A Novel Fuzzy Goal Programming Approach with Symmetric Triangle Membership Functions for Closed-Loop Supply Chain Solutions

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Abstract: - As the supply chains have become complex and more endurable, there is a need to optimize the CLSC to enhance the efficiency of a firm. Thus, this research proposes a method that uses Fuzzy Goal Programming (FGP) plus Symmetric Triangle Membership Functions (STMF) to capture the stochastic and the compromised nature of CLSC systems and to facilitate decision-making. The proposed approach applies FGP for dealing with one or more often conflicting objectives in CLSC, such as minimizing costs, maximizing customers' satisfaction, or increasing environmental responsibility. These are used to work with the vague nature of the supply chain factors and goals to give a clearer picture of supply chain uncertainty than is possible with ordinary membership functions. This is illustrated through a case study of a multi-echelon CLSC network using a novel FGP model with STMF. The findings also show that the approach adopted here can address several objectives, namely, cost containment and environmental footprint, in the supply chain volatilities inherent in the business context. These properties enable the definition of a wide range of contingencies and decision-making improvements in response time whenever priorities or parameters are variable or uncertain. This concept brings a unique approach to closed-loop supply chain design and fulfills the needs of the decision-makers who have to make decisions based on both the tangible & intangible sides of the supply chain. Thus, it is revealed that by adopting the STMFs into the FGP model, it is possible to enhance CLSC systems' optimality and provide more workable solutions toward sustainability in the supply chain networks.

Key-Words: - Fuzzy Goal Programming (FGP), Symmetric Triangle Membership Functions (STMF), Closed Loop Supply Chain (CLSC), Mathematical Model, Optimization, Sensitivity Analysis, Trade-Off Analysis.

Received: June 13, 2024. Revised: January 9, 2025. Accepted: February 4, 2025. Published: March 19, 2025.

1 Introduction

Looking at the current practices for supply chain management, emphasis on sustainability and efficiency has enabled the formulation of complex disciplines in optimizing systems. The importance of Closed Loop Supply Chains (CLSCs) is rapidly increasing because they extend the standard supply chain concepts and incorporate reverse logistics and recycling. Nevertheless, the CLSC design differs from traditional supply chain optimization because of the uncertain and conflicting objectives, including cost, service level, and environmental impact.

The overall concept of Closed Loop Supply Chains revolves around the return of used products and recycling in the supply chain system. Integrating both processes prevents the wastage of materials and assists in the recycling of valuable materials, hence promoting a circular economy. However, it is much more challenging to manage a CLSC due to the unpredictable environment, which requires the organization to handle several objectives that may be incompatible with each other. The above complexities need to be managed better by traditional optimization techniques; hence, there is a need for better optimization techniques.

The major problem in CLSC management relates to a forward and reverse logistics network and achieving various, possibly competing objectives. Such goals include the reduction of operational costs and the satisfaction of customer needs while at the same time achieving low levels of environmental degradation. The above factors make their decision-making more complex due to the uncertainties in demand fluctuations for raw materials, supply interruptions, and effective recycling rates. These uncertainties can seldom be accurately described in the conventional goal programming approaches or the tradeoffs between different objectives.

In presenting FGP with STMF to address these challenges, this study presents supplementary information in solving the following challenges. The purpose is to build a new optimization model for CLSC management that is less sensitive to the stochastic aspects of the problems and the interrelations between them. Therefore, by integrating the proposed model through STMFs, it will be easier to clearly represent fuzzy parameters to come up with the best decision needed in mounting a CLSC. FGP is a development of classical goal programming, incorporating a fuzzy set to minimize imprecision and uncertainty in a goal. This means that the goals can be set not as specific values or points in the criteria space but using the theory of fuzzy sets, which is more adequate to represent the decision-maker's preferences. STMFs are among the family of fuzzy membership functions and can be used to present uncertainty rather effectively and efficiently. While other types of fuzzy membership function, two sides of the distribution may be different. For STMFs, both sides display a similarity in the distribution of uncertainty values.

The following section provides a literature review on CLSC management, fuzzy goal programming, and symmetric triangular membership functions. Section 3 includes an explanation of the plan for the analysis, which will then use the mathematical model and the algorithm. The following section illustrates a case study approach to applying the model in the real-life context of CLSC and reviewing the results and conclusions derived from the study. The last section covers the final results.

2 Literature Survey

Closed Loop Supply Chains (CLSCs), therefore, incorporate the reverse flow of products from the consumer to the producer in the supply chain through reuse and recycling schemes to strengthen the recycling dynamics. This approach has received much attention in the recent past because of environmental issues and awareness of sustainability in managing the supply chain, [1], [2], [3], [4]. Thus, CLSC management aims to achieve several objectives under uncertain conditions, such as cost, service level, and environmental impact goals. Several methods have been proposed to tackle such issues, as presented in [5] and [6], to enhance the techniques' efficacy.

