Solution of Three-Dimensional Mboctara Equation Via Gamar Transform

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Abstract: - The objective of present paper is to introduce and apply a novel general triple integral transform named Gamar Transform. First, we outline the basic properties and then establish same important results such as the existence and triple convolution theorems as well as the derivatives properties. Furthermore, the current transform is applied to solve a wide range of partial differential equations including homogeneous and nonhomogeneous Mboctara partial differential equations. Figures are utilized to clarify and exemplify the solutions.

Key-Words: - Laplace transform; Gamar transform; general triple convolution theorem; partial derivatives; Mboctara equation.

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1 Introduction

Integral transform methods have proved their value among the most powerful and effective ways in solving partial differential equations (PDEs). Their applications cover a wide range of phenomena in the disciplines of mathematics, physics, engineering, to mention only a few scientific specializations [1-13]. Thereby, it is feasible to transform PDEs in terms of algebraic equations and thus obtain exact solutions of PDEs.

Many scholars have focused great labor both to develop and enhance these new methods prior to applying them to resolve a wide spectrum of problems in the realm of mathematics. These methods are best represented by Fourier and Laplace transforms, to cite only few illustrative examples [14-25].

In more recent times, triple Laplace transforms have been extensively used to solve PDEs, specifically those with unknown function of three variables, with the ultimate goal of obtaining more satisfactory solutions accompanying the new method [26-28]. Furthermore, many enhancements and extensions have been devised by researchers to the original triple Laplace transform. These include triple Sumudu transform [29], triple Elzaki transform [30], triple Aboodh transform [31] ,triple Shehu transform [32] , triple Natural transform [33] ,triple Kamal transform [34] and triple Laplace-ARA -Sumudu transform [35], all of which are extensions and modifications to the original Laplace transform.

For his part, Jafri has proposed a new general integral transform, namely Jafri transform [36], which is illustrated as follows:

$$\mathbb{J}[f(t);s] = \mathbb{F}_I(v)$$
$$= \alpha(v)\int_0^\infty f(t)\, e^{-\vartheta(v)y} dy. \qquad (1)$$

where $\alpha(v)$ and $\vartheta(v)$ are regular complex functions such that $\alpha(v) \neq 0,$ for all v belongs to complex number.

Recently, Meddahi et.al have proposed a general double transform [37] defined by:

$$\mathbb{M}[z(u,s);(r,v)] = G_D(r,v)$$
$$= \mathcal{E}(r)\mathcal{G}(v)\int_0^\infty \int_0^\infty z(u,s)\, e^{-(\varphi(r)u+w(v)s)}duds. \quad (2)$$

where $\varphi(r)$and $w(v)$ are the transform functions for u and s respectively.

More recently, Abdelilah [38] is introduced a novel triple general integral transform known as Gamar transform is defined as:

$$G[f(x,y,t),(r,s,v)] = F(r,s,v)$$
$$= \mathbb{F}_x\big[\mathbb{F}_y[\mathbb{F}_t[f(x,y,t); t \to v]y \to s]x \to r\big], r,s,v > 0,$$

$$= \mathcal{E}(r)\int_0^\infty e^{-\alpha(r)x}\left(\mathcal{G}(s)\int_0^\infty e^{-\beta(s)y}\left(\mathcal{C}(s)\int_0^\infty e^{-\theta(v)t} f(x,y,t)\, dt\right)dy\right)dx$$

$$= \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\int_0^\infty\int_0^\infty\int_0^\infty e^{-\alpha(r)x-\beta(s)y-\vartheta(v)t} f(x,y,t)dxdy\, dt. \quad (3)$$

provided that all integrals exist for some $\alpha(r), \beta(s)$ and $\theta(v)$, where $\mathbb{F}_x, \mathbb{F}_y$ and $\mathbb{F}_t$ are general transform functions for x, y and t, respectively.

The inverse Gamar transform is defined by

$$G^{-1}[F(r,s,v)] = \mathbb{F}_r{}^{-1}\left[\mathbb{F}_s{}^{-1}\left[\mathbb{F}_v{}^{-1}[F(r,s,v)]\right]\right]$$
$$= f(x,y,t)$$

$$= \frac{1}{2\pi i}\int_{j-i\infty}^{j+i\infty}\frac{1}{\mathcal{E}(r)}e^{\alpha(r)x}dr\,\frac{1}{2\pi i}\int_{h-i\infty}^{h+i\infty}\frac{1}{\mathcal{G}(s)}e^{\beta(s)y}\,ds\,\frac{1}{2\pi i}\int_{w-i\infty}^{w+i\infty}\frac{1}{\mathcal{C}(v)}e^{\theta(v)t}F(r,s,v)dv, \quad (4)$$

where j, h and w are real constants.

Considered as a special kind of both third-order homogeneous and nonhomogeneous partial differential equations, the Mboctara equation, this equation is mainly utilized to investigate the nature of collective motion concerning micro-particles in materials. It is possible to solve this equation by means of any triple integral transform. Though been a comparatively new operator, the Gamar transform has proved its effectiveness in solving various differential equations [38]. Consequently, it has been widely adopted in different fields, including physics, engineering and material science. For instance, in engineering, it has greatly helped to model the nature of behavior of certain fluids and to analyze the dynamics of some kinds of waves.

In this study, we consider the Mboctara partial differential equations of the following form:

$$m_{xyt}(x,y,t) + m(x,y,t) = \aleph(x,y,t).$$

Subject to the boundary and initial conditions

$$\begin{cases} m(x,y,0) = f(x,y) \quad , \quad m(x,0,0) = F(x), \\ m(x,0,t) = h(x,t) \quad , \quad m(0,y,0) = H(y), \\ m(0,y,t) = z(y,t) \quad , \quad m(0,0,t) = Z(t)\,. \end{cases}$$

and $m(0,0,0) = 0$, where $m(x,y,t)$ is an unknown function, $\aleph(x,y,t)$ is the source term.

