A New Fuzzy Positive and Negative Ideal Solution for Fuzzy TOPSIS

MEHDI AMIRI-AREF, NIKBAKHSH JAVADIAN, MOHAMMAD KAZEMI
Department of Industrial Engineering
Mazandaran University of Science & Technology
Azadi Ave, Babol, PO Box 734
IRAN
{m.a.aref, nijavadian, m.kazemi}@ustmb.ac.ir

Abstract: - This paper presents a fuzzy multiple criteria decision making (FMCDM) problem with the Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) based on the new concept of positive and negative ideal solution. Whereas decision making is a process which accuracy of data play major role to select the best alternative, considering decision making problems in fuzzy environments motivate the authors to search in this field. Triangular fuzzy numbers (TFN) among decision making process are used to evaluate the weighted different alternatives versus various criteria and a fuzzy group weight is made by different experts. In this paper, additionally, a new fuzzy distance formula is applied to compute distance between each alternative and positive as well as negative ideal solution. There is a flexibility to consider general fuzzy numbers (such as triangular, trapezoidal, interval), though. Then, new fuzzy TOPSIS to determine the ranking order of the alternatives is also presented. Finally, a numerical example from the literature is solved to demonstrate applicability of the proposed model. The comparison of illustrated algorithm with the three methods in the literature on various examples proved the efficiency of our decisions.
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Key-Words: - TOPSIS, Fuzzy Numbers, Fuzzy Distance, Positive Ideal Solution, Negative Ideal Solution.

1 Introduction

Decision-making is the procedure to find the best alternative among a set of feasible alternatives. Sometimes, decision-making problems considering several criteria are called multi-criteria decision-making (MCDM) problems [2,4] and often require the decision makers to provide qualitative/quantitative assessments for determining the performance of each alternative with respect to each criterion, and the relative importance of evaluation criteria with respect to the overall objective of the problems. So, Multi-criteria decision making (MCDM) refers to screening, prioritizing, ranking, or selecting a set of alternatives (also referred to as ''candidates'' or ''actions'') under usually independent, incommensurate or conflicting criteria [5]. These problems will usually result in uncertain, imprecise, indefinite and subjective data being present, which makes the decision-making process complex and challenging. In other words, decision-making often occurs in a fuzzy environment where the information available is imprecise/ uncertain. Therefore, the application of fuzzy set theory to multi-criteria evaluation methods under the framework of utility theory has proven to be an effective approach [9]. The overall utility of the alternatives with respect to all criteria is often

represented by a fuzzy number, which is named the fuzzy utility and is often referred to by fuzzy multi-criteria evaluation methods. The ranking of the alternatives is based on the comparison of their corresponding fuzzy utilities [10]. The technique for order preference by similarity to ideal solution (TOPSIS) proposed is one of the well-known methods for classical MCDM. The underlying logic of TOPSIS is to define the ideal solution and negative ideal solution. The ideal solution is the solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution is the solution that maximizes the cost criteria and minimizes the benefit criteria. In short, the ideal solution consists of all of best values attainable of criteria, whereas the negative ideal solution is composed of all worst values attainable of criteria. The optimal alternative is the one which has the shortest distance from the ideal solution and the farthest distance from the negative ideal solution. Since TOPSIS is a well-known method for classical MCDM, many researchers have applied TOPSIS to solve FMCDM and FMCGDM problems in the past with different approaches [4]. Because of different comments of different experts for weighting the criteria a fuzzy group weight is needed.

In fuzzy TOPSIS, in addition, the technique of positive and negative ideal solution is easily used to find the best alternative, considering that the chosen alternative should simultaneously have the shortest distance from the positive ideal point and the longest distance from the negative ideal point [6, 12]. The positive ideal solution is composed of all best criteria values attainable, and the negative ideal solution is composed of all worst criteria values attainable. This technique can also obtain the gap between the ideal alternative and each alternative, and the ranking order of alternatives, so it can be used widely in many fields. Extension the TOPSIS for group decision-making in a fuzzy environment [2] and incorporation the fuzzy set theory and the basic concepts of positive and negative ideal to expand multi-criteria decision-making in a fuzzy environment [4] and fuzzy pair wise comparison and the basic concepts of positive ideal and negative ideal points to expand multi-criteria decision-making in a fuzzy environment [13]. A fuzzy multi-criteria decision-making method based on concepts of positive ideal and negative ideal points to evaluate bus companies' performance is also proposed [7].

The main aim of this paper is to extend the TOPSIS in the fuzzy environment. Considering the fuzziness in the decision data and group decision-making process, linguistic variables are used to assess the weights of all criteria and the ratings of each alternative with respect to each criterion. According to the concept of TOPSIS, we define the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS). And then, in this paper, a method to calculate the distance between two generalized fuzzy numbers is used.

