On Semi Generalization of Compatible Ideals Regarding Semi Generalised Open Sets

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Abstract: -In this paper, we use the exiting semi-generalized open set to define semi generalised local function as intersection between any subset of a topological space X and semi generalized-open neighboorhood of any point of X that is not belong to ideal I and investigate its properties in ideal topological space. It is a generalization of the existing generalized local function. We also use the exiting semi-generalized open set to define semi generalised compatible ideals as for every $A \subseteq X$ such that for every $x \in A$, there exist semi generalized-open set U containing x such that $U \cup A \in I$, then $A \in I$. It is a generalization of the existing generalized compatible ideal. We characterized relationship between semi generalised local function and semi generalised compatible ideal.

Key-Words: - Dense, Ideal topological space, Semi generalised open sets, semi-generalized - neighbourhood, Semi generalised local function and Semi generalised compatible ideal.

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1 Introduction

[1] was the first person that presented the concept of ideal in topological space in the year 1930 as a nonempty collection of subset of a topological space (X,τ) that is closed under heridity and finite additive properties. It is denoted by I. The notion of ideal in topological space was introduced in two ways:The first way was concerned with the study of local properties of topological space that may be extended to global properties. While the second way was concerned with ideals to generalize certain properties of topological space, such as compactness and separation axioms [2]. When a topological space is endowed with an ideal, it is called an ideal topological space denoted by (X, τ, I). In 1933 [3] first presented the notion of local function $()^* : P(X) \longrightarrow P(X)$ defined such as $A^\star(I, \tau) = \{x \in X : U \cap A \notin I\}$ where U is open neighborhood of x and A is any subset of X. When there is no uncertainty, we use A^* instead of $A^\star(I, \tau)$. In 1944 [4] studies the local function introduced by Kuratowski and investigate some of its properties. [5] in 1987 gave the concept of semi generalized closed and open sets. In 2010 [6] presented the notion of semi-local function and semi-compatible ideals by using ideal and semi-open sets in an ideal topological space. In 2013 [7] presented the notion of local closure function as generalization of local function and explored the closure compatible ideal in ideal topological space. In 2013 [8] used generalised open sets to gave the concept of generalised local function in ideal topological space. In 2014 [9] used a-open sets to presented the concept of a-local function and characterized a-compatible ideal with a-local function. In 2017 [10] presented a new class of closed sets called semi generalized ωα-closed sets in topological spaces which properly lies between the class of semi-closed sets and the class of gs-closed sets. Further defined sgωα-closure and sgωα-interior in topological spaces and obtained some of their properties. In 2019, [11] presented the notion of generalized compatible ideal with generalized-open sets in codense ideal topological space. Therefore, in this paper we presented semi generalised local function and semi generalised compatible ideal.
The followings are very important in this paper.

1.1 Definition[1]

An ideal in topological space is defined as a collection I of nonempty subset of a topological space (X,τ) satisfying the following conditions.

1. If $A \in I$ and $B \subseteq A$, then $B \in I$ $(heredity)$
2. If $A \in I$ and $B \in I$, then $A \cup B \in I$ $(finite\ additivity)$

1.2 Definition [12]

A topology on a nonempty set X is a collection τ of subsets of X, that satisfies the following.

1. $\emptyset$ and X belong to τ;

2. The union of any collection of sets in τ belong to τ;
3. The intersection of any finite number of sets in τ also belong to τ.

1.3 Remark
The space (X, τ) is called a topological space and also the space (X, τ, I) is called an ideal topological space. The members of topology τ are called open subsets of X. If $A \subseteq X$, the interior of A denoted as $int(A)$ is the union of all open subsets of X contained in A.

1.4 Definition [13]
Let (X, τ, I) be defined as an ideal topological space. An ideal I is said to be compatible with respect to open subsets of X, denoted by $\tau \sim I$ if the following condition is satisfied for every $A \subseteq X$: if for every $x \in A$ there exists an open set U containing x such that $U \cap A \in I$, then $A \in I$.

