Covid in Italy: Comparison between Models

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Abstract: - The text aims to address the topic of the SARS-CoV-2 pandemic from the point of view of mathematical models. Numerous studies refer to the impact of COVID-19 on student behavior, the use of e-learning, and harm reduction policies. The analysis presented in the text has a different objective: to develop, among students, knowledge of some mathematical models starting from data detected in a phenomenon of notable impact. In this case, the focus is on the aspects that could be covered in the last class of some upper secondary school courses or the first year of university in mathematics and statistics courses. The data refers to the Italian case, and the analysis considers data from 02/24/2020 to 5/5/2023, the date the WHO declared the end of the pandemic.

Key-Words: - SARS-CoV-2, GeoGebra, Spreadsheet, Models, Markov chains, Correlation.

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1 Introduction

Especially during the first phase of the SARS-CoV-2 epidemic, also known as COVID-19, a very heated debate developed between virologists, epidemiologists, mathematicians, and statisticians on the role of forecasting models because, very often, they proved to be unsuccessful and unable to provide a precise indication.

The researchers used known models, but the values of the phenomenon could have been more precise, or they tried to adapt others unsuitable to describe the development of this pandemic.

The controversies have not been lacking. One involved the epidemiologist Guido Silvestri of the Emory School in Atlanta, according to which these models highlighted their inadequacy to predict the natural trend of the epidemic.

UMI (Italian Mathematical Union) responded: «A mathematical model is not a crystal ball. It is a tool that makes it possible to objectively calculate the consequences of what we know about the transmission of the virus; there is undoubtedly a substantial margin of uncertainty linked to the estimate of real data and to everything we do not know, but the models, if one knows how to read them, also provide estimates on what the margin of error might be.

Furthermore, indeed all models, by definition, can be improved. However, giving up their use to rely totally on the sensations of experts (often contradicting each other, among other things) or perhaps on haruspices does not seem like a great idea to us».

According to the epidemiologist Donato Greco "the first mathematical models had the right defect of being too early, there were too few elements available, and the models are used to interpret, not to predict the future".

The problem is that the more the models are complex and linked to uncertain parameters, the more the predictions can diverge from reality.

It would be better to use more of the famous Occam's razor, a methodological principle that indicates the importance of choosing the most straightforward hypothesis. That is: «Entia non sunt multiplicanda praeter necessitatem», namely «Do not multiply elements more than necessary».

As stated in an article in Nature: «Mathematical models are a great way to explore questions. They are also a dangerous way to assert answers», [1].

An interesting aspect is that the data are numerous, easily available, and can be presented to students in the last class of some upper secondary school addresses or the first year of university in mathematics and statistics courses.

Silvia Benvenuti affirms in an article available online (https://it.pearson.com/aree-disciplinari/scienze-matematica/articoli/matematica-come-strumento-consapevolezza-sociale-lezioni-covid.html): «Well, the COVID-19 – which has taken and is taking so much from us in terms of personal freedom, serenity and, unfortunately, human lives – has provided mathematics teachers of all levels and degrees a virtually inexhaustible sample of reality problems which, being so close to the everyday experience of our students, are sure to succeed in attracting their attention».

Naturally, this does not console us, but we might as well take advantage of it". One of the biggest problems is that there are no certainties about the exact number of people infected. The only "sufficiently" reliable figure is that of deaths, at least in countries capable of keeping serious accounts.

However, the adverb “sufficiently” should be considered with some caution. To define death as due to COVID-19, all of the following criteria must be present, according to a study by ISS INAIL ISTAT Working Group May 2020:

1. Death occurred in a patient definable as a confirmed case of COVID-19.
2. Presence of a clinical and instrumental picture suggestive of COVID-19
   - Fever
   - Cough
   - Dyspnoea
   - Chills
   - Tremor
   - Muscle pains
   - Headache
   - Sore throat
   - Acute loss of smell or taste
3. Absence of an apparent cause of death other than COVID-19 or not attributable to SARS-CoV-2 infection (for example, trauma). To evaluate this criterion, pre-existing pathologies that may have favored or predisposed to an unfavorable course of the infection should not be considered among the apparent causes of death other than COVID-19.
4. Absence of complete clinical recovery period between illness and death. A complete clinical recovery period means the documented complete remission of the clinical and instrumental picture of the Sars-CoV-2 infection.

Under these rules, if a person dies in a road accident while testing positive, they are not counted as a COVID-19 death. However, if he suffers from a severe pathology and ceases to live while positive, he is included in the COVID-19 death account.

This official position has raised several controversies based on two questions:

1. Did the patient die of COVID-19 or from COVID-19?
2. Are the data on mortality due to the pandemic overestimated?

On the first point, assuming a definitive position not tainted by ideological presuppositions is challenging. The available data refer only to a sample of medical records from which only the role played by comorbidities can be deduced.