A novel concept termed Gamar transform is introduced in the current study which is specifically directed to functions with three variables. As a foundation, we first establish and prove some key theorems that comprise existence and triple convolution, among other properties. Subsequently, we obtain the Gamar transform regarding some basic functions. Likewise, the Gamar transform of some partial differential derivatives is established and obtained. The findings prove that the new general triple transform indeed implies the original triple Laplace transform. The effectiveness of

Gamar transforms to solve Mboctara partial differential equations upon applications is firmly proved.

2 Some Properties and Theorems of Gamar transform [38]

In this section, we proceed to prove some basic properties and theorems such as existence, triple convolution.

Property 2.1. **(Linearity).** If $G[f(x,y,t)] = \mathbb{F}(r,s,v)$ and $G[g(x,y,t)] = \mathbb{G}(r,s,v)$, then for any constants $\mathcal{M}$ and $\mathcal{N}$, we have

$$G[\mathcal{M}f(x,y,t) + \mathcal{N}g(x,y,t)] = \mathcal{M}\,\mathbb{F}(r,s,v) + \mathcal{N}\,\mathbb{G}(r,s,v). \quad (5)$$

Proof of Property 2.1. From the definition of Gamar transform, we obtain

$$G[\mathcal{M}f(x,y,t) + \mathcal{N}g(x,y,t)] = \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\int_0^\infty\int_0^\infty\int_0^\infty e^{-\alpha(r)x-\beta(s)y-\theta(v)t}[\mathcal{M}f(x,y,t) + \mathcal{N}g(x,y,t)]dxdy\,dt.$$

$$= \mathcal{M}\,\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\int_0^\infty\int_0^\infty\int_0^\infty e^{-\alpha(r)x-\beta(s)y-\theta(v)t}[f(x,y,t)]dxdy\,dt + \mathcal{N}\,\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\int_0^\infty\int_0^\infty\int_0^\infty e^{-\alpha(r)x-\beta(s)y-\theta(v)t}[g(x,y,t)]dxdy\,dt.$$

$$= \mathcal{M}\,\mathbb{F}(r,s,v) + \mathcal{N}\,\mathbb{G}(r,s,v).$$

Thus, Gamar transform is linear integral transformation. Similarly, we can prove that the inverse Gamar transform is also linear.

Property 2.2.Let $m(x,y,t) = \hbar(x)f(y)g(t)$, $x > 0, y > 0$ and $t > 0$. Then

$$G[m(x,y,t)] = \mathbb{F}_x[\hbar(x)]\mathbb{F}_y[f(y)]\mathbb{F}_t[g(t)]. \quad (6)$$

where $\mathbb{F}_x$,$\mathbb{F}_y$ and $\mathbb{F}_t$ are general integral transform for $\hbar(x)$,$f(y)$ and $g(t)$ respectively.

Proof of Property 2.2. From the definition of Gamar transform, we obtain

$$G[m(x,y,t)] = \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\int_0^\infty\int_0^\infty\int_0^\infty e^{-\alpha(r)x-\beta(s)y-\theta(v)t}[\hbar(x)f(y)g(t)]dxdy\,dt$$

$$= \left(\mathcal{E}(r)\int_0^\infty e^{-\alpha(r)x}[\hbar(x)]\,dx\right)\left(\mathcal{G}(s)\int_0^\infty e^{-\beta(s)y}[f(y)]\,dy\right)\left(\mathcal{C}(v)\int_0^\infty e^{-\theta(v)t}[g(t)]\,dt\right)$$

$$= \mathbb{F}_x[\hbar(x)]\mathbb{F}_y[f(y)]\mathbb{F}_t[g(t)].$$

Definition 2.1. If $m(x,y,t)$ defined on $[0,X]\times[0,Y]\times[0,T]$ satisfies the condition $|m(x,y,t)| \le \mathcal{N}e^{\kappa x+\delta y+\lambda t}$, $\exists\mathcal{N} > 0$, $\forall x > X$, $\forall y >$Y and $\forall t > T$. Then, $m(x,y,t)$ is called a function of exponential orders κ,δ and λ $as\ x,y,t \to \infty$.

Theorem 2.1. The existence condition of Gamar transform of the continuous function $m(x,y,t)$ defined on $[0,X]\times[0,Y]\times[0,T]$is to be of exponential orders κ,δ and λ , for $\mathrm{Re}[\alpha(r)] > \kappa$, $\mathrm{Re}[\beta(s)] > \delta$ and $\mathrm{Re}[\theta(v)] > \lambda$.

Proof of Theorem 2.1. From the definition of Gamar transform, we get

$$|\mathbb{M}(r,s,v)| = |G[m(x,y,t)]| = \left|\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\int_0^\infty\int_0^\infty\int_0^\infty e^{-\alpha(r)x-\beta(s)y-\theta(v)t}[m(x,y,t)]dxdy\,dt\right|$$

$$\le \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\int_0^\infty\int_0^\infty\int_0^\infty e^{-\alpha(r)x-\beta(s)y-\theta(v)t}|m(x,y,t)|dxdy\,dt$$

$$\le \mathcal{N}\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\int_0^\infty e^{-(\alpha(r)-\kappa)x}dx\int_0^\infty e^{-(\beta(s)-\delta)y}dy\int_0^\infty e^{-(\theta(v)-\lambda)t}dt$$

$$= \frac{\mathcal{N}\,\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)}{(\alpha(r)-\kappa)(\beta(s)-\delta)(\theta(v)-\lambda)}.$$

where $\mathrm{Re}[\alpha(r)] > \kappa$, $\mathrm{Re}[\beta(s)] > \delta$and $\mathrm{Re}[\theta(v)] > \lambda$.

