The use of Markov chain model for rating location based turbine performance

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Abstract: - It is shown how a 12-state Markov chain model can be used in rating turbine performance in a given location. A 1.8 MW wind turbine exposed to wind speed in San Angelo, USA is used for illustration. The model fits the data. As such, features of the model are used in providing indices for rating the performance of the turbine in this location. Probability distribution of the wind speed in this location is introduced into each of the traditional methods of computing average power output from the turbine power curve and expected extract-able power. The estimates obtained are 937 kW and 826 kW, respectively.

Keywords/phrase: - Markov chain model, turbine performance rating, turbine power curve, wind energy, wind speed data

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1. Introduction

Since the mid-20th century to date, renewable power sources have attracted so much attention, partly due to the increasing power needs by the fast growing world population in the face of fast depleting fossil fuel sources, making fossil fuels consumption unsustainable. The other part of the push for renewable power sources is the concern over large volumes of greenhouse gas (GHG) emission from fossil fuel consumption, which is the key player in temperature increases that causes global warming. These concerns have made global renewable power supply to sore geometrically over the last two decades. For instance, renewable power contributed up to 26.2 % of global electricity generation in 2018 and is projected to increase to 45% by 2040 [21]. Similarly, the global non-hydro renewable power generation capacity grew by a record 184 GW in 2019, an increase of 20 GW (12%) over the generation capacity of 2018 [1].

Wind power is a major player in the renewable power market. Wind and hydroelectricity made up two-thirds of the total renewable power generation in 2018 in the 27-member EU countries [22]. Wind power installed capacity in 2018 exceeded 563 GW and accounted for approximately 24% of the world's total renewable power generation capacity [20]. In 2020 wind power generation capacity grew by 53 %, an increase of 93 GW of new installations [33]. The year 2021 saw global grid connected increase in wind generated energy of 94.3 GW, giving a cumulative total of 838.9 GW [30].

The intermittent nature of renewable power sources poses a major challenge in the planning and operation of their power systems. Hence, probability models come in handy to characterize renewable power sources in the face of this intermittency. A mixture probability density provided the modeling capability for determining uncertainty in loads on onshore and offshore wind turbines [25]. There are many other extensive literature on the use of parametric probability models such as the Weibull, Raleigh, lognormal, generalized extreme value, exponentiatedepsilon and gamma distributions in the modeling of renewable power potentials of many locations. See for examples [12, 13, 23, 29].

One of the intermittent properties of wind speed as a major renewable power contributor is that it changes abruptly over short time period. It is hard to tell whether a particular rate of wind speed will decrease, remain same or increase within any time interval. The stochastic nature of this intermittency can be captured by Markov chain models. Consequently, time homogeneous Markov chain models are used [19] in the literature for modeling wind speed time series data mainly for the purpose of forecasting. Markov chain models are also useful in the determination of the reliability indices of wind power systems [9]. A Markov-based Back Propagation (BP) neural network optimized by the Particle Swarm Optimization (PSO) used for wind power prediction showed a non-significant improvement over the traditional Markov chain [31]. A semi-Markov model was also used to evaluate the performance of wind turbine systems [26].

In order to characterize wind power more accurately forecast [2, 10, 15] and simulation [18, 24] are two methods mainly used. Discrete time Markov chain has been used for the generation of synthetic wind speed and wind power time series [24, 27] as well as for short term Wind Power Forecasting (WPF) with good performance. Other methods for wind speed series can be found in Mycielski Algorithm [5, 6].

These WPF methods use only onsite information in forecasting targeted wind farms. Currently there is ever-increasing number of wind farms over a region prompting researchers to explore the spatio-temporal-interdependence structure between wind farms in improving WPF performance [28]. Towards the end different artificial intelligence methods [3, 7], regime-switching space-time methods [11], multichannel adaptive filters [8], sparse vector autoregressive (VAR)-based model [4, 34], and so on, have been developed. The idea of spatial Markov chain used in geo-statistical modeling has inspired the development of a first order discrete spatio-temporal Markov chain model for short term wind power forecasting (WPF) [32].

