A High Order P-type Iterative Learning Control Scheme for Unknown Multi Input Multi Output Nonlinear Systems with Unknown Input Saturation

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Abstract: In this paper, a new high order P-type iterative learning control scheme is presented to solve the trajectory tracking problem of Multi Input Multi Output (MIMO) nonlinear systems with unknown input saturation. It is well known that most systems can be affected by input uncertainties such as saturation. This undesirable input has the potential to destabilize the system. Thus, it is important to mitigate the effect of saturation. In this paper, we take this problem into account. In addition, the controller scheme is very simple, in which the control input in each trial is adjusted by using the tracking error signals obtained from previous and current trials. As the iterations continue, the control system eventually learns the task and follows the desired trajectory with little or no errors. The asymptotic stability of the closed loop system under unknown input saturation is guaranteed over the whole finite time by using the λ-norm method. Finally, to illustrate the effectiveness of the proposed method, simulation results are presented.

Keywords: Nonlinear Systems, Control, Simulation, Nonlinear Control

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1. Introduction

The trajectory tracking problem for repetitive nonlinear systems has been important subject research in the control theory field. In fact, Iterative Learning Control (ILC) is the best technique to deal with this problem [1], [2], [3], [4], [5]. The basic idea of this approach is to use the information from the previous iterations to generate a better controller through the iterations. In general, there are two approaches to studying the asymptotic stability of nonlinear systems based on the ILC scheme. The first is the theory of Lyapunov, which based on the construction of a scalar energy-like function and showing that to be monotonically decreasing under the control design scheme [6], [7], [8]. The second approach is the λ-norm. This method has been defined and used in the first publication of the ILC technique [9], and many works employ this method, see for example [10], [11], [12], [13]. In our work, we use λ-norm to prove the asymptotic stability of the closed loop system.

In addition, in accordance with the learning action type in the ILC schemes, we have mainly four types of ILC: P-type ILC [14], [15], D-type ILC [16], PD-type ILC [17] and PID-type ILC [18]. Indeed, numerical methods can be applied to obtain the error derivative in the implementation of a derivative action. However, the numerical error differentiation might be a source of several noises if the output is contaminated with measurement noise. In fact, due to this action (derivative), the measurement noise amplification accumulates and increases through iterations for repetitive systems. Thus, it is preferred to use only proportional action in the controller scheme. In our paper, we use P-type ILC. On the other hand, and according to the information used in the controller, the ILC is classified into two types: one order and high order. For the one order type, the information used in the controller comes only from one iteration. For the high order type, the information used in the controller comes from several iterations. In the literature, some studies have been done to compare the two types [19], [20]. It is shown that the performances of the systems are better with the high order type. In our work, we use the high order type, in which the information obtained from previous and current trials are used to improve the control input for next trial to achieve a fast convergence rate.

Furthermore, the most systems that existed in the industry can be affected by nonsmooth and non-affine input uncertainties such as the saturation. The presence of saturation may limit the performance of the system and even worse may lead the system unstable. However, owing to the difficulty of guaranteeing the stability of the closed-loop system in the presence of input saturation, there are little studies based on the ILC method to deal with this problem. For example, an iterative learning control for single input single output systems with input saturation is presented in [21], [22], [23]. There are other works on the ILC that have studied the input saturation for linear time-invariant system [24] and for differential and discrete linear systems [25]. Differently to these studies, we proposed a high order P-type ILC for multi-input multi-output nonlinear systems in the presence of unknown input saturation.

In this paper, we present a simple high order P-type ILC scheme to solve the trajectory tracking problem of multi input multi output (MIMO) non-linear systems with unknown input saturation. To achieve a fast convergence rate, the information obtained from previous and current trials are used to improve the control input for next trial. The asymptotic stability of the closed system under unknown input saturation over the whole finite time is guaranteed by using the λ-norm method. Finally, an illustrative example is presented to demonstrate the effectiveness of the proposed controller. The rest of this paper is organized as follows: the problem formulation of the nonlinear systems with unknown input saturation is presented in Section II. The high order

P-type ILC scheme for the system and its convergence analysis is proposed in Section III. An illustrative example is presented in Section IV and conclusions are discussed in Section V.