Our motivation to define new FPIS and FNIS is to present a more reliable and easier way which guarantees that the preferred alternative is closer to the positive ideal solution and farther from the final negative ideal solution. As a result, a compromise satisfactory solution can be found, so the closeness coefficient value of each alternative for the positive ideal solution and negative ideal solution can also be considered, while maintaining the objectivity with regard to the criteria of ups and downs of alternatives. Therefore, according to the closeness coefficient values, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives. The results show that our proposed method can be implemented as an effective decision aid in decision-making problems. As a matter of fact, both, reliability, because of calculating in approximately all parts of algorithm in fuzzy environments and easiness,

because of using data from available numbers and generating FPIS and FNIS so fast are the differences between our approach and other related works which are done in past. In other word, these are the advantages of our proposed methodology.

2 Fuzzy numbers and linguistic variables

The representation of multiplication operation on two or more fuzzy numbers is a useful tools for decision makers in the fuzzy multiple criteria decision-making environment for ranking all the candidate alternatives and selecting the best one.

In this section, some basic definitions of fuzzy sets, fuzzy numbers and linguistic variables are reviewed from Buckley [14] , Kaufmann and Gupta [15].

Definition 1. A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval [0, 1]. The function value $\mu_{\tilde{A}}(x)$ is termed as the grade of membership of x in $\tilde{A}$.

Definition 2. A triangular fuzzy number $\tilde{A}$ can be defined by a triplet (a_1, a_2, a_3). Its conceptual schema and mathematical form are shown by Eq. (1):

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 < x < a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 < x < a_3 \\ 0, & a_3 < x \end{cases} \quad (1)$$

A triangular fuzzy number $\tilde{A}$ in the universe of discourse X that conforms to this definition is shown in Fig. 1.
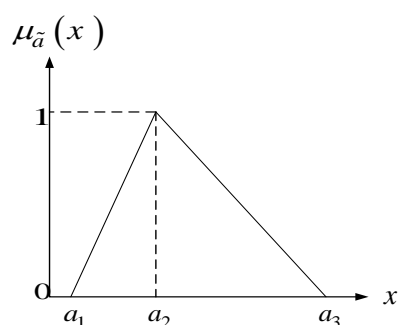

Fig. 1. A triangular fuzzy number $\tilde{A}$

Definition 3. A trapezoidal fuzzy number $\tilde{A}$ can be defined by a quadruplet (a_1, a_2, a_3, a_4). Its

conceptual schema and mathematical form are shown by Eq. (2):

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 < x < a_2 \\ 1, & a_2 < x < a_3 \\ \frac{a_3-x}{a_3-a_4}, & a_2 < x < a_3 \\ 0, & a_3 < x. \end{cases} \qquad (2)$$

A trapezoidal fuzzy number $\tilde{A}$ in the universe of discourse X that conforms to this definition is shown in Fig. 2.

Definition 4. The α_cut $\tilde{A}_\alpha$, and strong α_cut $\tilde{A}_{\alpha^+}$ of the fuzzy set $\tilde{A}$ in the universe of discourse X is defined by:

$$\tilde{A}_\alpha = \{x|\mu_{\tilde{A}}(x) \geq \alpha, x \in X\}, \alpha \in [0,1], \qquad (3)$$
$$\tilde{A}_{\alpha^+} = \{x|\mu_{\tilde{A}}(x) > \alpha, x \in X\}, \alpha \in [0,1]. \qquad (4)$$

The lower and upper points of any α_cut $\tilde{A}_\alpha$ are represented by $inf\tilde{A}_\alpha$ and $sup\tilde{A}_\alpha$, respectively, and we suppose that both are finite. For convenience, we denote $inf\tilde{A}_\alpha$ by $\tilde{A}_{\alpha^-}$ and $sup\tilde{A}_\alpha$ by $\tilde{A}_{\alpha^+}$ (see Fig. 3).
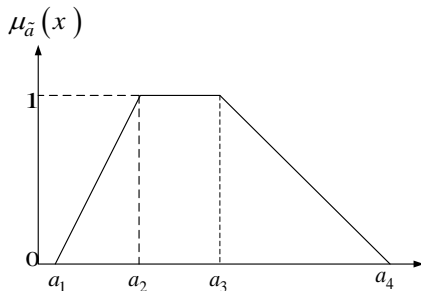

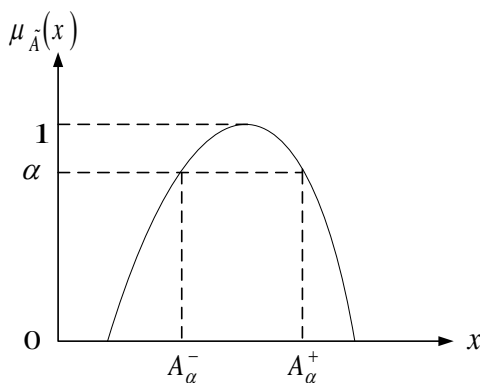**Fig. 2.** A trapezoidal fuzzy number $\tilde{A}$.

Fig. 3. An example of an α_cut.

