The Influence of Nonlinear Cannibalism to Logistic Equation

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Abstract: - A single species model with Holling II type cannibalism term is proposed and studied in this paper. Local and global stability property of the system are investigated. By applying the iterative method, we show that the system always admits the unique globally asymptotically stable positive equilibrium. A threshold value R_0, which depends on the cannibalism rate and the transform rate, is obtained. Depending on $R_0 > 1$, $R_0 = 1$ or $R_0 < 1$, the final density of the species will smaller or equal to or bigger than the system without cannibalism. Our study shows that if the cannibalism rate is too large, and transform rate is too small, then $R_0 > 1$ and cannibalism has negative effect on the final density of the species, which increase the extinction property of the species.

Key-Words: Single species, Cannibalism, Stability

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1 Introduction

The aim of this paper is to investigate the dynamic behaviors of the following single species model incorporating the nonlinear cannibalism rate

$$\frac{dx}{dt} = x(a + c - bx) - \frac{hx^2}{d + x}, \qquad (1.1)$$

where a is intrinsic rate of the species, a/b is the environment carrying capacity, h is the cannibalism rate. $C(x) = h \times x \times \frac{x}{x+d}$ is the the generic cannibalism term. cx is the new offsprings due to the cannibalism. Obviously, $c < h$.

During the last decades, mathematics biology becomes one of the important research area ([1]-[43]), specially, many scholars investigated the dynamic behaviors of the ecosystem with cannibalism, see [34]-[43] and the references cited therein. Cannibalism often occurs in plankton[34], fishes[35], spideres[36] and social insect populations[37]. It is a behavior that consumes the same species and helps to provide food sources.

In 2018, Basheer et al.[43] proposed the prey-predator model with both predator and prey cannibalism as follows:

$$\begin{aligned}
\frac{du}{dt} &= u(1 + c_1 - u) - \frac{uv}{u + \alpha v} - c\frac{u^2}{u + d}, \\
\frac{dv}{dt} &= \delta v\left(\beta - \frac{v}{\gamma u + \rho v}\right),
\end{aligned} \qquad (1.2)$$

where $c_1 < c$, u and v represent the densities of prey and predator at time t, respectively. The parameters $c_1, \alpha, c, d, \delta$ and β, γ, ρ are all nonnegative constants. Basheer et al.[43] used the Holling II type functional response to describe the cannibalism of prey species. Here the generic cannibalism term $C(u)$, is added in the prey equation, and is given by

$$C(u) = c \times u \times \frac{u}{u + d}, \qquad (1.3)$$

where c is the cannibalism rate. This term has a clear gain of energy to the cannibalistic prey. This gain results in an increase in reproduction in the prey, modeled via adding a $c_1 u$ term to the prey equation. Obviously, $c_1 < c$, as it takes depredation of a number of prey by the cannibal to produce one new offspring. The authors of [43] tried to investigated the local and global stability property of the equilibrium of the system (1.2). Indeed, they used the Iterative method to prove the global stability property of the positive equilibrium, they first applying differential inequality theory to the first equation of (1.2) and obtained

$$\limsup_{t \to +\infty} u(t) \le 1 + c_1. \qquad (1.3)$$

Hence, for $\varepsilon > 0$ enough small, there exists a $T_1 > 0$ such that

$$u(t) \le 1 + c_1 + \varepsilon \stackrel{\text{def}}{=} M_1^{(1)}. \qquad (1.4)$$

By using (1.4), from the second equation of (1.2), one could obtain

$$\limsup_{t \to +\infty} v(t) \le \frac{\beta \gamma M_1^{(1)}}{1 - \beta \rho}. \qquad (1.5)$$

Hence, for above $\varepsilon > 0$, there exists a $T_2 \ge T_1$, such that

$$v(t) \le \frac{\beta \gamma M_1^{(1)}}{1 - \beta \rho} + \varepsilon \stackrel{\text{def}}{=} M_2^{(1)}. \qquad (1.6)$$

Equation (1.6) together with the first equation of (1.2) leads to

$$\begin{aligned}
\frac{du}{dt} &\ge u(1 + c_1 - u) - v - cu \\
&\ge u\left[(1 + c_1 - M_2^{(1)}) - (1 + c)u\right].
\end{aligned} \qquad (1.7)$$

Here, in (1.7), the authors had used the fact

$$v \leq u M_2^{(1)} \qquad (1.8)$$

However, from the proof of the theorem, more precisely, from (1.6), we could only obtain the fact $v(t) \leq M_2^{(1)}$ for all $t \geq T_2$, hence, the deduction of (1.7) is incorrect, or at least is not strictly. Some other similar mistakes also happened in their deduction. Hence, the conclusion of Theorem 3.3 in [43] may not hold. One natural issue is to revisit the stability property of the system (1.2), and to give the right conditions to ensure the stability of the positive equilibrium. However, at present we have difficulty in dealing with this matter. So, we try to study some more simple model, i.e., single species model (1.1), we hope that our study will bring some light to this issue, and finally could solve the stability problem of system (1.2).