Definition 2.3. The triple convolution of $n(x,y,t)$ and $m(x,y,t)$ is denoted by $(n *** m)(x,y,t)$ defined by

$$(n *** m)(x,y,t) = \int_0^x\int_0^y\int_0^t n(x-\kappa, y-\delta, t-\lambda) m(\kappa,\delta,\lambda)\, d\kappa d\delta d\lambda. \quad (7)$$

Theorem 2.2. Let $G[m(x,y,t)] = \mathbb{M}(r,s,v)$. Then,

$$G[m(x-\kappa, y-\delta, t-\lambda)H(x-\kappa, y-\delta, t-\lambda)] = e^{-\alpha(r)\kappa-\beta(s)\delta-\theta(v)\lambda}\mathbb{M}(r,s,v). \quad (8)$$

where $H(x,y,t)$ denotes the unit step function defined by

$$H(x-\kappa, y-\delta, t-\lambda) = \begin{cases} 1, x > \kappa, y > \delta, t > \lambda \\ 0, \text{Otherwise.} \end{cases}$$

Theorem 2.3. (General Triple Convolution Theorem).

If G$[n(x,y,t)] = \mathbb{N}(r,s,v)$ and G$[m(x,y,t)] = \mathbb{M}(r,s,v)$,

then

$$G[(n *** m)(x,y,t)] = \frac{1}{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)}\mathbb{N}(r,s,v)\mathbb{M}(r,s,v). \quad (9)$$

3. Gamar Transform for Some Basic Functions

In this section, we introduce the Gamar transform for some basic functions.

i. Let $m(x,y,t) = 1$.Then

$$G[1] = \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\int_0^\infty\int_0^\infty\int_0^\infty e^{-\alpha(r)x-\beta(s)y-\theta(v)t}dxdy\,dt.$$

From **Property 2.2**, we have

$$G[1] = \mathbb{F}_x[1]\mathbb{F}_y[1]\mathbb{F}_t[1] = \mathcal{E}(r)\int_0^\infty e^{-\alpha(r)x}\,dx\ \mathcal{G}(s)\int_0^\infty e^{-\beta(s)y}\,dy\ \mathcal{C}(v)\int_0^\infty e^{-\theta(v)t}\,dt$$

Thus,

$$G[1] = \frac{\mathcal{E}(r)}{\alpha(r)}\frac{\mathcal{G}(s)}{\beta(s)}\frac{\mathcal{C}(v)}{\theta(v)}.$$

ii. Let $m(x,y,t) = x\,y\,t$ $,x > 0, y > 0$ and $t > 0$ .Then

$$G[xyt] = \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\int_0^\infty\int_0^\infty\int_0^\infty e^{-\alpha(r)x-\beta(s)y-\theta(v)t}[xyt]dx\,dy\,dt.$$

From **Property 2.2**, we obtain

$$G[xyt] = \mathbb{F}_x[x]\mathbb{F}_y[y]\mathbb{F}_t[t] = \mathcal{E}(r)\int_0^\infty e^{-\alpha(r)x}[x]\,dx\ \mathcal{G}(s)\int_0^\infty e^{-\beta(s)y}\,[y]dy\ \mathcal{C}(v)\int_0^\infty e^{-\theta(v)t}[t]\,dt.$$

By integrating by parts, we have

$$G[xyt] = \frac{\mathcal{E}(r)}{\alpha^2(r)}\frac{\mathcal{G}(s)}{\beta^2(s)}\frac{\mathcal{C}(v)}{\theta^2(v)}.$$

Thus, by induction, we prove

$$G[(xyt)^n] = \frac{\mathcal{E}(r)}{\alpha^{n+1}(r)}\frac{\mathcal{G}(s)}{\beta^{n+1}(s)}\frac{\mathcal{C}(v)}{\theta^{n+1}(v)}(\Gamma(n+1))^3,\ \ n \in \mathcal{R}.$$

iii. Let $m(x,y,t) = e^{ax+by+ct}$ $,x > 0, y > 0$ and $t > 0$ and a, b and c are constants. Then

$$G[e^{ax+by+ct}] = \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\int_0^\infty\int_0^\infty\int_0^\infty e^{-\alpha(r)x-\beta(s)y-\theta(v)t}[e^{ax+by+ct}]dx\,dy\,dt.$$

From **Property 2.2**, we obtain:

$$G[e^{ax+by+ct}] = \mathcal{E}(r)\int_0^\infty e^{-(\alpha(r)-a)x}\,dx\ \mathcal{G}(s)\int_0^\infty e^{-(\beta(s)-b)y}\,dy\ \ \mathcal{C}(v)\int_0^\infty e^{-(\theta(v)-c)t}\,dt.$$

Thus,

$$G[e^{ax+by+ct}] = \frac{\mathcal{E}(r)}{(\alpha(r)-a)}\frac{\mathcal{G}(s)}{(\beta(s)-b)}\frac{\mathcal{C}(v)}{(\theta(v)-c)}.$$

Similarly,

$$G\left[e^{(ax+by+ct)i}\right] = \frac{\mathcal{E}(r)}{(\alpha(r)-ia)}\frac{\mathcal{G}(s)}{(\beta(s)-ib)}\frac{\mathcal{C}(v)}{(\theta(v)-ic)}.$$