Definition 5. Assuming that both $\tilde{A}$ and $\tilde{B}$ are fuzzy numbers and $\lambda \in \mathbb{R}$, the notions of fuzzy sum, $\oplus$, fuzzy product by a real number, $\cdot$, and fuzzy product, $\otimes$, are defined as follows [13]:

$$\mu_{(\tilde{a}\oplus\tilde{b})}(z) = \sup\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)) : (x,y) \in \mathbb{R}^2 \text{ and } x + y = z\},$$

$$(\lambda \cdot \tilde{a})(z) = \begin{cases} \tilde{a}\left(\frac{z}{\lambda}\right), & \lambda \neq 0 \\ I_{\{0\}}(z), & \lambda = 0 \end{cases}$$

where $I_{\{0\}}(z)$ is the indicator function of ordinary set $\{0\}$, and

$$\mu_{(\tilde{a}\otimes\tilde{b})}(z) = \sup\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)) : (x,y) \in \mathbb{R}^2 \text{ and } x \times y = z\}$$

Let $\tilde{A}$ and $\tilde{B}$ be two positive fuzzy numbers and $\alpha \in [0,1]$. The basic operations on positive fuzzy numbers with α_cut operator are as follows:

$$(\tilde{a}\oplus\tilde{b})_\alpha = [a_\alpha^- + b_\alpha^-, a_\alpha^+ + b_\alpha^+]$$
$$(\tilde{a}\otimes\tilde{b})_\alpha = [a_\alpha^- \times b_\alpha^-, a_\alpha^+ \times b_\alpha^+]$$

and if $\lambda \in \mathbb{R} \setminus \{0\}$, then we have: $(\lambda \cdot \tilde{a})_\alpha = \lambda a_\alpha$, namely,

$$(\lambda \cdot \tilde{a})_\alpha = [\lambda a_\alpha^-, \lambda a_\alpha^+], \quad \text{if } \lambda > 0,$$
$$(\lambda \cdot \tilde{a})_\alpha = [\lambda a_\alpha^+, \lambda a_\alpha^-], \quad \text{if } \lambda < 0.$$

Definition 6. A linguistic variable is a variable the values of which are linguistic terms. Linguistic terms have been found intuitively easy to use in expressing the subjectiveness and/or qualitative imprecision of a decision maker's assessments [8].

Definition 7. A fuzzy MCDM problem with m alternatives and n criteria can be concisely expressed in a fuzzy decision matrix format as:

$$\widetilde{D} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \tilde{x}_{13} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \tilde{x}_{23} & \dots & \tilde{x}_{2n} \\ \tilde{x}_{31} & \tilde{x}_{32} & \tilde{x}_{33} & \dots & \tilde{x}_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \tilde{x}_{m3} & \dots & \tilde{x}_{mn} \end{bmatrix},$$

$$\widetilde{W} = [\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n]$$

where $\tilde{x}_{ij}, (i = 1,\dots,m, j = 1,\dots,n)$, and $\tilde{w}_j, (j = 1,\dots,n)$, are linguistic fuzzy numbers. Note that $\tilde{w}_j$ represents the weight of the jth criterion, $\tilde{C}_j$ and $\tilde{x}_{ij}$ is the performance rating of the ith alternative, A_i, with respect to the jth criterion, C_j, evaluated by k evaluators. We applies the method of average value to integrate the fuzzy performance score $\tilde{x}_{ij}$ for k evaluators concerning the same evaluation criteria, that is,

$$\bar{\tilde{x}}_{ij} = \frac{1}{k} \cdot (\tilde{x}_{ij}^1 \oplus \tilde{x}_{ij}^2 \oplus \dots \oplus \tilde{x}_{ij}^k), \qquad (6)$$

where $\tilde{x}_{ij}^p$ is the rating of alternative $\tilde{A}_i$ with respect to criterion C_j evaluated by pth evaluator. The weighted fuzzy decision matrix is:

$$\tilde{V} = \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \tilde{v}_{13} & \dots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \tilde{v}_{23} & \dots & \tilde{v}_{2n} \\ \tilde{v}_{31} & \tilde{v}_{32} & \tilde{v}_{33} & \dots & \tilde{v}_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \tilde{v}_{m3} & \dots & \tilde{v}_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} \tilde{w}_1 \otimes \tilde{x}_{11} & \tilde{w}_2 \otimes \tilde{x}_{12} & \dots & \tilde{w}_j \otimes \tilde{x}_{1j} & \dots & \tilde{w}_n \otimes \tilde{x}_{1n} \\ \tilde{w}_1 \otimes \tilde{x}_{21} & \tilde{w}_2 \otimes \tilde{x}_{22} & \dots & \tilde{w}_j \otimes \tilde{x}_{2j} & \dots & \tilde{w}_n \otimes \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{w}_1 \otimes \tilde{x}_{i1} & \tilde{w}_2 \otimes \tilde{x}_{i2} & \dots & \tilde{w}_j \otimes \tilde{x}_{ij} & \dots & \tilde{w}_n \otimes \tilde{x}_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{w}_1 \otimes \tilde{x}_{m1} & \tilde{w}_2 \otimes \tilde{x}_{m2} & \dots & \tilde{w}_j \otimes \tilde{x}_{mj} & \dots & \tilde{w}_n \otimes \tilde{x}_{mn} \end{bmatrix}$$