1.5 Definition [14]
Let (X, τ) be a topological space. A subset A of X is said to be generalized open (g-open) if $F \subseteq int(A)$ and $F \subseteq A$ whenever F is closed in (X, τ).
And A is said to be a generalized closed (g-closed) if $Cl(A) \subseteq U$ and $A \subseteq U$ whenever U is open in (X, τ).
Complement of a g-open set is g-closed and complement of g-closed set g-open. A collection of all generalized open sets in a topological space (X, τ) is denoted as τ_g.

1.6 Definition [15]
Let (X, τ) be defined as a topological space. A subset A of X is said to be Semi-open set if the following holds $A \subseteq cl(int(A))$. The family of all semi-open subsets of X is denoted by τ_s. The union of all semi-open subsets of X contained in A is called the semi-interior of A denoted by $_sint(A)$ or $int_s(A)$.

1.7 Definition [15]
Let (X, τ) be defined as a topological space. A subset A of X is said to be a Semi-closed set if $int(cl(A)) \subseteq A$. The family of all semi-closed subsets of X is denoted by τ_s-closed. The intersection of all semi-closed sets containing a subset A of X is known as semi-closure of A and is denoted by $_scl(A)$ or $cl_s(A)$.

1.8 Definition [5, 15, 16]
Let (X, τ) be defined as a topological space. A subset A of X is said to be a semi-generalized open (sg-open) set if $F \subseteq int_s(A)$ whenever $F \subseteq A$ and F is semi-closed set in (X, τ). The family of all semi generalized open subsets of X is denoted as τ_{sg}. The union of all semi-generalized open subsets of X contained in $A \subseteq X$ is called semi generalized-interior of A denoted by $int_{sg}(A)$.

1.9 Remark
The complement of sg-open set is sg-closed and compliment of sg-closed set is sg-open.

1.10 Definition [15, 16]
Let (X, τ) be a topological space. A subset A of X is said to be a semi-generalized closed (sg-closed) set if $Cl_s(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ). The family of all semi- generalized closed subsets of X is denoted by τ_{sg}-closed. The intersection of all semi-generalized closed subsets of X containing a subset A of X is called semi-generalized closure of A denoted by $cl_{sg}(A)$.

1.11 Definition
If (X, τ, I) is an ideal topological space. A subset A of X is said to be [17]

1. L^*-perfect if $A - A^* \in I$
2. R^*-perfect if $A^* - A \in I$
3. C^*-perfect if A is both L^*-perfect and R^*-perfect.

1.12 Definition [13, 16]
Let X be a topological space and $x \in X$. A subset N of X is said to be semi-generalized - neighbourhood (sg - neighbourhood) of x if there exist sg -open set G such that $x \in G \subset N$.

1.13 Definition [3]
Let (X, τ) be a topological space and $A \subseteq X$, then a point $x \in A$ is said to be semi-generalized interior (sg - interior) point of A if A is sg - neighbourhood of x. The set of all Sg - interior point of A is called the sg - interior of A and is denoted by sg - $int(A)$ or $int_{sg}(A)$.

1.14 Definition [3]
Let (X ,τ) be a topological space and $A \subseteq X$, the intersection of all sg - closed sets containing A is called sg - closure of A denoted by sg-$cl(A)$. i.e. $cl_{sg}(A) = \cap \{F : A \subset F \in sgc(X)\}$.

1.15 Lemma [16]
If A is a subsets of X, then g-$int(A) \subset$ sg-$int(A)$, where g-$int(A)$ is given by g-$int(A) = \cup\{G : G\ is$ g-open$, G \subset A\}$.

1.16 Proof
Let A be the subset of X.
Let $x \in int(A) \Rightarrow x \in \cup\{G : G\ is$ g-open$, G \subset A\}$
$\Rightarrow$ there exists a g-open set G such that $x \in G \subset A$
$\Rightarrow$ there exists a sg-open set G such that $x \in G \subset A$, as every g-open set is sg-open set in X
$\Rightarrow x \in \cup\{G : G\ is$ sg-open$, G \subset A\}$. Hence g-$int(A) \subset$ sg-$int(A)$.