The rest of the paper is arranged as follows. In next section, we will investigate the existence and local stability of the equilibrium of the system (1.1). In Section 3, we will discuss the global stability of the equilibrium by using the iterative method. Numeric simulations are presented in Section 4 to show the feasibility of the main results. We end this paper by a briefly discussion.

2 The existence and local stability of the equilibria of system (1.1)

Concerned with the existence of the equilibria of system (1.1), we have the following result.

Theorem 2.1. *System (1.1) admits the boundary equilibrium $x_0 = 0$ and the unique positive equilibrium x_1^*, where*

$$\begin{aligned} x_1^* &= \frac{a + c - h - bd + \sqrt{\delta}}{2b}, \\ \delta &= (a + c - h - bd)^2 + 4bd(a + c). \end{aligned} \qquad (2.1)$$

Proof. The equilibria of system (1.1) satisfies the equation

$$x(a + c - bx) - \frac{hx^2}{d + x} = 0. \qquad (2.2)$$

Equation (2.2) has three solution $x_0 = 0$, and

$$\begin{aligned} x_1^* &= \frac{a + c - h - bd + \sqrt{\delta}}{2b}, \\ x_2^* &= \frac{a + c - h - bd - \sqrt{\delta}}{2b}. \end{aligned} \qquad (2.3)$$

where δ is defined by (2.1). Noting that

$$\begin{aligned} \sqrt{\delta} &= \sqrt{(a + c - h - bd)^2 + 4bd(a + c)} \\ &> |a + c - h - bd|, \end{aligned}$$

hence

$$\begin{aligned} a + c - h - bd + \sqrt{\delta} &> 0, \\ a + c - h - bd - \sqrt{\delta} &< 0. \end{aligned}$$

Therefore,

$$x_1^* > 0, \quad x_2^* < 0.$$

Hence, system (1.1) admits a unique positive equilibrium x_1^*.

Theorem 2.2. *$x_0 = 0$ is unstable equilibrium, and x_1^* is locally asymptotically stable equilibrium.*

Proof. Set

$$F = x(a + c - bx) - \frac{hx^2}{d + x}. \qquad (2.4)$$

Then

$$F' = a + c - 2bx - \frac{2hx}{d + x} + \frac{hx^2}{(d + x)^2}. \qquad (2.5)$$

Substituting x_0, x_1^* into F' leads to

$$F'|_{x=x_0} = a + c > 0. \qquad (2.6)$$

So, x_0 is unstable.

$$\begin{aligned} F'|_{x=x_1^*} &= -a - c + \frac{h(x_1^*)^2}{(d + x_1^*)^2} \\ &= \frac{x_1^*(a + c - bx_1^*)}{d + x_1^*} - a - c \qquad (2.7) \\ &< a + c - bx_1^* - a - c \\ &= -bx_1^* < 0. \end{aligned}$$

So, x_1^* is locally asymptotically stable. This ends the proof of Theorem 2.2.

3 Global attractivity

Concerned with the global attractivity of the positive equilibrium, we have the following result.

Theorem 3.1. *The positive equilibrium x_1^* is globally attractive.*

Proof. From (1.1) we have

$$\frac{dx}{dt} \leq x(a + c - bx), \qquad (3.1)$$

Hence,

$$\limsup_{t \to +\infty} x(t) \leq \frac{a + c}{b}. \qquad (3.2)$$

For $\varepsilon > 0$ enough small, without loss of generality, we may assume that $\varepsilon < \frac{1}{2} \frac{a + c}{b + \frac{h}{d}}$, it follows from (3.2) that there exists a $T_1 > 0$ such that

$$x(t) < \frac{a + c}{b} + \varepsilon \stackrel{\text{def}}{=} M_1. \qquad (3.3)$$

From (1.1) we also have

$$\frac{dx}{dt} \geq x\Big(a + c - (b + \frac{h}{d})x\Big), \qquad (3.4)$$

Hence,

$$\liminf_{t \to +\infty} x(t) \geq \frac{a+c}{b+\dfrac{h}{d}}. \qquad (3.5)$$

For above $\varepsilon > 0$, it follows from (3.5) that there exists a $T_2 > T_1$ such that

$$x(t) > \frac{a+c}{b+\dfrac{h}{d}} - \varepsilon \overset{\text{def}}{=} m_1. \qquad (3.6)$$

From (3.3), for $t \geq T_2$, we have

$$\frac{dx}{dt} \leq x\Big(a + c - (b + \frac{h}{d+M_1})x\Big), \qquad (3.7)$$

Hence,

$$\limsup_{t \to +\infty} x(t) \leq \frac{a+c}{b+\dfrac{h}{d+M_1}}. \qquad (3.8)$$

For above $\varepsilon > 0$, it follows from (3.2) that there exists a $T_3 > T_2$ such that

$$x(t) < \frac{a+c}{b+\dfrac{h}{d+M_1}} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_2. \qquad (3.9)$$