(7)

Definition 8. A fuzzy set $A = (a_1, a_2, a_3, a_4)$ is called a generalized left right fuzzy number (GLRFN) if its membership function satisfies the following:

$$\mu(x) = \begin{cases} L\left(\dfrac{a_2 - x}{a_2 - a_1}\right) & for \quad a_1 \le x \le a_2 \\ 1 & for \quad a_2 \le x \le a_3 \\ R\left(\dfrac{x - a_3}{a_4 - a_3}\right) & for \quad a_3 \le x \le a_4 \\ 0 & else \end{cases}$$

where L and R are strictly decreasing function defined on $[0,1]$ and satisfying the conditions:

$L(x) = R(x) = 1 \ \ if \ \ x \le 0,$
$L(x) = R(x) = 0 \ \ if \ \ x \ge 1.$

For $a_2 = a_3$, we have the classical definition of left right fuzzy numbers (LRFN) of Dubios and Prade. Trapezoidal fuzzy numbers (TrFN) are special cases of GLRFN with $L(x) = R(x) = 1 - x$. Triangular fuzzy numbers (TFN) are also special cases of GLRFN with $L(x) = R(x) = 1 - x$ and $a_2 = a_3$.

Definition 9. Distance measure between two interval numbers $A: (a_1, a_2)$ and $B: (b_1, b_2)$ give as follow [16]:

$$D^2(A, B) = \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} \left\{ \left[\left(\frac{a_1 + a_2}{2}\right) + x(a_2 - a_1)\right] - \left[\left(\frac{b_1 + b_2}{2}\right) + x(b_2 - b_1)\right] \right\}^2 dx\, dy = \left[\left(\frac{a_1 + a_2}{2}\right) - \left(\frac{b_1 + b_2}{2}\right)\right]^2 + \frac{1}{3}\left[\left(\frac{a_2 - a_1}{2}\right)^2 + \left(\frac{b_2 - b_1}{2}\right)^2\right].$$
(9)

Definition 10. Distance between two generalized left right fuzzy numbers (GLRFNs) A and B give as follow [16]:

$$D^2(A, B, f)$$
$$= \int_0^1 \left\{ \left(\frac{a_2 + a_3}{2} - \frac{b_2 + b_3}{2}\right)^2 \right.$$
$$+ \left(\frac{a_2 + a_3}{2} - \frac{b_2 + b_3}{2}\right)[(a_4 - a_3)R_A^{-1}(\propto)$$
$$- (a_2 - a_1)L_A^{-1}(\propto) - (b_4 - b_3)R_B^{-1}(\propto)$$
$$- (b_2 - b_1)L_B^{-1}(\propto)] + \frac{1}{3}\left(\frac{a_3 - a_2}{2}\right)^2 + \frac{1}{3}\left(\frac{a_3 - a_2}{2}\right)$$
$$\times [(a_4 - a_3)R_A^{-1}(\propto) + (a_2 - a_1)L_A^{-1}(\propto)]$$
$$+ \frac{1}{3}\left(\frac{b_3 - b_2}{2}\right)^2 + \frac{1}{3}\left(\frac{b_3 - b_2}{2}\right)$$
$$\times [(b_4 - b_3)R_B^{-1}(\propto) + (b_2 - b_1)L_B^{-1}(\propto)]$$
$$+ \frac{1}{3}[(a_4 - a_3)^2(R_A^{-1}(\propto))^2 + (a_2 - a_1)^2$$
$$\times (L_A^{-1}(\propto))^2 + (b_4 - b_3)^2(R_B^{-1}(\propto))^2 + (b_2 - b_1)^2$$
$$\times (L_B^{-1}(\propto))^2]$$
$$- \frac{1}{3}[(a_2 - a_1)(a_4 - a_3)L_A^{-1}(\propto)R_A^{-1}(\propto)$$
$$+ (b_2 - b_1)(b_4 - b_3)L_B^{-1}(\propto)R_B^{-1}(\propto)]$$
$$+ \frac{1}{2}[(a_4 - a_3)(b_2 - b_1)R_A^{-1}(\propto)L_B^{-1}(\propto)$$
$$+ (a_2 - a_1)(b_4 - b_3)L_A^{-1}(\propto)R_B^{-1}(\propto)$$
$$- (a_4 - a_3)(b_4 - b_3)R_A^{-1}(\propto)R_B^{-1}(\propto)$$
$$- (a_2 - a_1)(b_2 - b_1)L_A^{-1}(\propto)L_B^{-1}(\propto)] \right\}$$
$$\times f(\propto)d \propto / \int_0^1 f(\propto)d \propto .$$
(8)