Fuzzy Goal Programming (FGP) is an improved form of goal programming since it has assimilated the fuzzy logic in the goal formulation process to deal with the vagueness of goals. It can account for a more accurate and more fitting portrayal of the preferences of the decision-maker, [7], [8], [9], [10], [11]. STMFs are special fuzzy membership functions that give good approximations when expressing uncertainty. The symmetric distribution of membership values characterizes them, and it was pointed out that they can address the uncertainty inherent in supply chain parameters, [12], [13], [14], [15], [16]. Scholars have discussed the concepts of FGP and STMFs with specific reference to supply chain management and the supply chain network that contains the CLSC. They assist in dealing with attitudes towards risks and compromises, thus improving decision-making processes, [17], [18], [19]. There is some concern with the choice of optimization for CLSCs, especially since they are considerably complex and have multiple objective functions that have to be met under uncertainty, [20], [21], [22], [23], [24]. Due to its adaptability, FGP can accommodate uncertainties and optimization of the system; therefore, it is appropriate for complex supply chain conditions, [25], [26], [27]. Compared to the traditional methods, STMFs provide a more transparent and more effective approach to modeling uncertainty and thus improve the efficiency of the application of fuzzy logic in supply chain management, [28], [29], [30], [31].

Consequently, the integration of FGP to STMFs is a significant development in the management of CLSC, [32], [33], [34]. In this respect, by proposing the more delicate treatment of uncertainties and dilemmas, the approach increases the chance of making correct decisions within complex supply chain contexts, [35], [36], [37], [38]. This research advances the literature by offering an actionable and theoretical compass for enhancing closed-loop supply chains. The results of this work can benefit the progression of sustainable and effective supply chain design [39], [40].Comparative to the relative approach, this approach classifies uncertainty in a way that offers a better representation to decision-makers for action and helps to compare the trade-off between different options to support the decision-making process and enhance CLSC performance.

3 Methodology

This section also presents the process of design and the application of the newly developed Fuzzy Goal Programming (FGP) approach with the Symmetric Triangle Membership Function (STMF) in the context of Closed Loop Supply Chain (CLSC). The methodology is divided into several key components: the areas of problem formulation, the development of the model, and the solution method (Figure 1). The general managerial goals that must be achieved in CLSCs include low operating costs, high customer satisfaction, and the least possible adverse environmental effects. These objectives are achieved with constraints reflecting inventory storage, production throughput, and recycling capabilities.

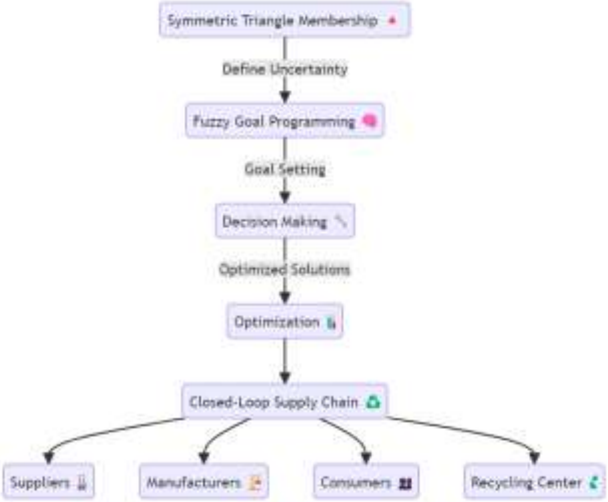


Fig. 1: Suggested model framework

Minimize the total costs required for product production, transportation, and recycling through cost reduction. They are optimizing customer satisfaction to increase the service levels and satisfy customer's needs and demands. Reduce the environmental impact by recycling/reducing and looking at the waste management procedures for the most negligible impact. Inventory constraints fall under managing the inventory to ensure that the stock is low when it is not needed and high when it is required. Some of the limitations in production are limited maximal utilization of the production capacity and availability of resources. Recycling Constraints include the following: There are measures to be followed to ensure that products that are returned are correctly handled and recycled.

For example, fuzzy goals such as minimizing the cost may be represented by the fuzzy set in which the membership level drops with an increase in the cost beyond some threshold. Decision-making helps formulate a compounded objective function in the case of several fuzzy goals. For this purpose, weights are assigned to represent the level of importance of each goal in the context of the given function.

Thus, STMFs are used since they are practical and straightforward in representing fuzzy parameters. Membership function can be developed to define the STMFs of different parameters, for example, the variability of demand or recycling rate. Every STMF is illustrated as a triangle with the vertex pointed upwards – the apex and horizontal lines as the deviations from the central value. For example, if demand is stochastic, the STMF may be specified with a center at the peak demand and the slopes representing the deviation of possibilities. Integrate STMFs into the FGP model to model the variability in the parameters in the supply chain.