2. Problem Formulation

Now, we consider the unknown MIMO nonlinear system with unknown input saturation:

$$\dot{x}_k = f\big(x_k(t), t\big) + B\big(t\big)\,\mathrm{sat}(u_k(t), u^*) \tag{1}$$

where: $t \in [0, T]$, k denotes the iteration index. $x_k \in \mathbb{R}^n$ is the state of the system.$f(x_k) \in \mathbb{R}^n$ is an unknown function, $B(t) \in \mathbb{R}^{n\times n}$ is an unknown input matrix. $\mathrm{sat}(u_k(t), u^*)$ is the vector-valued of the saturation function, which can be defined as follows

$$\mathrm{sat}(u_k(t), u^*) = \begin{cases} u_k(t) & |u_k(t)| \leqslant u^* \\ \mathrm{sgn}(u_k(t))u^* & \text{else} \end{cases} \tag{2}$$

Lemma 1: [26] For saturation function $w = \mathrm{sat}(u, m) + d$, we can obtain

$$\big(\mathrm{sat}(w, m) - w\big)^T(\mathrm{sat}(w, m) - w) \leqslant d^T d \tag{3}$$

where w, u, m and $d \in \mathbb{R}^m$

Lemma 2: [27] Let $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ be defined for $t \in [0, T]$, then we have

$$\Big(\int_0^t \|x(\tau)\|\, d\tau\Big)e^{-\lambda t} \leqslant \frac{1}{\lambda}\, \|x(t)\|_\lambda, \qquad \lambda > 0. \tag{4}$$

Lemma 3: [Gronwall-Bellman] [26] Suppose that $f(t)$ and $g(t) \geqslant 0$ are real and locally integrable scalar functions, L is a constant. If $f(t)$ satisfies

$$f(t) \leqslant L + \int_0^t g(\tau)f(\tau)d\tau, \qquad t \in [a, b] \tag{5}$$

then, on the same interval, $f(t)$ satisfies

$$f(t) \leqslant L \exp\Big(\int_0^t g(\tau)d\tau\Big). \tag{6}$$

The following assumptions for system (1) are made.

Assumption 1: The function $f(x_k(t), t)$ satisfies the Lipschitz condition in x for $t \in [0, T]$ that means

$$\|f(x_1(t), t) - f(x_2(t), t)\| \leqslant \alpha\, \|x_1(t) - x_2(t)\| \tag{7}$$

where α is the Lipschitz constant.

Assumption 2: The identical initialisation condition is satisfied, i.e., $x_k(0) = x_d(0)$.

Our objective in this work is to find a sequence of updating control along with iteration such that, the real state trajectory $x_k(t)$ follows exactly the desired trajectory $x_d(t)$ when the number of iteration k tends to infinity.

3. Main Results

The high order P-type ILC scheme at the $(k+1)^{\text{th}}$ iteration is developed as follows

$$u_{k+1}(t) = \mathrm{sat}(u_k(t), u^*) + P_1 e_k(t) + P_2 e_{k+1}(t) \tag{8}$$

where P_1 and P_2 are positive gains, $e_k(t) = x_d(t) - x_k(t)$ and $e_{k+1}(t) = x_d(t) - x_{k+1}(t)$.

Theorem 1: Applying the controller law (8) to the MIMO nonlinear systems (1). Under Assumptions 1 and 2, we get,

$$\lim_{k\to\infty} x_k(t) = x_d(t), \qquad \forall t \in [0, T].$$

Proof: From (1), the state vector at the $(k+1)$th iteration can be written as

$$x_{k+1}(t) = x_{k+1}(0) + \int_0^t \big(f(x_{k+1}) + B(t)\,\mathrm{sat}(u_{k+1}, u^*)\big)d\tau \tag{9}$$

Applying (8) and (9), we obtain

$$x_{k+1}(t) = x_k(0) + \int_0^t \Big(f(x_{k+1}) + B(t)\; \mathrm{sat}(\mathrm{sat}(u_k, u^*) + P_1 e_k + P_2 e_{k+1}, u^*)\Big)d\tau \tag{10}$$

Adding and subtracting $f(x_k) + B(t)(\mathrm{sat}(u_k, u^*) + P_1 e_k + P_2 e_{k+1})$, we get

$$\begin{aligned} x_{k+1}(t) = x_k(0) &+ \int_0^t (f(x_{k+1}) - f(x_k))d\tau + \int_0^t \Big(f(x_k) \\ &+ B(t)(\mathrm{sat}(u_k, u^*) + P_1 e_k + P_2 e_{k+1})\Big)d\tau \\ &+ \int_0^t B(t)\Big(\mathrm{sat}(\mathrm{sat}(u_k, u^*) + P_1 e_k + P_2 e_{k+1}, u^*) \\ &- (\mathrm{sat}(u_k, u^*) + P_1 e_k + P_2 e_{k+1})\Big)d\tau \end{aligned} \tag{11}$$

