Conditional Inference on the Weibull Distribution Parameters Using Generalized Progressive Hybrid Censored Data

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Abstract: - It is well known that conditional confidence intervals for unknown distribution parameters are often as efficient as Bayesian confidence intervals based on non-informative priors. Accordingly, the main objective of this work is to derive conditional point estimates for the unknown parameters of the Weibull distribution using pivotal functions, and to compare them with Bayesian point estimates through Monte Carlo simulations. The simulation results indicate that, under the generalized progressive hybrid-censoring scheme, the conditional point estimates are highly efficient and outperform their Bayesian counterparts. Finally, the proposed methods are applied to real data to illustrate their practical effectiveness.

Key-Words: - Bayes estimations; Conditional Point estimations; Informative prior; Kernel prior.

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1 Introduction

In statistical inference, conditional inference is known to be highly efficient, comparable to Bayesian inference based on non-informative priors, particularly when sample sizes are small or when data are heavily censored. However, conditional inference is generally less efficient than Bayesian inference that incorporates an informative prior. Therefore, the main objective of this work, addressed for the first time in the literature, is to derive conditional point estimators for distribution parameters that achieve efficiency comparable to Bayesian point estimators based on informative priors.

To illustrate the proposed approach, we apply it to the Weibull distribution, one of the most widely used lifetime distributions in reliability theory. The Weibull distribution is valued for its flexibility in modelling lifetime data characterized by constant as well as non-constant hazard rates, and it has proven useful in medical, biological, and engineering sciences for analyzing failure time data. Additionally, the Weibull distribution includes two well-known special cases, the exponential and Rayleigh distributions, and admits various prior distributions for its parameters, which have been extensively studied in the literature. The probability density function (pdf) and cumulative distribution function (cdf) of the Weibull distribution are given, respectively, as follows:

$$f(x;\alpha,\beta)=\alpha\beta x^{\alpha-1}\,exp(-\beta x^{\alpha}),\quad x>0 \quad (1)$$

$$F(x;\alpha,\beta)=1-exp(-\beta x^{-\alpha}),\quad x>0 \quad (2)$$

$\alpha,\beta>0$are the shape and scale parameters respectively.

Many authors have used non-informative priors for the Weibull model parameters. Among them, [24], [28] derived an informative conjugate prior by assuming that each parameter follows a gamma distribution. [5, 6. 7] proposed a different prior based on prior information about the reliability level or the hazard rate at a given time, converting this information into knowledge about the model parameters. [26] derived confidence intervals via pivotal quantities based on progressively censored samples. [1] obtained estimates of the parameters using classical and Bayesian approaches. [14] presented reliability and quantile analysis for the Weibull distribution. [3] derived maximum likelihood estimates (MLEs) for the Weibull parameters under complete and censored data. [18] applied several methods for estimating the Weibull parameters. [25] obtained MLEs based on Type-II progressively censored samples. [25] employed MLE and Bayesian methods for estimating the Weibull parameters from censored samples, while [21] developed empirical Bayes inference for the Weibull model parameters. As a continuation of these efforts, the purpose of this paper is to derive point estimates for the Weibull parameters using conditional and Bayesian methods.

In reliability analysis, experiments are often terminated before all units on test fail, owing to cost and time constraints, additionally, units may be lost or removed from the test prior to failure. Consequently, censored sampling schemes arise in such life testing experiments. A general framework for studying these experiments is the progressive censoring scheme, which is one of the familiar schemes useful in both industrial life testing and clinical trials. It allows the removal of some surviving experimental units at various stages before the test is terminated. The progressive Type-II censoring scheme is the most commonly applied in life test experiments, although the experimental time can be very long due to the presence of highly reliable units. To address this, [15] recently proposed the Type-II progressive hybrid censoring scheme, which combines progressive Type-II and hybrid censoring schemes.

However, a drawback of the progressive hybrid censoring scheme is that very few failures may occur before the time point T. To provide assurance regarding both the number of observed failures and the time needed to complete the test, [8], [9] introduced the generalized progressive hybrid censoring scheme (GPHCS). This scheme modifies the progressive hybrid censoring scheme by allowing the experiment to continue beyond time T if the number of observed failures is less than a pre-specified value, thereby ensuring that a desired minimum number of failures is observed. The scheme can be described as follows:

Consider n identical items are placed on a test with considering $R_1, R_2, \ldots, R_m$ are the random removal units, which are fixed at the beginning of the experiment with $m < n$ such that $n = m + \sum_{i=1}^{m} R_i$. The terminated time T is also fixed beforehand with the integers k and m are pre-fixed such that $k < m$. In general, at the time of the i^{th} failure, R_i units will be removed randomly from the remaining surviving units $S_i = n - i - \sum_{j=1}^{i-1} R_j$, where $i = 1,2, \ldots, m$. Thus, we have three scenarios:

1. If the time of the m^{th} failure occurs before the time point T, then the experiment will stop at the time point $X_{m:m:n}$ and all the remaining surviving units $R_m = n - m - \sum_{j=1}^{m-1} R_j$ will be removed.
2. If the m^{th} failure does not occur before the time point T and only k failures occur after the time point T, where $X_{m:m:n} > T$. Then at the time point $X_{k:m:n}$ the experiment terminates and all the remaining surviving units $R_k = n - k - \sum_{j=1}^{k-1} R_j$ will be removed.
3. If the m^{th} failure does not occur before the time point $T$and only D failures occur at the time point T, where $X_{k:m:n} < T < X_{m:m:n}$. Then at the time point $X_{D:m:n}$ the experiment terminates and all the remaining surviving units $R_T^* = n - D - \sum_{j=1}^{D} R_j$ will be removed.

Thus, given a generalized progressive hybrid censored sample, the likelihood function for the three different cases can be written in a unified form as follows:

$L(\bar{X};\theta) = C\prod_{i=1}^{n} f(x_i)\,[1 - F(x_i)]^{R_i}[1 - F(T)]^{\delta R_T^*}$, (3)

where $C = \prod_{i=1}^{n} \sum_{j=i}^{m} (R_j + 1)$,

$$n = \begin{cases} m, & \delta = 0 \quad , \text{ if } \quad X_{k:m:n} \leq \quad X_{m:m:n} < T \\ k, & \delta = 0 \quad , \text{ if } \quad T < \quad X_{K:m:n} \leq \quad X_{m:m:n} \quad , \\ D, & \delta = 1 \quad , \text{ if } \quad X_{k:m:n} < \quad T < \quad X_{m:m:n} \end{cases}$$

where $\underline{X} = (X_1, X_2, \ldots, X_n)$ and R_T^* is the number of surviving units that are removed at the stopping time $T^* = \max\{X_{k:m:n}, \min\{X_{m:m:n}, T\}\}$.

The GPHCS has been applied for some distributions such as the Weibull distribution, see [10], the inverse Weibull distribution, see [22], the exponential distribution, see [9], the Rayleigh distribution, see [8], and the shape-scale family, see [23].

1.1 Conditional inference

This section outlines conditional inference, which transforms the standard likelihood function into a function that depends on pivotal quantities and ancillary statistics. For further details, see [16], [17], [18], [19], [20],where pivotal quantities for the parameters are employed to construct confidence intervals for the unknown parameters based on complete and progressive censored samples.