On the second point, it is perhaps worth referring to what Milena Gabanelli reported in her DATAROOM in the newspaper Corriere della Sera: «From March to December 2020, the positive deaths from COVID-19 included in the bulletin were around 78 thousand, compared to the deaths from all causes in the years 2015-2019, 108 thousand more people died: the difference is 30 thousand. It means fewer deaths from COVID-19 than the real ones have been counted – reflects ISPI researcher Matteo Villa -. Between January and October 2021, however, the deaths included in the COVID-19 bulletin were around 54,000, while the difference from the average mortality of previous years was around 49,000 people».

This data might suggest an overestimation of the COVID-19 deaths; in reality, one must consider that the flu has disappeared. If we exclude the flu deaths of previous years from the comparison, the confirmed deaths more than expected were around 63,000 in 2021, that is, nine thousand more than the COVID-19 deaths declared in the bulletin. It is believable to think that there are no extra counts but that the deaths in the bulletin are a good approximation of the people for whom COVID-19 was the determining cause of death in the last two years.

As can be seen, there are few certainties, and therefore, one must be content with approximate but, in any case, sufficiently reliable estimates. More prepared researchers will be able to build sophisticated models and make predictions, often and unfortunately quite different from the real behavior of the phenomenon due to the intrinsic nature of the phenomenon; as, [2], state: « Predicting the future of epidemics and pandemics is much more difficult as the number of cases to be studied can be measured in one hand. At one end of the scale is the case of SARS, where the fear of becoming a pandemic was overblown, resulting in overspending and the application of restrictive measures to be contained, which turned out to be unnecessary. At the other end is the Spanish flu that turned out to be a serious pandemic with catastrophic consequences, arguably in a different era when communication and the ability to raise public awareness (and possibly exaggerated fear) were limited».

Mathematical models (with limits already mentioned) make it possible to make projections and explore future scenarios, but this differs from the text's aim. The focus is to verify some curves' adaptation to the pandemic's trend. For our purposes, we can examine the problem from different points of view that can be presented to students in the last class of scientific high school or the first year of university:

1. Comparison between some classic models, from the simplest to the most complex.
2. Analysis of two other models, less known to the general public.
3. Modeling using Markov chains.

The first point of the proposed path can be found, in part, in a paper by, [3]. Anyone wishing to examine more sophisticated models could refer to some models present in the literature, such as the Covasim (COVID-19 Agent-based Simulator), [4], the ANN (artificial neural network), [5], or the SIDARTHE Mathematical Model, [6]. The citation of these three models expresses only a part of the leading models. It would need to be completed but represents only an indicator of the variety of the research.

2 Comparison of Some Classic Models

Based on the data from 24/02/2020 to 5/5/2023, four particularly significant pandemic waves followed by a couple with less evident peaks can be seen from the count of cases of Italian deaths. A growth, a peak, and a decline phase characterize each of them. Looking at the graph below, one can see the differences between the various waves.

For example, the first wave is more "ordered", i.e., with fewer abrupt changes. In contrast, the second one shows a slower decline and more significant variability due (perhaps) to a less stringent lockdown. Any epidemic is affected by political, medical, and social factors that intervene in the phenomenon.

As Alessandro Vespignani states: «… the company made aware of the results obtained with predictive methods can initiate changes in behavior that lead to distorting predictions. A classic example is the prediction of the spread of a devastating pandemic, which, once brought to the public's attention, will determine rations (absenteeism from work, isolation at home, travel restrictions) that will change the evolution of the pandemic itself. It seems like a paradox, but predictions become part of the system they try to predict.", [7]. The forecast can influence the phenomenon when reactions come into play (use of masks, vaccinations, lockdowns, etc.) that change the trajectory of the epidemic.

The graph of deaths (Figure 1), day by day, effectively shows the trend of the phenomenon (there is a single anomaly on 15 August 2020 due, most likely, to a lag in the communication of data to the Higher Institute of Health).

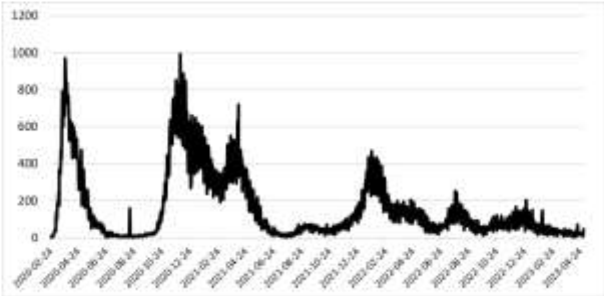

Fig. 1: Deaths

The graph of cumulative deaths is shown in Figure 2. The cumulative frequencies provide us with a clear idea of the speed of the infection and the plateaux, but above all, the disturbing data on the total number of deaths.

