Effect of Rented Warehouses for Deteriorating Items Under Stock Dependent Demand, Partial Backlogging

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Abstract: - In the area of two warehouse-based inventory models, various studies have been carried out, and the authors presented their models under different parameters and environmental conditions. In most of the study, it can be seen that one warehouse was assumed to be its own warehouse and other was assumed to be rented. Due to different facilities and conditions, both warehouses have different rates of demand and deterioration, along with different storage capacities. A study has already been done by considering both warehouses as ranted. It was also assumed in the study that both warehouses had the same capacity. In addition, it has also been assumed that the items in the second ranted warehouse start decaying after some time, whereas the items in the first warehouse were used first to meet the demand of the customers. After critically reviewing the model published on two ranted warehouses, it can be observed that the storage techniques of items need to be further explored, as the model used the items from the first warehouse to satisfy the demand of the customers, while the items whose deterioration was about to start were stored in the second warehouse. This concept of storage has to change, as items in the second warehouse totally deteriorate until their turn comes up. Keeping this fact in mind, we have developed an inventory model based on two different warehouses for deteriorated items whose rate of deterioration is not so high, but after a very short period of time, these items start decaying. According to the current model, the demand of the customers has been satisfied in the first warehouse; to avoid deterioration in the second warehouse, as a result, transfer such items whose deterioration is about to start from the second warehouse to the first warehouse so that deterioration of the items must be minimize. In the current model, it has been assumed both warehouses are rented, which is suitable for the manufacturer whose product demand is very high and space is not enough. It is obvious that the demand for the items in the stock increases due to the stock reaching a certain level. In the model, the demand for the first warehouse is assumed to be stock-dependent. When, there is extra stock, the second warehouse is used for the products, and author also assumed that the demand has become constant. The model is also instigated under sensitivity analysis, and a brief numerical example with real data has been illustrated.

Key-Words: - warehouses; stock-based demand, partial backlogging, deterioration.

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1 Introduction

In the past few years, the concept of space restriction is also introduced as manufactures always wish to purchase more and more items. The first study based on two warehouses was developed by [1], in which the rate of deterioration was considered as a constant in both the warehouse. After some time, the concept of constant deterioration was ignored and new models were developed by using more parameters, in the same way [2], introduced the concept of bulk release pattern and developed a two-warehouse inventory model for decaying items. Few more study based on the constant demand was developed such as, [3] provided a two-storage inventory model with constant demand and shortages under inflationary environment. Based on inflation, [4] presented a two-warehouse inventory model under a constant demand rate and variable deterioration. After some more research papers the concept of taking constant demand was no longer more applicable as it is not feasible to have a constant demand all the time, by considering the stock level dependent demand, [5] developed a stock level dependent demand model under quantity-based transportation cost without shortage. Where, [8] presented an updated form of the study which was being developed by [5] by introducing an inventory model under two-warehouse by taking stock dependent demand for deteriorating items where shortages of items are also assumed. In all previous study made by various researches, the demand was mainly assumed as constant and some time it was considered as time dependent. It is not correct that the demand fixed as constant always, in case of the items with high rate of deteriorated products the concept of constant demand is not correct. Based on newsvendor-type products, [8] developed a two-warehouse inventory model for the deteriorated items where two ordering opportunities has been carried out. By considering the condition-based delay in payment, [10] presented a model on two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment.

A new approach of volume flexibility was introduced by [7], where, a production model for different demands under volume flexibility being developed.

By considering trade credits on inventory model, [14] developed a comprehensive model lot- sizing decisions for deteriorating items with two warehouses, in this model the order- size- dependent trade credits were assumed. In the mid of 2015, a new concept of assuming secondary location in the problem of two warehouses inventory models was introduced which was very useful for the items whose demands are seasonal. The parameters such as perishable products with trade credit also improve the profit of the inventory as customer need to pay latter in the trade credit policy. A model with replenishment of the items was assumed by [15] and developed optimal dynamic pricing and replenishment policy for perishable items with inventory-level-dependent demand.

In the above model the inflationary conditions were not covered, which play a major important role in the inventory management. [16], presented a model based on replenishment policy for non-instantaneous deteriorating items in two storage facilities under inflationary conditions. By using a different approach on two warehouse-based model, [18] presented two-stage stochastic optimization techniques for warehouse configuration and inventory policy of deteriorating items. Based on multi-trip location-transportation problems a model by [19] has been developed where in this model an effective two-stage stochastic multi-trip location-transportation model with social concerns in relief supply chains was developed.