For the Weibull distribution, the likelihood function corresponding to (2) based on the GPCS given in (3) can be expressed as follows:

$$L(\alpha, \beta; x) = C(\alpha\beta)^n \prod_{i=1}^{n} x_i^{\alpha-1} e^{-\beta x_i^{\alpha}}\, e^{-R_i \beta x_i^{\alpha}}\, e^{-R_T^* \delta\beta T^{\alpha}}$$
$$= C(\alpha\beta)^n \prod_{i=1}^{n} x_i^{\alpha-1} \exp[-\beta(\textstyle\sum_{i=1}^{n}(1 + R_i)x_i^{\alpha} + \delta R_T^* T^{\alpha})].$$

(4)

Since

$\beta x_i^{\alpha} = (\beta^{\hat{\alpha}/\alpha}\hat{\beta} x_i^{\hat{\alpha}}/\hat{\beta})^{\frac{\alpha}{\hat{\alpha}}} = (z_2 a_i)^{z_1}$

Thus, $Z_1 = \alpha/\hat{\alpha}$, and $Z_2 = \beta^{1/z_1}/\hat{\beta}$ are pivotal quantities and $a_i = \hat{\beta} x_i^{\hat{\alpha}}$, for $i = 1,2, \ldots, n-2$, are the ancillary statistics.

Theorem:

Let $\hat{\alpha}$ and $\hat{\beta}$ be the maximum likelihood estimators of α and β based on the generalized progressive hybrid censored sample from the Weibull distribution (1). Thus, the joint conditional PDF of

$Z_1 = \alpha/\hat{\alpha}$and $Z_2 = \beta^{1/z_1}/\hat{\beta}$ given the ancillary statistics $\underline{A} = (a_1, a_2 \ldots, a_{n-2})$ can be derived in the form

$$f(z_1, z_2 | \underline{A}) = D z_1^{n-1} z_2^{Nn-1} \prod_{i=1}^{n} a_i^{z_1}$$
$$\times \exp[-z_2^{z_1}(\textstyle\sum_{i=1}^{n}(1 + R_i)\, a_i^{z_1} + \delta R_T^* a_T^{z_1})], \quad (5)$$

where D is the normalizing constant.

Proof:

Make the change of variables from $(x_1, x_2, x_3, \dots, x_n)$ that has joint density function (4) to $(\hat{\alpha}, \hat{\beta}, a_1, a_2 \dots, a_{n-2})$. This transformation can be written as follows:

$X_i = (a_i/\hat{\beta})^{\frac{1}{\hat{\alpha}}}$, $i = 1,2, \dots, n-2$,

$x_{n-1} = (a_{n-1}/\hat{\beta})^{\frac{1}{\hat{\alpha}}}$, and $X_n = (a_n/\hat{\beta})^{\frac{1}{\hat{\alpha}}}$, where a_n and a_{n-1} can be expressed in terms of $a_1, a_2 \dots, a_{n-2}$.

The Jacobin of this transformation is defined as follows:

$|J| = \left|\frac{\partial(x_1, x_2, \dots\dots, x_n)}{\partial(\hat{\alpha}, \hat{\beta}, a_1, a_2, \dots, a_{n-2})}\right|$, which can be derived in the form:

$$|J| = \prod_{i=1}^{n} a_i^{\frac{1}{\hat{\alpha}}} \hat{\beta}^{-\frac{n}{\hat{\alpha}} - n} \hat{\alpha}^{-n} K(A, n),$$

Which, is independent of Z_1 and Z_2. Therefore the joint PDF of $\hat{\alpha}, \hat{\beta}, \underline{A}$ can be derived as follows:

$$f(\hat{\alpha}.\hat{\beta};\ \underline{A}) = C(\alpha\beta)^n \prod_{i=1}^{n} (a_i/\hat{\beta})^{\frac{\alpha-1}{\hat{\alpha}}}$$

$$\times \exp[-\sum_{i=1}^{n} (1+R_i)(\beta^{\frac{\hat{\alpha}}{\alpha}} \hat{\beta} x_i^{\hat{\alpha}}/\hat{\beta})^{\frac{\alpha}{\hat{\alpha}}} - \delta R_T^*(\beta^{\frac{\hat{\alpha}}{\alpha}} \hat{\beta} T^{\alpha}/\hat{\beta})^{\frac{\alpha}{\hat{\alpha}}}]|J|$$

Making further change of variables from $(\hat{\alpha}, \hat{\beta}; \underline{A})$ into $(Z_1, Z_2; \underline{A})$, the Jacobin of this transformation can be derived as follows:

$$|J| = \left|\frac{\partial(\hat{\alpha}, \hat{\beta})}{\partial(z_1, z_2)}\right| = \begin{vmatrix} -\frac{\alpha}{z_1^2} & 0 \\ * & -\frac{\beta^{1/z_1}}{Z_2^2} \end{vmatrix} = \frac{\alpha\beta^{1/z_1}}{z_1^2 Z_2^2}$$

$$= \frac{\hat{\alpha}\hat{\beta}}{Z_1 Z_2} \propto \frac{1}{Z_1 Z_2}$$

Thus,

$$f(z_1, z_2;\ \underline{A}) = D z_1^{n-1} z_2^{nZ_1 - 1} \prod_{i=1}^{n} a_i^{z_1}$$

$$\times \exp[-z_2^{z_1}(\sum_{i=1}^{n} (1+R_i)\, a_i^{z_1} + \delta R_T^* a_T^{Z_1})]$$

Finally, the joint conditional distribution function of (Z_1, Z_2) given $\underline{A}$ can be derived as in (5)■

Thus, based on (5), we can derive the marginal conditional distribution for each Z_1 and Z_2 given $\underline{A}$ respectively as follows:

$$f(z_1|\underline{A}) = K \int_0^{\infty} z_1^{n-1} z_2^{nZ_1 - 1} \prod_{i=1}^{n} a_i^{z_1}$$

$$\times \exp[-z_2^{z_1}(\textstyle\sum_{i=1}^{n} (1+R_i)\, a_i^{z_1} + \delta R_T^* a_T^{Z_1})] dz_2. \quad (6)$$

$$f(z_2|\underline{A}) = K \int_0^{\infty} z_1^{n-1} z_2^{nZ_1 - 1} \prod_{i=1}^{n} a_i^{z_1}$$

$$\times \exp[-z_2^{z_1}(\textstyle\sum_{i=1}^{n} (1+R_i)\, a_i^{z_1} + \delta R_T^* a_T^{Z_1})] dz_1. \quad (7)$$

K is normalizing constant and can be evaluated as follows:

$$K^{-1} = \Gamma(n) \int_0^{\infty} z_1^{n-2} \prod_{i=1}^{n} a_i^{z_1}$$

$$\times [\sum_{i=1}^{n} (1+R_i)\, a_i^{z_1} + \delta R_T^* a_T^{Z_1}]^{-n} dz_1.$$

The confidence intervals for the pivotal Z_1 and Z_2 can be derived from (6) and (7) respectively and transformed fiducially to the parameters α and β separately. Therefore, the statistical significance of this work is finding the point estimators for the unknown parameters. Thus, for finding the point estimator for the parameter α say, we integrate (6) with respect to Z_1 from z_i to z_{i+1}, we have

$$F(z_{i+1}|A) - F(z_i|A) = \gamma - W(z_i), \quad (8)$$

Where

$$W(z_i) = D\Gamma(n) \int_0^{z_i} z_1^{n-2} \prod_{i=1}^{n} a_i^{z_1}$$

$$\times [\textstyle\sum_{i=1}^{n} (1+R_i)\, a_i^{z_1} + \delta R_T^* a_T^{Z_1}]^{-n} dz_1,$$

and γ is a uniform random number $U(0,1)$.

It is known that the second order approximation of the first derivative, which is defined by the centered differencing, can be written as follows:

$$\frac{dF(Z_1^*|\underline{A})}{dZ_1} = \frac{F(z_{i+1}|\underline{A}) - F(Z_i|\underline{A})}{z_{i+1} - z_i} = f(z_1^*|\underline{A}), \quad (9)$$

$z_i < z_1^* < z_{i+1}$.