Thus, one can obtain

$$G\left[e^{(ax+by+ct)i}\right] = \frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)[\alpha(r)\beta(s)\theta(v)-ab\theta(v)-bc\alpha(r)-ac\beta(s)]}{(\alpha^2(r)+a^2)(\beta^2(s)+b^2)(\theta^2(v)+c^2)} + i\frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)[b\alpha(r)\theta(v)+a\beta(s)\theta(v)+c\alpha(r)\beta(s)-abc]}{(\alpha^2(r)+a^2)(\beta^2(s)+b^2)(\theta^2(v)+c^2)}.$$

Using Euler’s formulas:

$$\sin(ax+by+ct) = \frac{e^{(ax+by+ct)i}-e^{-(ax+by+ct)i}}{2i},$$

$$\cos(ax+by+ct) = \frac{e^{(ax+by+ct)i}+e^{-(ax+by+ct)i}}{2}.$$

And the formulas:

$$\sinh(ax+by+ct) = \frac{e^{ax+by+ct}-e^{-(ax+by+ct)}}{2},$$

$$\cosh(ax+by+ct) = \frac{e^{ax+by+ct)}+e^{-(ax+by+ct)}}{2}.$$

Then, we find the Gamar transform of the following functions:

$$G[\cos(ax+by+ct)] = \frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)[\alpha(r)\beta(s)\theta(v)-ab\theta(v)-bc\alpha(r)-ac\beta(s)]}{(\alpha^2(r)+a^2)(\beta^2(s)+b^2)(\theta^2(v)+c^2)},$$

$$G[\sin(ax+by+ct)] = \frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)[b\alpha(r)\theta(v)+a\beta(s)\theta(v)+c\alpha(r)\beta(s)-abc]}{(\alpha^2(r)+a^2)(\beta^2(s)+b^2)(\theta^2(v)+c^2)},$$

$$G[\cosh(ax+by+ct)] = \frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)[\alpha(r)\beta(s)\theta(v)-ab\theta(v)-bc\alpha(r)-ac\beta(s)]}{(\alpha^2(r)-a^2)(\beta^2(s)-b^2)(\theta^2(v)-c^2)},$$

$$G[\sinh(ax+by+ct)] = \frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)[b\alpha(r)\theta(v)+a\beta(s)\theta(v)+c(r)\beta(s)+abc]}{(\alpha^2(r)-a^2)(\beta^2(s)-b^2)(\theta^2(v)-c^2)}.$$

4. Gamar transform for Partial Differential Derivatives [38]

In this section, we present some results related to the Gamar transform of partial derivatives. We begin by obtaining partial derivatives with respect to x, y and t.

Theorem 4.1. (*Derivative properties with respect tox*)**.** Let $\mathbb{M}(r,s,v)$ is Gamar transform of $m(x,y,t)$and $G_D(0,s,v)$ is general double transform of $m(0,y,t)$, then:

a) $G\left[\frac{\partial m(x,y,t)}{\partial x}\right] = \alpha(r)\mathbb{M}(r,s,v) - \mathcal{E}(r)G_D(0,s,v).$

b) $G\left[\frac{\partial^2 m(x,y,t)}{\partial x^2}\right] = \alpha^2(r)\mathbb{M}(r,s,v) - \mathcal{E}(r)\alpha(r)G_D(0,s,v) - \mathcal{E}(r)\mathbb{F}_y\mathbb{F}_t\left[\frac{\partial m(0,y,t)}{\partial x}\right].$

c) $G\left[\frac{\partial^n m(x,y,t)}{\partial x^n}\right] = \alpha^n(r)\mathbb{M}(r,s,v) - \mathcal{E}(r)\sum_{i=0}^{n-1}\alpha^{n-1-i}(r)\mathbb{F}_y\mathbb{F}_t\left[\frac{\partial^i m(0,y,t)}{\partial x^i}\right].$

Theorem 4.2. (*Derivative properties with respect to* y)**.** Let $\mathbb{M}(r,s,v)$ is Gamar transform of $m(x,y,t)$and $G_D(r,0,v)$ is general double transform of $m(x,0,t)$, then:

a) $G\left[\frac{\partial m(x,y,t)}{\partial y}\right] = \beta(s)\mathbb{M}(r,s,v) - \mathcal{G}(s)G_D(r,0,v).$

b) $G\left[\frac{\partial^2 m(x,y,t)}{\partial y^2}\right] = \beta^2(s)\mathbb{M}(r,s,v) - \mathcal{G}(s)\beta(s)G_D(r,0,v) - \mathcal{G}(s)\mathbb{F}_x\mathbb{F}_t\left[\frac{\partial m(x,0,t)}{\partial y}\right].$

c) $G\left[\frac{\partial^n m(x,y,t)}{\partial y^n}\right] = \beta^n(s)\mathbb{M}(r,s,v) - \mathcal{G}(s)\sum_{i=0}^{n-1}\beta^{n-1-i}(s)\mathbb{F}_x\mathbb{F}_t\left[\frac{\partial^i m(x,0,t)}{\partial y^i}\right]$.

