1) [17,18].

$$P_{ij}^k = \begin{cases} \frac{(\tau_{ij})^\alpha \cdot (\eta_{ij})^\beta}{\sum_{l \in J_i^k} (\tau_{il})^\alpha \cdot (\eta_{il})^\beta} & \text{if } j \in J_i^k \\ 0 & \text{if } j \notin J_i^k \end{cases} \quad (1)$$

where J_i^k is the set of neighbors of vertex i of the k^{th} ant, τ_{ij} is the amount of pheromone trail on edge (i, j) , α and β are weightings that control the pheromone trail and the visibility value, i.e. η_{ij} , which expression is given by (2).

$$\eta_{ij} = \frac{1}{d_{ij}} \quad (2)$$

d_{ij} is the distance between vertices i and j .

The pheromone values are updated each iteration by all the m ants that have built a solution in the

iteration itself. The pheromone τ_{ij} , which is associated with the edge joining vertices i and j , is updated as follows:

$$\tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \sum_{k=1}^m \Delta \tau_{ij}^k \quad (3)$$

where ρ is the evaporation rate, m is the number of ants, and $\Delta \tau_{ij}^k(t)$ is the quantity of pheromone laid on edge (i, j) by ant k :

$$\Delta \tau_{ij}^k = \begin{cases} \frac{Q}{L^k} & \text{if ant } k \text{ used edge } (i, j) \\ & \text{in its tour,} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Q is a constant and L^k is the length of the tour constructed by ant k .

The pseudo code of the ACO procedure is as follows:

```

Random initialization of the pheromone
value
Do
  For each iteration
    For each ant
      Compute of the probability P
      according to (1)
      Determine the Pmax
    End
    Compute OF
  End
  Deduce the best OF
  Update pheromone values
  according to (3)
End
Report the best solution
END
    
```

Algorithm 1. Pseudo code of ACO

3 Application to the Optimal Design of the Butterworth low pass filter

Analog active filters are extensively used in the separation and demodulation of signals, frequency selection decoding, estimation of a signal from noise; they are the key components in mixed-signal circuit design [19].

The considered circuit is a fourth order low pass Butterworth filter formed by two operational amplifiers, four resistors and four capacitors. The schematic of this filter is given in Figure 2.

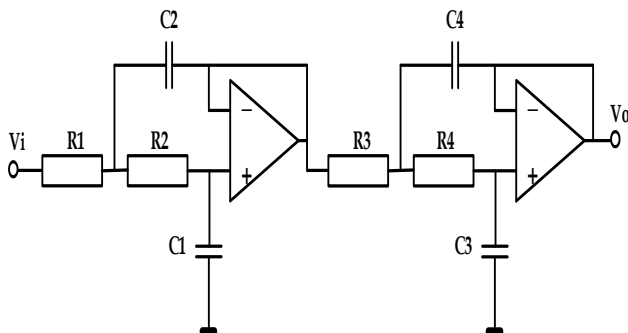


Fig. 2 Butterworth fourth order low-pass filter

The transfer function of this filter can be obtained as follows:

$$H(s) = \frac{\omega_{c1}^2}{s^2 + \frac{\omega_{c1}}{Q_1}s + \omega_{c1}^2} \times \frac{\omega_{c2}^2}{s^2 + \frac{\omega_{c2}}{Q_2}s + \omega_{c2}^2} \quad (5)$$

$$H(s) = \frac{1}{s^2 R_1 R_2 C_1 C_2 + s(R_1 C_1 + R_2 C_1) + 1} \times \frac{1}{s^2 R_3 R_4 C_3 C_4 + s(R_3 C_3 + R_4 C_3) + 1} \quad (6)$$

The cutoff frequency (ω_{c1}, ω_{c2}) and the selectivity factor (Q_1, Q_2) of filter, which depend only on the values of the passives components, are given as follows:

$$\omega_{c1} = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad (7)$$

$$\omega_{c2} = \frac{1}{\sqrt{R_3 R_4 C_3 C_4}} \quad (8)$$

$$Q_1 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 C_1 + R_2 C_1} \quad (9)$$

$$Q_2 = \frac{\sqrt{R_3 R_4 C_3 C_4}}{R_3 C_3 + R_4 C_3} \quad (10)$$

For comparison reasons, the specification chosen here is [20]:

- $\omega_{c1} = \omega_{c2} = 10\,000$ rad/s (1591.55 Hz)
- $Q_1 = 0.7654$
- $Q_2 = 1.8478$