From (8) and (9) we can derive the following recurrence relation for Z_1 as follows:

$$z_{i+1} = z_i + C[\gamma - W(z_i)], \quad (10)$$

for i = 1,2,3, ...

The convergence of (10) is guaranteed by the condition $C < \frac{1}{f(z_1^*|\underline{A})}$.

The iterative process is continued until two consecutive numerical solutions almost the same, that is if $|z_{i+1} - z_i| < 10^{-5}$. Thus, we can get the successive approximation for the pivotal Z_1 from (10) and transforming fiducially to α as follows: $\alpha^* = \hat{\alpha}\, z_{i+1}$. Similarly, the estimators for β can be derived from (7).

2 Bayes Estimations

In this section, the Bayes estimators for the parameters α and β will be derived using the informative gamma prior and the non-parametric kernel prior, based on two different loss functions.

Firstly, the squared error loss function (SQEL), $L(g(\theta),\hat{g}(\theta)) = (g(\theta)-\hat{g}(\theta))^2$. For this loss function the Bayes estimator that minimizes the risk function is given by $\hat{g}(\theta) = E_\theta(g(\theta)|x)$.

Secondly, the compound LINEX loss function (LNXL) which is defined as follows:

$$L(\Delta) = L_\delta(\Delta) + L_{-\delta}(\Delta) = e^{\delta\Delta} + e^{-\delta\Delta} - 2, \delta > 0.$$

It is named LINEX-based loss function, where $L_\delta(\Delta) = \exp[\delta\Delta] - \delta\Delta - 1,\ \Delta = \hat{g}(\theta) - g(\theta),\ \delta \neq 0,$

is the LINEX loss function (LLF) that has been introduced in [27]. The Bayes estimator of the parameter θ that minimizes the risk function can be derived as follows:

$$\hat{g}_L(\theta) = \frac{1}{2\delta}\log\left(\frac{E(e^{\delta g(\theta)}|X)}{E(e^{-\delta g(\theta)}|X)}\right).$$

2.1 Informative Gamma Prior

We consider the unknown parameters α and β have independent gamma prior distributions with the joint probability density function, which is given by:

$$h(\alpha,\beta) \propto \alpha^{a-1}\beta^{c-1}e^{-b\alpha-d\beta}, \quad (11)$$

where the hyper-parameter a, b, c and d are assumed to be known non-negative real numbers and chosen to reflect the prior belief about the unknown parameters.

2.2 Informative Kernel Prior

For deriving the kernel prior, we introduce the bivariate kernel density estimator for the unknown probability density function $g(\alpha,\beta)$ with support on $(0,\infty)$, which is defined as follows:

$$\hat{g}(\alpha,\beta) = \frac{1}{nh_1h_2}\sum_{i=1}^{n} K\left(\frac{\alpha-\alpha_i}{h_1},\frac{\beta-\beta_i}{h_2}\right), \quad (12)$$

$h_i, i = 1,2$are called the bandwidths or smoothing parameters. Bases on the properties of the maximum likelihood estimates (MLEs) for the parameters, which converge in probability to the original parameters. The kernel prior has been used in [2] and [23]. Thus, using the joint priors of (11) and (12) with the likelihood function of the GPHCS (3) the posterior density for the parameters α and β can be written in a unified form as follows:

$f(\alpha,\beta|\underline{X}) = Kq(\alpha,\beta)L(\underline{X};\alpha,\beta)$, where

$$q(\alpha,\beta) = \hat{g}(\alpha,\beta)h(\alpha,\beta)$$
$$= \hat{g}_1^{p_1}(\alpha)\hat{g}_2^{p_2}(\beta)\,\alpha^{a-1}\beta^{c-1}e^{-b\alpha-d\beta}, \quad (13)$$

is the general prior distribution function with $p_1 = p_2 = 0$ for the informative prior (11), and $p_1 = p_2 = 1$, $a = c = 1$, and $b = d = 0$ for the kernel prior (12).

Using the general prior (13) and the likelihood function of the GPHCS (3) that can be written as follows:

$$L(\underline{X};\alpha,\beta) = C(\alpha\beta)^n\prod_{i=1}^{n} x_i^{\alpha-1}$$
$$\times \exp[-\beta(\textstyle\sum_{i=1}^{n}(1+R_i)x_i^\alpha + \delta R_T^* T^\alpha)]. \quad (14)$$

Thus, the posterior density can be written as follows:

$$f(\alpha,\beta|\underline{X}) = C\hat{g}_1^{p_1}(\alpha)\hat{g}_2^{p_2}(\beta)\alpha^{n+a-1}\beta^{n+c-1}$$
$$\times \exp[-\alpha(b - \sum_{i=1}^{n}\ln(x_i))$$
$$-\beta(d + \textstyle\sum_{i=1}^{n}(1+R_i)x_i^\alpha + \delta R_T^* T^\alpha)].$$

3 Simulation Study

The purpose of this simulation study is to compare the performance of estimators obtained via Bayesian and conditional methods under two different loss functions. The comparison is based on the absolute average bias (AVB) and the mean squared error (MSE), defined as follows:

$AVB = \frac{1}{L}\sum_{i=1}^{L}|\hat{\theta}_i - \theta|$, $MSE(\theta^*) = \frac{1}{L}\sum_{i=1}^{L}(\theta-\theta^*)^2$

$\hat{\theta}$ is the estimate of θ and L is the number of replications.

In our simulation study we choose the distribution hyperparameters of α and β as: $(a,b,c,d) = (5,3,5,3)$, and the values for the parameters $\alpha = (1,2)$ and $\beta = (1.5,3)$. Using these values of the parameters for generating different samples from the Weibull distribution with sizes $n = 20, 40$ and 60 to represent small, moderate and large sizes. To assess the performance of these estimates, the AVB and MSEs for each one were calculated using 1000 replications.

The simulation results are presented in Tables 3, 4, 5, and 6. From these tables, several key observations can be made and are summarized as follows:

1. In general, the point estimates obtained from the conditional method yield the smallest AVB and MSE values compared to those from the Bayesian method under the different loss functions.
2. The estimated AVB and MSE increase as the value of α increases, and decrease as the value of β increases.
3. Smaller hyper parameters of the informative priors lead to smaller estimated AVB and MSE values.
4. As expected, the estimated AVB and MSE decrease with increasing termination time T of the experiment and with increasing sample size.

5. For both parameters α and β, the estimated AVB and MSE based on the conditional method are consistently smaller than those based on the Bayesian method.

In conclusion, the point estimates derived from the conditional method appear to outperform those obtained from the Bayesian method under the considered loss functions.

4 Real Data Applications

In this section, we analyze two real-world datasets to evaluate the performance of the proposed methods under the Weibull model, a widely used and highly desirable lifetime distribution in reliability and survival analysis. This distribution has found extensive applications across various fields, including emerging areas such as biomedical sciences and survival analysis, where it is commonly employed to model lifetimes characterized by specific mortality and failure patterns. To assess the adequacy of the Weibull model for these datasets, we apply several goodness-of-fit tests, namely the Kolmogorov–Smirnov (K-S), Anderson–Darling (A-D), and Chi-Square (χ^2) tests, at a significance level of 0.05. A comprehensive overview of these tests can be found in [11], [12], [13].

4.1 Vinyl Chloride Data

Vinyl chloride is classified as a known human carcinogen; therefore, exposure should be minimized to the greatest extent practicable, and concentrations should be maintained as low as technically feasible. It has been estimated that a vinyl chloride concentration of 0.5 mg/L in drinking-water is associated with an excess risk of liver and brain tumors for exposure beginning in adulthood, and that continuous exposure from birth could double the lifetime cancer risk.