Theorem 4.3. (*Derivative properties with respect to t*)**.** Let $\mathbb{M}(r,s,v)$ is Gamar transform of $m(x,y,t)$ and $G_D(r,s,0)$ is general double transform of $m(x,y,0)$, then:

a) $G\left[\frac{\partial m(x,y,t)}{\partial t}\right] = \theta(v)\mathbb{M}(r,s,v) - \mathcal{C}(v)G_D(r,s,0)$.

b) $G\left[\frac{\partial^2 m(x,y,t)}{\partial t^2}\right] = \theta^2(v)\mathbb{M}(r,s,v) - \mathcal{C}(v)\theta(v)G_D(r,s,0) - \mathcal{C}(v)\mathbb{F}_x\mathbb{F}_y\left[\frac{\partial m(x,y,0)}{\partial t}\right]$.

c) $G\left[\frac{\partial^n m(x,y,t)}{\partial t^n}\right] = \theta^n(v)\mathbb{M}(r,s,v) - \mathcal{C}(v)\sum_{i=0}^{n-1}\theta^{n-1-i}(v)\mathbb{F}_x\mathbb{F}_y\left[\frac{\partial^i m(x,y,0)}{\partial t^i}\right]$.

Theorem 4.4. (*Derivative properties with respect to x, y and t*)**.** Let $\mathbb{M}(r,s,v)$ is Gamar transform of $m(x,y,t)$, then

$$
\begin{aligned}
G\left[\frac{\partial^3 m(x,y,t)}{\partial x\partial y\partial t}\right]
&= \alpha(r)\beta(s)\theta(v)\mathbb{M}(r,s,v) \\
&- \alpha(r)\beta(s)\mathcal{C}(v)G_D[m(x,y,0)] \\
&- \alpha(r)\theta(v)\mathcal{G}(s)G_D[m(x,0,t)] \\
&-\beta(s)\theta(v)\mathcal{E}(r)G_D[m(0,y,t)] \\
&+ \alpha(r)\mathcal{G}(s)\mathcal{C}(v)\mathbb{F}_x[m(x,0,0)] \\
&+ \beta(s)\mathcal{E}(r)\mathcal{C}(v)\mathbb{F}_y[z(0,y,0)] \\
&+ \theta(v)\mathcal{E}(r)\mathcal{G}(s)\mathbb{F}_t[m(0,0,t)] \\
&- \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)m(0,0,0).
\end{aligned}
$$

Corollary 4.1.(*Gamar transform of integral*). Let $\alpha(r), \beta(s)$ and $\theta(v)$ be positive functions and let $\mathbb{M}(r,s,v)$ the Gamar transform of $m(x,y,t)$, then

$$
G\left[\int_0^x\int_0^y\int_0^t m(\ell,p,n)d\ell\, dp\, dn\right] = \frac{1}{\alpha(r)\beta(s)\theta(v)}\mathbb{M}(r,s,v).
$$

where $\alpha(r)\beta(s)\theta(v) \neq 0$ for all $r,s,v \in \Re^+$.

Theorem 4.5. Let $\mathbb{M}(r,s,v)$ is Gamar transform of $m(x,y,t)$, then

a) $G[x^n m(x,y,t)] = (-1)^n\frac{\mathcal{E}(r)}{\alpha'(r)}\frac{\partial^n}{\partial r^n}\left(\frac{\mathbb{M}(r,s,v)}{\mathcal{E}(r)}\right)$.

b) $G[y^n m(x,y,t)] = (-1)^n\frac{\mathcal{G}(s)}{\beta'(s)}\frac{\partial^n}{\partial s^n}\left(\frac{\mathbb{M}(r,s,v)}{\mathcal{G}(s)}\right)$.

c) $G[t^n m(x,y,t)] = (-1)^n\frac{\mathcal{C}(v)}{\theta'(v)}\frac{\partial^n}{\partial v^n}\left(\frac{\mathbb{M}(r,s,v)}{\mathcal{C}(v)}\right)$.

5. Applications

In this section, we apply the properties associated with Gamar transform established above to solve homogeneous and nonhomogeneous three-dimensional Mboctara partial differential equations. All the following figures of the selected examples were obtained using Mathematica software 13.

Example 5.1

Consider the following homogeneous three-dimensional Mboctara partial differential equation

$$m_{xyt}(x,y,t) + m(x,y,t) = 0. \qquad (10)$$

Subject to the boundary and initial conditions

$$
\begin{cases}
m(x,y,0) = e^{x+y} \ , & m(x,0,0) = e^x, \\
m(x,0,t) = e^{x-t} \ , & m(0,y,0) = e^y. \\
m(0,y,t) = e^{y-t} \ , & m(0,0,t) = e^{-t}.
\end{cases} \qquad (11)
$$

Applying Gamar transform on both sides of Eq. (10), we have

$$G\left[m_{xyt}(x,y,t) + m(x,y,t)\right] = 0. \qquad (12)$$

By linearity property and partial derivative properties of Gamar transform, we get

$$\alpha(r)\beta(s)\theta(v)\mathbb{M}(r,s,v) - \alpha(r)\beta(s)\mathcal{C}(v)G_D[m(x,y,0)] - \alpha(r)\theta(v)\mathcal{G}(s)G_D[m(x,0,t)] - \beta(s)\theta(v)\mathcal{E}(r)G_D[m(0,y,t)] + \alpha(r)\mathcal{E}(s)\mathcal{C}(v)\mathbb{F}_x[m(x,0,0)] + \beta(s)\mathcal{E}(r)\mathcal{C}(v)\mathbb{F}_y[m(0,y,0)] + \theta(v)\mathcal{E}(r)\mathcal{G}(s)\mathbb{F}_t[m(0,0,t)] - \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)m(0,0,0) + \mathbb{M}(r,s,v) = 0. \quad (13)$$