In this model the advance payment scheme was not covered although with the almost same assumptions, [20], developed an inventory model in which customer can pay in advance so that they can have the inventory items at the earliest also. The model developed by [21], considered the concept of warehouse inventory model for deteriorating items of a supply chain with nonlinear demand function and permissible delay in payment under inflation. The trade credits policy was not covered in the above studies where the trade credits policy can also increase the profit of any model, [20] considered the concept of trade credits in the inventory model and presented optimizing theory on a two-warehouse system where shortage backordering and trade credit was taken into consideration. A model by [12] has been presented the continuous resupply policy for deteriorating items with stock-dependent observable demand in a two-warehouse and two-echelon supply chain in the most literatures resented by various researches, the demand was assumed as a single parameter dependent but it is also essential to take it two or more variable dependent which can improve the total profit. It is also seen that the holding cost related to holding the inventory of the model, is also assumed as constant but it is also important to assumed it variable. By considering all these aspects, first time, a model based on two parameters demand was considered by [25]. In this study the demand was considered as stock and price of the items with quantity dependent trade credit and variable holding cost under partial backlogging. One more facility over warehouse was developed by [23], in warehouse selection facility, where anyone can select the mode of the warehouse which was very useful for the inventory models. In this study an inventory model for non-perishable items with partial backlogging and trapezoidal-type demand was assumed. In the same direction by making some more realistic changes in the parameters like preservation technology, [24] developed a partially backlogged two-warehouse EOQ model with non-instantaneous deteriorating items, where the demand depending on the expiration date with limited storage capacity. Recently, a two-warehouse based model with two variables selling and time dependent demand and variable holding cost-based inventory control model was developed by [26]. The space restriction was also a problem faced by hole sealer as well as manufacturer, if proper space for the deteriorated items was not handle, the cost on deterioration incenses. When the items stored in the warehouses their preservation becomes important, [28], developed a model in which two-warehouse supply chain model considered and a new approach preservation technology was implemented, the model has been assumed under stochastic demand with shortages. In most of the model discussed above, the demand was considered single parameters dependent, the concept of the multi variable demand also come to the light, even a multi variable demand can also increase the profit as it can handle a model in different dimension. In the same way, [27], presented an optimizing two multi-echelon inventory model where the demand was considered with price and stock dependent, also the model was developed for the perishable products. A case study by [29] was also presented, in this study a multi-warehouse, multi-product inventory control model for agri-fresh products was considered.

Optimization of a non-instantaneous deteriorating inventory problem with time and price dependent demand over finite time horizon via hybrid DESGO algorithm was presented by [30]. [31], presented a study on two-warehouse lot sizing problem for defective items with a completely backlogged shortage under limited storage capacity for rented warehouses, in which a limited storage capacity was taken. Where, [32], developed a study of a two-storage single product inventory system with ramp type demand, N-Phase prepayment and purchase for exigency.

1. **Comparisons and need of the study:**
   The present model has been developed according to the present market situation and present environmental conditions.

- In the present situation, market demand is very unlicenced therefore the shortage and overstock both can affect the cost and profit as well, keeping these factors in the mind, current model work on the maximum utilization of the inventory.
- Dou to high demand fluctuations author have assumed both the warehouses as ranted, which is a key approach toward the manufactures space restriction issues. Also, in the past study related to the warehouse problems, demand was considered as stock dependent so there was no such need of the extra ranted warehouses.
- Author have considered a different strategy for the rented warehouses, have considered that when they have to store the items in the warehouse then obviously, they have to store the items in the warehouse whose deterioration rate is high as compare with the items whose deterioration rate is low for. It can avoid the unwanted deterioration and hence author can improve the profit.
- The demand rate of the items in both warehouses assumed is assumed stock dependent up to a certain level only after that it is constant.

It is also assumed that the deterioration rate is different in both the warehouse as both the warehouses have different storage facility. The shortages are also entertained in the models and treated as partially backlogged when they are out of stoke. The aim of this study is to find out the optimal total cost and shortage period in the case when both the warehouses are rented and product is deteriorating in nature. The model is also discussed with a suitable numerical example and the sensitivity analysis is also carried out.

2 Assumptions and Notations

2.1 Assumptions

Present model has been developed under the following assumptions: -

(1) The model is developed under both rented warehouses.
(2) For the proper running of the inventory, they have assumed that the demand in the first warehouse is stoke based, where in the second warehouse the demand is assumed as constant throughout.
(3) In the current study they have assumed that there is no deterioration takes place in the second(reserve) warehouse.
(4) In the first warehouse, a time depend rate of deterioration assumed.
(5) The fixed capacity of the warehouses assumed W units and capacity is assumed higher than the ordered quantity.
(6) LIFO policy introduced for the storage of the goods.
(7) For the proper formulation of the study, author have named the warehouse as W_1 and W_2 also, it is assumed that the stock is transferred to warehouse W_2 only after filling the warehouse W_1.
(8) The inventory holding cost per unit time in W_2 is higher than those in W_1.
(9) The shortages are allowed only in W1.
(10) The period when the items are not available in the stock are assumed partially backlogged with a constant rate of δ.

2.2 Notations

W = Denote the capacity of the warehouse.

t_1= Used to represent the time at which the level of the inventory in warehouse W_2 becomes zero

t_2 = Used to represent the time at which the level of the inventory in warehouse W_1 becomes zero.

T = Used to represent cycle of the replenishment.

θ = Used to represent deterioration in the warehouse W_2.

a, b = Used to represent the demand parameters.

0 = Used to represent the cost of ordering cost per cycle.

h_1 = Denote the cost of holding the items per unit in warehouse W_1.

h_2 = Denote the cost of holding the items per unit in warehouse W_2.

c_h = Denote the shortage cost per unit.

l = Denote the cost being paid by lost sale per unit.

d = Represent the cost due to deterioration per unit in warehouse W_2.

$I_{w_1}(t)$ = Represent the inventory level at any time t in warehouse W_1.

$I_{w_2}(t)$ = Represent the inventory level at any time t in warehouse W_2.

S = Denote the cost due to ordering quantity.