Using Lemma 1, (11) is transformed into

$$\begin{aligned} x_{k+1}(t) \leqslant x_k(0) &+ \int_0^t \big(f(x_{k+1}) - f(x_k)\big)d\tau \\ &+ \int_0^t \big(f(x_k) + B(t)\,\mathrm{sat}(u_k, u^*)\big)d\tau \\ &+ \int_0^t \big(P_1 e_k + P_2 e_{k+1}\big)d\tau \\ &+ \int_0^t \mu\Big(P_1\, \|e_k\| + P_2\, \|e_{k+1}\|\,\Big)d\tau \end{aligned} \tag{12}$$

where $\mu = \max(\|B(t)\|)$.

From (1) and using Assumption 2, we have

$$\begin{aligned} x_{k+1}(t) \leqslant x_k(t) &+ \int_0^t \big(f(x_{k+1}) - f(x_k)\big)d\tau \\ &+ \int_0^t \big(P_1 e_k + P_2 e_{k+1}\big)d\tau \\ &+ \int_0^t \mu\big(P_1\, \|e_k\| + P_2\, \|e_{k+1}\|\,\big)d\tau \end{aligned} \tag{13}$$

Taking the norm of both sides of (13) and using the Assumption 1, we obtain

$$\begin{aligned} \|x_{k+1} - x_k\| \leqslant \alpha &\int_0^t \|x_{k+1} - x_k\|\, d\tau \\ &+ \int_0^t \big(P_1\, \|e_k\| + P_2\, \|e_{k+1}\|\,\big)d\tau \\ &+ \int_0^t \mu\big(P_1\, \|e_k\| + P_2\, \|e_{k+1}\|\,\big)d\tau \end{aligned} \tag{14}$$

Using Lemma 3, (14) can be rewritten as

$$\begin{aligned} \|x_{k+1} - x_k\| \leqslant \int_0^t &\Big(\big(P_1 + P_1\mu\big)\, \|e_k\| \\ &+ \big(P_2 + P_2\mu\big)\, \|e_{k+1}\|\,\Big)d\tau \exp(\alpha) \end{aligned} \tag{15}$$

Multiplying the both side of (15) by $e^{-\lambda t}$ and according to Lemma 2, we find

$$\begin{aligned} \|x_{k+1} - x_k\|_\lambda \leqslant \Bigg(&\Big(\frac{P_1 + P_1\mu}{\lambda}\Big)\, \|e_k l\|_\lambda \\ &+ \Big(\frac{P_2 + P_2\mu}{\lambda}\Big)\, \|e_{k+1}\|_\lambda\Bigg) \exp\Big(\frac{\alpha}{\lambda}\Big) \end{aligned} \tag{16}$$

Knowing that $e_{k+1} - e_k = x_k - x_{k+1}$. (13) can be written as:

$$-e_{k+1}-\int_0^t\big(P_1e_k+P_2e_{k+1}\big)d\tau-\int_0^t\mu\big(P_1\left\|e_k\right\|+P_2\left\|e_{k+1}\right\|\big)d\tau \leqslant -e_k-\int_0^t\Big(f(x_{k+1})-f(x_k)\Big)d\tau \tag{17}$$

According to Assumption 1 and Lemma 2, (17) becomes

$$\left(\frac{P_1+P_1\mu}{\lambda}\right)\left\|e_k\right\|_\lambda+\left(1+\frac{P_2+P_2\mu}{\lambda}\right)\left\|e_{k+1}\right\|_\lambda \leqslant \left\|e_k\right\|_\lambda+\frac{\alpha}{\lambda}\left\|x_{k+1}-x_k\right\|_\lambda \tag{18}$$