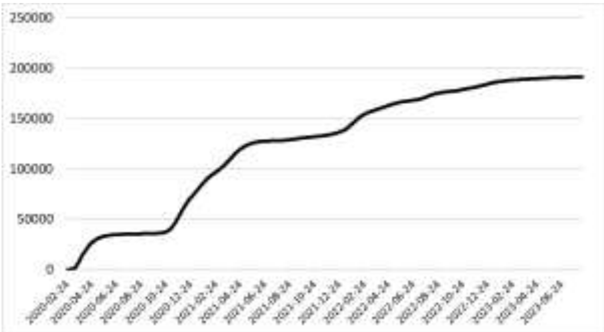

Fig. 2: Cumulative Deaths

Let us open a small parenthesis. We notice a statistically interesting phenomenon if we compare the trend of deaths and positives. In this second case (Figure 3), the fourth and subsequent waves abundantly exceed the previous peaks and reduce them to simple "hills".

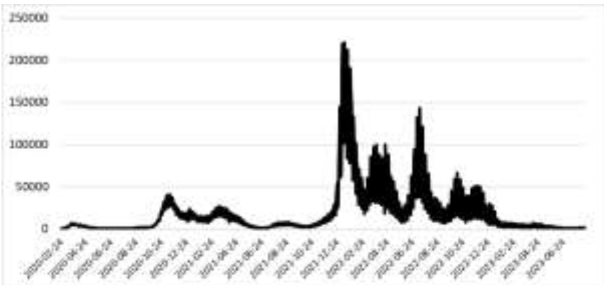

Fig. 3: Positives

Leaving aside the forecasting aspects but focusing only on what happened, we can ask ourselves: Can the tragic COVID-19 experience serve as an example to verify which model has proved to be "closer" to the death trend? In this period of pandemic, there have been many studies that have tried to build articulated models. Wishing to remain exclusively on a research plan of the best regression, understandable at the indicated level of education, we can compare five models with different complexity levels. In the first instance, the dataset will be examined, and then it will be compared with the first wave. The link with the phenomenon will become more evident.

2.1 The Models

1. Linear model: $y_t = a + b \cdot t$
2. Quadratic model: $y_t = a + b \cdot t + c \cdot t^2$
3. Logistic model: $y_t = \frac{L}{1+e^{(-k(t-x_0))}}$
4. 4PL: $y_t = k + \frac{a-k}{1+(\frac{t}{c})^b}$
5. Gompertz model: $y_t = k \cdot e^{-c \cdot e^{-r \cdot t}}$

y indicates the number of deaths, *t* the time, while the others are parameters to be estimated.

The first three models can be easily analyzed using GeoGebra, a popular mathematics learning and teaching software that provides tools for studying geometry, algebra, and analysis. Unfortunately, the limit of data that one can import into a GeoGebra spreadsheet depends on the capacity of the computer and the resources available. For this reason, it is advisable to limit the analysis to a single wave, which, in the text, refers to 120 days.

A brief guide to the steps in GeoGebra is the following:

1. Open the Spreadsheet
2. Enter or, preferably, paste the values from a source such as *.xlsx
3. Create Polyline (right mouse)
4. Scale the axes (Shift + left mouse)
5. Select the values
6. Use Bivariate Regression Analysis
7. Choose among the various options: linear, 2nd polynomial, or logistic.

For a more in-depth analysis, Excel proves to be superior. Even without resorting to specific software by exploiting widely used and easy-to-use software, it is possible to obtain highly satisfactory results. In addition to the ease with which one can view the graph, there are two advantageous features: the Trendline and the Solver. Spreadsheet users can add a trendline to a chart to visualize the general direction of the data over time. Less known to the general public is Solver, an Excel add-in that can perform what-if analysis to find an optimal value for a formula in a target cell, subject to constraints placed in other cells of a worksheet.

A brief guide to the steps to be performed in Excel to analyze the logistic, Gompertz, and 4PL functions is as follows, assuming that the available data is already stored in a Sheet in the first two columns:

1. Add two columns for Estimates and (Data - Estimates)^2
2. Provide the parameters of the functions to be studied (placed, for example, in column G) with initial estimates placed in the next column:
   - For Logistics, set the initial values L, k, x_0
   - For Gompertz, set initial values of K, c, r
   - For 4PL, set initial values k, a, c, b
3. Complete the column of estimates using the desired formula respecting a syntax such as:
   - For Logistics =$I$2/(1+EXP(-$I$3*(B2-$I$4)))
   - For Gompertz =$I$5+(($I$2-$I$5)/(1+(B2/$I$4)^$I$3)
   - For 4PL =$I$5+(($I$2-$I$5)/(1+(B2/$I$4)^$I$3))
4. Complete the squared deviation column (Data - Estimates)^2
5. Place AutoSum at the bottom of this column
6. Insert scatterplot of the data and predictions
7. From the Data menu, open the Solver to determine the parameters

1) *Linear model*: the estimate can be determined thanks to the regression line calculated with the LINEREG function in the linear model case. The function calculates the statistics using the least squares method to calculate the line that best represents the data and returns a matrix with the information that describes it.