Here, our focus is to use the time homogeneous Markov chain model in modeling the performance of a wind turbine in a specific location. We are interested in showing how to characterize the salient features in the power curve specific to a turbine using Markov chain method. This will help system operators and marketers make informed optimal decisions regarding its likely performance, particularly when it concerns investment in planning and development of wind farm in a location.

2 Materials and Methods

2.1 Markov chain model

For the purpose of illustration we use the power curve of a 1.8 MW wind turbine with 100 m rotor diameter at 100 m turbine hub height and 1.225 kg/m^3 air density. The details are given in Table 1 below.

Table 1 Power curve of a 1.8 MW wind turbine

Wind speed (m/s)	< 3	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12	> 20
Power, Pe, (kW)	0	51	175	346	584	913	1313	1660	1784	1799	1800	0

Definition 1

Notice, from Table 1, that the turbine has cut-in and cut-out wind speeds of 3 m/s and 20 m/s, respectively, for which times the turbine power generation is zero. Also, notice that the wind speed between 12 m/s and 20 m/s, inclusive, gives the optimal turbine performance.

As interest lies in a model that can capture the intermittent nature of the wind speed that give rise to this power curve, we employ a Markov chain model. Wind speed at specified time intervals are captured in order to create a discrete state space for the model. It should be noted that although wind speed is a continuous real-valued random variable, a discrete state space Markov model is chosen in order to make the model mathematically tractable. Also the states chosen are not sacrosanct but sufficiently reflective of the change of status of the Markov chain for modeling purpose. The construction of the states of the Markov chain are presented in Table 2 below.

Table 2 States of the Markov chain

State	1	2	3	4	5	6	7	8	9	10	11	12
Wind speed range (m/s)	< 3	3 – 4	4 – 5	5 – 6	6 – 7	7 – 8	8 – 9	9 - 10	10 – 11	11 – 12	12 – 20	> 20

Note: Upper bounds exclusive except state 11

The usual assumption of a Markov chain model obtains. That is, there is Markov dependence and time homogeneity of the Markov chain.

2.2 The problems

Pertinent questions that are of interest to wind power developers, investors and manufacturers, specific to the wind speed in a location for a given wind turbine, are given below.

i. What is the downtime of the wind turbine? That is, the estimate of the proportion of its idle time in the long run due to intermittency of wind speed.
ii. What time duration will it take a wind turbine to return to optimal, or near-optimal, power generation having moved to an idle state or low power generation state?
iii. What is the amount of power that a wind turbine will be able to generate in the long run, in a given location?
iv. What is the average extract-able power a given turbine can generate in a given location?

The model can provide answers to these questions thereby enabling interested persons make informed decisions regarding the location-specific performance of a wind turbine.

2.3 The Data

The wind speed time series data used in this study for illustration are real-time average wind speed recorded at every 5-minute interval from the 1st hour of January 1, 2010 to the 24th hour of December 31, 2012. They are 315,360 data values of wind speed time series for a location near San Angelo, Texas, with site ID 998289 on latitude 31.3983ºN and longitude 101.1654ºW. The data were recorded at 100 m above ground level with no missing value. They were obtained at http://www.wind.nrel.gov.

The data were processed for analysis as follows. The data were classified into 12 states of the Markov chain according to Table 2 above. The transition matrix is of the form

$$\mathbf{N} = \{n_{ij}\}, i, j = 1, 2, \ldots, 12 \qquad 1$$

where n_{ij} is the number of transitions from state i to state j. This is given in Table 3 below.

Table 1 Transition matrix of the wind speed data

State	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]
[1,]	23776	1004	47	8	7	1	1	1	0	1	0	0
[2,]	1011	17163	1540	49	6	3	5	2	0	0	0	0
[3,]	34	1532	23183	2050	57	17	7	1	0	0	0	0
[4,]	7	43	1997	27444	2536	61	16	6	1	0	0	0
[5,]	8	18	77	2449	31081	2876	56	9	2	1	0	0
[6,]	4	4	27	81	2790	32434	3024	67	14	4	0	0
[7,]	1	6	4	14	54	2938	31533	2980	62	11	10	0
[8,]	1	4	1	9	20	84	2884	28935	2459	42	17	0
[9,]	1	2	3	3	11	16	60	2360	21780	1784	62	0
[10,]	1	0	3	1	6	7	16	64	1690	14042	1188	0
[11,]	2	2	1	3	9	11	11	31	74	1132	20140	30
[12,]	0	1	0	0	0	0	0	0	0	0	29	71