From (1.1) we also have

$$\frac{dx}{dt} \geq x\Big(a + c - (b + \frac{h}{d+m_1})x\Big), \qquad (3.10)$$

Hence,

$$\liminf_{t \to +\infty} x(t) \geq \frac{a+c}{b+\dfrac{h}{d+m_1}}. \qquad (3.11)$$

For above $\varepsilon > 0$, it follows from (3.5) that there exists a $T_2 > T_1$ such that

$$x(t) > \frac{a+c}{b+\dfrac{h}{d+m_1}} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2. \qquad (3.12)$$

Repeating the above procedure, we get four sequences $m_i, M_i, i = 1, 2, \ldots$ such that

$$\begin{aligned}
M_i &= \frac{a+c}{b+\dfrac{h}{d+M_{i-1}}} + \frac{\varepsilon}{i}, \\
m_i &= \frac{a+c}{b+\dfrac{h}{d+m_{i-1}}} - \frac{\varepsilon}{i}.
\end{aligned} \qquad (3.13)$$

From the deduction process, for $t > \max\{T_{2i}\}$, we have

$$m_i < x(t) < M_i. \qquad (3.14)$$

We claim that sequences M_i is strictly decreasing, and sequences m_i is strictly increasing. To proof this claim, we will carry out by induction. Obviously, we have

$$M_1 = \frac{a+c}{b} + \varepsilon > \frac{a+c}{b+\dfrac{h}{d+M_1}} + \frac{\varepsilon}{2} = M_2. \quad (3.15)$$

$$m_1 = \frac{a+c}{b+\dfrac{h}{d}} - \varepsilon < \frac{a+c}{b+\dfrac{h}{d+m_1}} - \frac{\varepsilon}{2}. \quad (3.16)$$

(3.15) and (3.16) show that the conclusion holds for $i = 2$. Let us assume now that our claim is true for $i = k$, that is,

$$M_k < M_{k-1}, m_k > m_{k-1}, \qquad (3.17)$$

then

$$\begin{aligned}
\frac{h}{d+M_{k-1}} &< \frac{h}{d+M_k}, \\
\frac{h}{d+m_{k-1}} &> \frac{h}{d+m_k}.
\end{aligned} \qquad (3.18)$$

And so

$$\begin{aligned}
M_{k+1} &= \frac{a+c}{b+\dfrac{h}{d+M_k}} + \frac{\varepsilon}{k+1} \\
&< \frac{a+c}{b+\dfrac{h}{d+M_{k-1}}} + \frac{\varepsilon}{k} = M_k,
\end{aligned} \qquad (3.19)$$

$$\begin{aligned}
m_{k+1} &= \frac{a+c}{b+\dfrac{h}{d+m_k}} - \frac{\varepsilon}{k+1} \\
&> \frac{a+c}{b+\dfrac{h}{d+m_{k-1}}} - \frac{\varepsilon}{k} = m_k.
\end{aligned} \qquad (3.20)$$

Above analysis shows that M_i is strictly decreasing sequence, m_i is strictly increasing sequence. Set

$$\lim_{i \to +\infty} M_i = M, \quad \lim_{i \to +\infty} m_i = m. \qquad (3.21)$$

Setting $i \to +\infty$ in (3.13) leads to

$$\begin{aligned}
M &= \frac{a+c}{b+\dfrac{h}{d+M}}, \\
m &= \frac{a+c}{b+\dfrac{h}{d+m}}.
\end{aligned} \qquad (3.22)$$

(3.22) shows that M, m are all the positive solution of (2.2). By Theorem 2.1, (2.2) has a unique positive solution x_1^*. Hence, we conclude that $M = m = x_1^*$. that is

$$\lim_{t \to +\infty} x(t) = x_1^*. \qquad (3.23)$$

Thus, the unique interior equilibrium x_1^* is globally attractive. This completes the proof of Theorem 3.1.

4 The influence of cannibalism

It's well known that the Logistic equation

$$\frac{dx}{dt} = x(a - bx) \qquad (4.1)$$

admits a unique positive equilibrium $x^* = \dfrac{a}{b}$, which is globally asymptotically stable. Theorem 2.1, 2.2 and 3.1 shows that system (1.1) admits a unique positive equilibrium $x_1^* = \dfrac{a + c - h - bd + \sqrt{\delta}}{2b}$, which is also globally asymptotically stable. From this, we can draw the first conclusion:

(I) For the Logistic equation, nonlinear cannibalism has no influence on the persistent property of the system.

Next, let's compare the big or small of x^* and x_1^*. Noting that

$$x_1^* - x^* = \frac{-a + c - h - bd + \sqrt{\delta}}{2b}. \qquad (4.2)$$

From (4.2), by simple computation, we have

(i) If

$$\frac{b}{a} > \frac{h - c}{cd}, \qquad (4.3)$$

then $x_1^* > x^*$;

(ii) If

$$\frac{b}{a} = \frac{h - c}{cd}, \qquad (4.4)$$

then $x_1^* = x^*$;

(iii) If

$$\frac{b}{a} < \frac{h - c}{cd}, \qquad (4.5)$$

then $x_1^* < x^*$.