One can view the fundamental parameters and the curve more quickly with the trendline functionality. Choosing from six different regression types, one can add a trendline to a chart. A trendline is most reliable when the R^2 value (coefficient of determination automatically calculated by Excel) is close to 1. Given the type of phenomenon, we cannot expect a straight line to significantly represent the death curve trend, as seen from the following two graphs.

In the first case (Figure 4), we see the line that "cuts" the curve of cumulative deaths. With the data available, the function turns out to be y = 21340.85 + 167.06 · t with a coefficient of determination equal to R^2 = 0.938. This result is not negligible given the model's simplicity, even if the future projection of deaths would be significantly overestimated.

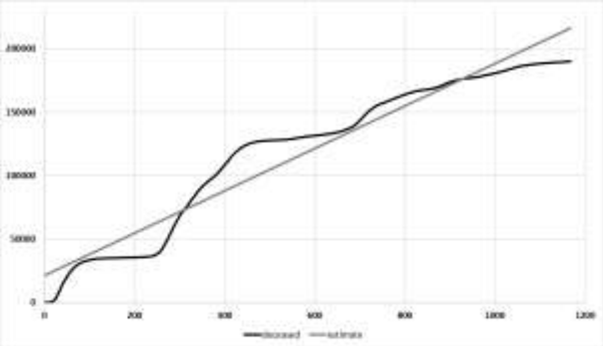

Fig. 4: Linear model

If we limit ourselves to the analysis of the first wave, R^2 gets worse but not so clearly (Figure 5). On the other hand, it is significant that the straight line's characteristics appear scarcely reliable. The intercept is negative, and in the end, it diverges significantly from the data.

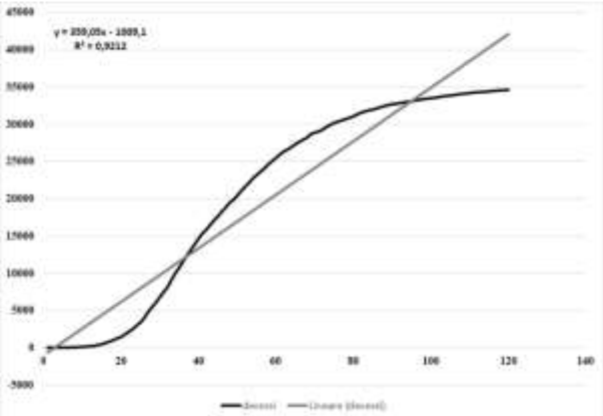

Fig. 5: Linear model first wave

With the data available, the function turns out to be y = -1009.1 + 359.05 · t with a coefficient of determination equal to R^2 = 0.9212.

2) *Quadratic model*: it is possible to estimate the parameters thanks to the trend line function; the choice of the quadratic curve (Figure 6) is due to simplicity. Considering all the data improves the estimate only very slightly compared to the previous model. The parabola equation is y = 5325.6 +306.83 · t – 0.1239 · t^2 with an R^2 = 0.946.

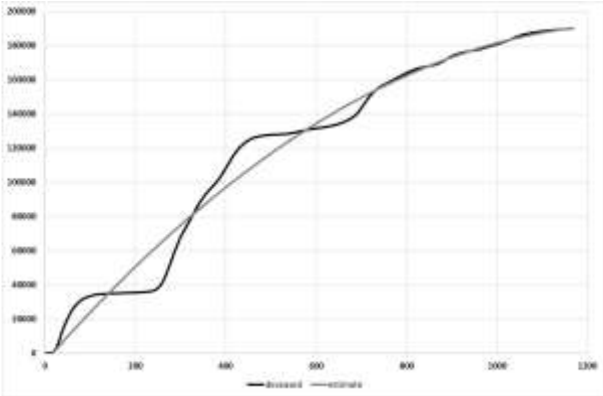

Fig. 6: Quadratic model

If we analyze the first period (Figure 7), we reach R^2 = 0.977 with y = - 4711.9 + 300.78 · t – 0.1145 · t^2. In reality, results with a slightly higher R^2 can be obtained using polynomials of a higher degree; with a fourth-degree function, 98% of the explained variability is reached.