Substituting

$$G_D[m(x,y,0)] = \frac{\mathcal{E}(r)\mathcal{G}(s)}{(\alpha(r)-1)(\beta(s)-1)},$$

$$\mathbb{F}_x[m(x,0,0)] = \frac{\mathcal{E}(r)}{(\alpha(r)-1)},$$

$$G_D[m(x,0,t)] = \frac{\mathcal{E}(r)\mathcal{C}(v)}{(\alpha(r)-1)(\theta(v)+1)},$$

$$\mathbb{F}_y[m(0,y,0)] = \frac{\mathcal{G}(s)}{(\beta(s)-1)},$$

$$G_D[m(0,y,t)] = \frac{\mathcal{G}(s)\mathcal{C}(v)}{(\beta(s)-1)(\theta(v)+1)},$$

$$\mathbb{F}_t[m(0,0,t)] = \frac{\mathcal{C}(v)}{(\theta(v)+1)},$$

$$m(0,0,0) = 1.$$

in Eq. (13) and simplifying, we obtain:

$$\mathbb{M}(r,s,v) = \frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)}{(\alpha(r)-1)(\beta(s)-1)(\theta(v)+1)}. \quad (14)$$

Taking inverse Gamar transform for Eq. (14), we get

$$m(x,y,t) = G^{-1}[\mathbb{M}(r,s,v)] = G^{-1}\left[\frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)}{(\alpha(r)-1)(\beta(s)-1)(\theta(v)+1)}\right] = e^{x+y-t}.$$

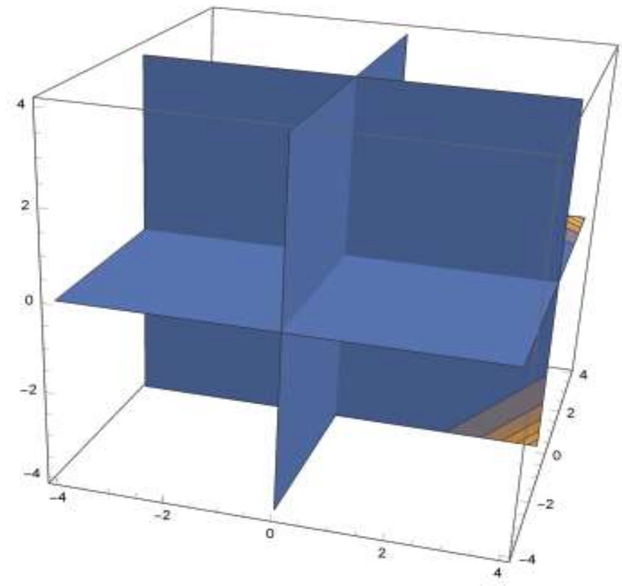

Figure 1. The exact solution of Example 5.1.

Example 5.2

Consider the following nonhomogeneous third-order Mboctara partial differential equation

$$m_{xyt}(x,y,t) + m(x,y,t) = 3e^{-x-2y+t}. \quad (15)$$

Subject to the boundary and initial conditions

$$\begin{cases} m(x,y,0) = e^{-x-2y}, & m(x,0,0) = e^{-x}, \\ m(x,0,t) = e^{-x+t}, & m(0,y,0) = e^{-2y}, \\ m(0,y,t) = e^{-2y+t}, & m(0,0,t) = e^{t}. \end{cases} \quad (16)$$

Applying Gamar transform on both sides of Eq. (15), we have

$$G\left[m_{xyt}(x,y,t) + m(x,y,t)\right] = G[3e^{-x-2y+t}]. \quad (17)$$

By linearity property and partial derivative properties of Gamar transform, we get

$$\alpha(r)\beta(s)\theta(v)\mathbb{M}(r,s,v) - \alpha(r)\beta(s)\mathcal{C}(v)G_D[m(x,y,0)] - \alpha(r)\theta(v)\mathcal{G}(s)G_D[m(x,0,t)] - \beta(s)\theta(v)\mathcal{E}(r)G_D[m(0,y,t)] + \alpha(r)\mathcal{G}(s)\mathcal{C}(v)\mathbb{F}_x[m(x,0,0)] + \beta(s)\mathcal{E}(r)\mathcal{C}(v)\mathbb{F}_y[m(0,y,0)] +$$

$$\theta(v)\mathcal{E}(r)\mathcal{G}(s)\mathbb{F}_t[m(0,0,t)] - \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)m(0,0,0) + \mathbb{M}(r,s,v) = \frac{3\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)}{(\alpha(r)+1)(\beta(s)+2)(\theta(v)-1)}. \quad (18)$$

Substituting

$$G_D[m(x,y,0)] = \frac{\mathcal{E}(r)\mathcal{G}(s)}{(\alpha(r)+1)(\beta(s)+2)},$$

$$\mathbb{F}_x[m(x,0,0)] = \frac{\mathcal{E}(r)}{(\alpha(r)+1)},$$

$$G_D[m(x,0,t)] = \frac{\mathcal{E}(r)\mathcal{C}(v)}{(\alpha(r)+1)(\theta(v)-1)},$$

$$\mathbb{F}_y[m(0,y,0)] = \frac{\mathcal{G}(s)}{(\beta(s)+2)},$$

$$G_D[m(0,y,t)] = \frac{\mathcal{G}(s)\mathcal{C}(v)}{(\beta(s)+2)(\theta(v)-1)},$$

$$\mathbb{F}_t[m(0,0,t)] = \frac{\mathcal{C}(v)}{(\theta(v)-1)},$$

$$m(0,0,0) = 1.$$

in Eq. (18) and simplifying, we obtain:

$$\mathbb{M}(r,s,v) = \frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)}{(\alpha(r)+1)(\beta(s)+2)(\theta(v)-1)}. \quad (19)$$

Taking inverse Gamar transform for Eq. (19), we get

$$m(x,y,t) = G^{-1}[\mathbb{M}(r,s,v)] = G^{-1}\left[\frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)}{(\alpha(r)+1)(\beta(s)+2)(\theta(v)-1)}\right] = e^{-x-2y+t}.$$