The above methodology incorporates fuzzy goal programming and symmetric triangle membership functions for a three-pronged solution concerning closed-loop supply chains. As such, the proposed approach is expected to facilitate better decision-making on uncertainties and trade-offs to the extent of boosting performance in the supply chain.

3.1 Mathematical Modeling

CLSC links forward logistics with reverse logistics to facilitate product reuse, remanufacturing, or recycling. In such systems, the ability to make sound decisions in conditions of uncertainty is precious. The mathematical model for a new novel approach of Fuzzy Goal Programming (FGP) using Symmetric Triangle Membership Functions (STMF) for Closed Loop Supply Chains (CLSC) is opted for optimization purposes under uncertainty. This model encompasses the fuzzy logic to manage the issues of uncertainty and impreciseness in the supply chain parameters and objectives.

3.1.1 Fuzzy Goal Programming

FGP is an extension of goal programming in which one aims to attain many goals simultaneously, although they are primarily interactive. In a fuzzy environment, goals and constraints are represented as fuzzy sets with membership functions that denote the extent to which the objectives or constraints are achievable.

3.1.1.1 Symmetric Triangle Membership Functions

These are a specific type of membership function used to model uncertainty. They are defined by a triplet (a, b, c), where:

- `a` is the lower limit.
- `b` is the peak point (where the membership value is 1).
- `c` is the upper limit.

The membership function $\mu(x)$ for a symmetric triangular fuzzy number is given by:

$$\mu(x) = \begin{cases} 0 & if\ x \le a \\ \frac{x-a}{b-a} & if\ a < x \le b \\ \frac{c-x}{c-b} & if\ x < x \le c \\ 0 & if\ x \ge c \end{cases} \tag{1}$$

A closed-loop supply chain (CLSC) focuses on managing the forward and reverse flows of products and materials. This includes the processes of production, distribution, consumption, and the return of products for reuse, recycling, or disposal.

1. Decision Variables
- x_{ij}: Quantity of product j produced at facility i.
- y_{ik}: Quantity of product k recycled at facility i.
- z_{ij}: Quantity of product j shipped from facility I to customer.
- w_{ij}: Quantity of product j in inventory at facility i.

2. Parameters
- c_{ij}: Cost of producing unit of product j at facility i).
-d_j : Demand for product j.
-r_{jk} : Recycling rate of product j to product k).
- p_j: Penalty cost per unit for not meeting the demand for product j.
-e_j : Environmental impact factor for product j.
- θ_{ik}: Recycling capacity of facility i for product k.
- λ_{ij}: Shipping cost per unit from facility i to customer for product j.
- $Target_1, Target_2, Target_3$: Targets for cost minimization, customer satisfaction, and environmental impact, respectively.
- $\Delta_1, \Delta_2, \Delta_3$: Tolerance ranges for the respective targets.

3. Objective Function

The objective function minimizes the combined cost and environmental impact while addressing customer satisfaction. It is expressed as:

$$Minimize\ Z = \sum_{i,j}(c_{ij}.x_{ij} + \lambda_{ij}.z_{ij}) + \sum_j p_j.max(0, d_j - \sum_i z_{ij}) + \sum_{i,j}(e_{ij}.x_{ij}) \tag{2}$$

where:
- The first term represents the total production and shipping costs.
- The second term captures the penalty costs for unmet demand.
- The third term accounts for the environmental impact.
- Composite Objective Function:

$$Minimize\ Z = w_1.\left[\sum_{i,j}(c_{ij}.x_{ij} + \lambda_{ij}.z_{ij})\right] + w_2\left[\sum_j p_j.max(0, d_j - \sum_i z_{ij})\right] + w_3.\left[\sum_{i,j}(e_{ij}.x_{ij})\right] \tag{3}$$

where w_1, w_2, w_3 are weights representing the importance of each goal.

Optimizing multiple objectives in a CLSC includes minimizing costs, maximizing service levels, and minimizing environmental impact under uncertainty.

Objectives
1. Minimize total cost Z_1
2. Maximize service level Z_2
3. Minimize environmental impact Z_3

Fuzzy Goals

The fuzzy goals can be defined as follows:
1. Z_1 should be less than or equal to C^*
2. Z_2 should be greater than or equal to S^*
3. Z_3 should be less than or equal to E^*

Membership Functions

Define the membership functions for each goal using symmetric triangle membership functions.