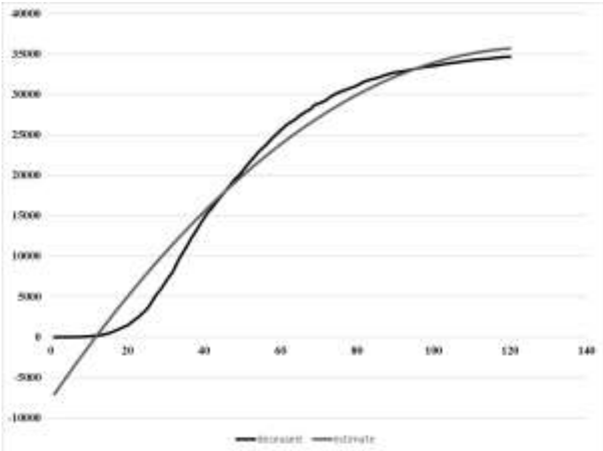

Fig. 7: Quadratic model first wave

With the data available, the function turns out to be y = -7286.8 + 659.26 · t – 2.4027 · t^2 with a coefficient of determination similar to that obtained with linear regression. Adherence is better except in the first phase.

3) *Logistic model*: a logistic function or logistic curve typically describes an S-curve of growth. Developed by, [8], it has found numerous fields of application, from economics to demography to biology.

At first, the growth is almost exponential; then, it slows down to reach an asymptotic position where no more growth exists. When the epidemic reaches its peak, this corresponds to the inflection point. The function has several formulations; among these, we will use the following (1):

$$y_t = \frac{L}{1+e^{(-k(t-x_0))}} \quad (1)$$

L = maximum value
k = growth rate
x0 = inflection point

To identify when a trend reversal in the spread of the pandemic began, we can work on the derivative. Suppose we superimpose the graph of this derivative function on the daily variations of the first wave (Figure 8). In that case, the peak appears, albeit not coinciding with the maximum value of deaths.

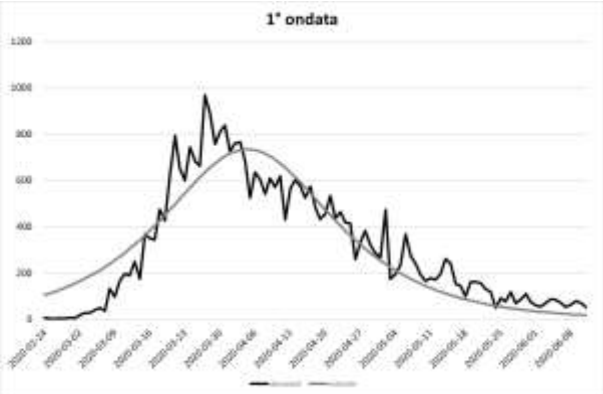

Fig. 8: Logistic model non-cumulative data

The maximum has been reached 41 days after February 24; since then, there has been a trend towards a decrease in the daily number of infections for the first phase. The decline will occur after about 45 days. An easy way to define the curve is to use Excel's Solver tool. With this tool, the cell containing the sum of the squares of the residuals (predicted deviations from the actual values of the empirical data) is set as the minimum of a nonlinear GRG optimization problem.

The program modifies the cells with the values of the parameters present in the model, determining the values of the three parameters. If one looks at the cumulative values, the data and logistics graphs show this trend (Figure 9):

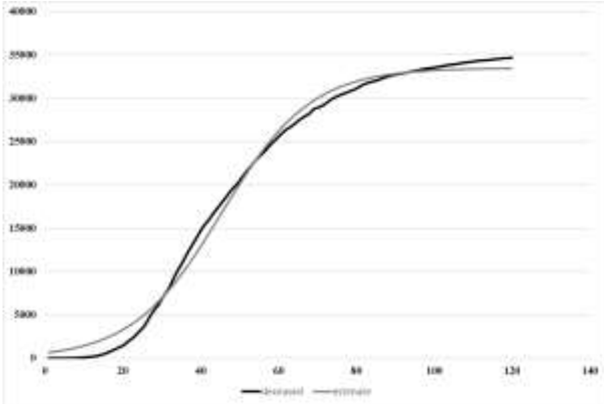

Fig. 9: Logistic model first wave

In this case, examining the first wave was deliberately brought forward to understand the curve type. In this case, the values L = 33509.205 are obtained; k = 0.087; x_0 = 45.373.

If we wanted to apply the same model to all the data, we would get the following (Figure 10):

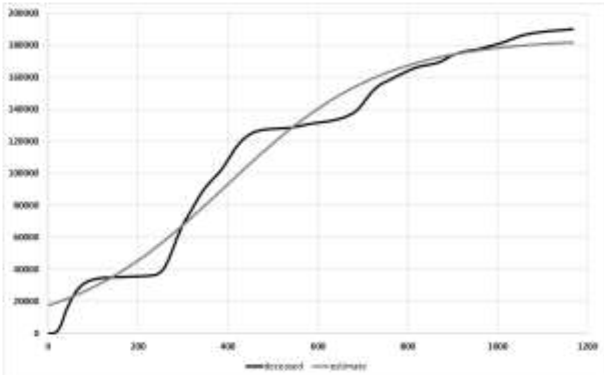

Fig. 10: Logistic model

4) *4PL (four parameters logistic)*: is an interesting variant. There is also the 5PL, which we omit for simplicity. This model also uses a sigmoidal graph logistic function. In this case, four parameters must be estimated as the function is as follows (2):

$$y_t = k + \frac{a-k}{1+(\frac{t}{c})^b} \quad (2)$$

Initially and at the end, the curves do not "fit", but the central part has a clear improvement (Figure 11). The four parameters are: a = 12983.52; b = 1.96; c = 82.39; k = 219777.10.