Without loss of generality, since we are interesting in the influence of cannibalism, we may assume that a, b, d are fixed positive constants, noting that h is the cannibalism rate and c can be denote by transform rate. We then have the following results.

(II) If c is enough small, then the inequality (4.5) holds, in this case, the cannibalism will decrease the final density of the species.

(III) If c is enough large, such that $h - c \to 0$, then the inequality (4.3) holds, in this case, the cannibalism will increase the final density of the species.

Remark 4.1. Set $R_0 = \dfrac{a(h - c)}{bcd}$, then R_0 can be seen as the threshold parameter of the system (1.1). If $R_0 < 1$, then $x_1^* > x^*$; If $R_0 > 1$, then $x_1^* < x^*$, and if $R_0 = 1$, then cannibalism has no influence on the final density of the species.

Finally, noting that x_1^* is the function of h and c, we have

$$\frac{\partial x_1^*}{\partial h} = -\frac{a + c - bd - h + \sqrt{\delta}}{2b\sqrt{\delta}} < 0. \qquad (4.6)$$

$$\frac{\partial x_1^*}{\partial c} = \frac{a + c + bd - h + \sqrt{\delta}}{2b\sqrt{\delta}} > 0. \qquad (4.7)$$

That is

(IV) The final density of the species is the decreasing function of cannibalism rate and the increasing function of the transform rate.

5 Numeric simulations

Example 5.1. Now let's fixed $a = b = d = 1, c = 0.01$, then

$$x_1^*(h) = 0.005 - 2 - \frac{h}{2} + \frac{1}{2}\sqrt{h^2 - 0.02h + 4.0401}.$$

Fig. 1 shows that x_1^* is the strictly decreasing function of h, also, if h is enough large, then $x_1^* \to 0$. That is, for the fixed transform rate, if the cannibalism coefficient h is enough large, the species may driven to extinction, though at first sight, for the fixed cannibalism rate, the system is permanent. The final density of the species may become very small if the cannibalism rate is enough large.

Example 5.2. Now let's fixed $a = b = d = 1, h = 2$, then

$$x_1^*(c) = -1 + \frac{1}{2}c + \frac{1}{2}\sqrt{c^2 + 8}.$$

Fig. 2 shows that x_1^* is the strictly increasing function of c.

Example 5.3. Now let's fixed $a = b = d = 1$, then we have $x^* = 1$, from (4.3)-(4.5), we know that if $h > 2c$, then $x_1^* > x^*$, if $h = 2c$, then $x_1^* = x^*$, if $h < 2c$, then $x_1^* < x^*$. Fig. 3 shows that $x_1^*(h, c)$ smaller or bigger than x^*, depending on the parameters lies below or above the line $h = 2c$.

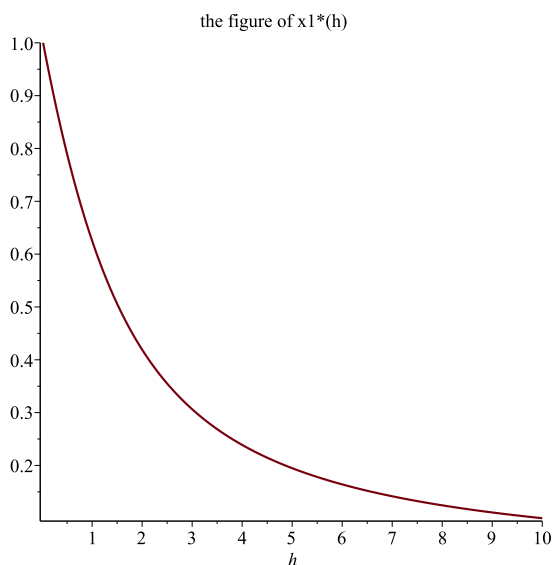Now let's fix $c = 0.5$, $h = 0.5, 1$ and 2, respectively. Then, for $h = 0.5$, $x_1^* > x^*$, for $h = 1$,
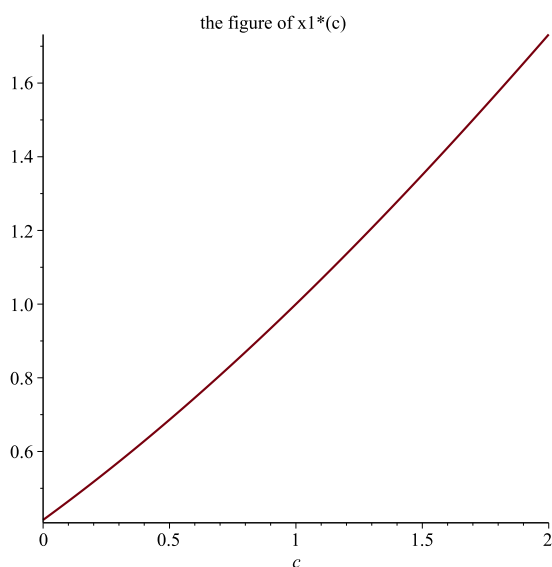
Figure 1: Relationship of x_1^* and h.