3. Mathematical Formulation

The current model is being developed for the deteriorated items stored in two rented warehouses, where the demand of the items is assumed as stock dependent. they have also assumed that the items those are having a short life are stored in the W_2 warehouse because they have to clear such items first. And the rate of deterioration in the W_1 warehouse is not assumed which is most important aspect of the study. Now initially, in the beginning of the cycle S units of inventory received out of which W units of the items are kept in warehouse W_1 and the remaining (S-W) units are kept in warehouse W_2. For proper running of the inventory the LIFO policy is used.

As, they have to store the low life items in the warehouse W_2 so they have used the stock of this warehouse, now between the time period [0, t_1] the stock level in warehouse W_2 is decreases because of the effect of demand of the item and deterioration. At the time t_1 the stock level becomes 0 in the W_2, now as author have assumed 0 deterioration in the second warehouse W_1 during the time period [0, t_1] so full stock of this warehouse is available for the customer's demand. Now, to satisfy the customer's demand, they have to start using stock from the warehouse W_1 during [t_1, t_2]. Now the stock start depletes in the warehouse W_1 due to combined effect of demand and deterioration. At the point t_2

entire stock is completed and shortages start at this point.

The graphical behaviours of the model are shown in Fig.-1

Case-1: For the warehouse W_1

The differential equations representing the behaviour in the warehouse W_1 are given as follow:

$$\frac{dI_{W_1}(t)}{dt} = W \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI_{W_1}(t)}{dt} = -K_1 t I_{W_1}(t) - (a + b(I_{W_1}(t)) \qquad t_1 \le t \le t_2 \qquad (2)$$

$$\frac{dI_{W_1}(t)}{dt} = -a \qquad t_2 \le t \le T \qquad (3)$$

With the help of the boundary conditions, the following solutions of the equation (1), (2) and (3) $I_{W_1}(0) = W$ and $I_{W_1}(t_2) = W$ (4)

$$I_{W_1}(t) = W \qquad 0 \le t \le t_1 \qquad (5)$$

$$I_{W_1}(t) = ae^{-bt-\frac{kt^2}{2}}[(t_2 - t) + \frac{b}{2}(t_2{}^2 - t^2) + \frac{K_1}{6}(t_2{}^3 - t^3)] \qquad t_1 \le t \le t_2 \qquad (6)$$

$$I_{W_1}(t) = a(t_2 - t) \qquad t_2 \le t \le T \qquad (7)$$

Case-2: For the warehouse W_2

The differential equations representing the behaviour in the warehouse W_2 are given as follow:

$$\frac{dI_{W_2}(t)}{dt} = -K_2 t I_{W_2}(t) - (a + b(I_{W_2}(t)) \qquad 0 \le t \le t_1 \qquad (8)$$

Under the boundary condition

$$I_{W_2}(t_1) = 0 \qquad (9)$$

By considering the boundary condition the solution of the equation (8) is given by

$$I_{W_2}(t) = [e^{(t_1-t)K_2} - 1]\left[\frac{(a+bW)}{K_2}\right] \quad 0 \le t \le t_1 \quad (10)$$

4. Calculation of the total inventory cost those are associated with the current models

4.1 Cost associated to the ordering cost of the items per cycle: They have to made order at the stating of each cycle hence

Ordering Cost: O

4.2 Cost associated with holding of the items.

4.2.1 For the warehouse W_1

As, they have used the items from the warehouse W_2 hence the inventory items in warehouse W_1 for the whole time period of positive inventory, hence the holding cost is given by

$$\text{H.}C_{W_1} = h_1\left[\int_0^{t_1} I_{W_1}(t)dt + \int_{t_1}^{t_2} I_{W_2}(t)dt\right]$$

$$= h_1\left[\int_0^{t_1} a(t_2 - t)\ dt + \int_{t_1}^{t_2}[e^{(t_1-t)K_2} - 1]\left[\frac{(a+bW)}{k_2}\right]dt\right]$$

$$\text{H.}C_{W_1} = h_1\left[\left(a\left(t_2 t_1 - \frac{t_1{}^2}{2}\right) + (a+bW)t_2 e^{t_1 K_2}\right]\right. \qquad (11)$$

4.2.2 For the warehouse W_2

It can be seen from the graphical representation of the model that the stock in warehouse W_2 is positive only the duration of [0, t1], so holding cost during this period in warehouse W_2 will be

$$\text{H.}C_{W_2} = h_2[\int_0^{t_1} I_{W_2}(t)dt]$$

$$= h_2[\int_0^{t_1}(e^{(t_1-t)K_2} - 1)\left(\frac{a+bW}{K_2}\right)dt]$$

$$= h_2\left[\frac{a+bW}{K_2})(1 - e^{t_1 K_2}) - t_1\right] \qquad (12)$$

4.3 Shortage cost: Shortage cost per cycle can be calculated as follow;

$$\text{SC} = c_h \int_{t_2}^{T} a\ dt$$

$=a\, c_h(T- t_2)$ (13)

4.4 Lost sale cost

When at time t_2 there is no stock and shortages occurs then it has been assumed that a fix fraction of occurring demand is turn into the backlogging and remaining fraction of the demand lost, so the lost sale cost can be calculated as:

$$LCS = m\int_{t_2}^{T}(1-\theta)a\, dt = am(1-\theta)(T- t_2) \quad (14)$$

4.5 Costs related to the deterioration in the warehouse

4.5.1 Cost in warehouse W_1

To calculate the units deteriorated author must have to find the difference between the initial stock level and total demand which occurring in warehouse W_1. But in the current study they have not assumed any deterioration takes place they must have stored such items in the warehouse W_2.