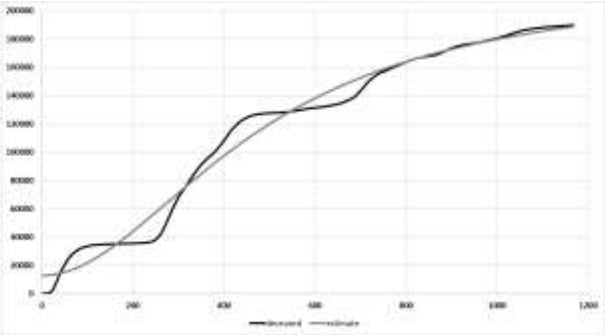

Fig. 11: 4PL model

The comparison between data and estimates in the case of the first wave is decidedly excellent (Figure 12). The four parameters are decidedly different: a = 499.69; b = 3.51; c = 45.97; k = 35661.56.

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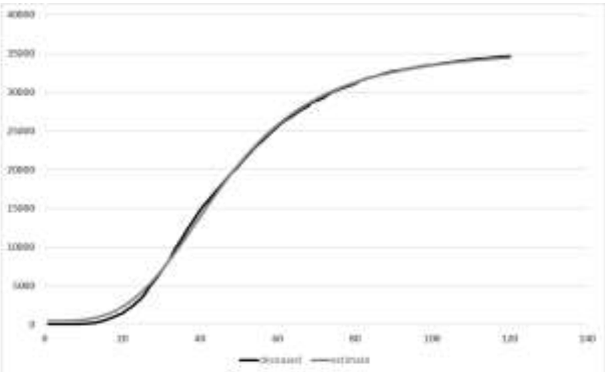

Fig. 12: 4PL model first wave

5) *Gompertz model*: a curve widely used by epidemiologists is the one that refers to the studies of, [9]. Also, in this case, we can compare the two graphs, total (Figure 13) and first wave (Figure 14). The Solver tool determines the parameters in the same way as previously.

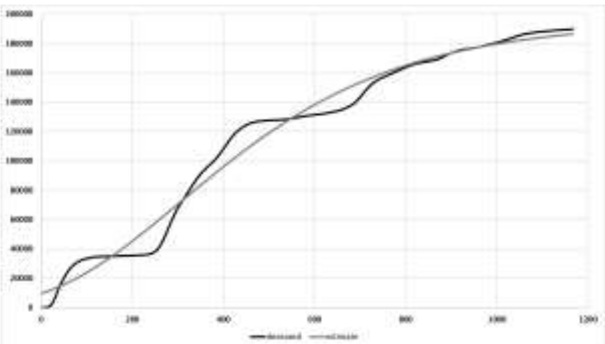

Fig. 13: Gompertz model

From the elaboration, we find k = 194886.55; c = 3.005; r = 0.004. The agreement is only discreet. If we look at the first period, things change (Figure 14). In this case, the Solver finds a bearing capacity k = 34775.3 similar to the empirical one, a value of the parameter c of 7.07, and a growth rate r = 0.052.

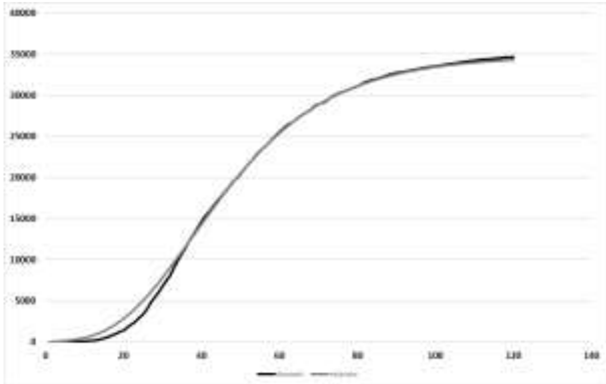

Fig. 14: Gompertz model first wave

2.2 Analysis of Two other Models

The first is that of, [10]. The function, which depends on two parameters, k and α, is (3):

$$y_t = k^{\left\{(ln^{\left(\frac{y}{k}\right)})(e^{-\alpha(t-t_0)})\right\}} \quad (3)$$

Considering the entire period, we have the following graph (Figure 15). As can be seen, the interpretation of the phenomenon is excellent.