1.17 Example [10]

Let $X = \{a, b, c\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$. Then $\tau_{sg} = \{X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $\tau_g = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$. If $A = \{b, c\}$, then $sg\text{-}int(A) = \{b, c\}$ and $g\text{-}int(A) = \{c\}$. It follows $g\text{-}int(A) \subset sg\text{-}int(A)$.

1.18 Remark

Every g-open set is sg-open set in X. But the converse is not true.

2 Semi-generalized local function

2.1 Definition

Let (X,τ,I) be an ideal topological space. A set operator $()^*_{sg} : P(X) \longrightarrow P(X)$ is called semi-generalized local (sg-local) function of A with respect to τ and I is defined as $A^*_{sg}(I, \tau) = \{x \in X : U \cap A \notin I for\ every\ U \in N(sg)_x\}$ where $N(sg)_x$ is called a semi-generalized neighbourhood (sg-neighbourhood) system of x. i.e the collection of all semi generalized open (sg-open) sets containing x. $N(sg)_x = \{U \in \tau_{sg} : x \in U\}$. Where τ_{sg} is the set of all semi generalized open sets in (X, τ, I).

2.2 Lemma

Let (X, τ, I) be an ideal topological space and $A, B \subseteq X$. Then the following holds:

1. $\emptyset^*_{sg} = \emptyset$;
2. $A \subseteq B \Rightarrow A^*_{sg} \subseteq B^*_{sg}$;
3. $A^*_{sg} \subseteq A$;
4. If $I_1 \subseteq I_2 \subseteq I$, then $A^*_{sg}\ (I_2) \subseteq A^*_{sg}\ (I_1)$;
5. $(A^*_{sg})^*_{sg} \subseteq A^*_{sg}$;
6. $A^*_{sg} \cup B^*_{sg}$= $(A \cup B)^*_{sg}$;
7. (A^*_{sg}-B^*_{sg}) ⊆ (A-B)$^*_{sg}$-B^*_{sg}= (A-B)$^*_{sg}$;
8. If $I_1, I_2 \subseteq I$, then $A^*_{sg}(I_1) \cup A\,^*_{sg}(I_2) \subseteq A^*_{sg}(I_1 \cap I_2)$;
9. $A^*_{sg} = cl_{sg}(A^*_{sg}) \subseteq cl_{sg}(A)$.

2.3 Proof

Let (X, τ, I) be an ideal topological space. If $A, B \subseteq X$.