2.4 Test of the time homogeneity of the model

The maximum likelihood estimator of the elements of the one-step transition probability matrix of the Markov chain can be obtained from Table 3, and is computed from

$$\hat{\mathbf{P}} = \left\{ \hat{p}_{ij} \right\} i, j = 1, 2, ..., 12 \qquad 2$$

where $\hat{p}_{ij} = \dfrac{n_{ij}}{n_{i.}}$ and $n_{i.} = \sum_{j=1}^{12} n_{ij}$.

The result is given as matrix $\boldsymbol{P}$ below.

$$\boldsymbol{P} = \begin{pmatrix}
.95694 & .04041 & .00189 & .00032 & .00028 & .00004 & .00004 & .00004 & .00000 & .00004 & .00000 & .00000 \\
.05112 & .86774 & .07786 & .00248 & .00030 & .00015 & .00025 & .00010 & .00000 & .00000 & .00000 & .00000 \\
.00126 & .05699 & .86240 & .07262 & .00212 & .00067 & .00026 & .00004 & .00000 & .00000 & .00000 & .00000 \\
.00022 & .00134 & .06219 & .85466 & .07897 & .00190 & .00050 & .00019 & .00003 & .00000 & .00000 & .00000 \\
.00022 & .00049 & .00211 & .06695 & .84974 & .07863 & .00153 & .00025 & .00005 & .00003 & .00000 & .00000 \\
.00010 & .00010 & .00070 & .00211 & .07257 & .84356 & .07865 & .00174 & .00037 & .00010 & .00000 & .00000 \\
.00003 & .00016 & .00011 & .00037 & .00143 & ..07811 & .83835 & .07923 & .00165 & .00029 & .00027 & .00000 \\
.00003 & .00011 & .00003 & .00026 & .00058 & .00244 & .08370 & .83977 & .07137 & .00122 & .00049 & .00000 \\
.00004 & .00008 & .00011 & .00012 & .00042 & .00061 & .00230 & .09048 & .83506 & .06840 & 00238 & .00000 \\
.00006 & .00000 & .00017 & .00006 & .00035 & .00041 & .00094 & .00376 & .09931 & .82513 & .06981 & .00000 \\
.00009 & .00009 & .00005 & .00014 & .00042 & .00051 & .00051 & .00145 & .00345 & .05279 & .93910 & .00140 \\
.00000 & .00990 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .28713 & .70297
\end{pmatrix}$$

To test for the assumption of time homogeneity, the entire dataset is divided into three subsamples, one for each of the three years (2010, 2011, 2012) for which the data were generated. The transition probability matrix for each of these years are similarly estimated and given as $\boldsymbol{P}_1$, $\boldsymbol{P}_2$ and $\boldsymbol{P}_3$ below.

$$\boldsymbol{P}_1 = \begin{pmatrix}
.96069 & .03746 & .00123 & .00037 & .00012 & .00000 & .00000 & .00000 & .00000 & .00012 & .00000 & .00000 \\
.04803 & .87024 & .07984 & .00126 & .00031 & .00000 & .00016 & .00016 & .00000 & .00000 & .00000 & .00000 \\
.00097 & .05335 & .87223 & .07182 & .00162 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 \\
.00037 & .00119 & .05758 & .86120 & .07766 & .00155 & .00018 & .00018 & .00009 & .00000 & .00000 & .00000 \\
.00017 & .00050 & .00215 & .06739 & .84736 & .08128 & .00091 & .00017 & .00000 & .00008 & .00000 & .00000 \\
.00000 & .00008 & .00045 & .00181 & .07147 & .84333 & .08082 & .00158 & .00038 & .00008 & .00000 & .00000 \\
.00000 & .00008 & .00008 & .00015 & .00108 & ..07909 & .83875 & .07886 & .00138 & .00031 & .00023 & .00000 \\
.00000 & .00017 & .00000 & .00026 & .00078 & .00259 & .08558 & .83394 & .07478 & .00130 & .00060 & .00000 \\
.00000 & .00011 & .00000 & .00000 & .00011 & .00100 & .00177 & .09165 & .83413 & .06934 & 00189 & .00000 \\
.00000 & .00000 & .00037 & .00000 & .00075 & .00037 & .00056 & .00522 & .10916 & .81265 & .07091 & .00000 \\
.00000 & .00016 & .00000 & .00016 & .00033 & .00082 & .00098 & .00230 & .00345 & .05857 & .93273 & .00049 \\
.00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .33333 & .66667
\end{pmatrix}$$