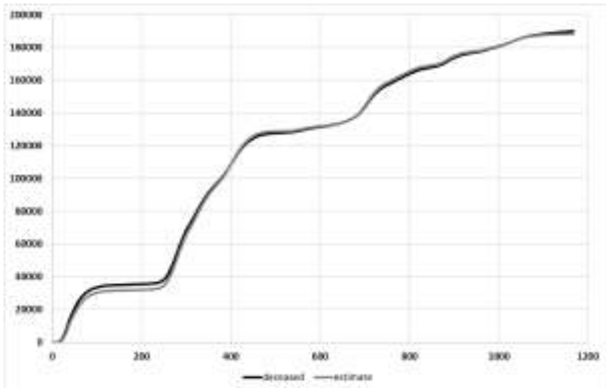

Fig. 15: Perez model

Continuing to refer to the deceased in the first wave, we report the graphical representation (Figure 16) of the deaths and the model used.

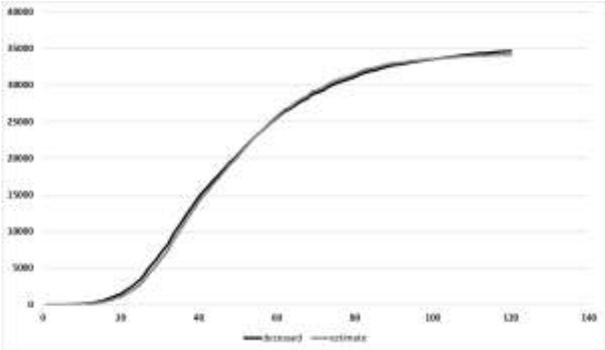
Fig. 16: Perez model first wave

The second model (4) is due to, [11], revisited by, [12].

$$y_t = k \cdot t^a \cdot e^{\frac{-t}{t_0}} \quad (4)$$

Some Italian scholars added a further linear parameter to the model established by the adaptation operation, [13]. If we keep the original model, we can observe its behavior regarding the total (Figure 17) and the first wave (Figure 18).

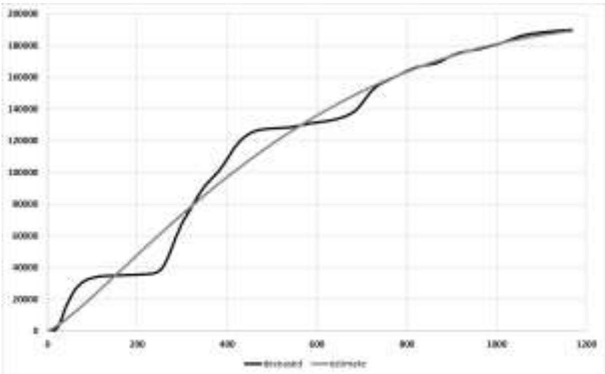
Fig. 17: Vazquez-Ziff model

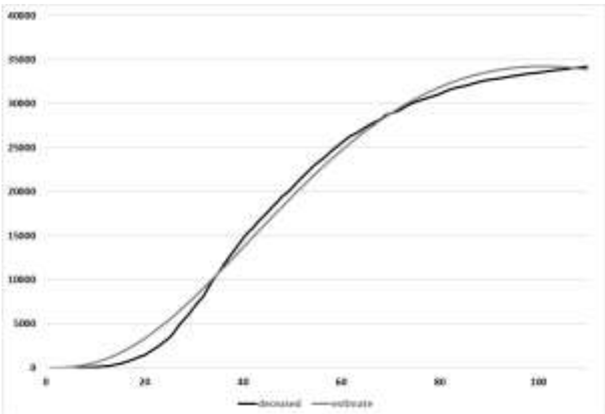
Fig. 18: Vazquez-Ziff model first wave

2.3 Modelling using Markov Chains

This case shows the utility of Markovian models in the epidemiological field based on the hypothesis that epidemic development is a random process in which the transition probability from one system state to another depends only on the immediately preceding system state.

An interesting modeling case is the one presented by, [14]. He defines a Markov chain on five states: Susceptible, Infected, Hospitalised, Intensive care, Deceased.

Based on the Italian data, one can reconstruct the initial matrix with a good approximation (Table 1):

Table 1. Initial matrix

S	I	H	I C	D
0.973	0.027	0	0	0
0.08	0.8	0.1	0.02	0
0	0.135	0.845	0.02	0
0	0	0.518	0.48	0.002
0	0	0	0	1

The values in Table 1 can be interpreted as follows: the daily probability that a person becomes infected is 2.5% (97.5% remain uninfected). No other transition is possible from this state. The probability of daily recovery of the Infected, without hospitalization, is 9%, while 11% need hospitalization (2% are so ill as to be hospitalized directly in intensive care); 78% remain infected at home. 13% of those admitted to the hospital improve and are sent home for further treatment; 85% remain hospitalized, and 2% get so sick that they must be moved to intensive care. 2.5% of those in intensive care improve and return to the ward again; 74.7% stay in intensive care for another day. Unfortunately, 0.3% died.