The values of the resistors and capacitors to choose must be able to generate $\omega_{c1}, \omega_{c2}, Q_1$ and Q_2 approaching the specified values. For this, we define the Total Error (TE) [20] which expresses the offset values, of the cut-off frequency and the selectivity factor, compared to the desired values, by:

$$TE = 0.5\Delta\omega + 0.5\Delta Q \quad (11)$$

were

$$\Delta\omega = \frac{|\omega_{c1} - \omega| + |\omega_{c2} - \omega|}{\omega} \quad (12)$$

and

$$\Delta Q = \left| Q_1 - \frac{1}{0.7654} \right| + \left| Q_2 - \frac{1}{1.8478} \right| \quad (13)$$

The objective function considered is the Total Error (13). The decision variables are the resistors and capacitors forming the circuit. Each component must have a value of the standard series (E12, E24, E48, E96, E192). The resistors have values in the range of 10^3 to $10^6\Omega$., similarly, each capacitor must have a value in the range of 10^{-9} to $10^{-6}F$.

$$TE = \left(0.5 \left(\left| \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} - \omega \right| + \left| \frac{1}{\sqrt{R_3 R_4 C_3 C_4}} - \omega \right| \right) + 0.5 \left(\left| \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 C_1 + R_2 C_1} - \frac{1}{0.7654} \right| + \left| \frac{\sqrt{R_3 R_4 C_3 C_4}}{R_3 C_3 + R_4 C_3} - \frac{1}{1.8478} \right| \right) \right) \tag{13}$$

4 Simulations and comparison results

In this section we applied ACO algorithm to perform optimization of a low-pass Butterworth fourth order filter.

The studied algorithm parameters are given in Table 1 with a generation algorithm of 1000. The optimization techniques work on MATLAB codes and are able to link SPICE to measure performances.

Table 1. The ACO algorithm parameters

Evaporation rate (ρ)	0.1
Quantity of deposit pheromone (Q)	0.2
Pheromone Factor (α)	1
Heuristics Factor (β)	1

The optimal values of resistors and capacitors forming the considered filter and the performance associated with these values for the different series are shown in Tables 2.

Table 2. Values of components and related filter performances

	E12	E24	E48	E96	E192
R1 (K Ω)	4.70	4.70	4.64	4.64	4.70
R2 (K Ω)	4.70	5.10	4.87	4.99	4.99
C1 (nF)	8.20	8.20	7.87	7.87	7.87
C2 (nF)	56.0	56.0	53.6	53.6	54.2
R3 (K Ω)	1.00	1.00	1.00	1.00	1.00
R4 (K Ω)	2.20	2.40	2.37	2.32	2.34
C3 (nF)	56.0	56.0	56.2	54.9	54.9
C4 (nF)	82.0	75.0	78.7	78.7	77.7
$\Delta\omega$	0.0122	0.0508	0.0475	0.0131	0.0011
ΔQ	0.0198	0.0148	0.0026	0.0106	0.0087
TE	0.0160	0.0328	0.0251	0.0118	0.0049

Notice that the selection of the optimal own parameters is very crucial on minimizing total design error value thus effecting filter performance. Indeed, the values of the E192 series are the smallest total error compared to other series.

The ACO average computing time equals 98s¹ due to the search of exact component values compatible with each series.

In order to check the validity of the results, the following figure shows the PSPICE simulation in the filter gain for the optimal values of the E192 series. The practical cut off frequency are equal to 1593 Hz.

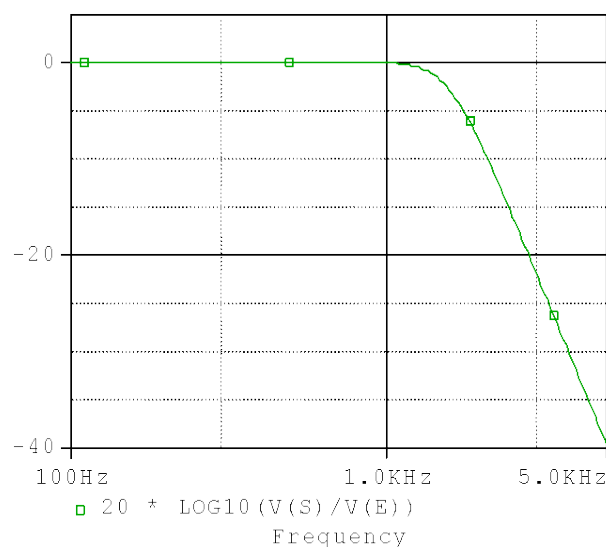


Fig. 3 Frequency responses of low-pass filter

The following table shows the comparison between the theoretical values and those practices for the error on the cut-off frequency for different series.