4.5.2 Cost in warehouse W_2

$$D.CW_2 = d\left\{I_{W_2}(0) - \int_0^{t_1} a + bW\, dt\right\}$$

$$=d\left\{(e^{(t_1K_2} - 1)\frac{(a+bW)}{K_2} - \int_0^{t_1}(a + bW)\, dt\right\} = d\,\frac{(a+bW)}{K_2}\left\{(e^{(t_1K_2)} - 1) - (t_1K_2)\right\} \quad (15)$$

5. Total average cost of the system:

The total average cost involved in the model is the sum of all associated costs:

T.A.C. = $\frac{1}{T}$[O.C.+ H. C.+ S.C.+ L.S.C. +D.C.]

Now, it can easily find the minimum value of t_1, t_2 and T by using any maximization procedure.

6. Numerical example: Following data has been taken to validated the model

The capacity of the warehouse (W) = 2,000 units, Demand parameters (a = 40 units, b = 0.01), h_1 = 0.5 rs/unit, h_2 = 0.8 rs/units, θ = 0.6, K = 0.02, d = 14 rs/unit, s = 5 rs/unit, l = 7 rs/unit, O = 400 rs/order. The output of the model and the optimal value of t_1 , t_2 , T and T.A.C. are as follow t_1 = 850.552 days, t_2 = 950.659 days, T = 1,276.15 days, T.A.C. = Rs. 89,783

7. Sensitivity analysis

The effect of each parameter as a sensitivity analysis is also carried out taking one at a time.

8. Comparison of the model with previous study

In this section they have compared the current study with the study those are already done in the two warehouses-based models. Based on the data available they have compared the total average cost with respect to the warehouse capacity.

Fig.-2, represent the graphical representation of total average cost with respect to the warehouse capacity for the current model and Fig.-3, represent the graphical representation of total average cost with respect to the warehouse capacity.

9. Observations

The model is examining through sensitivity analysis for the parameters, following main observations has been observed.

(i) It can be observed from the Table 1 that total average cost T.A.C decreases by making increment in warehouse capacity 'W', even the same increment also seen in the figure 2. But in figure 2(a) the total average cost T.A.C increases by making increment in warehouse capacity 'W'. this is the main outcome of the study.

(ii) It can also be observed from the Table 2 that total average cost T.A.C and Q both found increased by increasing the demand parameter 'a' where there are no changes noted in the parameter t_1. From figure 2 and 2(a) it can be seen that

It is also noted from the Table 3 that the same changes are found for the demand parameter 'b'

(iii) From Table 4, it can be observed that when the rate of deterioration parameter $'\boldsymbol{\theta}'$ increased its reverse changes are noted on t_1 and Q.

(iv) Table 5 shows that for increased values of backlogging parameters $'\boldsymbol{k}'$ and '$\boldsymbol{\beta}$', its reverse effect is noted on T.A.C. and Q, which show that as they have increased the backlogging time its directly affect the demand of the items and retailers can also purchase the items from the different market.

10. Future extension of the model

Current study focused on the two-warehouse based inventory control model where both the warehouse is assumed ranted and the items with low life are transferred to the warehouse whose items are used first. Also, there are some future scopes of the study like trade credits on the items can also be use. The model is further extended by applying Fuzzy algorithm.

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11. Conclusions

Present model being developed for the deteriorating items in which they have assumed both the warehouses as rented. It is observed the assumption of transferring the items whose deterioration rate is high in the second warehouse so that they can sale such items before the spoilage was a very good idea.

Following major high list of the study found;

- Model also suggested that as the capacity of the storage increased its means it can also increases the costs associated with the model.
- The current model also developed under different demand rate for the different warehouse where they have assumed that for the first warehouse, they have taken the demand rate as stock dependent where they have assumed it constant for the second warehouse which make this study more attractive and up to the mark as per the current market situation.
- The sensitivity analysis of the total average cost at each of the parameters they have used in the model and the clear views of the effect of the sensitivity carried out.
- As they have assumed that the shortages are allowed only in first warehouse which was partially backlogged so that the retailer can also purchase more items at the lowest cost from wholesaler and stocks it in two different warehouses.
- In the current model they have presented the updated form of the model being developed
- They have used the different concept as a result the current model play important role for the items with low life.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of interest

There is no conflict of interest with any financial organization regarding the material discussed in the manuscript

Fig.-1 Graphical behaviour of the inventory with respect to time.