$$\boldsymbol{P}_2 = \begin{pmatrix}
.95600 & .04098 & .00191 & .00041 & .00027 & .00014 & .00014 & .00014 & .00000 & .00000 & .00000 & .00000 \\
.04657 & .87193 & .07886 & .00186 & .00031 & .00031 & .00000 & .00016 & .00000 & .00000 & .00000 & .00000 \\
.00141 & .05471 & .86356 & .07641 & .00250 & .00109 & .00033 & .00000 & .00000 & .00000 & .00000 & .00000 \\
.00009 & .00117 & .06229 & .85882 & .07485 & .00197 & .00063 & .00018 & .00000 & .00000 & .00000 & .00000 \\
.00042 & .00042 & .00218 & .06813 & .85387 & .07298 & .00184 & .00008 & .00008 & .00000 & .00000 & .00000 \\
.00008 & .00000 & .00081 & .00242 & .06875 & .85200 & .07385 & .00170 & .00032 & .00008 & .00000 & .00000 \\
.00000 & .00018 & .00027 & .00062 & .00178 & ..07970 & .83400 & .08147 & .00142 & .00036 & .00009 & .00000 \\
.00000 & .00009 & .00000 & .00009 & .00035 & .00220 & .07855 & .84748 & .07045 & .00053 & .00026 & .00000 \\
.00000 & .00011 & .00023 & .00011 & .00034 & .00000 & .00191 & .08678 & .83692 & .07158 & 00203 & .00000 \\
.00000 & .00000 & .00000 & .00016 & .00031 & .00063 & .00094 & .00220 & .09496 & .83205 & .06875 & .00000 \\
.00011 & .00000 & .00000 & .00000 & .00057 & .00011 & .00023 & .00057 & .00260 & .04789 & .94690 & .00102 \\
.00000 & .00990 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .32143 & .67857
\end{pmatrix}$$

$$\boldsymbol{P}_3 =$$
$$\begin{pmatrix}
.95451 & .04251 & .00234 & .00021 & .00043 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 \\
.05811 & .86160 & .07514 & .00415 & .00029 & .00014 & .00057 & .00000 & .00000 & .00000 & .00000 & .00000 \\
.00143 & .06350 & .85030 & .08098 & .00226 & .00095 & .00048 & .00012 & .00000 & .00000 & .00000 & .00000 \\
.00020 & .00170 & .06713 & .84287 & .08501 & .00220 & .00070 & .00020 & .00000 & .00000 & .00000 & .00000 \\
.00008 & .00056 & .00199 & .06542 & .84811 & .08145 & .00183 & .00048 & .00008 & .00000 & .00000 & .00000 \\
.00023 & .00023 & .00086 & .00211 & .07738 & .83565 & .08105 & .00195 & .00039 & .00016 & .00000 & .00000 \\
.00007 & .00022 & .00000 & .00037 & .00150 & ..07582 & .84163 & .07761 & .00210 & .00022 & .00045 & .00000 \\
.00009 & .00009 & .00009 & .00043 & .00061 & .00251 & .08689 & .83802 & .06885 & .00182 & .00061 & .00000 \\
.00000 & .00000 & .00012 & .00024 & .00086 & .00086 & .00330 & .09323 & .83419 & .06391 & 00330 & .00000 \\
.00019 & .00000 & .00019 & .00000 & .00000 & .00019 & .00132 & .00416 & .09455 & .82943 & .06997 & .00000 \\
.00015 & .00015 & .00015 & .00031 & .00031 & .00077 & .00046 & .00184 & .00460 & .05400 & .93450 & .00276 \\
.00000 & .01562 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .00000 & .26563 & .71875
\end{pmatrix}$$

To test for the time homogeneity of the model, the following notations are adopted.