Figure 2: Relationship of x_1^* and c.

$x_1^* = x^*$, and for $h = 2$, $x_1^* < x^*$. Fig.4-6 supports this assertion. From Fig.4-6 we could also find that x_1^* is the decreasing function of h.
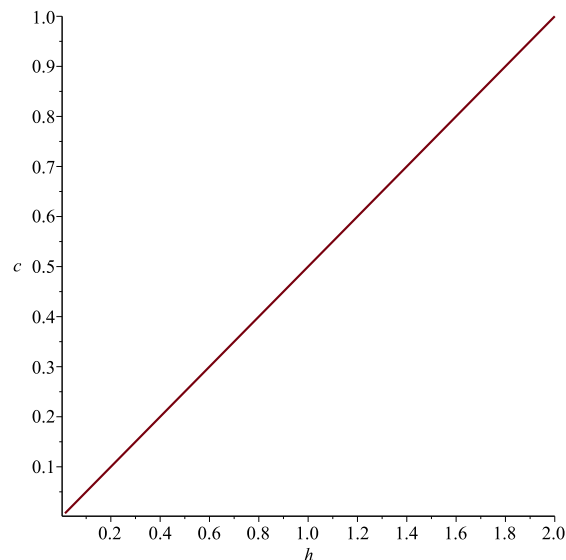
Now let's fix $h = 1$, $c = 0.3, 0.5$ and 0.7, respectively. Then, for $c = 0.7$, $x_1^* > x^*$, for $c = 0.5$, $x_1^* = x^*$, and for $c = 0.3$, $x_1^* < x^*$. Fig.7-9 supports this assertion. From Fig.7-9 we could also find that x_1^* is the increasing function of c.

Figure 3: Relationship of h and c. $x_1^*(h, c)$ smaller or bigger than x^*, depending on the parameters lies above or below the line $h = 2c$.

6 Conclusion

Based on the traditional Logistic equation and the works of Basheer et al.[42, 43], we proposed a single species model incorporating the nonlinear cannibalism. Already, Basheer et al.[42] incorporated the cannibalism to the Holling-Tanner model with ratio-dependent functional response (i.e., system (1.2)). They showed that cannibalism in the prey cannot stabilize the unstable interior equilibrium in the ODE case, but can destabilize the stable interior equilibrium, leading to a stable limit cycle. In this paper, we focus our attention to the single species model, our study shows that the system with cannibalism always admits a unique globally asymptotically stable equilibrium, which means that the cannibalism has no influence on the persistent property of the system. However, we could show that depending on the parameter regime, the final density of the species maybe larger or smaller or equal to the final density of the the system without cannibalism. It's in this sense, that
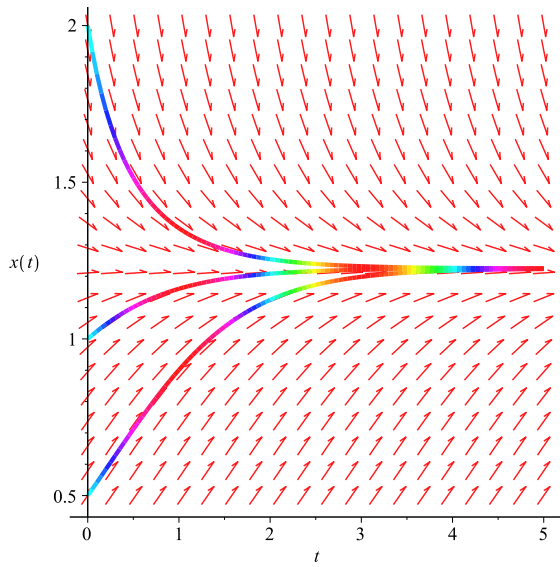
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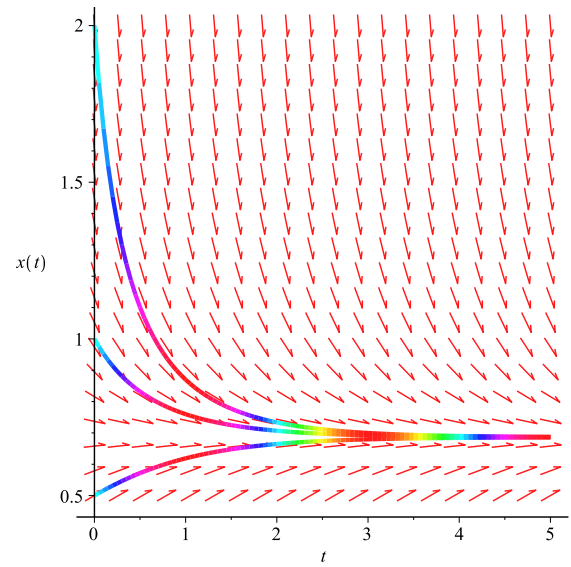Figure 4: Dynamic behaviors of the system (1.1), with $a = b = d = 1, c = 0.5, h = 0.5$.