For triangular fuzzy numbers $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ the above distance with $\propto= 1$ is calculated as:

$$(a_2 - b_2)^2 + \frac{1}{2}(a_2 - b_2)[(a_3 + a_1) - (b_3 + b_1)] + \frac{1}{9}[(a_3 - a_2)^2 + (a_2 - a_1)^2 + (b_3 - b_2)^2 + (b_2 - b_1)^2] - \frac{1}{9}[(a_2 - a_1)(a_3 - a_2) + (b_2 - b_1)(b_3 - b_2)] + \frac{1}{6}(2a_2 - a_1 - a_3)(2b_2 - b_1 - b_3).$$
(11)

And if $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ are trapezoidal fuzzy numbers, the distance with $\propto= 1$ is calculated as:

$$\left(\frac{a_2 + a_3}{2} - \frac{b_2 + b_3}{2}\right)^2 + \frac{1}{2}\left(\frac{a_2 + a_3}{2} - \frac{b_2 + b_3}{2}\right) \times [(a_4 - a_3) - (a_2 - a_1) - (b_4 - b_3) - (b_2 - b_1)] + \frac{1}{3}\left(\frac{a_3 - a_2}{2}\right)^2 + \frac{1}{6}\left(\frac{a_3 - a_2}{2}\right) \times [(a_4 - a_3) + (a_2 - a_1)] + \frac{1}{3}\left(\frac{b_3 - b_2}{2}\right)^2 + \frac{1}{6}\left(\frac{b_3 - b_2}{2}\right) \times [(b_4 - b_3) + (b_2 - b_1)] + \frac{1}{9}[(a_4 - a_3)^2 + (a_2 - a_1)^2 + (b_4 - b_3)^2 + (b_2 - b_1)^2] - \frac{1}{9}[(a_2 - a_1)(a_4 - a_3) + (b_2 - b_1)(b_4 - b_3)] +$$

$$\frac{1}{6}[(a_4 - a_3)(b_2 - b_1) + (a_2 - a_1)(b_4 - b_3) - (a_4 - a_3)(b_4 - b_3) - (a_2 - a_1)(b_2 - b_1)]. \quad (12)$$

3 The proposed algorithm

According to Section 2, an algorithm for solving the MCDM problem using our proposed fuzzy TOPSIS model is as follow.

Algorithm: Fuzzy TOPSIS algorithm.

Step 1: The linguistic ratings or fuzzy values $\tilde{x}_{ij}, (i = 1, \ldots, m, j = 1, \ldots, n)$, for alternatives with respect to criteria and then, the appropriate linguistic variables $\tilde{w}_j, (j = 1, \ldots, n)$ as weights of the criteria must be chosen.

Step 2: The raw data are normalized to eliminate anomalies with different measurement units and scales in several MCDM problems. However, the purpose of linear scales transform normalization function used in this study is to preserve the property that the ranges of normalized triangular fuzzy numbers to be included in $[0, 1]$. Suppose $\tilde{R}$ denotes normalized fuzzy decision matrix, then

$$\tilde{R} = [\tilde{r}_{ij}], i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n,$$
$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+}\right), \quad j \in B,$$
$$c_j^+ = \max_i c_{ij} \ if \ j \in B,$$
$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right), \quad j \in C, \qquad a_j^-$$
$$= \min_i a_{ij} \ if \ j \in C, \quad (13)$$

where B is the benefit criteria set, C is the cost criteria set.

Step 3: by using Eq. (7), the weighted normalized fuzzy decision matrix $\tilde{V} = [\tilde{v}_{ij}]_{m \times n}$ will be generated.

Step 4: Herein we propose a new simple comparison between two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ to find the maximum and minimum fuzzy numbers, which is the novelty of our algorithm, is as follows
Suppose $\tilde{A}^i = (a_1^i, a_2^i, a_3^i), \ i = 1, 2, \ldots, n$ are n TFN. By our proposed method for determining Min (Max), follow the below procedure:

Step 4-1: List all $a_j^i, i = 1, 2, \ldots, n; j = 1, 2, 3.$

Step 4-2: Sort increasingly a_j^i.

Step 4-3: Select the first (last) three a_j^i as minimum (maximum) TFN of $\tilde{A}^i, \ i = 1, 2, \ldots, n,$.

Step 4-4: Record this as $\tilde{A}_{min} (\tilde{A}_{max})$ Which:

$$\tilde{A}_{min} = \wedge_i \tilde{A}_i, i = 1, 2, \ldots, n \ \ (\tilde{A}_{max} = \vee_i \tilde{A}_i, i = 1, 2, \ldots, n) \quad (14)$$

As matter of fact, $\tilde{A}_{min}$ and $\tilde{A}_{max}$ show the minimum and maximum TFN $\tilde{A}^i$, $i = 1, 2, \ldots, n$ according to our discussion about comparing the triangular fuzzy numbers.

It is worth noting that, individually for benefit criterion, the new simple fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) can be calculated by $\tilde{A}_{max}, \tilde{A}_{min}$, respectively. On the other hand, for cost criterion, the new simple fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) can be calculated by $\tilde{A}_{min}, \tilde{A}_{max}$, respectively.