Let $\boldsymbol{N}_t$ be the matrix of the number of one step transitions at the time $t = 1, 2, 3$ with elements, ${}_t n_{ij}$ which are the number of one step transitions from state i to state j, $i, j = 1, 2, \dots, 12$; and ${}_t n_{i.}$ is the i^{th} row total of $\boldsymbol{N}_t$.

Let $\boldsymbol{P}_t$ denote the matrix of one step transition probabilities at time t, with elements ${}_t p_{ij}$. Then the estimates of ${}_t p_{ij}$ is given by

$$ {}_t \hat{p}_{ij} = \frac{{}_t n_{ij}}{{}_t n_{i.}}, t = 1, 2, 3; i, j = 1, 2, \dots, 12 \qquad 3$$

The hypothesis for the test is given by

$$H_0 : {}_t p_{ij} = \hat{p}_{ij}; i, j = 1, 2, \dots, 12$$

against

$$H_1 : {}_t p_{ij} \neq \hat{p}_{ij}; t = 1, 2, 3$$

And the test statistic is given by

$$Q = \sum_{t=1}^{3} \sum_{i=1}^{12} \sum_{j=1}^{12} {}_t n_{i.} \frac{\left({}_t \hat{p}_{ij} - \hat{p}_{ij} \right)^2}{\hat{p}_{ij}} \qquad 4$$

Q is asymptotically chi-square distributed with $t(d - 12)$ degrees of freedom, where $d = \sum_{i=1}^{12} \sum_{j=1}^{12} d_{ij}$ is the number of positive entries, $n_{ij} > 0$ (that is, $d_{ij} = 1$ if $n_{ij} > 0$, and $d_{ij} = 0$, otherwise).

Here, $t = 3$ and $d = 112$, hence the degrees of freedom for the test is 300. The computed value of $Q = 330.72$ with a p-value of 0.107, suggesting that the null hypothesis of time homogeneity should not be rejected.

3 Results and discussion

3.1 Computation of measures for assessing turbine performance

Some of the properties of Markov chain can be used in assessing turbine performance particularly when directed in answering the questions posed in section 3. These are computed as follows.

3.1.1 Limiting probability distribution vector and mean recurrence times

The limiting probability distribution vector gives the proportion of times spent in each of the states of the Markov chain in the long run.
Let $\mathbf{x}$ denote this vector. Then

$$\mathbf{x} = \mathbf{x}\hat{\mathbf{P}} \qquad 5$$

provides estimates for the elements of this vector. These estimates are given below.

$$\widehat{\boldsymbol{X}} = [.07886 \quad .06278 \quad .08533 \quad .10189 \quad .11602 \quad .12192 \quad .11924 \quad .1092 \quad .08263 \quad .05389 \quad .06792 \quad .00032]$$

The inverse of the elements of this vector gives the mean recurrence time vector denoted by $\mathbf{r}$. The estimate of $\mathbf{r}$ is given below.

$$\hat{\boldsymbol{r}} = (13 \quad 16 \quad 12 \quad 10 \quad 9 \quad 8 \quad 8 \quad 9 \quad 12 \quad 19 \quad 15 \quad 3125)$$

By definition, the mean recurrence time gives the length of time it takes to return to a state S_j for the first time given it started in S_j.

3.1.2 Expected first passage times

Definition 2

If an ergodic Markov chain starts from state S_i, the expected (or average) number of steps to reach state S_j for the first time is called the expected first passage (or visit) time to state S_j.

It should be noted that when $i = j$, the expected first passage time becomes the mean recurrence time. The mean (or expected) first passage time matrix, $\mathbf{M}$, for an ergodic Markov chain is determined from the fundamental matrix, $\mathbf{F}$. This is given [17] by

$$\mathbf{F} = (\mathbf{I} - \mathbf{P} + \mathbf{X})^{-1} \qquad 6$$

where $\mathbf{I}$ is an identity matrix, $\mathbf{P}$ is the one-step transition probability matrix for the Markov chain and $\mathbf{X}$ is the matrix whose rows are the vectors of the limiting state transition probability distribution.