1. $\emptyset^*_{sg} = \emptyset$ is obvious.
2. Let $A \subseteq B$, then we want to show that $A^*_{sg} \subseteq B^*_{sg}$. Thus, if $x \in A^*_{sg}$, then there exist sg-open set U which contain x such that $U \cap A \notin I$. Since $A \subseteq B$, then clearly $U \cap A \subseteq U \cap B$ by heredity property of ideal $U \cap B \notin I$. Hence $A^*_{sg} \subseteq B^*_{sg}$.
3. Let $A \subseteq X$, if $x \in A^*_{sg}$, then there exist sg-open set U which contain x such that $U \cap A \notin I$. Since $U \cap A \subseteq A$, then clearly $A^*_{sg} \subseteq A$.
4. Let $I_1, I_2 \subseteq I$ and $I_1 \subseteq I_2$, if $x \in A^*_{sg}(I_1)$, then there exist sg-open set containing x such that $U \cap A \notin I_1$ and also $x \in A^*_{sg}(I_2)$, implies here exist sg-open set containing x such that $U \cap A \notin I_2$, but $I_1 \subseteq I_2$ implies $U \cap A \notin I_2 \subseteq U \cap A \notin I_1$. Hence $A^*_{sg}(I_2) \subseteq A^*_{sg}(I_1)$.
5. Let $A \subseteq X$ and $x \in A^*_{sg}$, then there exist sg-open set U containing x such that $U \cap A \notin I$. Since by our lemma 2.2 (3) $A^*_{sg} \subseteq A$ implies $U \cap A^*_{sg} \subseteq U \cap A \notin I$ by considering the heredity property of ideal $U \cap A^*_{sg} \notin I$. Hence $(A^*_{sg})^*_{sg} \subseteq A^*_{sg}$.
6. Let $A, B \subseteq X$. Then since $A \subset A \cup B$ and $B \subset A \cup B$. By lemma 2.2 (2) $A^*_{sg} \subset (A \cup B)^*_{sg}$ and $B^*_{sg} \subset (A \cup B)^*_{sg}$. Clearly,
$A^*_{sg} \cup B^*_{sg} \subset (A \cup B)^*_{sg}$. conversely, Suppose $x \in (A \cup B)^*_{sg}$, then there exist sg-open set containing x such that $U \cap (A \cup B) \notin I$. Thus,
$U \cap (A \cup B) \subset U \cap A \cup (U \cap B)$ by considering the heredity and finite additive properteis of ideal $U \cap (A \cup B) \subset U \cap A \cup (U \cap B) \notin I$ implies $U \cap A \notin I$ and $U \cap B \notin I$. Clearly, $(A \cup B)^*_{sg} \subset A^*_{sg} \cup B^*_{sg}$. consequently,
$A^*_{sg} \cup B^*_{sg}$ = $(A \cup B)^*_{sg}$.
7. Suppose $A, B \subseteq X$, since $A \subset (A - B) \cup B$ by our
lemma 2.2 (2) $A^*_{sg} \subset (A - B)^*_{sg} \cup B^*_{sg}$ implies
A^*_{sg} - $B^*_{sg} \subset (A - B)^*_{sg} \cup B^*_{sg}$ - B^*_{sg} implies
A^*_{sg} - $B^*_{sg} \subset (A - B)^*_{sg}$. conversely, since $A - B \cup B \subset A$ by
lemma 2.2 (2) $(A - B)^*_{sg} \cup B^*_{sg} \subset A^*_{sg}$
implies $(A - B)^*_{sg} \cup B^*_{sg}$ - $B^*_{sg} \subset A^*_{sg}$ - B^*_{sg} implies
$(A - B)^*_{sg} \subset A^*_{sg}$ - B^*_{sg}. Therefore,
A^*_{sg} - B^*_{sg} = $(A - B)^*_{sg}$.
8. Suppose $I_1, I_2 \subseteq I$, then we want to show that $A^*_{sg}(I_1) \cup A^*_{sg}(I_2) \subseteq A^*_{sg}(I_1 \cap I_2)$. Since $I_1 \cap I_2 \subseteq I_1$ and $I_1 \cap I_2 \subseteq I_2$ by lemma 2.2 (4) $A^*_{sg}(I_1) \subseteq A^*_{sg}(I_1 \cap I_2)$ and $A^*_{sg}(I_2) \subseteq A^*_{sg}(I_1 \cap I_2)$. Clearly $A^*_{sg}(I_1) \cup A^*_{sg}(I_2) \subseteq A^*_{sg}(I_1 \cap I_2)$.
9. Let $A \subseteq X$ and since $A^*_{sg} \subset cl_{sg}(A^*_{sg})$ hold in general. Let $x \in cl_{sg}(A^*_{sg})$, then there exist sg-open set U which contain x such that $A^*_{sg} \cap U \neq \emptyset$. Therefore, there exist some $y \in A^*_{sg} \cap U$ and $U \in \tau_{sg}(x)$. Since
$y \in A^*_{sg}\ A \cap U \notin I$ and so $x \in A^*_{sg}$. Thus $cl_{sg}(A^*_{sg}) \subset A^*_{sg}$. Hence

$A^*_{sg} = cl_{sg}(A^*_{sg})$ since by lemma 2.2 (3) $A^*_{sg} \subseteq A$ implies
$cl_{sg}(A^*_{sg}) \subseteq cl_{sg}(A)$. Consequently $A^*_{sg} = cl_{sg}(A^*_{sg}) \subseteq cl_{sg}(A)$.