Figure 6: Dynamic behaviors of the system (1.1), with $a = b = d = 1, c = 0.5, h = 2$.

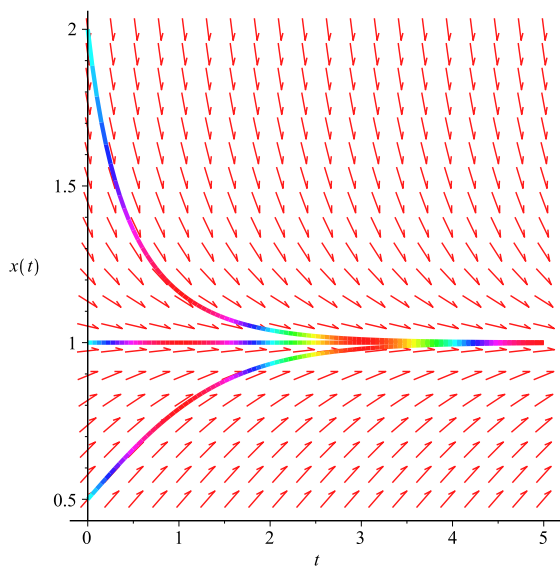

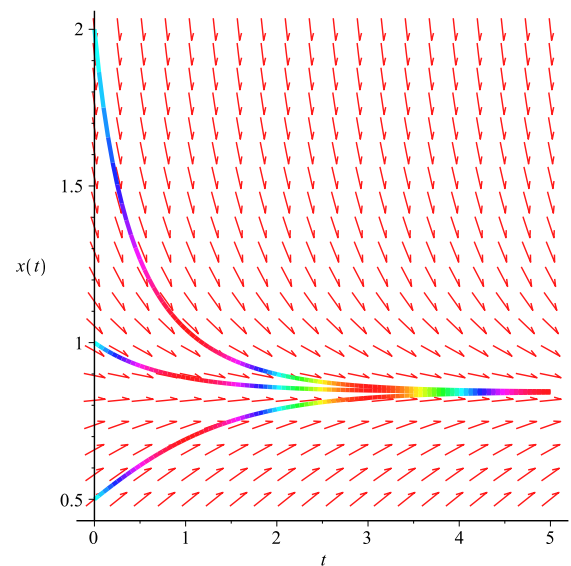Figure 5: Dynamic behaviors of the system (1.1), with $a = b = d = 1, c = 0.5, h = 1$.

Figure 7: Dynamic behaviors of the system (1.1), with $a = b = d = 1, h = 1, c = 0.3$.

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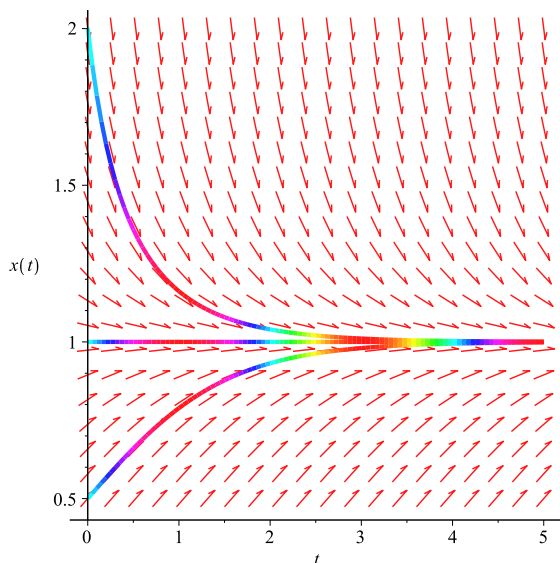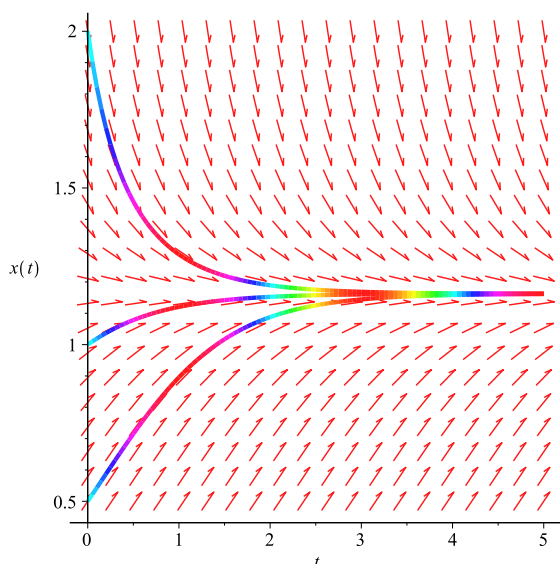Figure 8: Dynamic behaviors of the system (1.1), with $a = b = d = 1, h = 1, c = 0.5$.