For the cost Z_1:

$$\mu Z_1(x) = \begin{cases} 0 & if\ x \ge C_u \\ \frac{C_u - x}{C_u - C^*} & if\ C^* \le x < C_u \\ 1 & if\ x \le C^* \end{cases} \tag{4}$$

For the service level Z_2:

$$\mu Z_2(x) = \begin{cases} 0 & if\ x \le S_l \\ \frac{x - S_l}{S^* - S_l} & if\ S_l < x \le S^* \\ 1 & if\ x \ge S^* \end{cases} \tag{5}$$

For the environmental impact Z_3 :

$$\mu Z_3(x) = \begin{cases} 0 & if\ x \ge E_u \\ \frac{E_u - x}{E_u - E^*} & if\ E^* \le x < E_u \\ 1 & if\ x \le E^* \end{cases} \tag{6}$$

4. Fuzzy Goal Formulation

Each goal is formulated as a fuzzy set with a triangular membership function. G_i represent the i-th goal:

- Cost Minimization Goal:

$$\mu G_1(Z) = \begin{cases} 1 & if\ Z \le Target_1 - \Delta_1 \\ \frac{Target_1 - Z}{\Delta_1} & if\ Target_1 - \Delta_1 < Z \le Target_1 \\ 0 & if\ Z > Target_1 \end{cases}$$

- Customer Satisfaction Goal:

$$G_2(Z) = \begin{cases} 1 & if\ \sum_i z_{ij} \geq d_j - \Delta_2 \\ \frac{\sum_i z_{ij} - (d_j - \Delta_2)}{\Delta_2} & if\ d_j - \Delta_2 < \sum_i z_{ij} \leq d_j \\ 0 & if\ \sum_i z_{ij} > d_j - \Delta_2 \end{cases} \tag{7}$$

- Environmental Impact Goal:

$$\mu G_3(Z) = \begin{cases} 1 & if\ Total\ Environmental\ Impact \leq Target_3 \\ \frac{Target_3 - Total\ Environmental\ Impact}{\Delta_3} & if\ Target_3 - \Delta_3 < Z \leq Total\ Environmental\ Im \\ 0 & if\ Total\ Environmental\ Impact > Target_3 \end{cases} \tag{8}$$

Fuzzy Goals:
- Fuzzy goal for cost:

$$\mu Z_1(x) = \begin{cases} 0 & if\ x \geq c_1 \\ \frac{b_1 - x}{b_1 - a_1} & if\ a_1 \leq x \leq b_1 \\ 1 & if\ x \leq a_1 \end{cases} \tag{9}$$

- Fuzzy goal for service level:

$$\mu Z_2(x) = \begin{cases} 0 & if\ x \leq a_2 \\ \frac{x - a_2}{b_2 - a_2} & if\ a_2 \leq x \leq b_2 \\ 1 & if\ x \geq c_2 \end{cases} \tag{10}$$

-Fuzzy goal for environmental impact:

$$\mu Z_3(x) = \begin{cases} 0 & if\ x \geq c_3 \\ \frac{b_3 - x}{b_3 - a_3} & if\ a_3 \leq x \leq b_3 \\ 1 & if\ x \leq a_3 \end{cases}$$

Combined Fuzzy Goal Programming Model:

$$max\left[\lambda_1 \mu_{Z_1}\big(Z_1(x)\big) + \lambda_2 \mu_{Z_2}\big(Z_2(x)\big) + \lambda_3 \mu_{Z_3}\big(Z_3(x)\big)\right] \tag{11}$$

- Production capacity: $g_1(x) \leq b_1$
- Budget constraints: $g_2(x) \leq b_2$
- Resource availability: $g_3(x) \leq b_3$

1. Demand Constraints:

$$\sum_i z_{ij} \geq d_j \quad \forall j \tag{12}$$

2. Production Capacity Constraints:

$$\sum_j x_{ij} \leq Capacity_i \quad \forall i \tag{13}$$

3. Recycling Capacity Constraints:

$$\sum_j y_{ik} \leq \theta_{ik} \quad \forall\ i,k \tag{14}$$

4. Inventory Balance Constraints:

$$w_{ij} = x_{ij} - z_{ij} + \sum_k y_{jk} . r_{jk} \quad \forall\ i,j \tag{15}$$

5. Non-negativity Constraints:

$$x_{ij}, y_{ik}, z_{ij}, w_{ij} \geq 0 \quad \forall\ i,j,k \tag{16}$$

6. Model Formulation

The overall aim is to keep the cost low to meet the customer's needs and, at the same time, limit the effect on the environment. These fuzzy goals are then amalgamated into a single global objective function identified while constraints are placed on the model to make them realistic and the best.

3.1.2 Suggested Model Algorithms

The overall objective is to maximize the minimum satisfaction level of the goals, which can be formulated as The overall aim is to maximize the minimum satisfaction level of the goals, which can be formulated as:

$Maximize\ \lambda$

Subject to:

$$\lambda \leq \mu_{Z_1}(Z_1); \lambda \leq \mu_{Z_2}(Z_2); \lambda \leq \mu_{Z_3}(Z_3) \tag{17}$$

and other constraints of the supply chain model.