2.4 Example

Let $X = \{a,b,c\}$, $\tau = \{X, \emptyset, \{b\}, \{c\}, \{a,b\}, \{b,c\}\}$
$\tau - closed = \{\emptyset, X, \{a,c\}, \{a,b\}, \{c\}, \{a\}\}$.
$P(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$
$\tau_s = \{X, \emptyset, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$
τ_s-closed= $\{X, \emptyset, \{a,c\}, \{a,b\}, \{c\}, \{a\}, \{b\}\}$.
$\tau_{sg} = \{\emptyset, X, \{b\}, \{c\}, \{a,c\}, \{b,c\}, \{a,b\}\}$.
Taking $I = \{\emptyset, \{b\}, \{a,b\}\}$, $A = \{c\}$ and $B = \{a,c\}$ then we have
$A^*_{sg} = \{c\}$ and $B^*_{sg} = \{a,c\}$. Hence $A^*_{sg} \subseteq B^*_{sg}$

2.5 Example

Let $X = \{a,b,c,d\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$
$\tau - closed = \{\emptyset, X, \{b,c,d\}, \{a,c,d\}, \{c,d\}, \{d\}\}$.
$P(X) = \{X, \emptyset, \{a,\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{c,d\}, \{a,c\}, \{a,d\}, \{b,d\}, \{a,b,c\}, \{b,c,d\}, \{c,d,a\}, \{a,b,d\}\}$
$\tau_s = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{b,c,d\}, \{a,c,d\}\}$
τ_s-closed= $\{\emptyset, X, \{b,c,d\}, \{a,c,d\}, \{c,d\}, \{d\}, \{a\}, \{b\}\}$.
$\tau_{sg} = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{b,c,d\}, \{a,c,d\}\}$.
Taking I = {∅, {a}, {b }, {a, b}} and A = {a, b, c}, then A^*_{sg} = {c}, clearly $A^*_{sg} \subseteq A$.

2.6 Example

Let $X = \{a,b,c\}$, $\tau = \{X, \emptyset, \{b\}, \{c\}, \{a,b\}, \{b,c\}\}$
$\tau - closed = \{\emptyset, X, \{a,c\}, \{a,b\}, \{c\}, \{a\}\}$.
$P(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$
$\tau_s = \{X, \emptyset, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$
τ_s-closed= $\{X, \emptyset, \{a,c\}, \{a,b\}, \{c\}, \{a\}, \{b\}\}$.
$\tau_{sg} = \{\emptyset, X, \{b\}, \{c\}, \{a,c\}, \{b,c\}, \{a,b\}\}$.
Taking I_1 = {∅, {a}}, I_2 = {∅, {a}, {b}, {a, b}} and A = {b, c}, then $A^*_{sg}(I_1) = X$ and $A^*_{sg}(I_2)$ = {c}. Hence $A^*_{sg}(I_2) \subseteq A^*_{sg}(I_1)$

2.7 Example

Let $X = \{a,b,c\}$, $\tau = \{X, \emptyset, \{a,b\}, \{b,c\}, \{b\}\}$
$\tau - closed = \{\emptyset, X, \{c\}, \{a,\}, \{a,c\}\}$.
$P(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$
$\tau_s = \{X, \emptyset, \{a,b\}, \{b,c\}, \{b\}\}$
τ_s-closed= $\{X, \emptyset, \{c\}, \{a,\}, \{a,c\}\}$.
$\tau_{sg} = \{\emptyset, X, \{b,c\}, \{b\}, \{a,b\}\}$.
Taking I = {∅, {a}, {b }, {a, b}} and A = {b, c}, then A^*_{sg} = {c} and also $(A^*_{sg})^*_{sg}$ = {c}. Therefore, $(A^*_{sg})^*_{sg} \subseteq A^*_{sg}$.