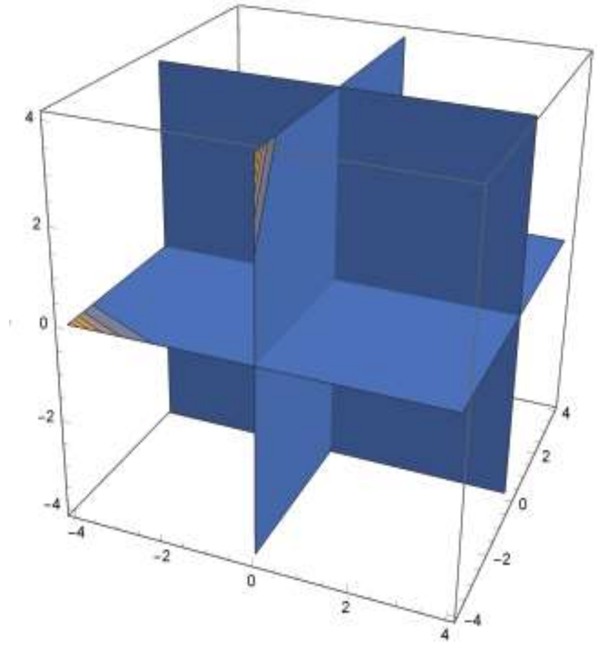

Figure 2. The exact solution of Example 5.2.

Example 5.3

Consider the following nonhomogeneous third-order Mboctara partial differential equation

$$m_{xyt}(x,y,t) + m(x,y,t) = \cos x \cos y \cos t - \sin x \sin y \sin t \quad (20)$$

Subject to the boundary and initial conditions

$$\begin{cases} m(x,y,0) = \cos x \cos y \ , m(x,0,0) = \cos x, \\ m(x,0,t) = \cos x \cos t \, , \, m(0,y,0) = \cos y, \quad (21) \\ m(0,y,t) = \cos y \cos t \ , m(0,0,t) = \cos t \, . \end{cases}$$

Applying Gamar transform on both sides of Eq. (21), we have

$$G\left[m_{xyt}(x,y,t)+m(x,y,t)\right] = G[\cos x\cos y\cos t - \sin x\sin y\sin t]. \quad (22)$$

By linearity property and partial derivative properties of Gamar transform, we get

$$\alpha(r)\beta(s)\theta(v)\mathbb{M}(r,s,v) - \alpha(r)\beta(s)\mathcal{C}(v)G_D[m(x,y,0)] - \alpha(r)\theta(v)\mathcal{G}(s)G_D[m(x,0,t)] - \beta(s)\theta(v)\mathcal{E}(r)G_D[m(0,y,t)] + \alpha(r)\mathcal{G}(s)\mathcal{C}(v)\mathbb{F}_x[m(x,0,0)] + \beta(s)\mathcal{E}(r)\mathcal{C}(v)\mathbb{F}_y[m(0,y,0)] + \theta(v)\mathcal{E}(r)\mathcal{G}(s)\mathbb{F}_t[m(0,0,t)] - \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)m(0,0,0) + \mathbb{M}(r,s,v) = \frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\alpha(r)\beta(s)\theta(v) - \mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)}{(\alpha^2(r)+1)(\beta^2(s)+1)(\theta^2(v)+1)}. \quad (23)$$

Substituting

$$G_D[m(x,y,0)] = \frac{\mathcal{E}(r)\mathcal{G}(s)\alpha(r)\beta(s)}{(\alpha^2(r)+1)(\beta^2(s)+1)},$$

$$\mathbb{F}_x[m(x,0,0)] = \frac{\mathcal{E}(r)\alpha(r)}{(\alpha^2(r)+1)},$$

$$G_D[m(x,0,t)] = \frac{\mathcal{E}(r)\mathcal{C}(v)\alpha(r)\theta(v)}{(\alpha^2(r)+1)(\theta^2(v)+1)},$$

$$\mathbb{F}_y[m(0,y,0)] = \frac{\mathcal{G}(s)\beta(s)}{(\beta^2(s)+1)},$$

$$G_D[m(0,y,t)] = \frac{\mathcal{G}(s)\mathcal{C}(v)\beta(s)\theta(v)}{(\beta^2(s)+1)(\theta^2(v)+1)},$$

$$\mathbb{F}_t[m(0,0,t)] = \frac{\mathcal{C}(v)\theta(v)}{(\theta^2(v)+1)},$$

$$m(0,0,0) = 1.$$

in Eq. (23) and simplifying, we obtain:

$$\mathbb{M}(r,s,v) = \frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\alpha(r)\beta(s)\theta(v)}{(\alpha^2(r)+1)(\beta^2(s)+1)(\theta^2(v)+1)}. \quad (24)$$

Taking inverse Gamar transform for Eq. (24), we get

$$m(x,y,t) = G^{-1}[\mathbb{M}(r,s,v)] = G^{-1}\left[\frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)\alpha(r)\beta(s)\theta(v)}{(\alpha^2(r)+1)(\beta^2(s)+1)(\theta^2(v)+1)}\right] = \cos x\cos y\cos t.$$