In this study, we analyze the dataset used by [4], which consists of 34 vinyl chloride concentration measurements (in mg/L) obtained from clean-up monitoring wells. The data are as follows: 5.1, 1.2, 1.3, 0.6, 0.5, 2.4, 0.5, 1.1, 8.0, 0.8, 0.4, 0.6, 0.9, 0.4, 2.0, 0.5, 5.3, 3.2, 2.7, 2.9, 2.5, 2.3, 1.0, 0.2, 0.1, 0.1, 1.8, 0.9, 2.0, 4.0, 6.8, 1.2, 0.4, 0.2.

The Weibull model was found to provide a good fit for this dataset, as illustrated in Table 2 and Figures (1a) and (1b). To analyze vinyl chloride concentrations in groundwater from these wells, we estimated the scale (α) and shape (β) parameters of the Weibull distribution using our model. These parameters characterize the distribution of concentrations and allow estimation of the average concentration.

The conditional and Bayes estimates for α were approximately one, indicating that the dataset is moderately right-skewed and that vinyl chloride concentrations tend to decrease over time, see Figure (1b). Additionally, the conditional and Bayes estimates for β were close to 0.5, further confirming the right-skewed nature of the data and supporting the trend of decreasing concentration over time. These findings underscore the importance of continued monitoring of these wells.

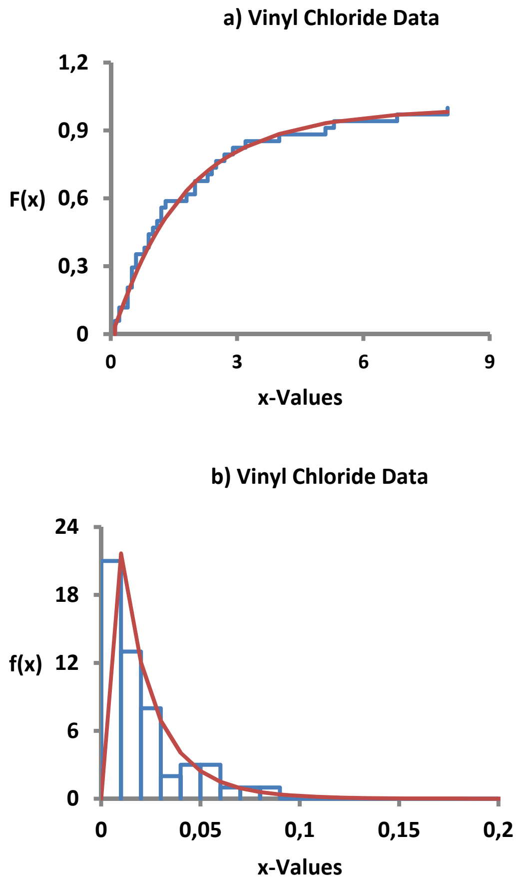
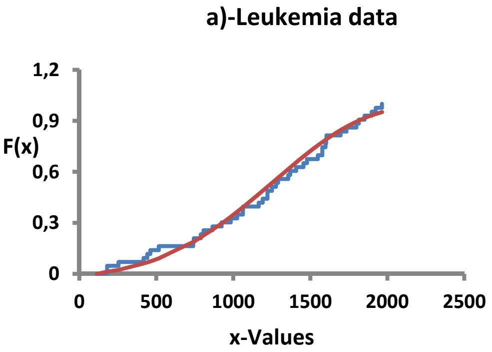


Fig. 1: a) The Empirical CDF and the fitted CDF. b) The Histogram and the fitted PDF

4.2 Leukemia Data

In the field of healthcare, leukemia is a malignancy that affects blood-forming tissues, and it is commonly diagnosed using a Complete Blood Count (CBC). Patients with leukemia often undergo chemotherapy as a primary treatment. In this study, we investigate the effect of chemotherapy on patient survival using a dataset collected from the Ministry of Health Hospital in Saudi Arabia, previously analyzed in [1]. The dataset comprises survival times (in days) for 43 leukemia patients following chemotherapy: 115, 181, 255, 418, 441, 461, 516, 739, 743, 789, 807, 865, 924, 983, 1025, 1062, 1063, 1165, 1191, 1222, 1222, 1251, 1277, 1290, 1357, 1369, 1408, 1455, 1478, 1549, 1578, 1578, 1599, 1603, 1605, 1696, 1735, 1799, 1815, 1852, 1899, 1925, 1965.

We found that the Weibull distribution provides the best fit for this dataset, as shown in Table 2 and Figure (2a). To assess the impact of chemotherapy, we estimated the distribution parameters and



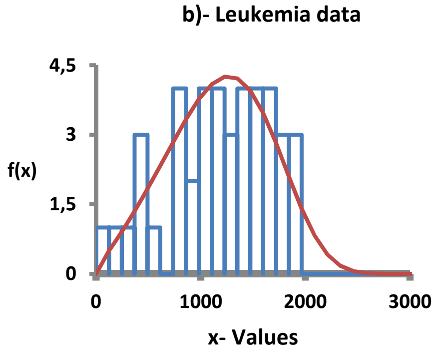


Fig. 2: a) The Empirical CDF and the fitted CDF. b) The Histogram and the fitted PDF

Table 1: The critical and calculated values for the K-S, A-D and CH2 tests and their powers (p-values) for the inverse Weibull model. The MLE's for the parameters for these data sets have been calculated.

Data	The Tests	Critical value	Calculated value	The P-values	$\hat{\alpha}$	$\hat{\beta}$
Chloride N=34	K-S	0.8624	0.5355	0.6525	1.0102	0.5263
	A-D	0.7504	0.2826	0.6708		
	CH2	15.428	4.9912	0.4474		
Leukemia N=43	K-S	0.8699	0.7285	0.1915	2.5493	1.0E-08
	A-D	0.7598	0.9159	0.0206		
	CH2	15.399	12.409	0.0528		

Table 2: The estimate (Est.) and the mean squared errors (MSEs) for the parameters α, β based on the Conditional and Bayes methods using the Gamma and Kernel priors under squared error Loss function with the hyperparameters: a=5, b=3, c=5, d=3, and $m = n/2$ and $k = m/2$.

Samples	T	Parameters	Cond. Estamite		Gamma Prior		Kernel Prior	
			Est.	MSE	Est.	RMSE	Est.	RMSR
Vinyl. Chl. Data N=34	0.5	α	0.9153	0.0091	1.3326	0.3224	1.2707	0.0678
		β	0.4537	0.0053	0.3568	0.0287	0.2813	0.0601
	3.5	α	0.9122	0.00962	0.8943	0.0134	0.9512	0.00349
		β	0.4541	0.00521	0.2181	0.09495	0.1723	0.1253
Leukemia Data N=43	250	α	2.3008	0.06178	2.9762	0.4547	2.8695	0.3853
		β	2.0E-08	5.1E-06	0.0501	0.0501	0.0038	0.00521
	850	α	2.3008	0.0618	1.8857	0.5368	2.1937	0.0863
		β	3.6E-09	5.3E-10	0.0223	4.5E-04	1.1E-06	9.9E-07

The results in Table 1 indicate that the Weibull model provides a good fit for these datasets, as the power of the tests exceeds the significance level, see Figures (1a-2a). Additionally, the results in Table 2 show that the estimated MSEs for the parameters obtained using the conditional method are smaller than those obtained using the Bayesian method, under the assumption that the MLEs are the true parameter values. Thus, the findings from these datasets corroborate the simulation results.

5 Conclusion

It is well known that Bayesian method incorporating informative priors are often more efficient than most frequentist approaches in statistical inference. However, the use of pivotal functions can render conditional inference more efficient than Bayesian method with informative priors in certain contexts.