2.8 Example

Let $X = \{a,b,c\}$, $\tau = \{X, \emptyset, \{b\}, \{c\}, \{a,b\}, \{b,c\}\}$
$\tau - closed = \{\emptyset, X, \{a,c\}, \{a,b\}, \{c\}, \{a\}\}$.
$P(X) = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$
$\tau_s = \{X, \emptyset, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$
τ_s-closed= $\{X, \emptyset, \{a,c\}, \{a,b\}, \{c\}, \{a\}, \{b\}\}$.
$\tau_{sg} = \{\emptyset, X, \{b\}, \{c\}, \{a,c\}, \{b,c\}, \{a,b\}\}$.
Taking I = {∅, {a}, {b }, {a, b}} and A = {c}, B = {a, c}, then A^*_{sg} = {c}, B^*_{sg} = {c} and $(A \cup B)^*_{sg}$ = $(\{a,c\})^*_{sg}$ = {c}. Hence $A^*_{sg} \cup B^*_{sg} = (A \cup B)^*_{sg}$.

2.9 Example

Let $X = \{a,b,c\}$, $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a,c\}\}$
$\tau_{sg} = \{\emptyset, X, \{a,c\}, \{b,c\}, \{a,b\}, \{a\}, \{c\}\}$.
Taking I = {∅, {a}, {b }, {a, b}} and A = {a, b}, B = {c}, then A^*_{sg} = ∅,
B^*_{sg} = {c} implies A^*_{sg} - B^*_{sg} = ∅ and since $(A - B)$ = ∅ implies $(A - B)^*_{sg}$ = $\emptyset^*_{sg}$ = ∅. Hence A^*_{sg} - B^*_{sg} = $(A - B)^*_{sg}$.

2.10 Example

Let $X = \{a,b,c\}$, $\tau = \{X, \emptyset, \{b\}, \{c\}, \{a,b\}, \{b,c\}\}$
τ_s-closed= $\{X, \emptyset, \{a,c\}, \{a,b\}, \{c\}, \{a\}, \{b\}\}$.
$\tau_{sg} = \{\emptyset, X, \{b\}, \{c\}, \{a,c\}, \{b,c\}, \{a,b\}\}$.
Taking I_1 = {∅, {a}}, I_2 = {∅,} and A = {a, c}, then $A^*_{sg}(I_1)$ = {c} and
$A^*_{sg}(I_2)$ = {a, c}, since $I_2 \subseteq I_1$ implies $I_1 \cap I_2 = I_2$, thus,
$A^*_{sg}(I_1 \cap I_2) = A^*_{sg}(I_2)$ = {a, c}. Hence $A^*_{sg}(I_1) \cup A^*_{sg}(I_2) \subseteq A^*_{sg}(I_1 \cap I_2)$.

2.11 Definition

A subset A of an ideal topological space (X,τ,I) is said to be

1. τ^*_{sg} - closed if $A^*_{sg} \subseteq A$;
2. *-sg-dense in itself if $A \subseteq A^*_{sg}$;
3. I - sg-dense if $A^*_{sg} = X$;
4. *-sg-perfect if $A = A^*_{sg}$;
5. L^*_{sg} - perfect if A - $A^*_{sg} \in I$;
6. R^*_{sg} - perfect if A^*_{sg} - $A \in I$;
7. C^*_{sg} - perfect if A is both L^*_{sg} - perfect and R^*_{sg} - perfect.

3 Semi-generalized compatible (Sg-compatible) ideal

3.1 Definition

Let (X, τ, I) be an ideal topological space. An ideal I is said to be semi generalized compatible (sg-compatible) with respect to semi-generalized open (sg-open) subsets of X, denoted by $\tau \sim_{sg} I$ if the following condition is satisfied for every $A \subseteq X$: if for every $x \in A$, there exist sg-open set U containing x such that $U \cup A \in I$, then $A \in I$.

3.2 Theorem

Let (X, τ, I) be an ideal topological space and A be any subset of X, then the following are equivalent.

1. $\tau \sim_{sg} I$;
2. If $\mathcal{U} = \{U_\alpha : \alpha \in \Delta\}$ is semi-generalized open cover for A such that $cl(U_\alpha) \cap A \in I, \forall \alpha \in \Delta$, then $A \in I$;
3. $A \cap A^*_{sg} = \emptyset$, then $A \in I$;
4. A is L^*_{sg} - perfect set;
5. If A is R^*_{sg}- perfect set, then $A \, \Delta \, A^*_{sg} \in I$;
6. For every τ^*_{sg} -closed subset of A, then $A - A^*_{sg} \in I$.