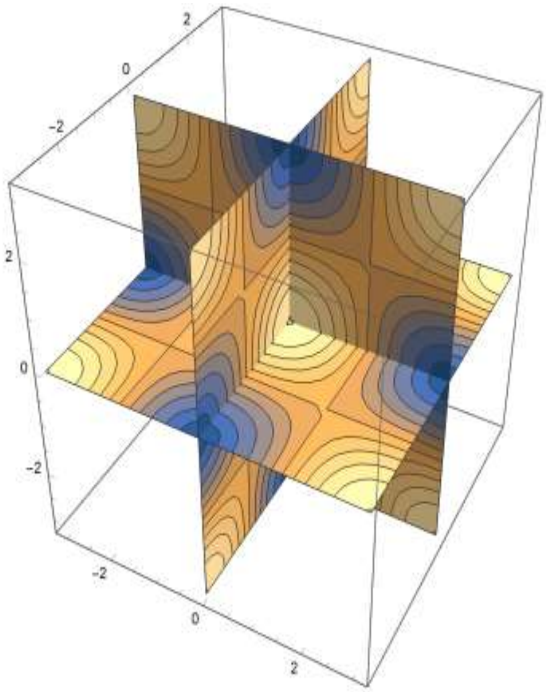

Figure 3. The exact solution of Example 5.3.

4 Conclusion

Greatly inspired by the work carried out in the single integral transform related to one-dimensional spaces and double integral transform in two-differential spaces, the new notion of Gamar transform is introduced. This novel transform is characterized by its capacity to both refine and imply its original model, namely Laplace triple transform in positive quadrant places. The next goal was to prove some major properties concerning the proposed Gamar transform, which include, among other properties, triple convolution theorem. In order to judge and evaluate the effectiveness of this transform, it is utilized to solve a selection of PDEs under standard conditions. As an extension of the of the present study, we recommend that scholars pursue investigations dealing with the feasibility of applying this transform to solve both differential and functional differential equations.

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Appendix

Table 1: *Here we present a list of the previous results Gamar transform of some special function and general properties:*

$\boldsymbol{m(x,y,t)}$	$G[\boldsymbol{m(x,y,t)}] = \mathbb{M}(\boldsymbol{r,s,v})$
$x^n y^n t^n$	$\frac{\mathcal{E}(r)}{\alpha^{n+1}(r)}\frac{\mathcal{G}(s)}{\beta^{n+1}(s)}\frac{\mathcal{C}(v)}{\theta^{n+1}(v)}(\Gamma(n+1))^3$
$e^{ax+by+ct}$	$\frac{\mathcal{E}(r)}{(\alpha(r)-a)}\frac{\mathcal{G}(s)}{(\beta(s)-b)}\frac{\mathcal{C}(v)}{(\theta(v)-c)}$
$\cos(ax+by+ct)$	$\frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)[\alpha(r)\beta(s)\theta(v)-ab\theta(v)-bc\alpha(r)-ac\beta(s)]}{(\mathcal{W}^2(r)+a^2)(\psi^2(s)+b^2)(\varphi^2(v)+c^2)}$
$\sin(ax+by+ct)$	$\frac{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)[b\alpha(r)\varphi(v)+a\beta(s)\theta(v)+c\alpha(r)\beta(s)-abc]}{(\alpha^2(r)+a^2)(\beta^2(s)+b^2)(\theta^2(v)+c^2)}$
$m(x-\kappa,y-\delta,t-\lambda)H(x-\kappa,y-\delta,t-\lambda)$	$e^{-\alpha(r)\kappa-\beta(s)\delta-\theta(v)\lambda}\mathbb{M}(r,s,v).$
$(n *** m)(x,y,t)$	$\frac{1}{\mathcal{E}(r)\mathcal{G}(s)\mathcal{C}(v)}\mathbb{N}(r,s,v)\mathbb{M}(r,s,v)$
$\frac{\partial^n m(x,y,t)}{\partial x^n}$	$\alpha^n(r)\mathbb{M}(r,s,v)-\mathcal{E}(r)\sum_{i=0}^{n-1}\alpha^{n-1-i}(r)\mathbb{T}_y\mathbb{T}_t\left[\frac{\partial^i m(0,y,t)}{\partial x^i}\right]$
$\frac{\partial^n m(x,y,t)}{\partial y^n}$	$\beta^n(s)\mathbb{M}(r,s,v)-\mathcal{G}(s)\sum_{i=0}^{n-1}\beta^{n-1-i}(s)\mathbb{T}_x\mathbb{T}_t\left[\frac{\partial^i m(x,0,t)}{\partial y^i}\right]$
$\frac{\partial^n m(x,y,t)}{\partial t^n}$	$\theta^n(v)\mathbb{M}(r,s,v)-\mathcal{C}(v)\sum_{i=0}^{n-1}\theta^{n-1-i}(v)\mathbb{T}_x\mathbb{T}_y\left[\frac{\partial^i m(x,y,0)}{\partial t^i}\right]$
$x^n m(x,y,t)$	$(-1)^n\frac{\mathcal{E}(r)}{\alpha'(r)}\frac{\partial^n}{\partial r^n}\left(\frac{\mathbb{M}(r,s,v)}{\mathcal{E}(r)}\right)$
$y^n m(x,y,t)$	$(-1)^n\frac{\mathcal{G}(s)}{\beta'(s)}\frac{\partial^n}{\partial s^n}\left(\frac{\mathbb{M}(r,s,v)}{\mathcal{G}(s)}\right)$
$t^n m(x,y,t)$	$(-1)^n\frac{\mathcal{C}(v)}{\theta'(v)}\frac{\partial^n}{\partial v^n}\left(\frac{\mathbb{M}(r,s,v)}{\mathcal{C}(v)}\right)$
$\int_0^x\int_0^y\int_0^t m(\ell,p,n)\,d\ell\,dp\,dn$	$\frac{1}{\alpha(r)\beta(s)\theta(v)}\mathbb{M}(r,s,v)$

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