Then the $(i, j)^{th}$ component of $\mathbf{M}$ is given by

$$m_{ij} = \frac{f_{jj} - f_{ij}}{x_j} \qquad 7$$

where f_{ij} is the $(i, j)^{th}$ component of $\mathbf{F}$ and x_j is the j^{th} component of limiting state transition probability vector, $\mathbf{x}$.

The estimated values for the matrix of expected first passage times are computed from equations (6 and 7) and tabulated below.

Table 4 Matrix of expected first passage times

State	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]
[1,]	13	36	57	84	115	153	201	260	345	472	666	11161
[2,]	260	16	33	62	94	132	180	240	325	451	645	11141
[3,]	406	156	12	36	69	108	156	216	302	428	622	11118
[4,]	501	255	107	10	39	79	128	188	274	400	594	11090
[5,]	565	320	176	77	9	45	95	156	241	368	562	11057
[6,]	613	368	225	129	59	8	54	115	200	327	521	11017
[7,]	648	404	262	167	101	45	8	64	151	278	472	10967
[8,]	673	429	287	193	125	74	35	9	90	218	413	10909
[9,]	691	448	307	213	146	96	60	31	12	133	330	10826
[10,]	705	462	321	228	161	113	78	52	31	19	208	10704
[11,]	715	472	332	239	173	126	93	69	54	41	15	10496
[12,]	703	460	325	237	174	129	99	78	66	58	25	3125

3.1.3 Expected Wind Power Generation

Wind power is the amount of power derivable from the wind. Theoretically, only 59% of the power in the wind is extract-able [14]. However, in practical applications, the extract-able capacity of a wind turbine is 30% of the power in the wind [16]. Consequently, the 1.8 MW wind turbine power rating is not attainable in any given location. Like the power curve given in Table 1, it is more useful as a manufacturer's index of specification of the performance of a wind turbine.

The intermittent nature of the wind is a known factor that compounds wind power generation. This has been aptly described by the Markov chain model for the location here and its salient features captured in the limiting probability distribution vector, $\mathbf{x}$, of the

Markov chain. It might be pertinent to state that $\mathbf{x}$ gives the limiting probability of finding the chain in state S_j ($j = 1, 2, ..., 12$). Notice the correspondence between the wind speed and the states of the Markov chain as shown in Table 2 and the correspondence between the wind speed and the power from the turbine as shown in Table 1. Hence we can input the values of $\mathbf{x}$ into the calculation of power generated by a turbine in a specified location, and consequently, make it more informative to marketers, developers and buyers. This computation is new and is described below. Here, we estimate the wind power generated from a turbine using this new method in two ways.

i Based on the 1.8 MW turbine power curve

The power curve output is given in Table 1. We create a new row to allow multiplication with the corresponding limiting probability values. This allows the characteristics in the intermittent nature of the wind in this location to be captured in the power curve. This is shown in the table below.

Table 5 Location specific power curve of a 1.8 MW win turbine

Wind speed (m/s)	< 3	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12	> 20
Power, Pe_j (kW)	0	51	175	346	584	913	1313	1660	1784	1799	1800	0
x_j	.07886	.06278	.08533	.10189	.11602	.12192	.11924	.1092	.08263	.05389	.06792	.00032

Hence, the expected power that can be generated from the power curve for this location is given by

$$PE = \sum_{j=1}^{12} x_j Pe_j \qquad 8$$

$$= 937kW$$

$$= 0.937MW$$

ii Based on extract-able power

Theoretically, the power in the wind is computed as

$$P_w = \frac{1}{2}\rho C_\rho A u^3 \qquad 9$$

where ρ is the air density, C_ρ is the capacity factor that gives the practical amount of power a wind turbine can generate, A is the swept area of the wind turbine blade and u is the average wind speed. As explained earlier, there is a limited amount of power a turbine can generate from the wind. Allowing for the wind characteristics in this location as reflected by x, the expected extract-able power is given by

$$EP_w = \sum_{j=1}^{12} \frac{1}{2}\rho C_f A x_j u_j^3 \qquad 10$$

where u_j is the mid-interval of the states of the Markov model. We assume $u_1 = u_{12} = 0$ m/s, $u_{11} = 12$ m/s, $\rho = 1.225 kg/m^3$, $C_f = 0.3$ and $A = 7854$. Substituting these values into equation (10) gives the estimate of the extract-able power of

$$EP_w = 826kW$$

$$= 0.826MW$$

3.2 Discussion

The test results show that a discrete state space discrete time Markov chain model fits the wind speed data. This enables the use of the features of this model to answer questions that may be of interest to a wind turbine marketer, investor or manager.