3.3 Proof

1. (1) $\Rightarrow$ (2) Suppose $A \subseteq X$ and for every $x \in A$, there exist sg -open set U which contain x such that $U \cap A \in I$, then $A \in I$. Implies (2)
2. (2) $\Rightarrow$ (3) Suppose $A \subseteq X$ has sg-open cover whose intersection with A belongs to I. If $x \in A$, there exist $U \in \tau_{sg}(x)$ such that $U \cap A \in I$. Since $x \in A$ clearly $x \notin A^*_{sg}$ and so $A \cap A^*_{sg} = \emptyset$, then $A \in I$. Implies (3)
3. (3) $\Rightarrow$ (4) Suppose for every $A \subseteq X$ and $A \cap A^*_{sg} = \emptyset$, then $A \in I$. Since $A - A^*_{sg} \subseteq A$, then by lemma 2.2 (2) $(A - A^*_{sg})^*_{sg} \subseteq A^*_{sg}$ clearly $(A - A^*_{sg}) \cap (A - A^*_{sg})^*_{sg} \subseteq (A - A^*_{sg}) \cap A^*_{sg}$ since by lemma 2.2 (3) $A^*_{sg} \subseteq A$ implies $(A - A^*_{sg}) \cap A^*_{sg} = \emptyset$. Hence $(A - A^*_{sg}) \cap (A - A^*_{sg})^*_{sg} = \emptyset$, then $A - A^*_{sg} \in I$. Therefore, A is L^*_{sg} perfect set. Implies (4)
4. (4) $\Rightarrow$ (5) Suppose for every $A \subseteq X$, A is L^*_{sg}-perfect set. i.e $A - A^*_{sg} \in I$. If A is R^*_{sg}-perfect set, then $A^*_{sg} - A \in I$. Therefore, by finite additive property of ideal $(A - A^*_{sg}) \cup (A^*_{sg} - A) \in I$. Hence $AA^*_{sg} \in I$. Implies (5)
5. (1) $\Rightarrow$ (5) Suppose for every $A \subseteq X$: If for every $x \in A$, there exist sg-open set which contain x such that $U \cap A \in I$, then $A \in I$. Since $x \in A$ and $A \in I$ clearly $x \notin A^*_{sg}$ and $A \cap A^*_{sg} = \emptyset$ also we all know that $A - A^*_{sg} \subseteq A$ and by lemma 2.2 (2) implies $(A - A^*_{sg})^*_{sg} \subseteq A^*_{sg}$. Thus,$(A - A^*_{sg}) \cap (A - A^*_{sg})^*_{sg} \subseteq A \cap A^*_{sg} = \emptyset$ implies $(A - A^*_{sg}) \cap (A - A^*_{sg})^*_{sg} = \emptyset$ and $A - A^*_{sg} \in I$. If A is R^*_{sg}-perfect set, then $A^*_{sg} - A \in I$. By finite additive property of ideal $(A - A^*_{sg}) \cup (A^*_{sg} - A) \in I$. Hence AA^*_{sg}. Implies (5)
6. (5) $\Rightarrow$ (6) Suppose for every $A \subseteq X$ and A is R^*_{sg}-perfect set, then $AA^*_{sg} \in I$ implies $(A - A^*_{sg}) \cup (A^*_{sg} - A) \in I$. Since by lemma 2.2 (3) $A^*_{sg} \subseteq A$ clearly $A^*_{sg} - A \subseteq A - A^*_{sg}$ implies $(A - A^*_{sg}) \cup (A^*_{sg} - A) \in I = A - A^*_{sg} \in I$ implies $A - A^*_{sg} \in I$. Hence A is τ^*_{sg}-closed subset. Implies (6)
7. (1) $\Rightarrow$ (6) Suppose for every $A \subseteq X$ and for every $x \in A$ there exist sg-open set U which contain x such that $U \cap A \in I$, then $A \in I$. Since $x \in A$ and $A \in I$ clearly $x \notin A^*_{sg}$ and so $A \cap A^*_{sg} = \emptyset$. Since $A - A^*_{sg} \subseteq A$, then by lemma 2.2 (2) $(A - A^*_{sg})^*_{sg} \subseteq A^*_{sg}$ implies $(A - A^*_{sg}) \cap (A - A^*_{sg})^*_{sg} \subseteq A \cap A^*_{sg} = \emptyset$ implies $(A - A^*_{sg}) \cap (A - A^*_{sg})^*_{sg} = \emptyset$, then $A - A^*_{sg} \in I$. Hence for every τ^*_{sg}-closed subset of A, then $A - A^*_{sg} \in I$ Implies (6)