Step 5: From Eq. (10), separately, distance between the possible alternative $\tilde{v}_{ij}$ and the positive ideal solution $\tilde{A}_{max}$ and the negative ideal solution $\tilde{A}_{min}$ can be calculated respectively by using:

$$L_i^+ = \sum_{j=1}^{n} D(f, \tilde{v}_{ij}, \tilde{A}_{max}), \qquad i = 1, 2, \ldots, m,$$
$$L_i^- = \sum_{j=1}^{n} D(f, \tilde{v}_{ij}, \tilde{A}_{min}), \qquad i = 1, 2, \ldots, m.$$

Step 6: The closeness coefficient represents the distances to the fuzzy positive ideal solution (FPIS or $\tilde{A}_{max}$) and the fuzzy negative ideal solution (FNIS or $\tilde{A}_{min}$) simultaneously by taking the relative closeness to the fuzzy positive ideal solution. The closeness coefficient (CC_i) of each alternative is calculated as:

$$CC_i = \frac{L_i^-}{L_i^- + L_i^+}, \qquad i = 1, 2, \ldots, m.$$

While $L_i^- \geq 0$ and $L_i^+ \geq 0$, then, $CC_i \in [0,1]$, clearly.

Step 7: According to the descending order of CC_i, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

4 Numerical illustration

In this section, first we work out a numerical example to illustrate our TOPSIS approach for decision making problems with triangular fuzzy data, and then use six problems with known results, taken from [9], to compare the performance of our method with the methods of Chen and Hwang [10], Li [17], Chen [2].

This example presents the theoretical case of a distribution center (DC) location selection problem.

A logistic company desires to select a suitable city for establishing a new DC. The hierarchical structure of this decision problem is shown in Fig. 4.

The evaluation is done by a committee of five judges $D_1, D_2, ..., D_5$. First, we search for three possible alternatives A_1, A_2 and A_3 to remain for further evaluation after preliminary screening. The company considers six criteria for selecting the most suitable possible alternatives. The six estimation criteria are considered as follows:

(1) Benefit criteria:
(a) Expansion possibility (C_1),
(b) Availability of acquirement material (C_2),
(c) Closeness to demand market (C_3),
(d) Human resources (C_4),
(e) Square measure of area (C_5).

(2) Cost criterion:
(a) Investment cost (C_6).

Proposed algorithm is now applied to solve this problem. The computational procedure is explained next. Judges' subjective judgments develop the fuzzy criteria and use the linguistic variables (as shown in Table 1) to evaluate the ratings of alternatives with respect to each criterion as presented in Table 2.

We obtain the decision matrix of fuzzy ratings of possible alternatives with respect to criteria as in Eq. (7) above and the weights of criteria, and construct the fuzzy decision matrix $\widetilde{D}$ and the fuzzy weight matrix $\widetilde{W}$ (shown in Table 3) as given by (6).

Fig. 4: Hierarchical structure of decision problem.

Table 1

Linguistic variables for the ratings.

Very poor (VP)	(0; 0; 1)
Poor (P)	(0;1; 3)
Medium poor (MP)	(1; 3; 5)
Fair (F)	(3; 5;7)
Medium good (MG)	(5; 7; 9)
Good (G)	(7; 9; 10)
Very good (VG)	(9; 10; 10)

Table 2

The ratings of the three candidates by judges under all criteria

Criteria	Candidate	Decision makers				
		D_1	D_2	D_3	D_4	D_5
	A_1			8 million		
C_1	A_2			5 million		
	A_3			5.2 million		
	A_1	MP	F	MG	MP	F
C_2	A_2	F	F	MG	F	MG
	A_3	F	F	MG	MG	MG
	A_1	MG	F	G	MG	MG
C_3	A_2	G	G	VG	MG	G
	A_3	G	G	VG	G	G
	A_1	MP	MP	F	MP	P
C_4	A_2	G	G	G	G	MG
	A_3	G	G	G	G	MG
	A_1	MP	MP	F	MP	P
C_5	A_2	MG	MG	G	MG	MG
	A_3	G	G	G	G	MG
	A_1			5 hectare		
C_6	A_2			3.5 hectare		
	A_3			3.05 hectare		

Table 3

Fuzzy decision matrix and fuzzy weights of three candidates.

	C_1	C_2	C_3
A_1	(8, 8, 8)	(2.6, 4.6, 6.6)	(5, 7, 8.8)
A_2	(5, 5, 5)	(5, 3.8, 5.8)	(7, 8.8, 9.8)
A_3	(5.2, 5.2, 5.2)	(4.2, 6.2, 8.2)	(7.4, 9.2, 10)
Weight	(0.164, 0.232, 0.327)	(0.08, 0.113, 0.159)	(0.096, 0.136, 0.191)
	C_4	C_5	C_6
A_1	(1.2, 3, 5)	(1.2, 3, 5)	(5, 5, 5)
A_2	(6.6, 8.6, 9.8)	(5.4, 7.4, 9.2)	(3.5, 3.5, 3.5)
A_3	(6.6, 8.6, 9.8)	(6.6, 8.6, 9.8)	(3.05, 3.05, 3.05)
Weight	(0.130, 0.188, 0.274)	(0.096, 0.139, 0.202)	(0.137, 0.192, 0.273)

Table 4

Fuzzy normalized weighted decision matrix of three candidates.