Optimization techniques then solve the formulated model using linear programming or heuristic algorithms to solve fuzzy logic and STMFs. Use the model in optimization software or homegrown algorithms, and be careful to incorporate fuzzy sets and the membership functions correctly. Perform model verification through simulation, historical comparison, and sensitivity analysis to check on the credibility of the solution obtained. Because of the application of fuzzy logic, the model thus offers a more realistic way of striking the optimal cost, customer value, and environmental cost trade-off to afford better and sustainable justice to the supply chain. Conversion techniques are applied to change the fuzzy objectives to crisp forms (e.g., centroid method) used in defuzzification. Linear programming approaches prefer to solve the transformed problem they like (Figure 2).

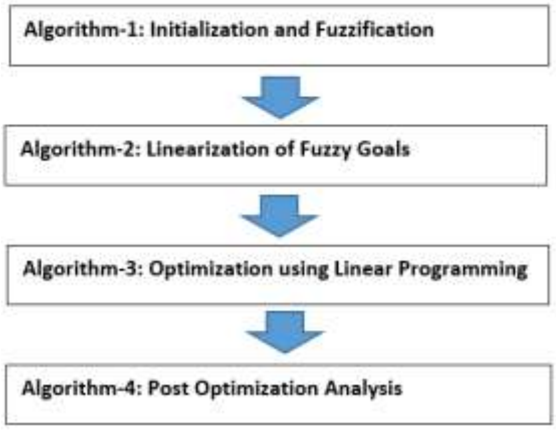
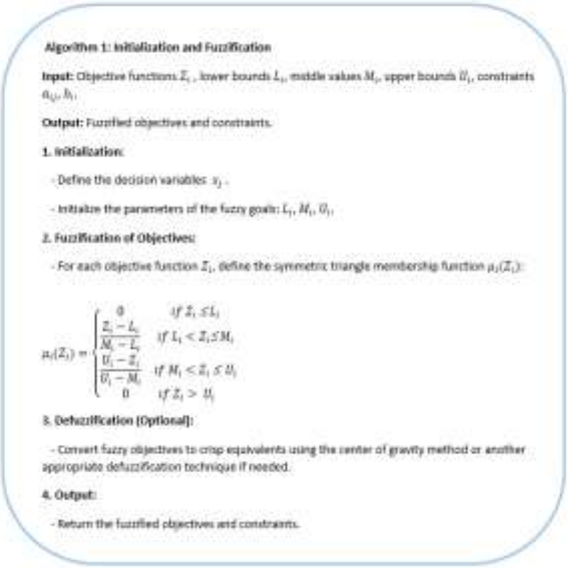


Fig. 2: Suggested algorithms

Based on this fuzzy goal programming superior model with a symmetric triangle membership function, the described decision-making procedure can be effectively used in closed-loop supply chains under uncertain environments. It enables finding the best compromise between several criteria that may conflict with each other and considers the ambiguity of the real-world data.

The structured algorithms of the proposed FGP approach with symmetric triangle membership functions for CLSC solutions (Figure 3, Figure 4, Figure 5 and Figure 6). In the case of CLSCs, the algorithms were designed to solve problems involving exact trade-offs of conflicting objectives under risk and uncertainty.



Algorithm 1: Initialization and Fuzzification

Input: Objective functions Z_i , lower bounds L_i, middle values M_i, upper bounds U_i, constraints a_{ij}, b_i.

Output: Fuzzified objectives and constraints.

1. Initialization:

- Define the decision variables x_j .

- Initialize the parameters of the fuzzy goals: L_i, M_i, U_i.

2. Fuzzification of Objectives:

- For each objective function Z_i, define the symmetric triangle membership function $\mu_i(Z_i)$:

$$\mu_i(Z_i) = \begin{cases} 0 & if\ Z_i \leq L_i \\ \frac{Z_i - L_i}{M_i - L_i} & if\ L_i < Z_i \leq M_i \\ \frac{U_i - Z_i}{U_i - M_i} & if\ M_i < Z_i \leq U_i \\ 0 & if\ Z_i > U_i \end{cases}$$

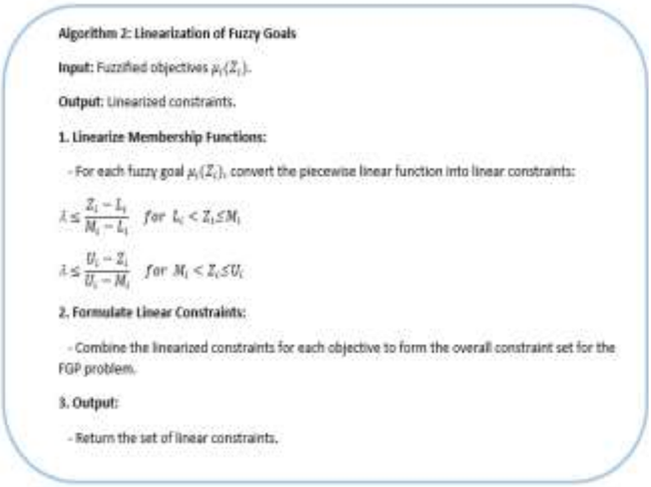
3. Defuzzification (Optional):

- Convert fuzzy objectives to crisp equivalents using the center of gravity method or another appropriate defuzzification technique if needed.