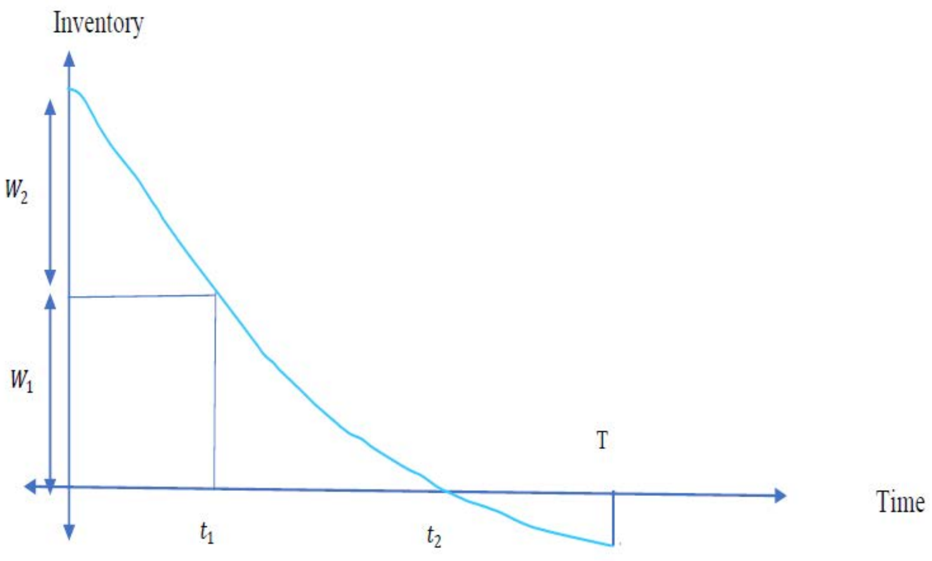

Fig. – 2 T.A.C. vs. First Warehouse Capacity

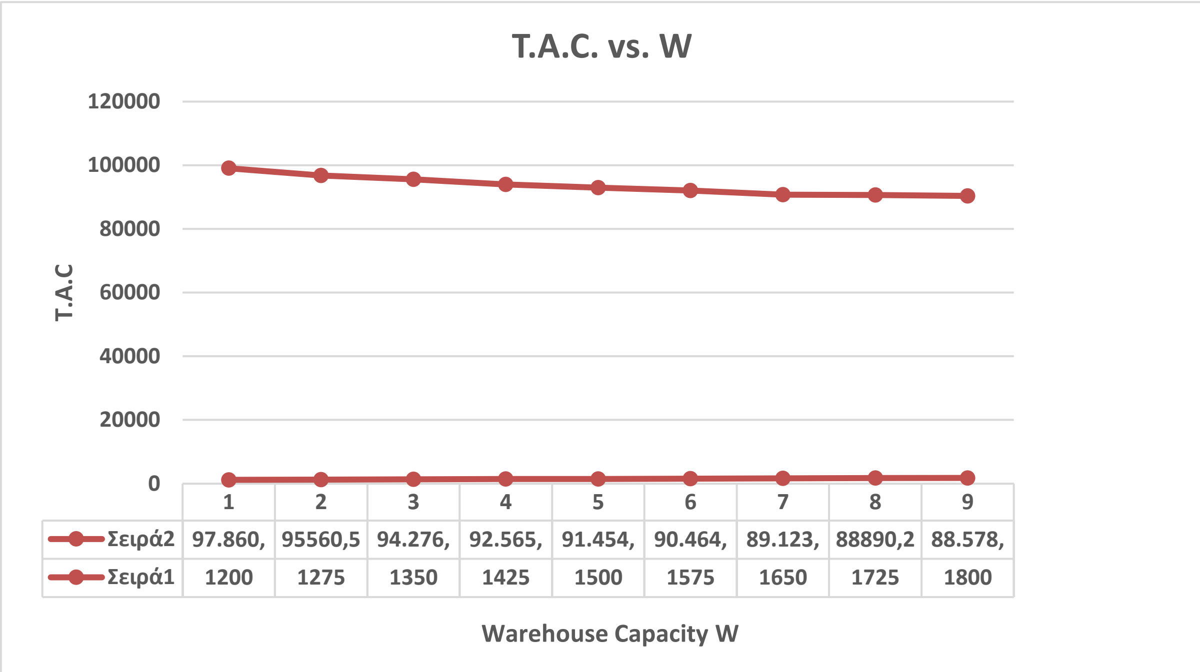

Fig.- 3 T.A.C. vs. Second Warehouse Capacity

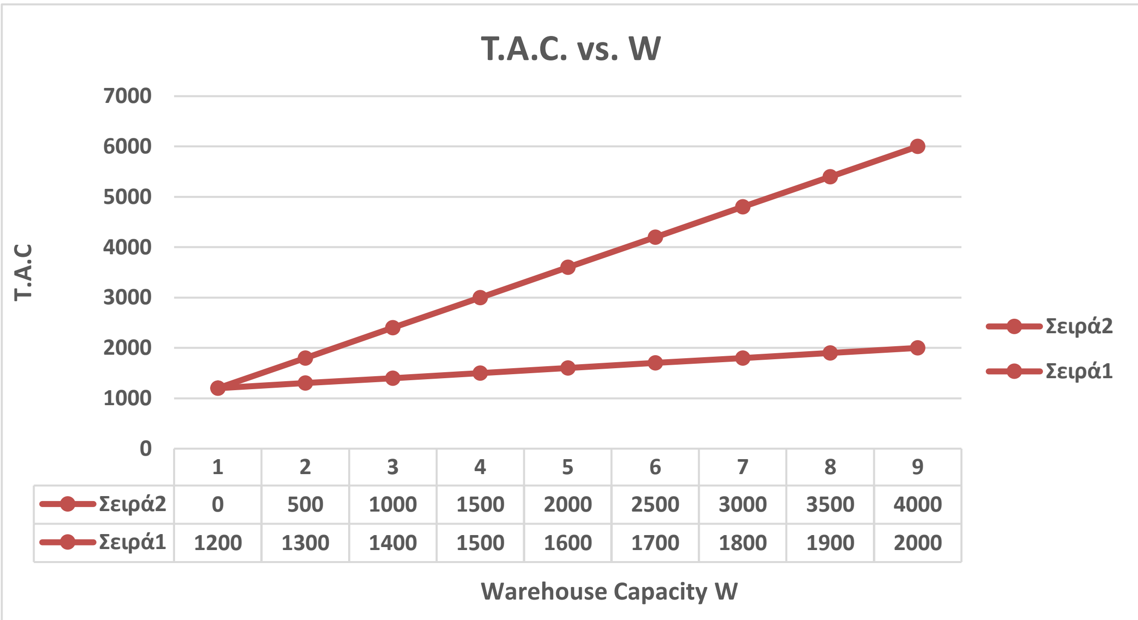

Fig- 4. T.A.C. vs. a

	