Substituting (15) in (18), we get

$$\left(\frac{P_2+P_2\mu}{\lambda}+1\right)\left\|e_{k+1}\right\|_\lambda \leqslant \left(\frac{P_1+P_1\mu}{\lambda}-1\right)\left\|e_k\right\|_\lambda + \frac{\alpha}{\lambda}\left(\left(\frac{P_1+P_1\mu}{\lambda}\right)\left\|e_k\right\|_\lambda + \left(\frac{P_2+P_2\mu}{\lambda}\right)\left\|e_{k+1}\right\|_\lambda\right)\exp\left(\frac{\alpha}{\lambda}\right) \tag{19}$$

Equation (19) can be simplified as

$$\left\|e_{k+1}\right\|_\lambda \leqslant \left(\frac{1-\dfrac{P_1+P_1\mu}{\lambda}}{1+\dfrac{P_2+P_2\mu}{\lambda}}\right)\left\|e_k\right\|_\lambda + \frac{\alpha\exp\left(\dfrac{\alpha}{\lambda}\right)}{\lambda\left(1+\dfrac{P_2+P_2\mu}{\lambda}\right)}\left(\left(\frac{P_1+P_1\mu}{\lambda}\right)\left\|e_k\right\|_\lambda + \left(\frac{P_2+P_2\mu}{\lambda}\right)\left\|e_{k+1}\right\|_\lambda\right) \tag{20}$$

Choosing $\lambda>0$ widely great, we have

$$\left\|e_{k+1}\right\|_\lambda \leqslant \left(\frac{1-\dfrac{P_1+P_1\mu}{\lambda}}{1+\dfrac{P_2+P_2\mu}{\lambda}}\right)\left\|e_k\right\|_\lambda \tag{21}$$

This implies

$$\left\|e_{k+1}\right\|_\lambda \leqslant \eta\left\|e_k\right\|_\lambda, \qquad \eta<1 \tag{22}$$

Thus, it is clear that

$$\lim_{k\to+\infty}\left\|e_{k+1}(t)\right\|_\lambda=0, \qquad \forall t\in[0,T] \tag{23}$$

This completes the proof.

4. Simulation Results

4.1 First Example

We consider the nonlinear systems with the following dynamics [29]:

$$\begin{bmatrix}\dot{x}_{1k}(t)\\ \dot{x}_{2k}(t)\end{bmatrix}=\begin{bmatrix}x_{2k}\\ -J_m^{-1}\,S\,\sin(x_{1k})\end{bmatrix}+\begin{bmatrix}0\\ -J_m^{-1}\end{bmatrix}\mathrm{sat}(u_k(t),u^*) \tag{24}$$

where: $t\in[0,2]$, $J_m=14$ kg m^2, $S=6$ kg m and $g=9.8$ m/s^2. Let the desired trajectory be $x_d(t)=\sin(2\pi t)$ and $y(t)=x_2(t)$.

By applying the high order P-type ILC (3) to the system (19), we get the simulation results shown in Figures 1, 2 and 3. where the parameters of the controller are chosen as $P_1=100$, $P_2=25$ and $u^*=9$.

From Figure 1, it is easily seen that the real tracking trajectory follows exactly the desired trajectory after just 10[th] iterations. Figure 2 presents the variations of the maximum errors bounds through the iteration numbers, it is clear that the maximum errors decreasing through the iterations and tends to 0 after only 5[th] iterations. The control signal input with saturation is presented in Figure 3, we can see that the high order P-type ILC satisfy the constraints and can be adapted gradually into the chosen boundary ($u^*=9$) along with the iteration numbers.

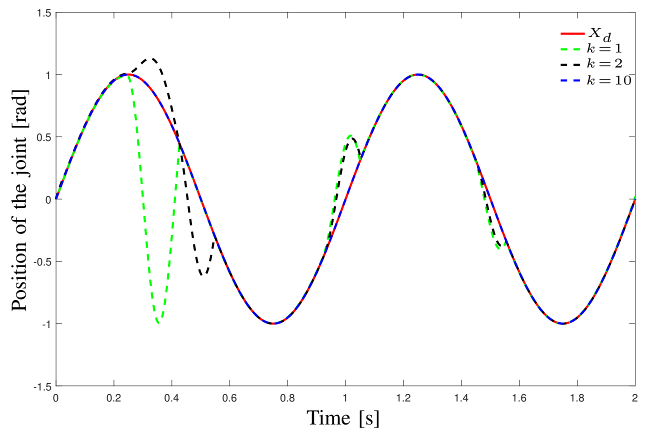

Fig. 1. The real and desired trajectories of the 1^{st}, 2^{nd} and 10^{th} iterations