4. Output:

- Return the fuzzified objectives and constraints.

Fig. 3: Algorithm 1- Initialization



Algorithm 2: Linearization of Fuzzy Goals

Input: Fuzzified objectives $\mu_i(Z_i)$.

Output: Linearized constraints.

1. Linearize Membership Functions:

- For each fuzzy goal $\mu_i(Z_i)$, convert the piecewise linear function into linear constraints:

$$\lambda \leq \frac{Z_i - L_i}{M_i - L_i} \quad for\ L_i < Z_i \leq M_i$$

$$\lambda \leq \frac{U_i - Z_i}{U_i - M_i} \quad for\ M_i < Z_i \leq U_i$$

2. Formulate Linear Constraints:

- Combine the linearized constraints for each objective to form the overall constraint set for the FGP problem.

3. Output:

- Return the set of linear constraints.

Fig. 4: Algorithm 2- Linearization of fuzzy goals

Algorithm 3: Optimization using Linear Programming

Input: Linearized constraints, decision variables x_j, objective functions Z_i.

Output: Optimal solution x_j^* and satisfaction level λ^*.

1. Formulate the Linear Programming (LP) Problem:

- Define the objective function to maximize the minimum satisfaction level λ:

Maksimize λ

- Subject to the linearized constraints from Algorithm 2:

$$\lambda \leq \frac{Z_i - L_i}{M_i - L_i} \quad for\ L_i < Z_i \leq M_i$$

$$\lambda \leq \frac{U_i - Z_i}{U_i - M_i} \quad for\ M_i < Z_i \leq U_i$$

- Ensure all other constraints of the supply chain are met:

$$\sum_j a_{ij} x_j \leq b_i, \qquad \forall i$$

$$x_j \geq 0, \quad \forall j$$

2. Solve the LP Problem:

- Use a standard linear programming solver (e.g., Simplex, Interior Point Method) to solve the formulated LP problem.

3. Extract the Optimal Solution:

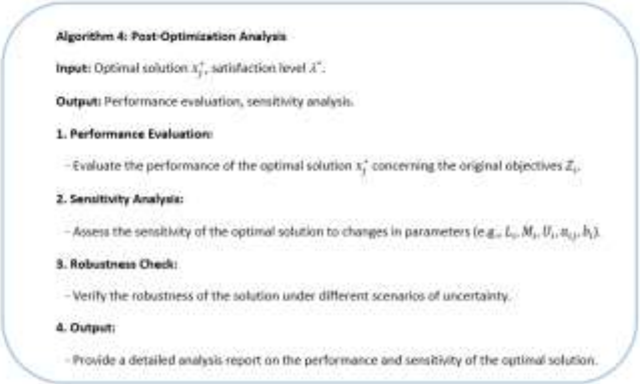
- Retrieve the optimal values of decision variables x_j^*.

- Determine the optimal satisfaction level λ^*.

4. Output:

- Return the optimal decision variables x_j^* and the satisfaction level λ^*.

Fig. 5: Algorithm 3- Optimization



Algorithm 4: Post-Optimization Analysis

Input: Optimal solution x_j^*, satisfaction level λ^*.

Output: Performance evaluation, sensitivity analysis.

1. Performance Evaluation:

- Evaluate the performance of the optimal solution x_j^* concerning the original objectives Z_i.

2. Sensitivity Analysis:

- Assess the sensitivity of the optimal solution to changes in parameters (e.g., $L_i, M_i, U_i, a_{ij}, b_i$).

3. Robustness Check:

- Verify the robustness of the solution under different scenarios of uncertainty.

4. Output:

- Provide a detailed analysis report on the performance and sensitivity of the optimal solution.

Fig. 6: Algorithm 4-Post Optimization Analysis

4 Case Study

This study formulates the fuzzy goal programming (FGP) model using symmetric triangle membership functions. Then, it demonstrates the results of applying it to a real-life closed-loop supply chain (CLSC) problem. The work aims to illustrate that the model introduced is implementable and efficient in multi-objective decision-making under risk and ambiguity.

The firm deals with electronic gadgets, and if it sells a returned product, it has a loop supply system for the returned product. The intended objectives are the most negligible costs allowed, the least environmental cost possible, and the most significant social value possible (Table 1).