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APPENDIX

Table 3: The Average bias (ABS) and Mean Squared Errors (MSEs) in parentheses for the Weibull parameter α using the Conditional and Bayes methods with m = (n/2 and 3n/4) and k=(m/2 and 3m/4) at T=1.5 and $\delta = 2$ for LINEX loss function.

n	m	k	α	β	Conditional Estimations	Gamma Prior		Kernel Prior	
						SQEL	LNXL	SQEL	LNXL
20	10	5	1	1.5	0.0683(0.0050)	0.2868(0.1470)	0.2244(0.0868)	0.1753(0.0517)	0.1769(0.0527)
				3	0.0639(0.0044)	0.1403(0.0321)	0.1205(0.0227)	0.1239(0.0246)	0.1242(0.0248)
			2	1.5	0.1680(0.0283)	0.6295(0.6896)	0.2773(0.1307)	0.2350(0.0883)	0.2379(0.0904)
				3	0.1661(0.0277)	0.2829(0.1272)	0.2932(0.1216)	0.1971(0.0596)	0.1975(0.0599)
		8	1	1.5	0.0681(0.0050)	0.3082(0.1711)	0.2400(0.0990)	0.1828(0.0575)	0.1845(0.0586)
				3	0.0628(0.0042)	0.1393(0.0322)	0.1236(0.0237)	0.1232(0.0246)	0.1234(0.0247)
			2	1.5	0.1671(0.0281)	0.6177(0.6808)	0.2849(0.1354)	0.2407(0.0906)	0.2432(0.0925)
				3	0.1656(0.0275)	0.2775(0.1273)	0.3060(0.1276)	0.1963(0.0600)	0.1965(0.0602)
	15	8	1	1.5	0.0684(0.0050)	0.3084(0.1740)	0.2350(0.0960)	0.1808(0.0562)	0.1824(0.0573)
				3	0.0633(0.0043)	0.1394(0.0324)	0.1243(0.0237)	0.1224(0.0249)	0.1226(0.0250)
			2	1.5	0.1674(0.0282)	0.6219(0.6840)	0.2753(0.1242)	0.2345(0.0862)	0.2369(0.0881)
				3	0.1662(0.0277)	0.2830(0.1342)	0.3116(0.1345)	0.1986(0.0630)	0.1990(0.0633)
		11	1	1.5	0.0680(0.0050)	0.2869(0.1495)	0.2306(0.0924)	0.1800(0.0550)	0.1815(0.0560)
				3	0.0641(0.0044)	0.1387(0.0336)	0.1213(0.0239)	0.1222(0.0253)	0.1225(0.0255)
			2	1.5	0.1688(0.0286)	0.5569(0.5786)	0.2838(0.1324)	0.2329(0.0852)	0.2350(0.0868)
				3	0.1655(0.0275)	0.2702(0.1244)	0.2880(0.1152)	0.1895(0.0573)	0.1898(0.0576)
40	20	10	1	1.5	0.0664(0.0047)	0.2033(0.0707)	0.1687(0.0474)	0.1338(0.0295)	0.1343(0.0297)
				3	0.0628(0.0041)	0.1019(0.0170)	0.0964(0.0145)	0.0894(0.0129)	0.0895(0.0129)
			2	1.5	0.1680(0.0283)	0.4025(0.2828)	0.2353(0.0928)	0.1943(0.0602)	0.1954(0.0608)
				3	0.1655(0.0274)	0.2036(0.0661)	0.2416(0.0827)	0.1528(0.0361)	0.1528(0.0361)
		15	1	1.5	0.0661(0.0046)	0.2045(0.0696)	0.1676(0.0465)	0.1359(0.0297)	0.1363(0.0299)
				3	0.0618(0.0040)	0.1074(0.0182)	0.0994(0.0152)	0.0942(0.0138)	0.0942(0.0138)
			2	1.5	0.1683(0.0284)	0.3973(0.2840)	0.2406(0.0966)	0.1951(0.0616)	0.1959(0.0621)
				3	0.1655(0.0274)	0.2055(0.0678)	0.2468(0.0864)	0.1561(0.0381)	0.1561(0.0381)
	30	15	1	1.5	0.0660(0.0046)	0.1968(0.0660)	0.1631(0.0444)	0.1308(0.0280)	0.1312(0.0282)
				3	0.0621(0.0040)	0.1003(0.0160)	0.0953(0.0139)	0.0888(0.0123)	0.0888(0.0123)
			2	1.5	0.1679(0.0283)	0.3976(0.2671)	0.2380(0.0905)	0.1961(0.0598)	0.1970(0.0603)
				3	0.1656(0.0275)	0.2030(0.0650)	0.2488(0.0864)	0.1539(0.0366)	0.1539(0.0366)
		23	1	1.5	0.0687(0.0049)	0.1676(0.0492)	0.1500(0.0383)	0.1262(0.0259)	0.1265(0.0260)
				3	0.0629(0.0041)	0.1029(0.0176)	0.0964(0.0145)	0.0911(0.0136)	0.0912(0.0136)
			2	1.5	0.1700(0.0290)	0.3433(0.2060)	0.2328(0.0896)	0.1942(0.0603)	0.1951(0.0608)
				3	0.1665(0.0278)	0.2099(0.0707)	0.2203(0.0701)	0.1583(0.0384)	0.1584(0.0385)
60	30	15	1	1.5	0.0663(0.0046)	0.1458(0.0367)	0.1283(0.0278)	0.1074(0.0190)	0.1076(0.0190)
				3	0.0617(0.0039)	0.0882(0.0121)	0.0864(0.0111)	0.0771(0.0090)	0.0771(0.0090)
			2	1.5	0.1677(0.0282)	0.3121(0.1706)	0.2148(0.0769)	0.1748(0.0491)	0.1753(0.0494)
				3	0.1658(0.0275)	0.1734(0.0486)	0.1959(0.0568)	0.1307(0.0273)	0.1307(0.0273)
		23	1	1.5	0.0666(0.0046)	0.1446(0.0350)	0.1280(0.0269)	0.1069(0.0183)	0.1071(0.0183)
				3	0.0623(0.0040)	0.0878(0.0121)	0.0841(0.0107)	0.0773(0.0092)	0.0773(0.0092)
			2	1.5	0.1676(0.0281)	0.2938(0.1550)	0.2050(0.0714)	0.1679(0.0462)	0.1684(0.0464)
				3	0.1657(0.0275)	0.1760(0.0495)	0.2061(0.0611)	0.1366(0.0289)	0.1366(0.0289)
	45	23	1	1.5	0.0683(0.0048)	0.1434(0.0355)	0.1287(0.0278)	0.1096(0.0194)	0.1098(0.0194)
				3	0.0622(0.0040)	0.0896(0.0133)	0.0865(0.0117)	0.0775(0.0098)	0.0775(0.0098)
			2	1.5	0.1679(0.0282)	0.2844(0.1359)	0.2059(0.0671)	0.1705(0.0453)	0.1709(0.0455)
				3	0.1657(0.0275)	0.1805(0.0513)	0.1918(0.0552)	0.1347(0.0283)	0.1347(0.0283)
		34	1	1.5	0.0683(0.0048)	0.1339(0.0305)	0.1231(0.0253)	0.1053(0.0180)	0.1055(0.0180)
				3	0.0622(0.0040)	0.0869(0.0122)	0.0822(0.0106)	0.0756(0.0091)	0.0756(0.0091)
			2	1.5	0.1694(0.0287)	0.2603(0.1192)	0.1984(0.0653)	0.1652(0.0442)	0.1656(0.0444)
				3	0.1657(0.0275)	0.1826(0.0541)	0.1862(0.0526)	0.1394(0.0307)	0.1394(0.0307)