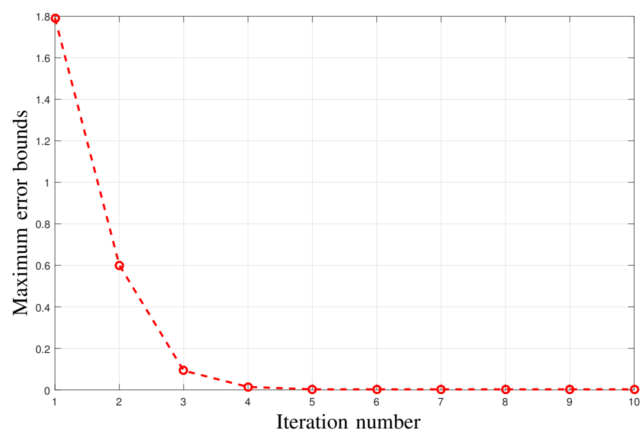

Fig. 2. The Sup-norm tracking errors through the iterations

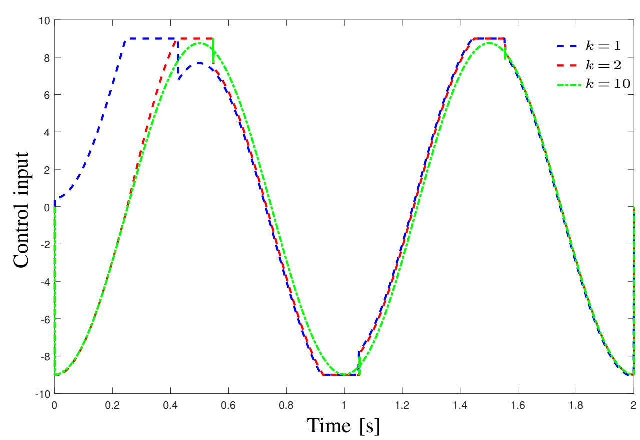

Fig. 3. Control signals input through the iterations

4.2 Second Example

We consider the MIMO nonlinear systems with inpu iteration [30]:

$$\begin{bmatrix}\dot{x}_{1k}(t)\\ \dot{x}_{2k}(t)\end{bmatrix} = \begin{bmatrix} f_1(x_k)\\ f_2(x_k)\end{bmatrix} + \begin{bmatrix}1.5+t & 0\\ 0 & 2-0.3t\end{bmatrix}\operatorname{sat}(u_k, u^*) \quad (2\!\!$$

where

$$\begin{aligned} f_1(x_k) &= (2+0.3t)x_{1,k} + (2.2+0.5t)x_{1,k}x_{2,k}\\ &\quad + \big(0.8-0.1\cos(4t)\big)x_{1,k}^2 \qquad (26\!\!\\ f_2(x_k) &= (2+0.3t) + \big(0.7+0.08\sin(3t)\big)x_{2,k}^2\\ &\quad + (2.2+0.5t)\big(x_{2,k}-x_{1,k}\big)\\ &\quad + \big(0.8-0.1\cos(4t)\big)\big(x_{1,k}-x_{2,k}\big)^2 \qquad (26b) \end{aligned}$$

The desired trajectories are given as

$$\begin{bmatrix}x_{1d}(t)\\ x_{2d}(t)\end{bmatrix} = \begin{bmatrix}\sin(2\pi t)\\ \cos(2\pi t)\end{bmatrix} \qquad (2\!\!$$

Applying the high order P-type ILC (3) to the system (2 where the parameters of the controller are chosen as $P_1 = 1$ $P_2 = 9$ and $u^* = 5$. The simulation results are shown Figures 4-7.

Figures 4 and 5 present the maximum tracking err through the iterations for the first and the second output respectively. It is clear that the maximum errors decreasi through the iteration, in which, after 30th iteration the max mum error for the first and the second outputs are less th 4×10^{-3} and 1.3×10^{-2}, respectively.

The control signal for the first and the second inputs are presented in Figures 6 and 7, respectively. We can see that the high order P-type ILC satisfy the constraints and can be adapted gradually into the chosen boundary ($u^* = 5$) for both inputs along with the iteration numbers.