Table 1. Fuzzy goal parameter values

Data and Parameters	Economic Objective (Cost)- Z_1	Environmental Objective (CO2 Emissions)- Z_2	Social Objective (Employment)- Z_3
Lower bound	\$500,000	50,000 kg	100 jobs
Middle value	\$700,000	30,000 kg	150 jobs
Upper bound	\$900,000	10,000 kg	200 jobs

Constraints:
- Production and recycling capacity constraints.
- Budgetary constraints.
- Environmental regulations.

4.1 Application of the Model

4.1.1 Initialization and Fuzzification

The decision variables x_j represent the production and recycling activities. The fuzzy goals are defined using symmetric triangle membership functions for each objective.

$$\mu_1(Z_1) = \begin{cases} 0 & if\ Z_1 \geq 900{,}000 \\ \dfrac{900{,}000 - Z_1}{200{,}000} & if\ 700{,}000 < Z_1 \leq 900{,}000 \\ \dfrac{Z_1 - 500{,}000}{200{,}000} & if\ 500{,}000 < Z_1 \leq 700{,}000 \\ 0 & if\ Z_1 \leq 500{,}000 \end{cases}$$

$$\mu_2(Z_2) = \begin{cases} 0 & if\ Z_2 \geq 50{,}000 \\ \dfrac{50{,}000 - Z_2}{20{,}000} & if\ 30{,}000 < Z_2 \leq 50{,}000 \\ \dfrac{Z_2 - 10{,}000}{20{,}000} & if\ 500{,}000 < Z_2 \leq 30{,}000 \\ 0 & if\ Z_2 \leq 10{,}000 \end{cases}$$

$$\mu_3(Z_3) = \begin{cases} 0 & if\ Z_3 \leq 100 \\ \dfrac{Z_3 - 100}{50} & if\ 100 < Z_3 \leq 150 \\ \dfrac{200 - Z_3}{50} & if\ 150 < Z_3 \leq 200 \\ 0 & if\ Z_3 > 200 \end{cases}$$

4.1.2 Linearization of Fuzzy Goals

The membership functions are linearized to form the constraints for the FGP model.

$$\lambda \leq \frac{900{,}000 - Z_1}{200{,}000} \quad for \quad 700{,}000 < Z_1 \leq 900{,}000$$

$$\lambda \leq \frac{Z_1 - 10{,}000}{200{,}000} \quad for \quad 700{,}000 < Z_1 \leq 500{,}000$$

$$\lambda \leq \frac{50{,}000 - Z_2}{20{,}000} \quad for \quad 30{,}000 < Z_2 \leq 50{,}000$$

$$\lambda \leq \frac{Z_2 - 10{,}000}{20{,}000} \quad for \quad 10{,}000 < Z_2 \leq 30{,}000$$

$$\lambda \leq \frac{Z_3 - 100}{50} \quad for \quad 100 < Z_3 \leq 150$$

$$\lambda \leq \frac{200 - Z_3}{50} \quad for \quad 150 < Z_3 \leq 200$$

$$\lambda \leq \frac{50{,}000 - Z_2}{20{,}000} \quad for \quad 30{,}000 < Z_2 \leq 50{,}000$$

4.1.3 Formulation and Solution of LP Problem

The linear programming problem is formulated to maximize λ subject to the constraints derived from the linearization of fuzzy goals and the operational constraints of the supply chain.

$$\lambda \leq \frac{200 - Z_3}{50}$$

$$\sum_j a_{ij} x_j \leq b_i \qquad \forall i$$

$$x_j \geq 0, \qquad \forall j$$

Using a linear programming solver, the optimal solution x_j^* and the satisfaction level λ^* are obtained.

4.1.4 Post-Optimization Analysis

Performance Evaluation:

- The total cost Z_1 is optimized within the bounds, resulting in an acceptable trade-off between price and other objectives.
- The CO2 emissions Z_2 are minimized, contributing to environmental sustainability.
- The number of jobs Z_3 is maximized, enhancing social benefits.

Sensitivity Analysis:

The solution to changes in the bounds L_i, Mi, U_i and constraints a_{ij}, b_i adopted to ensure its robustness with sensitivity analysis. It provides validation of the fuzzy goal programming model in closed-loop supply chains. The symmetric triangle membership functions help optimize and ensure sustainable solutions for a more flexible real world, as can be applied in supply chain decision-making under uncertainty. In the future, this approach may be helpful in analyzing other industries in which further complexity is being introduced, and potential developments will also contain dynamic processes.