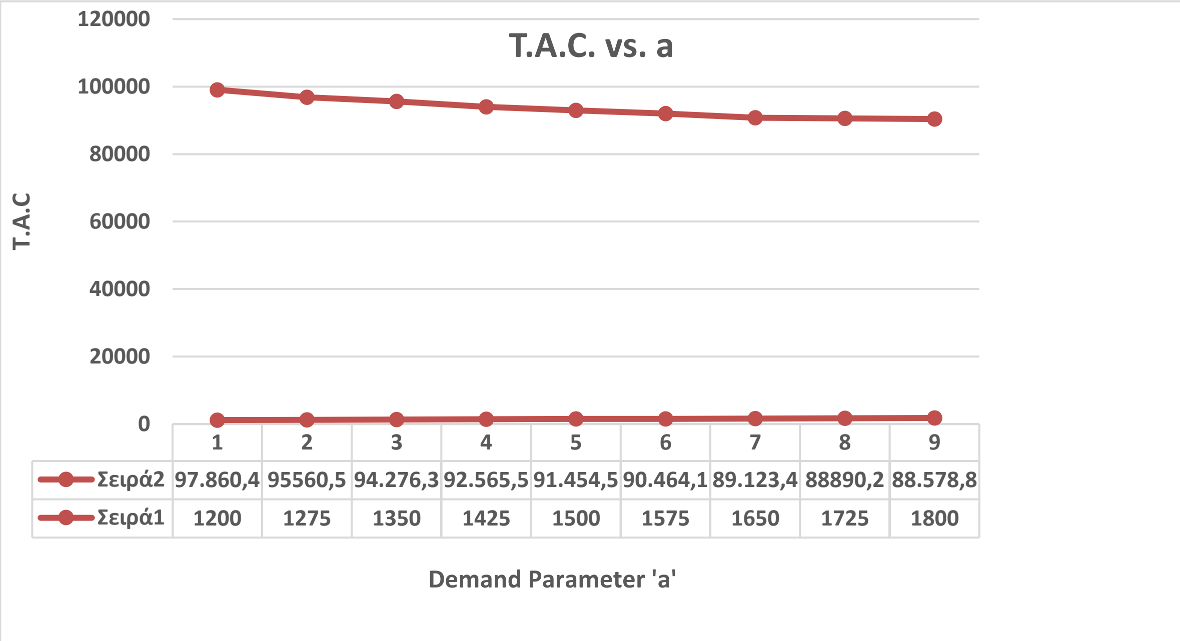

Fig.- 5 T.A.C. vs. b

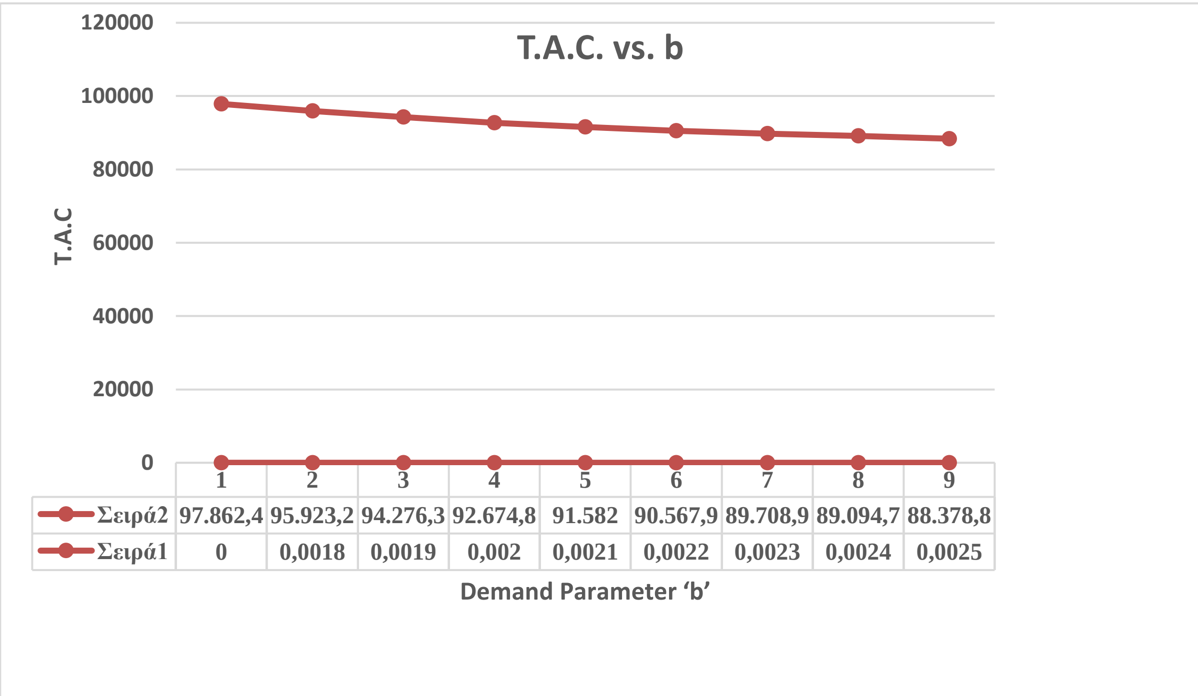

Fig- 6 T.A.C. vs. θ

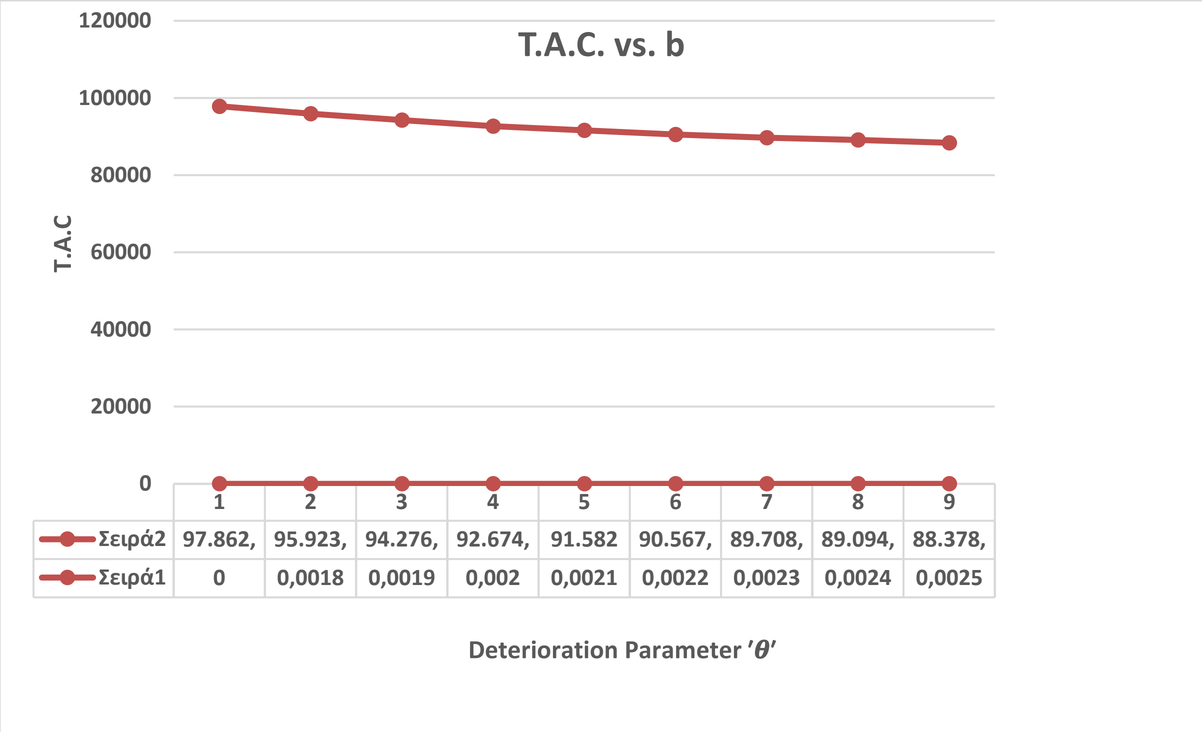

Fig.- 7 T.A.C. vs. *K*

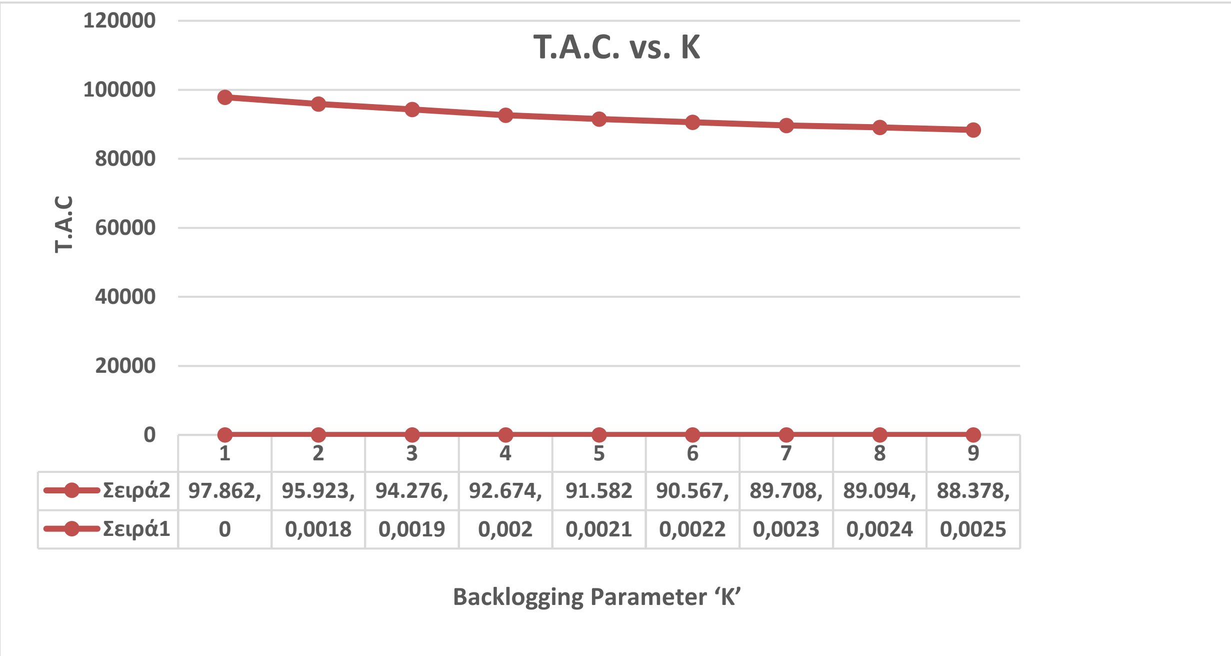

Table 1 **Analysis of Sensitivity for the warehouse capacity W**

% Variation in W	W	t_1	v	T	T.A.C
–20%	1,175	1,068.06	1,212.06	1,269.76	96,860.4
–15%	1,275	1,000.5	1,088.25	1,205.76	95,523.6
–10%	1,375	1,015.78	1,042.78	1,220.77	92,276.3
–5%	1,475	981.144	1,009.64	1,217.84	92,564.8
0%	1,575	949.978	979.978	1,176.21	91,642.9
5%	1,675	921.785	953.285	1,141.27	90,697.9
10%	1,775	896.161	929.161	1,109.51	89,608.9
15%	1,875	872.77	907.27	1,080.52	89,074.7
20%	1,975	851.333	887.333	1,053.95	88,238.8

Table 2.0 Analysis of Sensitivity for the demand parameter 'a'

% Variation in W	**a**	**t_1**	**v**	**T**	**T.A.C**
–20%	50	1,098.06	1,122.06	1,359.76	97,860.4
–15%	52.5	1,054.5	1,080	1,305.76	95,923.5