Table 4: The Average bias (ABS) and Mean Squared Errors (MSEs) in parentheses for the Weibull parameter α using the Conditional and Bayes methods with m = (n/2 and 3n/4) and k=(m/2 and 3m/4) at T=3 and $\delta = 2$ for LINEX loss function.

n	m	k			Conditional	Gamma Prior		Kernel Prior	
			α	β	Estimations	SQEL	LNXL	SQEL	LNXL
20	10	5	1	1.5	0.0683(0.0050)	0.3055(0.1633)	0.2397(0.0962)	0.1861(0.0571)	0.1878(0.0582)
				3	0.0639(0.0044)	0.1458(0.0345)	0.1252(0.0242)	0.1288(0.0265)	0.1291(0.0266)
			2	1.5	0.1680(0.0283)	0.5937(0.6315)	0.2836(0.1300)	0.2386(0.0867)	0.2408(0.0883)
				3	0.1661(0.0277)	0.2743(0.1252)	0.2993(0.1237)	0.1919(0.0590)	0.1923(0.0592)
		8	1	1.5	0.0682(0.0050)	0.2986(0.1595)	0.2298(0.0911)	0.1793(0.0533)	0.1807(0.0543)
				3	0.0626(0.0042)	0.1399(0.0316)	0.1250(0.0235)	0.1237(0.0243)	0.1239(0.0244)
			2	1.5	0.1682(0.0284)	0.6057(0.6195)	0.2749(0.1233)	0.2348(0.0848)	0.2377(0.0868)
				3	0.1657(0.0275)	0.2884(0.1396)	0.3057(0.1301)	0.2029(0.0644)	0.2032(0.0647)
	15	8	1	1.5	0.0720(0.0055)	0.2282(0.0940)	0.2024(0.0704)	0.1650(0.0452)	0.1661(0.0459)
				3	0.0636(0.0043)	0.1401(0.0328)	0.1210(0.0234)	0.1230(0.0245)	0.1233(0.0246)
			2	1.5	0.1720(0.0297)	0.4661(0.3979)	0.2691(0.1232)	0.2243(0.0802)	0.2264(0.0816)
				3	0.1660(0.0277)	0.2706(0.1241)	0.2634(0.0977)	0.1884(0.0560)	0.1886(0.0562)
		11	1	1.5	0.0718(0.0055)	0.2545(0.1201)	0.2187(0.0840)	0.1746(0.0520)	0.1759(0.0528)
				3	0.0637(0.0043)	0.1395(0.0334)	0.1203(0.0235)	0.1216(0.0247)	0.1219(0.0249)
			2	1.5	0.1703(0.0291)	0.4956(0.4515)	0.2822(0.1282)	0.2304(0.0824)	0.2322(0.0838)
				3	0.1663(0.0277)	0.2776(0.1339)	0.2662(0.1013)	0.1900(0.0578)	0.1904(0.0581)
40	20	10	1	1.5	0.0671(0.0047)	0.1903(0.0607)	0.1599(0.0419)	0.1287(0.0267)	0.1291(0.0269)
				3	0.0615(0.0040)	0.1005(0.0164)	0.0971(0.0145)	0.0888(0.0128)	0.0889(0.0128)
			2	1.5	0.1679(0.0283)	0.3888(0.2575)	0.2427(0.0957)	0.1991(0.0627)	0.1999(0.0633)
				3	0.1658(0.0275)	0.2128(0.0736)	0.2347(0.0792)	0.1599(0.0401)	0.1600(0.0401)
		15	1	1.5	0.0673(0.0048)	0.1978(0.0650)	0.1676(0.0461)	0.1352(0.0297)	0.1357(0.0299)
				3	0.0616(0.0040)	0.1014(0.0168)	0.0971(0.0147)	0.0897(0.0130)	0.0897(0.0130)
			2	1.5	0.1680(0.0283)	0.3719(0.2490)	0.2328(0.0894)	0.1905(0.0582)	0.1912(0.0587)
				3	0.1658(0.0275)	0.2129(0.0712)	0.2323(0.0771)	0.1593(0.0389)	0.1594(0.0389)
	30	15	1	1.5	0.0709(0.0052)	0.1529(0.0406)	0.1432(0.0342)	0.1223(0.0242)	0.1226(0.0243)
				3	0.0625(0.0041)	0.0997(0.0163)	0.0919(0.0132)	0.0882(0.0125)	0.0882(0.0125)
			2	1.5	0.1712(0.0294)	0.2990(0.1569)	0.2202(0.0791)	0.1852(0.0542)	0.1859(0.0546)
				3	0.1663(0.0277)	0.2151(0.0763)	0.2062(0.0637)	0.1595(0.0405)	0.1597(0.0406)
		23	1	1.5	0.0720(0.0054)	0.1394(0.0347)	0.1332(0.0307)	0.1142(0.0223)	0.1145(0.0224)
				3	0.0626(0.0041)	0.1071(0.0185)	0.0963(0.0146)	0.0950(0.0143)	0.0951(0.0143)
			2	1.5	0.1725(0.0298)	0.2915(0.1530)	0.2175(0.0804)	0.1835(0.0553)	0.1842(0.0558)
				3	0.1665(0.0278)	0.2121(0.0725)	0.2024(0.0611)	0.1584(0.0390)	0.1585(0.0391)
60	30	15	1	1.5	0.0670(0.0047)	0.1551(0.0399)	0.1359(0.0300)	0.1121(0.0199)	0.1122(0.0200)
				3	0.0622(0.0040)	0.0857(0.0118)	0.0839(0.0108)	0.0749(0.0088)	0.0749(0.0089)
			2	1.5	0.1676(0.0281)	0.3002(0.1566)	0.2136(0.0727)	0.1724(0.0468)	0.1727(0.0470)
				3	0.1657(0.0275)	0.1810(0.0511)	0.2006(0.0588)	0.1388(0.0294)	0.1389(0.0294)
		23	1	1.5	0.0661(0.0045)	0.1481(0.0383)	0.1293(0.0285)	0.1070(0.0191)	0.1072(0.0192)
				3	0.0625(0.0040)	0.0874(0.0122)	0.0848(0.0111)	0.0755(0.0090)	0.0755(0.0090)
			2	1.5	0.1680(0.0283)	0.3075(0.1654)	0.2112(0.0750)	0.1727(0.0487)	0.1732(0.0490)
				3	0.1659(0.0276)	0.1774(0.0495)	0.1951(0.0564)	0.1335(0.0277)	0.1336(0.0277)
	45	23	1	1.5	0.0723(0.0053)	0.1052(0.0191)	0.1038(0.0184)	0.0943(0.0150)	0.0943(0.0150)
				3	0.0634(0.0041)	0.0871(0.0125)	0.0799(0.0103)	0.0764(0.0095)	0.0765(0.0095)
			2	1.5	0.1744(0.0305)	0.2135(0.0794)	0.1801(0.0546)	0.1565(0.0402)	0.1567(0.0404)
				3	0.1671(0.0279)	0.1767(0.0510)	0.1642(0.0406)	0.1340(0.0286)	0.1341(0.0286)
		34	1	1.5	0.0729(0.0054)	0.1042(0.0184)	0.1027(0.0178)	0.0926(0.0142)	0.0926(0.0143)
				3	0.0640(0.0042)	0.0888(0.0124)	0.0804(0.0101)	0.0780(0.0094)	0.0780(0.0094)
			2	1.5	0.1734(0.0301)	0.2166(0.0774)	0.1840(0.0536)	0.1596(0.0395)	0.1599(0.0396)
				3	0.1670(0.0279)	0.1716(0.0476)	0.1602(0.0392)	0.1307(0.0269)	0.1308(0.0269)