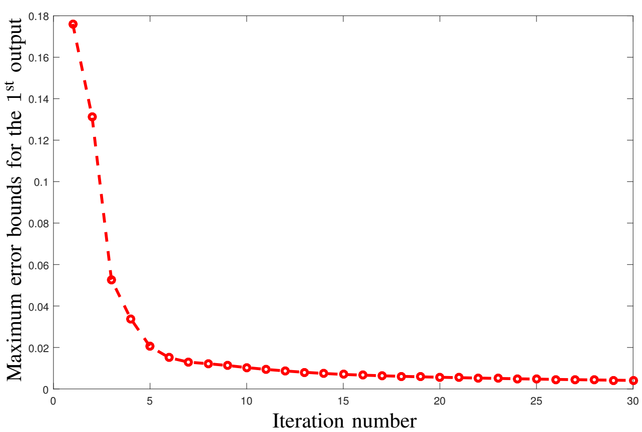

Fig. 4. The Sup-norm of the first tracking errors through the iterations numbers

From these simulation results, it is clear that the proposed controller works well.

5. Conclusion

In this paper, the trajectory tracking problem for MIMO nonlinear systems with unknown input saturation is solved by introducing a novel hight order P-type ILC. In order to achieve a fast convergence rate, the information obtained from previous and current trials are used to improve the control input for next trial. The λ-norm method is used to guarantee the asymptotic stability of the closed-loop system over the whole finite time. Finally, in order to evaluate the effectiveness of the method proposed, simulation results are presented, in which we conclude that the proposed controller works well.

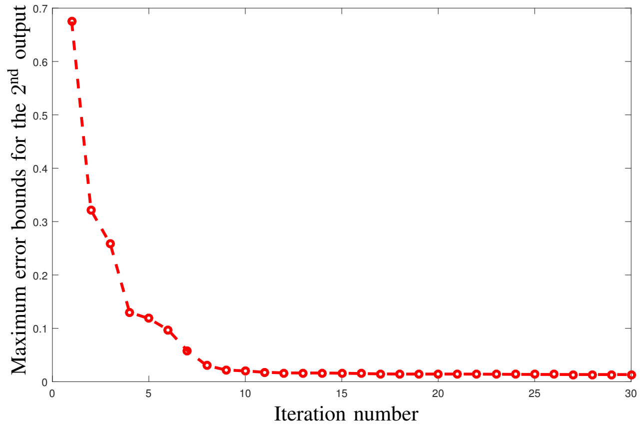

Fig. 5. The Sup-norm of the second tracking error through the iterations

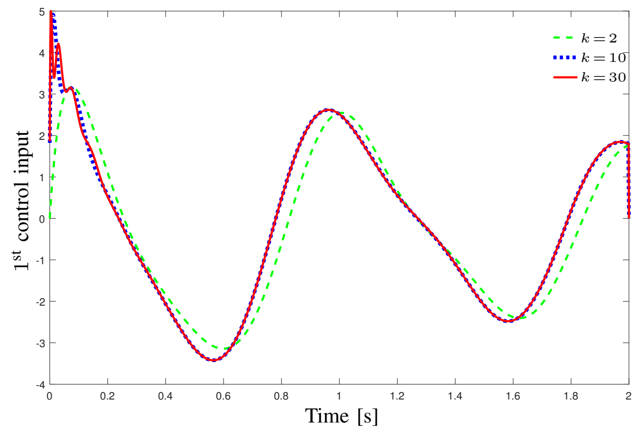

Fig. 6. The first control input signals through the iterations

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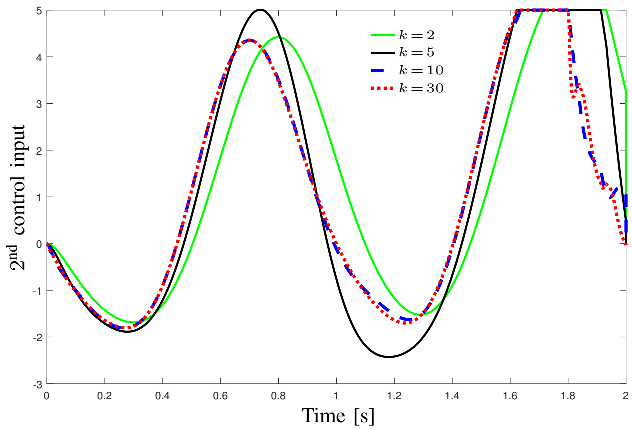

Fig. 7. The Second control input signals through the iterations

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The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

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