3.4 Theorem

Let (X, τ, I) be an ideal topological space. If $\tau \sim_{sg} I$, then the following properties are equivalent.

1. If for every $A \subseteq X$ and $A \cap A^*_{sg} = \emptyset$, then $A^*_{sg} = \emptyset$;
2. If for every $A \subseteq X$, then $(A - A^*_{sg})^*_{sg} = \emptyset$;
3. If for every $A \subseteq X$, then $(A \cap A^*_{sg})^*_{sg} = A^*_{sg}$.

3.5 Proof

1. (1) $\Rightarrow$ (2)Suppose $\tau \sim_{sg} I$ and if for every $A \subseteq X$, $A \cap \ A^*_{sg} = \emptyset$, then $A^*_{sg} = \emptyset$. Since we all know that $A - A^*_{sg} \subseteq A$, then by lemma 2.2 (2) $(A - A^*_{sg})^*_{sg} \subseteq A^*_{sg}$ implies $(A - A^*_{sg}) \cap (A - A^*_{sg})^*_{sg} \subseteq A \cap A^*_{sg} = \emptyset$ implies $(A - A^*_{sg}) \cap (A - A^*_{sg})^*_{sg} = \emptyset$, then $(A - A^*_{sg})^*_{sg} = \emptyset$ implies (2).
2. (2) $\Rightarrow$ (3) Suppose $\tau \sim_{sg} I$ and if for every $A \subseteq X$, then $(A - A^*_{sg})^*_{sg} = \emptyset$. Since by theorem 3.2(3) $A \cap \ A^*_{sg} = \emptyset$, then $A^*_{sg} = \emptyset$ clearly $A = (A - A^*_{sg}) \cup (A \cap A^*_{sg})$ by lemma 2.2 (2) $A^*_{sg} = ((A - A^*_{sg}) \cup (A \cap A^*_{sg}))^*_{sg}$ implies $A^*_{sg} = (A - A^*_{sg})^*_{sg} \cup (A \cap A^*_{sg})^*_{sg}$ since $(A - A^*_{sg})^*_{sg} = \emptyset$, then clearly $A^*_{sg} = (A \cap A^*_{sg})^*_{sg}$ implies (3).
3. (1) $\Rightarrow$ (3) Suppose $\tau \sim_{sg} I$ and if for every $A \subseteq X$, $A \cap A^*_{sg} = \emptyset$, then $A^*_{sg} = \emptyset$. Since by 4.2.1(3) $A^*_{sg} \subseteq A$, then clearly $A \cap A^*_{sg} \subseteq A$ by lemma 2.2 (2) $(A \cap A^*_{sg})^*_{sg} \subseteq A^*_{sg}$. Since $A \cap \ A^*_{sg} = \emptyset$, then $A^*_{sg} = \emptyset$ by lemma 2.2 (1) $\emptyset^*_{sg} = \emptyset$ and $A^*_{sg} = \emptyset^*_{sg}$ implies $(A \cap A^*_{sg})^*_{sg} \subseteq A^*_{sg} = \emptyset$. Hence $(A \cap A^*_{sg})^*_{sg} = A^*_{sg}$ implies (3).

4 Conclusion

It is concluded that every generalized local function is semi generalized local function but the converse is not true. Every generalized compatible ideals is semi generalized compatible ideals but the converse is not true.

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