The transition probability matrix $\boldsymbol{P}$ shows that whenever the Markov chain is in any of the twelve states, it has a higher chance of remaining there and a very small chance of moving out to an immediate neighborhood of the given state. It is also very noticeable that it is almost impossible to move from any of the states to any state farthest away in few transitions. This aptly describes the intermittent nature of the wind speed that can be experienced by a turbine in this location. That is, the wind speed variability is highly likely to be confined to within the speed limits defining a specified state and at every transition time, there is a very little chance of a drastic change in wind speed that is characteristic of a state farthest away from the given state. This is clearly exhibited in the simulated transitions presented for 10,000 realizations in Fig. 1. Two ideas are quickly identified from Fig. 1; (1) transitions from any state gravitates to the central states and their immediate neighborhoods, (2) the cut-out state (state 12) is rarely visited. These information corroborates the results of the matrix of first passage times in Table 4.

The results from the limiting probability distribution show that the turbine works for about 92 % of the time and is down about 8 % of the time. The cut-in speed (state 1) is mainly responsible for 7.8 % of the down-time while the cut-out speed (state 12) is responsible for only 0.03 %. Note that at down-times, the power generated by the turbine is zero. States with the highest proportion of working times are states 5, 6, 7 and 8. Each contributes slightly above 10 % of the total working time of the turbine in the long run.

The model allows only 5 minutes of dueling in any state before a change in status. That is, a transition time in 5 minutes only is allowed. The result of the mean recurrence times shows that when in state 1 (cut-in speed) it takes 13 transitions, or 65 minutes, before the next return to state 1. That is, when down because of cut-in speed (state 1), it works for about 65 minutes before the next cut-in speed down-time. Whereas when down because of a cut-out speed (state 12), it takes 10.9 days before experiencing the next cut-out down time. It is worthy of note that state 12 has the highest value of mean recurrence time while states 6 and 7 have the lowest, 40 minutes each. That is, state 12 is the least visited state and states 6 and 7 are the most visited states, as seen in Fig. 1. Consequently, power output of the turbine is mostly at the levels provided by the wind speeds for the states 6 and 7. These are 111.3 kW and 156.56 kW, respectively.

The results of the first passage times show a clear pattern; lowest times are recorded from any given state to its immediate neighborhood states. The values become higher the farther away the states get to the given state. Again this clearly depicts the intermittent nature of the wind speed in this location as mentioned earlier.

Table 6 below shows clearly the working times of the turbine before any down time. It is clear from this table that the turbine will work for an average of 2.05 days starting from any of the states 2 to 12 before a first experience of a down-time due to cut-in speed. On the other hand, it takes an average of about 38 days, starting from any of the states 1 to 11, before a first experience of a cut-out speed down-time.