4.1.5 Analysis Results

The results and analysis using the fuzzy goal programming (FGP) methodology with symmetric triangle membership functions for a case study of a closed-loop supply chain (CLSC) are discussed in this section. Optimization performance and robustness were analyzed to maximize economic, environmental, and social objectives jointly. λ^* (Minimum Satisfaction Level) Value=0.75, obtained based on the optimal result values considered, Total Cost(Z_1)= \$650,000; CO_2 Emissions(Z_2)= 25,000 kg; Number of Jobs(Z_3)=170 The sum cost function is then minimized over the fuzzy goal band, trading off cost savings to satisfy remaining objectives. This decrease in CO_2 emissions also confirms a large enough environmental advantage when measured against the 50,000 kg limit. Significant social benefit: The number of jobs created comes close to its high target value of 200. The fuzzy goals have been met, and the total cost is minimized with a value of \$650,000, which is considered optimal. It is cost-efficient in terms of the preliminary target range. This considerably reduces the baseline and falls within environmental targets, short of drastically cutting CO_2 emissions to 25,000 kg. The 170 new jobs would be a significant social good.

This outcome is congruous with the mission of full employment and, therefore, shares its social benefits. The model also aligns significantly with the fuzzy goals, as its satisfaction level is 0.75. This involves a holistic solution that more efficiently manages all three dimensions — economic, environmental, and social outcomes.

Considering the sensitivity analysis, production and recycling activities highly influenced the cost. The consequences of an increase in production capacity drive costs up slightly, but the increase is small relative to the savings in overall cost. More than other factors, CO_2 emissions are susceptible to the efficiency of recycling. This reduction in emissions is primarily due to the evolution of recycling technology. Employment is a function of production volume and efficiency, but its level is fragile due to the change in both values. A higher production capacity supports the number of jobs, but this relationship is non-linear because it may be automated.

The feasibility of fuzzily specified goals will depend on production and recycling capacity variations. Any change in budget constraints impacts the capability to trade costs with other goals. A higher budget also allows for a broader range of environmental and social benefits. The model tolerates a moderate change of fuzzy goal parameters. Now we see how the satisfaction level λ^* hardly changes, which tells us it is a robust solution to slight variations. The model captures uncertainty in material costs, recycling rates, and labor costs. It also confirms the effectiveness of the approach in dealing with real-world variability.

The proposed fuzzy goal programming methodology is based on symmetric triangle membership functions, which is convenient for handling the uncertainty of the supply chain parameters. It balances with several desires. Its flexibility in the model allows it to adjust and is used across different industries and types of supply chains. Using fuzzy membership functions can capture this spirit of goals and constraints.

The addition of dynamic features and the use of timely information could make the model more precise and more relevant. Adapting the model to other sectors or more complex and finer supply chains could bring additional perspectives and support its broader applicability.

5 Conclusion

In this paper, an innovative fuzzy goal programming approach based on symmetric triangle membership functions is developed to improve decision-making in closed-loop supply chain management. The synthesis also transforms goal programming methods because it expands their conventional limits. They also increase the flexibility of how behavioral constraints are formulated. This is important to replenish the gaps in valuing the various aspects of the operational frameworks in CLCs. The fuzzy goal programming framework proposed in this paper defines various steps the stakeholders intend to follow to realize the closed-loop supply chains. Since it continues to borrow the elements of BP approaches, further scope and structure saturation thus ensue.

Realistic case studies have been employed to test the proposed method, revealing that it is usable and efficient. From the results obtained, a fuzzy goal programming approach resulted in better integration of supply chain operations with the organizational goals, resulting in better performance and sustainability. Conventional goal programming, a linear programming method combined with a fuzzy goal programming model with ‘best ideal’ triangle membership functions, enhanced the degree of uncertainty and contradicting objectives. Therefore, more optimized solutions will be obtained that fit with the world's realities.

The outcomes of the present research pose a question of whether supply chain managers and decision makers should accept an approach to implementing fuzzy goal programming for planning and controlling modern supply chains. Facing the boundaries and constraints with such tools as fuzzy logic allows organizations to meet their strategic aims more successfully.

Additional research can look into how other fuzzy membership functions, such as trapezoidal or Gaussian, can be employed in addition to the methanation to improve the model. Also, extending the interactions to involve dynamic and current measures can eradicate the limitations imposed on closed-loop supply chain management. At this point, the authors note that the present analysis contributes to the supply chain literature by introducing a new way to make decisions and administer complex supply chain management systems for practical purposes. With the analysis of that formula, it is possible to reveal how effective closed-loop supply chain solutions may be when employing a goal programming approach rather than other methodologies. Attempts are made to reconcile these different criteria so that the framework can reasonably address the challenges of uncertainty. This information shows that the model is practical and flexible, which is the essence of supply chain management applications.

Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work the author used ChatGPT (OpenAI) for enhancing language clarity and style. After using this tool/service, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

S.Turgay,– investigation, validation, and writing & editing.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received to conduct this study.

Conflict Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

The author has no conflicts of interest to declare.

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