	C_1	C_2	C_3
A_1	(.164, .232, .327)	(.02537,.06339,.12798)	(.048,.0952, 8.8)
A_2	(.1025, .145, .20438)	(.04878, .05237, .1124)	(.0672, .11968, .18718)
A_3	(.1066, .1508, .21255)	(.04098, .08544, .159)	(.07104, .12512, .191)
Weight	(0.164, 0.232, 0.327)	(0.08, 0.113, 0.159)	(0.096, 0.136, 0.191)
	C_4	C_5	C_6
A_1	(.01592, .05755, .1398)	(.01176, .0425, .10306)	(.08357, .11712, .1665)
A_2	(.08755, .16498, .274)	(.0529, .10496, .18963)	(.11939, .16731, .2379)
A_3	(.08755, .16498, .274)	(.06465, .12198, .202)	(.137, .192, .273)
Weight	(0.130, 0.188, 0.274)	(0.096, 0.139, 0.202)	(0.137, 0.192, 0.273)

Table 5

Max and min of each column of three candidates.

	C_1	C_2	C_3
A_{max}	(.21255, .232, .327)	(.11246,.12798,.159)	(.16808,.18718, .191)
A_{min}	(.1025, .1066, .145)	(.02537, .0409, .04878)	(.048, .0672, .07104)
	C_4	C_5	C_6
A_{max}	(.16498, .274, .274)	(.12198, .18963, .202)	(.0835, .11712, .11939)
A_{min}	(.01592, .0575, .08755)	(.01176, .04255, .0529)	(.192, .2379, .273)

Table 6

The distance of each A_i $(i = 1, 2, 3)$ from A_{max}.

	C1			C2			C3		
	A_1	A_2	A_3	A_1	A_2	A_3	A_1	A_2	A_3
A_{max}	.001978	.019054	.01694	.008486	.010489	.003943	.015461	.008504	.007278
	C4			C5			C6		
	A_1	A_2	A_3	A_1	A_2	A_3	A_1	A_2	A_3
A_{max}	.078653	.017901	.017901	.037479	.011358	.007306	.027941	.009368	.004084

Table 7

The distance of each A_i $(i = 1, 2, 3)$ from A_{min}.

	C1			C2			C3		
	A_1	A_2	A_3	A_1	A_2	A_3	A_1	A_2	A_3
A_{min}	.032094	.003171	.004123	.001727	.000951	.005214	.002567	.00672	.007912
	C4			C5			C6		
	A_1	A_2	A_3	A_1	A_2	A_3	A_1	A_2	A_3
A_{min}	.000639	.026545	.026545	.000286	.010024	.014903	.000247	.006825	.01391

Table 8
Computations of L_i^+, L_i^- and CC_i.

	L_i^+	L_i^-	CC_i	rank
A$_1$	0.17	0.03756	0.18096	3
A$_2$	0.07667	0.05424	0.4143	2
A$_3$	0.05745	0.07261	0.55826	1

The normalized weighted fuzzy decision matrixes are constructed as in Table 4. By using step 4, maximum and minimum of each column are determined as FPIS and FNIS, respectively shown in Table 5. By using step 5, distance between the possible alternative $\tilde{v}_{ij}$ and the positive ideal solution $\tilde{A}_{max}$ and the negative ideal solution $\tilde{A}_{min}$ are calculated and are depicted in Tables 6 and 7, respectively. Finally, the values L_i^+ and L_i^- of the three possible suppliers A_i $(i = 1, 2, 3)$ and the closeness coefficient of each supplier is illustrated in Table 8. According to the closeness coefficient, ranking the preference order of these alternatives is A_3, A_2 and A_1. So the best selection is candidate A_3.

5. Conclusions and comments

This paper presents a novel decision method based on the concepts of comparing fuzzy numbers to find maximum and minimum fuzzy numbers and then, solve the multi-criteria group decision-making problem under fuzzy environment. During fuzzy TOPSIS method, we proposed a new simple method to find maximum or minimum fuzzy numbers which presented above and applied a general fuzzy distance. Finally, closeness coefficient for each alternative are obtained, separately. As known, the higher value of closeness coefficient the more preferred. The technique of positive ideal and negative ideal points easily produces satisfactory results which are composed of the overall best criteria values and overall worst criteria values attainable. Short considering, the fuzzy TOPSIS method is extended toward our proposed method.

Because of the complexity of selecting the best alternative, using various approaches causes various results which are clearly referred to Appendix .So, alternative ranking for an illustrated example shows that, our approach makes the specific order to select the preferred case. The algorithm presented in the previous section shows the practical advantages of the proposed method over other evaluation methods in terms of computational simplicity. The examination shows that the proposed method always produces satisfactory results for all the cases. To demonstrate how this method compares favorably with comparable methods, we present the ranking

results from Chen and Hwang [10], Li [17], Chen [2] and the proposed method which proves the accuracy of our methodology in Appendix. To this end, we have presented an extended approach for ranking fuzzy utilities in practical multi criteria decision making problems. The experimental performance of our proposed method shows its practical advantages for comparing fuzzy utilities in fuzzy decision problems which is a valuable methodology for general managers and decision makers in practice.