Table 5: The Average bias (ABS) and Mean Squared Errors (MSEs) in parentheses for the Weibull parameter β using the Conditional and Bayes methods with m = (n/2 and 3n/4) and k=(m/2 and 3m/4) at T=1.5 and and $\delta = 2$ for LINEX loss function.

n	m	k	α	β	Conditional Estimations	Gamma Prior		Kernel Prior	
						SQEL	LNXL	SQEL	LNXL
20	10	5	1	1.5	0.1581(0.0251)	0.6941(0.7101)	0.4080(0.2422)	0.2533(0.0916)	0.2636(0.0988)
				3	0.3326(0.1134)	0.2739(0.1550)	0.6858(0.5198)	0.1834(0.0634)	0.1910(0.0647)
			2	1.5	0.1523(0.0232)	0.7071(0.7152)	0.2937(0.1284)	0.2276(0.0752)	0.2362(0.0805)
				3	0.2571(0.0662)	0.2758(0.1507)	0.8330(0.7438)	0.1855(0.0625)	0.1902(0.0630)
		8	1	1.5	0.1607(0.0259)	0.7095(0.7318)	0.4135(0.2462)	0.2526(0.0924)	0.2638(0.0998)
				3	0.3437(0.1208)	0.2601(0.1351)	0.6731(0.4973)	0.1729(0.0546)	0.1806(0.0562)
			2	1.5	0.1540(0.0237)	0.7196(0.7566)	0.3068(0.1448)	0.2382(0.0844)	0.2471(0.0903)
				3	0.2616(0.0686)	0.2628(0.1439)	0.8287(0.7348)	0.1796(0.0620)	0.1849(0.0627)
	15	8	1	1.5	0.1587(0.0253)	0.7453(0.7814)	0.4210(0.2493)	0.2547(0.0925)	0.2668(0.1006)
				3	0.3429(0.1204)	0.2869(0.1704)	0.7307(0.5782)	0.1940(0.0685)	0.2070(0.0713)
			2	1.5	0.1540(0.0237)	0.7714(0.8213)	0.3087(0.1401)	0.2397(0.0836)	0.2503(0.0903)
				3	0.2616(0.0685)	0.2807(0.1600)	0.8807(0.8202)	0.1868(0.0625)	0.1962(0.0645)
		11	1	1.5	0.1567(0.0247)	0.6388(0.6267)	0.3864(0.2251)	0.2403(0.0851)	0.2498(0.0914)
				3	0.3240(0.1075)	0.2737(0.1546)	0.6512(0.4798)	0.1819(0.0648)	0.1857(0.0647)
			2	1.5	0.1510(0.0228)	0.6153(0.6052)	0.2901(0.1339)	0.2312(0.0804)	0.2378(0.0847)
				3	0.2539(0.0647)	0.2599(0.1348)	0.7808(0.6614)	0.1779(0.0604)	0.1796(0.0601)
40	20	10	1	1.5	0.1587(0.0252)	0.5154(0.4563)	0.3452(0.1946)	0.2202(0.0721)	0.2247(0.0752)
				3	0.3344(0.1131)	0.2794(0.1392)	0.5314(0.3533)	0.1832(0.0608)	0.1808(0.0591)
			2	1.5	0.1518(0.0231)	0.5199(0.4645)	0.2729(0.1229)	0.2009(0.0620)	0.2048(0.0644)
				3	0.2566(0.0659)	0.2834(0.1401)	0.6491(0.4941)	0.1769(0.0576)	0.1739(0.0558)
		15	1	1.5	0.1588(0.0253)	0.5309(0.4779)	0.3573(0.2060)	0.2267(0.0768)	0.2316(0.0801)
				3	0.3371(0.1150)	0.2872(0.1443)	0.5314(0.3561)	0.1865(0.0622)	0.1840(0.0603)
			2	1.5	0.1519(0.0231)	0.5271(0.4783)	0.2818(0.1308)	0.2084(0.0656)	0.2122(0.0680)
				3	0.2563(0.0657)	0.2764(0.1311)	0.6573(0.4992)	0.1782(0.0556)	0.1752(0.0539)
	30	15	1	1.5	0.1587(0.0252)	0.5098(0.4542)	0.3446(0.1978)	0.2198(0.0737)	0.2244(0.0768)
				3	0.3366(0.1148)	0.2792(0.1366)	0.5235(0.3459)	0.1828(0.0595)	0.1804(0.0578)
			2	1.5	0.1519(0.0231)	0.5190(0.4637)	0.2767(0.1275)	0.2049(0.0651)	0.2085(0.0675)
				3	0.2564(0.0658)	0.2892(0.1456)	0.6679(0.5194)	0.1881(0.0618)	0.1849(0.0598)
		23	1	1.5	0.1552(0.0241)	0.4031(0.3099)	0.2999(0.1591)	0.2111(0.0701)	0.2138(0.0720)
				3	0.3240(0.1062)	0.2893(0.1320)	0.4347(0.2643)	0.1818(0.0573)	0.1774(0.0551)
			2	1.5	0.1496(0.0224)	0.3767(0.2569)	0.2393(0.0948)	0.1902(0.0552)	0.1923(0.0565)
				3	0.2501(0.0626)	0.2938(0.1409)	0.5190(0.3523)	0.1806(0.0600)	0.1761(0.0578)
60	30	15	1	1.5	0.1587(0.0252)	0.3602(0.2402)	0.2724(0.1276)	0.1896(0.0550)	0.1916(0.0563)
				3	0.3377(0.1150)	0.2928(0.1344)	0.4184(0.2507)	0.1834(0.0578)	0.1794(0.0558)
			2	1.5	0.1520(0.0231)	0.3925(0.2980)	0.2482(0.1093)	0.1866(0.0547)	0.1886(0.0559)
				3	0.2564(0.0658)	0.2880(0.1297)	0.5184(0.3478)	0.1798(0.0548)	0.1760(0.0529)
		23	1	1.5	0.1586(0.0252)	0.3647(0.2495)	0.2761(0.1329)	0.1899(0.0567)	0.1922(0.0580)
				3	0.3360(0.1139)	0.2773(0.1201)	0.3911(0.2227)	0.1684(0.0497)	0.1647(0.0478)
			2	1.5	0.1519(0.0231)	0.3659(0.2486)	0.2328(0.0925)	0.1777(0.0488)	0.1796(0.0498)
				3	0.2567(0.0659)	0.2804(0.1201)	0.4941(0.3198)	0.1707(0.0483)	0.1669(0.0465)
	45	23	1	1.5	0.1570(0.0247)	0.3342(0.2163)	0.2589(0.1197)	0.1835(0.0534)	0.1854(0.0545)
				3	0.3363(0.1140)	0.2853(0.1283)	0.4035(0.2354)	0.1810(0.0555)	0.1772(0.0535)
			2	1.5	0.1520(0.0231)	0.3492(0.2278)	0.2305(0.0912)	0.1792(0.0502)	0.1808(0.0512)
				3	0.2564(0.0658)	0.2893(0.1276)	0.4946(0.3215)	0.1768(0.0530)	0.1730(0.0511)
		34	1	1.5	0.1555(0.0242)	0.2997(0.1762)	0.2397(0.1047)	0.1753(0.0506)	0.1768(0.0515)
				3	0.3270(0.1079)	0.2922(0.1260)	0.3546(0.1944)	0.1739(0.0519)	0.1702(0.0501)
			2	1.5	0.1499(0.0225)	0.3017(0.1791)	0.2138(0.0815)	0.1707(0.0467)	0.1719(0.0474)
				3	0.2517(0.0634)	0.2959(0.1241)	0.4246(0.2544)	0.1767(0.0503)	0.1729(0.0485)