Table 3: Comparisons between the theoretical and practices for the error on the cut-off frequency

	$\Delta\omega$ theoretical	$\Delta\omega$ practical
E12	0.0122	0.0182
E24	0.0508	0.0573
E48	0.0475	0.0516
E96	0.0131	0.0207
E192	0.0011	0.0018

¹ An Intel Core 2 Duo CPU T5800 - 2.00GHZ was used.

From the results presented in Table 3, we notice that simulation results are in good agreement with those obtained using the ACO algorithm. The slight difference between the two values is mainly due to imperfections of the op-amp which are considered perfect in the theoretical calculations.

Exact values of discrete components, deviations, and Total Error of GA, PSO, ABC-based design and the ACO method are tabulated in Table 4. A comparison between these four techniques, shows that the ACO algorithm achieved a smaller design error.

One can also notice that the techniques of the intelligence swarms (ABC, PSO and ACO) provide better results than those achieved by the Evolutionary algorithm (GA).

Table 4: Component Values and Performance of GA, ABC, PSO and ACO Techniques for Butterworth Filter Design

	GA[20]	ABC[20]	PSO[21]	ACO
R1 (K Ω)	6.80	4.70	4.58	4.70
R2 (K Ω)	6.80	4.70	4.70	4.99
C1 (nF)	5.60	8.20	8.20	7.87
C2 (nF)	39.0	56.0	56.0	54.2
R3 (K Ω)	39.0	1.00	1.10	1.00
R4 (K Ω)	1.00	39.0	1.00	2.34
C3 (nF)	4.70	4.70	87.6	54.9
C4 (nF)	56.0	56.0	102.2	77.7
$\Delta\omega$	0.0179	0.0201	0.0135	0.0011
ΔQ	0.0153	0.0024	0.0018	0.0087
TE	0.0166	0.0113	0.0076	0.0049

In order to check the convergence rate of the proposed algorithm, a robustness test was performed. *i.e.* the algorithm was applied several times for optimizing each objective function ($\Delta\omega$, ΔQ and TE). In Figure 4 we present, for hundred generations, the obtained results which relates the deviation errors for each objective with E192 series.

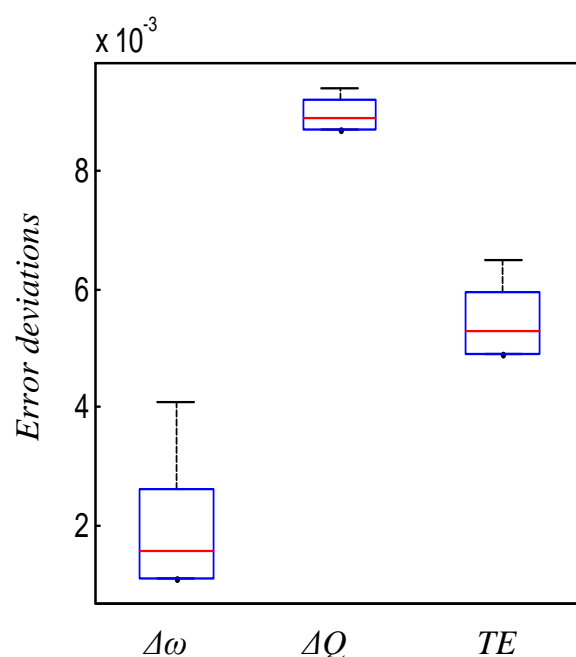


Fig. 4 Box plots $\Delta\omega$, ΔQ and TE performances for Butterworth filter design with E192 series for 100 runs of the ACO

The good convergence ratio to the (same) respective 'optimal' value can be easily noticed, despite the probabilistic aspect of the ACO algorithm.

5 Conclusion

We presented in this paper an application of the Ant Colony Optimization technique for optimal design of a fourth order Butterworth low pass analog filter. we investigated for the selection of passive components from different manufactured series. The design of the analog filter with high accuracy and short execution time is successfully realized using the ACO method. We have shown that the tolerances of the spacing values associated to the components affect significantly the performance of the filter.

SPICE simulation confirms the validity of the proposed algorithm. The comparison, with already published works, showed that the algorithm ACO achieved the best performance in terms of accuracy of results.

Now, we are focusing on applying the proposed ACO technique to dealing with the optimal design of complex analog circuits which include the CMOS transistor sizing and the selection of discrete components.

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