Figure 9: Dynamic behaviors of the system (1.1), with $a = b = d = 1, h = 1, c = 0.7$.

cannibalism may have positive or negative or has no influence on the final density of the species. Also, if the cannibalism rate is enough large while the transform rate is enough small, then the species may increase its probability of the extinction in the sense that the final density of the species may approach to zero.

We hope our findings could be applied to more complicated situation, such as the competition model or the mutualism model. Also, as was shown in the introduction section, the results about the global stability property of the positive equilibrium of system (1.2) may not right, is it possible for us to investigate the stability property of the positive equilibrium by using the iterative method? We leave these problem for future investigation.

References:

[1] Chen F. D., Chen W. L., et al, Permanece of a stage-structured predator-prey system, *Appl. Math. Comput.,* Vol. 219, No.17, 2013, pp.8856-8862.

[2] Chen F. D., Xie X. D., et al, Partial survival and extinction of a delayed predator-prey model with stage structure, *Appl. Math. Comput.* Vol. 219, No.8, 2012, pp. 4157-4162.

[3] Chen F. D., Wang H. N. , Lin Y. H., Chen W. L., Global stability of a stage-structured predator-prey system, *Appl. Math. Comput.* Vol. 223, No.1, 2013, 45-53.

[4] Chen F. D., Xie X. D., et al, Dynamic behaviors of a stage-structured cooperation model, *Commun. Math. Biol. Neurosci.* Vol 2015, 2015, Article ID 4.

[5] Li T. T., Chen F. D., et al, Stability of a mutualism model in plant-pollinator system with stage-structure and the Beddington-DeAngelis functional response, *J. Nonlinear Funct. Anal.* Vol.2017, 2017, Article ID 50.

[6] Li Z., Chen F. D., Extinction in periodic competitive stage-structured Lotka-Volterra model with the effects of toxic substances, *J. Comput. Appl. Math.* Vol. 231, No.1, 2009, pp. 143-153.

[7] Li Z., Han M. A., et al, Global stability of stage-structured predator-prey model with modified Leslie-Gower and Holling-type II schemes, *Int. J. Biomath.* Vol. 6, 2012, Article ID 1250057, 13pp.

[8] Li Z., Han M. , et al, Global stability of a predator-prey system with stage structure and mutual interference, *Discrete and Continuous Dynamical Systems-Series B (DCDS-B)*, Vol. 19, No.1, 2014, pp. 173-187.

[9] Lin X., Xie X., et al, Convergences of a stage-structured predator-prey model with modified Leslie-Gower and Holling-type II schemes, *Advances in Difference Equations*, Vol.2016, 2016, ARticle ID 181.

[10] Wu H. L., Chen H. L., Harvesting of a single-species system incorporating stage structure and toxicity, *Discrete Dynamics in Nature and Society* Volume 2009, 2009, Article ID 290123, 16 pages.

[11] Xiao Z., Li Z. , Zhu Z., et al. Hopf bifurcation and stability in a Beddington-DeAngelis predator-prey model with stage structure for predator and time delay incorporating prey refuge, *Open Mathematics*, Vol.17, No.1, 2019, pp.141-159.

[12] Yue Q., Permanence for a modified Leslie-Gower predator-prey model with Beddington-DeAngelis functional response and feedback controls[J]. *Advances in Difference Equations*, Vol.2015, 2015, ARticle ID 81.

[13] Yue Q., Permanence of a delayed biological system with stage structure and density-dependent juvenile birth rate, *Engineering Letters*, Vol.27, No.2, 2019, pp.1-5.

[14] Lei C., Dynamic behaviors of a stage-structured commensalism system, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 301.

[15] Lei C. Q., Dynamic behaviors of a stage structure amensalism system with a cover for the first species, *Advances in Difference Equations*, Vol. 2018, 2018, ARticle ID 272.

[16] Lin Q., Allee effect increasing the final density of the species subject to the Allee effect in a Lotka-Volterra commensal symbiosis model, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 196.

[17] Lin Q., Xie X., et al, Dynamical analysis of a logistic model with impulsive Holling type-II harvesting, *Advances in Difference Equations*, Vol.2018, 2018, ARticle ID 112.

[18] Xie X. , Xue Y., et al. Permanence and global attractivity of a nonautonomous modified Leslie-Gower predator-prey model with Holling-type II schemes and a prey refuge, *Advances in Difference Equations*, Vol. 2016, 2016, Article ID 184.

[19] Xie X. D., Chen F. D., et al, Note on the stability property of a cooperative system incorporating harvesting, *Discrete Dynamics in Nature and Society*, Volume 2014, 2014, Article ID 327823, 5 pages.

[20] Wu R., Li L., et al, A Holling type commensal symbiosis model involving Allee effect, *Communications in Mathematical Biology and Neuroscience*, Vol. 2018, 2018: Article ID 6.

[21] Wu R., Li L., Permanence and global attractivity of the discrete predator-prey system with Hassell-Varley-Holling III type functional response, *Discrete Dynamics in Nature and Society*, Volume 2013, 2013, Article ID 393729, 9 pages.