The matrix (Table 2) shows what happens after two years. There are 104 analysis periods (each period corresponds to one week), and the data are in millions.

Table 2. Final matrix

forecast	11.97	10.62	0.87	0.15
data	12.65	9.56	0.96	0.15

The following graph (Figure 19) clearly shows what happens. We recall that the matrices indicate probabilistic values. If one multiplies the data by the inhabitants of Italy (59,236,213 in 2020), it is possible to obtain the absolute values. The data on intensive care and deaths are "crushed" by the linear scale.

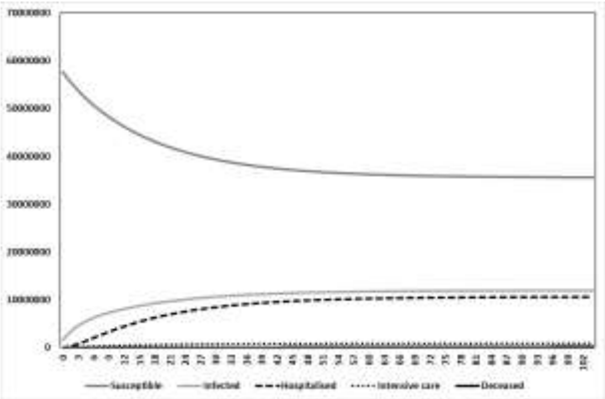

Fig. 19: Comparison of five states

On a logarithmic scale, the graph (Figure 20) is as follows:

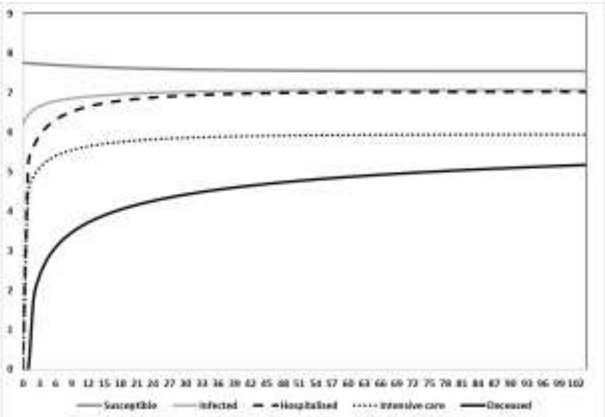

Fig. 20: Comparison on a logarithmic scale

3 Conclusions

Numerous studies refer to the impact of COVID-19 on student behavior, the use of e-learning, and harm reduction policies. The problem is that the authors of articles on the topic present in-depth studies and complex models that are not aimed at students. The exploitation of simpler models and the use of widespread IT tools represent that teaching aid, which is the basis of the motivations of this text.

An interesting point of view is to use the information on the pandemic to stimulate students' interest in modeling.

The results presented are sufficiently explanatory, but further information is possible. An interesting element may be to verify if there is a correlation between some data. For example (Table 3):

Table 3. Correlation

0.911	intensive care and newly deceased
0.957	hospitalized and intensive care
0.074	new positives and intensive care
0.365	total positive and deceased

It seems correct to compare the various models to assess their reliability. However, what does it mean to "compare"? It depends on the question.

Let us start from the known data: the total number of registered cases is 25,812,624, the deaths were 189,936, and therefore there is a lethality rate of 0.736%. If we limit ourselves to the first wave, there are 238,720 cases, 34,657 deaths, and a lethality rate of 14.518%. For comparison, during the 20th century, there were four influenza pandemics, the most severe being the so-called "Spanish flu" in 1918/1919, generated by a particularly virulent A/H1N1 subtype virus which throughout three waves caused 50 and 100 million deaths (lethality approx. 2-4%).

If we compare the five models, it turns out that if all the data are taken into consideration (Table 4), it is the straight line that best estimates the final values (but with a more significant quadratic error expressed in millions) if we limit ourselves to the first period (Table 5) the model PL4 is by far the best.

Table 4. Comparison of total

	|Δ| Deaths	|Δ| Fatality rates	Σe^2
1	26363	0,102%	245.938
2	426	0,002%	88.331
3	8164	0,032%	102.640
4	1192	0,005%	71.981
5	3437	0,013%	77.247

Table 5. Comparison of first-wave

	|Δ| Deaths	|Δ| Fatality rates	Σe^2
1	7420	3,108%	1.587
2	1030	0,432%	557
3	1198	0,502%	123
4	168	0,070%	18
5	362	0,152%	34

The difference between the two situations, referring to different periods, represents an additional intellectual stimulus.

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