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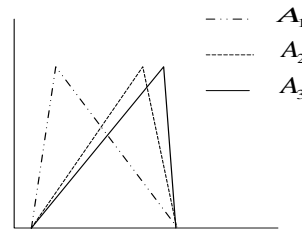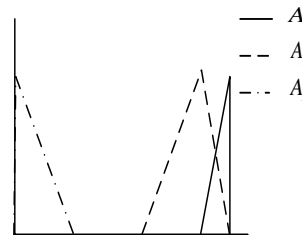
Appendix
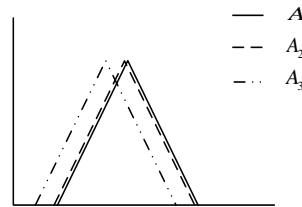
	C_1		C_2		Chen and Hwang [10]		Li[17]		Chen[2]		Proposed model	
weight	(.375, .511, .668)	Ranking [19]	(.489, .5, .511)	Ranking [19]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(.673, .878, 1)	1	(.15, .45, .9)	3	.70909	3	.01228	3	.37069	3	.444623	3
A_2	(.673, .878, 1)	1	(.499, .799, .999)	2	.84151	1	.99102	2	.42631	1	.578941	2
A_3	(.673, .878, 1)	1	(.5, .8, 1)	1	.84151	1	.99103	1	.42631	1	.578987	1
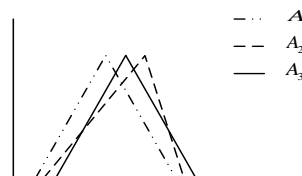
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weight	(.375, .511, .668)	Ranking [19]	(.489, .5, .511)	Ranking [19]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(.45, .65, .85)	3	(.15, .45, .90)	3	.60898	3	.00857	3	.32596	3	.244383	3
A_2	(.55, .9, .95)	1	(.45, .9, .95)	1	.81875	2	.98953	2	.41663	2	.777624	1
A_3	(.6, .8, 1)	2	(.5, .8, 1)	2	.81910	1	.99129	1	.41670	1	.679343	2

	C_1		C_2		Chen and Hwang [10]		Li[17]		Chen[2]		Proposed model	
weight	(.375, .511, .668)	Ranking [19]	(.489, .5, .511)	Ranking [19]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(.6, .75, 1)	3	(.7, .78, 1)	3	.83985	3	0	3	.42554	3	.295962	3
A_2	(.6, .9499, 1)	2	(.7, .9799, 1)	2	.90722	2	.99979	1	.46764	2	.666194	2
A_3	(.6, .98, 1)	1	(.7,.98, 1)	1	.90725	1	.99979	1	.46780	1	.666361	1

	C_1		C_2		Chen and Hwang [10]		Li[17]		Chen[2]		Proposed model	
weight	(.375, .511, .668)	Ranking [19]	(.489, .5, .511)	Ranking [19]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(9, 10, 10)	1	(9, 10, 10)	1	.9932	1	1	1	.4872	1	.907887	1
A_2	(7, 9, 10)	2	(7, 9, 10)	2	.8932	2	1	1	.4470	2	.892036	2
A_3	(0, 0, 1)	3	(0, 0, 1)	3	.06875	3	0	2	.2899	3	.000569	3

	C_1		C_2		Chen and Hwang [10]		Li[17]		Chen[2]		Proposed model	
weight (.375, .511, .668)		Ranking [19]	(.489, .5, .511)	Ranking [19]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(.673, .878, 1)	1	(.15, .45, .90)	3	.70909	3	.01228	3	.37069	3	.356423	3
A_2	(.673, .878, 1)	1	(.5001, .8, .999)	2	.84151	1	.99104	1	.42631	1	.695905	2
A_3	(.673, .878, 1)	1	(.5, .8, 1)	1	.84151	1	.99103	2	.42631	1	.695906	1
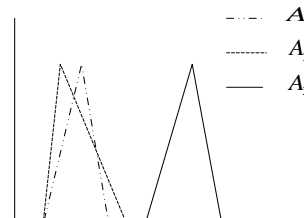
	C_1		C_2		Chen and Hwang [10]		Li[17]		Chen[2]		Proposed model	
weight (.375, .511, .668)		Ranking [19]	(.489, .5, .511)	Ranking [19]	Index value	Ranking	Index value	Ranking	Index value	Ranking	Index value	Ranking
A_1	(.673, .878, 1)	1	(.7, .77, .79)	2	.83334	2	0	2	.42023	2	.398572	2
A_2	(.673, .878, 1)	1	(.7, .75, .8)	3	.83171	3	0	2	.41946	3	.38934	3
A_3	(.673, .878, 1)	1	(.85, .95, 1)	1	.92356	1	1	1	.46407	1	.537007	1