[22] Xue Y., Xie X., et al. Global attractivity and extinction of a discrete competitive system with infinite delays and single feedback control, *Discrete Dynamics in Nature and Society*, Volume 2018, 2018, Article ID 1893181, 14 pages.

[23] Xue Y., Xie X., et al. Almost periodic solution of a discrete commensalism system, *Discrete Dynamics in Nature and Society*, Volume 2015, 2015, Article ID 295483, 11 pages.

[24] Lin Q., Allee effect increasing the final density of the species subject to the Allee effect in a Lotka-Volterra commensal symbiosis model, *Advances in Difference Equations*, Vol. 2018, 2018, Arctile ID 196.

[25] Lin Q., Stability analysis of a single species logistic model with Allee effect and feedback control, *Advances in Difference Equations*, Vol. 2018, 2018, Article ID 190.

[26] Chen L., Wang Y., et al, Influence of predator mutual interference and prey refuge on Lotka-Volterra predator-prey dynamics, *Communications in Nonlinear Science & Numerical Simulations*, Vol. 18, No.11, 2013, pp. 3174-3180.

[27] He Y., Chen F., Extinction and stability of an impulsive system with pure delays, *Applied Mathematics Letters*, Vol.91, No.1, 2019, pp. 128-136.

[28] He M., Li Z., et al, Dynamics of an impulsive model of plankton allelopathy with delays, *Journal of Applied Mathematics and Computing*, Vol.55, No.1-2, 2017, pp. 749-762.

[29] Zhao L., Qin B., et al, Permanence and global stability of a May cooperative system with strong and weak cooperative partners, *Advances in Difference Equations*, Vol.2018, 2018, Article ID 172.

[30] Yang K., Miao Z. S., et al, Influence of single feedback control variable on an autonomous Holling-II type cooperative system, *Journal of Mathematical Analysis and Applications*, Vol. 435, No.1, 2016, pp.874-888.

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Qun Zhu, Qianqian Li

[31] Chen F. , Xie X., et al. Extinction in two species nonautonomous nonlinear competitive system, *Applied Mathematics and Computation*, Vol. 274, No.1, 2016, pp.119-124.

[32] Chen B. , Dynamic behaviors of a non-selective harvesting Lotka-Volterra amensalism model incorporating partial closure for the populations, *Advances in Difference Equations*, Vol.2018, 2018, Article ID 111.

[33] Chen B., Permanence for the discrete competition model with infinite deviating arguments, *Discrete Dynamics in Nature and Society*, Volume 2016, 2016, Article ID 1686973, 5 pages.

[34] Smith C. , Reay P., Cannibalism in teleost fish, *Rev Fish Biol Fisheries*, Vol.1, No.1, 1991, pp. 41-54.

[35] Rickers S., Chen S., Cannibalism in Paradosa palustris (Araneae, Lycosidae): effects of alternative prey, habitat structure, and density, *Basic Appl Ecol* Vol.6, 2005, pp.471-478.

[36] Walters C., Christensen V., Fulton B., et al., Predictions from simple predator-prey theory about impacts of harvesting forage fishes, *Ecological modelling*, Vol.337, 2016, pp. 272-280.

[37] Petersen A. , Nielsen K. T. , Christensen C. B. , et al., The advantage of starving: success in cannibalistic encounters among wolf spiders, *Behavioral Ecology*, Vol.21, No.5, 2010, pp. 1112-1117.

[38] Kang Y., Rodriguez-Rodriguez M., Evilsizor S. , Ecological and evolutionary dynamics of two-stage models of social insects with egg cannibalism, *Journal of Mathematical Analysis and Applications*, Vol.430, No.1, 2015, pp. 324-353.

[39] Rodriguez-Rodriguez M. , Kang Y., Colony and evolutionary dynamics of a two-stage model with brood cannibalism and division of labor in social insects, *Natural Resource Modeling*, Vol.29, No.4, 2016, pp.633-662.

[40] Zhang L., Zhang C. , Rich dynamic of a stage-structured prey-predator model with cannibalism and periodic attacking rate, *Communications in Nonlinear Science and Numerical Simulation*,Vol.15, No.12, 2010, pp. 4029-4040.

[41] Zhang F. , Chen Y., Li J., Dynamical analysis of a stage-structured predator-prey model with cannibalism, *Mathematical Biosciences*, Vol. 307, 2019, pp. 33-41.

[42] Basheer A., Quansah E., Bhowmick S., et al., Prey cannibalism alters the dynamics of Holling-Tanner-type predator-prey models, *Nonlinear Dynamics*, Vol.85, No.4, 2016, pp. 2549-2567.

[43] Basheer A., Parshad R. D., Quansah E.,et al., Exploring the dynamics of a Holling-Tanner model with cannibalism in both predator and prey population, *International Journal of Biomathematics*, Vol.11, No.1, 2018, Article ID 1850010.

Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

Tingjie Zhou wrote the draft.
Qun Zhu and Qianqian Li carried out the simulation.
Fengde Chen proposed the issue and revise the paper.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.