–10%	55	1,015.78	1,042.78	1,257.77	94,276.3
–5%	57.5	981.144	1,009.64	1,214.84	92,874.8
0%	60	949.978	979.978	1,176.21	91,682.7
5%	62.5	921.785	953.285	1,141.27	90,667.9
10%	65	896.161	929.161	1,109.51	89,808.9
15%	67.5	872.77	907.27	1,080.52	89,087
20%	70	851.333	887.333	1,053.95	88,478.8

Table 3.0 Analysis of Sensitivity for the demand parameter 'b'

% Variation in b	b	t_1	v	T	T.A.C
–20%	0.017	1,098.06	1,122.06	1,359.76	97,860.4
–15%	0.018	1,054.5	1,080	1,305.76	95,923
–10%	0.019	1,015.78	1,042.78	1,257.77	94,276.3
–5%	0.020	981.144	1,009.64	1,214.84	92,874.8
0%	0.021	949.978	979.978	1,176.21	91,682
5%	0.022	921.785	953.285	1,141.27	90,667.9
10%	0.023	896.161	929.161	1,109.51	89,808.9
15%	0.024	872.77	907.27	1,080.52	89,084.7
20%	0.025	851.333	887.333	1,053.95	88,478.8

Table 4.0 **Analysis of Sensitivity for the deterioration parameter $'\theta'$**

% Variation in θ	θ	t_1	v	T	T.A.C