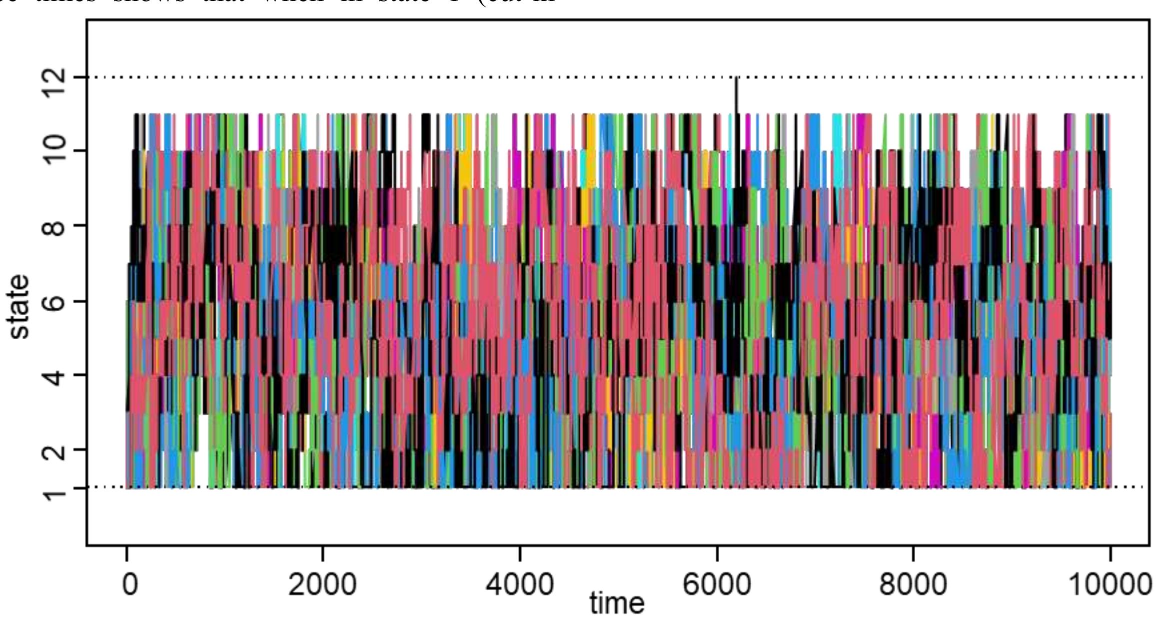

Fig. 1 Realizations of Transitions between Wind Speed Classifications

Table 6 First passage times to the two down-time states

First passage time from state	First passage time to state 1 (Cut-in) Days		First passage time to state 12 (Cut-out) Days	
1			38.75	Average = 38.03
2	0.90	Average = 2.05	38.68	
3	1.41		38.60	
4	1.74		38.51	
5	1.96		38.39	
6	2.13		38.25	
7	2.25		38.08	
8	2.34		37.88	
9	2.40		37.59	
10	2.45		37.17	
11	2.48		36.44	
12	2.44			

Two methods for power generated by a 1.8 MW turbine in the given location were computed and given below.

1. The expected power that can be generated from the power curve using the added information of the expected probability distribution of the wind speed in the location. This is 937 kW.
2. The expected extract-able power which is 826 kW. Again this involves using the estimated probability distribution of the wind speed in the location for its computation.

Both can be used as indices for rating turbine performance in a given location. The extract-able power is a more realistic estimate of power.

4 Conclusion

In this study, it is shown how a 12 states Markov chain can be used to model the wind speed exposed to a turbine in a given location so as to use the features of the model in rating the turbine performance. A 1.8 MW wind turbine exposed to wind speed data at a location near San Angelo, Texas, USA is used for illustration.
The model is compatible with the wind speed data in this location. Features of the model show that the turbine will, in the long run, work for 92 % of the time and experience 8 % down time. On the average it takes 2.05 days of working starting from any of its states before a first experience of a down-time due to a cut-in wind speed and 38 days of working before a first experience of a cut-out wind speed down-time. The turbine power outputs are most times characterized by wind speed in the range for states 6 and 7. That is, 111.31 kW and 156.56 kW, respectively.
Two estimates of the power generation potentials of the turbine in this location are obtained. They both introduced the long-run probability distribution of the wind speed experienced by the turbine in the traditional method of obtaining such estimates. This is new. These estimates are:

i. The expected power that can be generated from the turbine power curve; 937 kW.
ii. The expected extract-able power; 826 kW.

Taking an extract-able power of 826 kW as an hourly average, a turbine working at this capacity is able to generate 594,720 kWh of electric power in a 30 days a month basis. This is sufficient to power 685 average US homes per month on a consumption rate of 867 kWh per home. Consequently, a wind farm with 100 of such turbines can power 68,500 average US homes in a month.

These ratings of turbine performance are valuable indices that can provide informed policy decisions for any interested investors, marketers and wind turbine management planners in this location.

The model has provided an understanding of the pattern of transition of the wind speed within the states described by the 1.8 MW turbine using discrete Markov chain. The major assumption was that the wind speeds are stationary. A study assuming the wind speeds to be non-stationary will be appropriate, especially to compare with the procedures in this research, to guide better decisions. In addition, the combination of this modeling procedure, or others, with artificial intelligence could provide mode informative ideas. These are suggested as future research areas.

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