Table 6: The Average bias (ABS) and Mean Squared Errors (MSEs) in parentheses for the Weibull parameter β using the Conditional and Bayes methods with m = (n/2 and 3n/4) and k=(m/2 and 3m/4) at T=3 and $\delta =$for LINEX loss function.

n	m	k	α	β	Conditional Estimation	Gamma Prior		Kernel Prior	
						SQEL	LNXL	SQEL	LNXL
20	10	5	1	1.5	0.1581(0.0251)	0.6876(0.6950)	0.4024(0.2338)	0.2488(0.0885)	0.2596(0.0955)
				3	0.3326(0.1134)	0.2633(0.1406)	0.6756(0.5028)	0.1757(0.0566)	0.1832(0.0580)
			2	1.5	0.1523(0.0232)	0.6923(0.7185)	0.3024(0.1358)	0.2365(0.0801)	0.2447(0.0856)
				3	0.2571(0.0662)	0.2723(0.1486)	0.8340(0.7432)	0.1823(0.0614)	0.1879(0.0622)
		8	1	1.5	0.1584(0.0252)	0.7130(0.7416)	0.4173(0.2504)	0.2548(0.0940)	0.2662(0.1015)
				3	0.3352(0.1147)	0.2763(0.1535)	0.6840(0.5168)	0.1833(0.0622)	0.1902(0.0634)
			2	1.5	0.1523(0.0232)	0.7147(0.7521)	0.3094(0.1463)	0.2400(0.0858)	0.2489(0.0916)
				3	0.2571(0.0662)	0.2574(0.1379)	0.8203(0.7213)	0.1765(0.0593)	0.1813(0.0598)
	15	8	1	1.5	0.1507(0.0228)	0.4563(0.3752)	0.3135(0.1660)	0.2240(0.0779)	0.2282(0.0810)
				3	0.3106(0.0993)	0.2611(0.1156)	0.5053(0.3202)	0.1652(0.0505)	0.1611(0.0487)
			2	1.5	0.1472(0.0217)	0.4893(0.4292)	0.2784(0.1311)	0.2228(0.0762)	0.2268(0.0790)
				3	0.2446(0.0600)	0.2746(0.1324)	0.6442(0.4802)	0.1816(0.0618)	0.1777(0.0598)
		11	1	1.5	0.1516(0.0230)	0.5411(0.4965)	0.3612(0.2088)	0.2527(0.0949)	0.2580(0.0988)
				3	0.3148(0.1018)	0.2764(0.1447)	0.5672(0.3898)	0.1781(0.0627)	0.1756(0.0610)
			2	1.5	0.1484(0.0220)	0.5414(0.5146)	0.2898(0.1422)	0.2297(0.0822)	0.2344(0.0857)
				3	0.2471(0.0612)	0.2758(0.1370)	0.6899(0.5397)	0.1853(0.0634)	0.1828(0.0615)
40	20	10	1	1.5	0.1585(0.0251)	0.4946(0.4273)	0.3399(0.1909)	0.2198(0.0731)	0.2241(0.0760)
				3	0.3380(0.1156)	0.2842(0.1343)	0.4944(0.3185)	0.1786(0.0573)	0.1754(0.0553)
			2	1.5	0.1519(0.0231)	0.4802(0.3961)	0.2694(0.1170)	0.2054(0.0628)	0.2087(0.0648)
				3	0.2563(0.0658)	0.2749(0.1316)	0.6208(0.4568)	0.1781(0.0573)	0.1747(0.0554)
		15	1	1.5	0.1584(0.0251)	0.4779(0.3995)	0.3290(0.1787)	0.2141(0.0693)	0.2184(0.0720)
				3	0.3378(0.1156)	0.2758(0.1288)	0.4959(0.3167)	0.1769(0.0561)	0.1739(0.0543)
			2	1.5	0.1519(0.0231)	0.4830(0.4029)	0.2677(0.1181)	0.2026(0.0628)	0.2062(0.0649)
				3	0.2562(0.0657)	0.2681(0.1196)	0.6047(0.4343)	0.1692(0.0510)	0.1659(0.0492)
	30	15	1	1.5	0.1524(0.0233)	0.3484(0.2397)	0.2730(0.1349)	0.2046(0.0660)	0.2063(0.0672)
				3	0.3197(0.1036)	0.2894(0.1323)	0.3964(0.2330)	0.1813(0.0573)	0.1769(0.0552)
			2	1.5	0.1483(0.0220)	0.3341(0.2142)	0.2294(0.0916)	0.1868(0.0551)	0.1885(0.0562)
				3	0.2469(0.0610)	0.2859(0.1247)	0.4647(0.2913)	0.1735(0.0534)	0.1692(0.0514)
		23	1	1.5	0.1511(0.0229)	0.3085(0.1717)	0.2481(0.1037)	0.1920(0.0567)	0.1935(0.0577)
				3	0.3165(0.1017)	0.2894(0.1273)	0.4002(0.2305)	0.1836(0.0571)	0.1794(0.0551)
			2	1.5	0.1476(0.0218)	0.2993(0.1709)	0.2135(0.0790)	0.1773(0.0494)	0.1786(0.0502)
				3	0.2453(0.0602)	0.2931(0.1276)	0.4568(0.2850)	0.1842(0.0560)	0.1798(0.0539)
60	30	15	1	1.5	0.1585(0.0252)	0.3912(0.2877)	0.2934(0.1505)	0.1999(0.0618)	0.2022(0.0633)
				3	0.3362(0.1139)	0.2808(0.1246)	0.4130(0.2415)	0.1760(0.0543)	0.1723(0.0523)
			2	1.5	0.1520(0.0231)	0.3825(0.2755)	0.2429(0.1025)	0.1835(0.0528)	0.1854(0.0540)
				3	0.2563(0.0657)	0.2808(0.1212)	0.4867(0.3135)	0.1680(0.0481)	0.1641(0.0463)
		23	1	1.5	0.1589(0.0253)	0.3797(0.2714)	0.2845(0.1419)	0.1918(0.0576)	0.1942(0.0591)
				3	0.3354(0.1134)	0.2800(0.1228)	0.4121(0.2405)	0.1777(0.0530)	0.1740(0.0511)
			2	1.5	0.1519(0.0231)	0.3870(0.2740)	0.2442(0.1006)	0.1840(0.0520)	0.1861(0.0532)
				3	0.2563(0.0657)	0.2784(0.1240)	0.5016(0.3278)	0.1725(0.0514)	0.1687(0.0495)
	45	23	1	1.5	0.1498(0.0225)	0.2067(0.0753)	0.1820(0.0563)	0.1510(0.0363)	0.1515(0.0366)
				3	0.3078(0.0956)	0.2873(0.1172)	0.2825(0.1256)	0.1688(0.0454)	0.1662(0.0442)
			2	1.5	0.1459(0.0213)	0.2080(0.0784)	0.1731(0.0511)	0.1494(0.0356)	0.1498(0.0359)
				3	0.2409(0.0580)	0.2858(0.1193)	0.3283(0.1603)	0.1753(0.0483)	0.1727(0.0471)
		34	1	1.5	0.1498(0.0225)	0.2133(0.0813)	0.1875(0.0601)	0.1549(0.0382)	0.1555(0.0385)
				3	0.3074(0.0954)	0.2905(0.1218)	0.3035(0.1442)	0.1787(0.0509)	0.1759(0.0496)
			2	1.5	0.1463(0.0214)	0.2175(0.0842)	0.1784(0.0532)	0.1527(0.0366)	0.1533(0.0369)
				3	0.2415(0.0584)	0.2943(0.1211)	0.3322(0.1688)	0.1768(0.0492)	0.1742(0.0479)