–20%	0.45	1,098.06	1,122.06	1,359.76	97,860.4
–15%	0.48	1,054.5	1,080	1,305.76	95,923
–10%	0.51	1,015.78	1,042.78	1,257.77	94,276.3
–5%	0.54	981.144	1,009.64	1,214.84	92,874.8
0%	0.57	949.978	979.978	1,176.21	91,682
5%	0.60	921.785	53.285	1,141.27	90,667.9
10%	0.63	896.161	929.161	1,109.51	89,808.9
15%	0.66	872.77	907.27	1,080.52	89,084.7
20%	0.69	851.333	887.333	1,053.95	88,478.8

Table 5 **Analysis of Sensitivity for the backlogging parameter $'K'$**

% Variation in K	**K**	**t_1**	**v**	**T**	**T.A.C**
–20%	0.0017	1,198.62	1,120.0	1,358.76	97,862.4
–15%	0.0018	1,044.52	1,080.55	1,325.76	95,923.2
–10%	0.0019	1,015.78	1,042.78	1,257.77	94,276.3
–5%	0.0020	975.144	1,019.64	1,214.84	92,674.8
0%	0.0021	959.978	969.978	1,166.21	91,582
5%	0.0022	931.785	943.285	1,131.27	90,567.9
10%	0.0023	886.161	919.161	1,104.51	89,708.9
15%	0.0024	862.77	900.27	1,070.52	89,094.7
20%	0.0025	841.333	887.333	1,043.95	88,378.8