Inventory Optimization with Several Resources

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Abstract: - Inventory optimization is extended with additional constraints that affect production efficiency using estimated inventory. The extension of the inventory problem is in a special formal form with an additional optimization problem. The latter is aimed at minimizing production costs while using optimal volumes of material stocks. Such integration of inventory and production is formalized through a bi-level optimization problem. The case of simultaneous delivery of different types of goods is strongly related to the production process of final goods. A bi-level optimization problem is numerically defined and solved. Delivery costs are minimized for a given production plan. The bi-level problem is applied to prepare feed with the required nutrient content while minimizing the inventory costs of supplying the required raw materials. Empirical results give advantages for the obtained solution of the bi-level optimization problem.

Key-Words: - inventory modeling, inventory management, production optimization, business management, optimal nutrition content, bi-level optimization.

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1 Introduction

Inventory management is an important part of overall business policy. Its main goal is to minimize the costs of delivery, transportation, and storage of the resources used in the production of goods of the business entity. Quantifying the parameters involved in inventory management is a prerequisite for obtaining optimal solutions to minimize inventory costs. This minimization plays an important role in business results, as inventory resources are disbursements, resulting in a negative component of business profit. Effective inventory management affects the evaluation of [1]:

- the necessary level of spare parts;
- reducing the time for production of goods when the demand for them changes dynamically;
- reduction of losses from storage of excess quantities of raw materials;
- optimization of the workload of the production technology; reducing financial costs and increasing profit.

Inventory management uses a corresponding set of formal relations that are applied to define and solve optimization problems. The main objective of inventory management is to reduce inventory costs, [2]. In particular, the solution to the inventory problem yields optimal solutions for [3]: optimal supply size by minimizing costs; and reduction of material storage losses, given the limitation of storage capacity and the deadline for new delivery.

Inventory management is closely related to the requirements created by production technology. Therefore, the effects of inventory optimization are evaluated not only by reducing costs but also by its impact on additional processes related to production, [4]. Therefore, it is worthwhile to integrate into a common optimization problem the delivery of resources for the production of goods, since market behavior is dynamically applied and changes for the demand for goods. In [5], such an optimization problem from the class of linear integer optimization is defined. The problem simultaneously estimates the values of resources and the resulting quantities of goods. Such an approach for integrating inventory and production activities is formalized in [6]. The result of such a formalization gives the optimal supply values for the inventory. In [7], the formalization of inventory and production management is done with Markov chains, where the main goal of optimization is to maximize the quality of manufactured goods. In [8], the formalization of three types of activities that work in sequential order is illustrated: production, logistics operations, and delivery of resources. The defined optimization problem is a linear integer with total cost minimization. In [9], an optimization problem is defined to optimize the intensity of the production of goods, according to the changes that the market requires. The formalization that is applied in this problem belongs to the statistical estimates of the mean demands, the standard deviations of the demand, and the covariance between the volume of goods and the required resources. This optimization problem has a combinatorial nature and is solved by applying the ant colony algorithm. The relationships between inventory management and production intensity are analyzed in [10]. It evaluates the quantitative relationship between inventory management parameters and production intensity for the textile, flour, and food industries. In [11], the relationship between inventory management and the need for changes in administrative relations to be introduced in a business production unit is considered.

This review shows that inventory strongly influences the management of a business entity for its optimal production of goods. This is a prerequisite for the development of quantitative models and formalization that should estimate the optimal values of inventory and production parameters.

In this study, such integration of delivery and production activities is formalized as a common optimization problem. The peculiarity of this problem is its formal structure as a bi-level optimization. The solutions to the bi-level problem give the optimal delivery size according to the requirements of the production demands. The production of goods is maximized according to the profit that is obtained. The bi-level problem involves two optimization problems: minimization of inventory costs and maximization of production profit. Both problems are interconnected by common parameters regarding the volume of resources and the technological links between the production of goods and resources. The solutions to the bi-level problem are compared with the classic inventory management problem, which estimates the optimal amount of deliveries and their periodic fulfillment.

2 A Classic Resource Delivery Inventory Problem

Inventory management takes into account the storage of resources that are used for production processes, [12]. The purpose of inventory is to minimize the cost of inventory. Formally, these costs refer to [13]

- Costs for the purchase of raw materials;
- Transportation costs;
- Cost for preparing a delivery request K [BGN/per request];
- Storage costs h [BGN/per resource unit].
- Losses of materials during their storage.

The first two categories of costs are not affected by the actual activities related to inventory management. In particular, material losses can be included in storage costs h. Thus, total inventory costs are calculated from the delivery request and storage costs:

$$\text{Total costs} = K + hy,$$

where y is the delivery size.

The argument of the optimization is the size y to ship. This quantity must correspond to the demand D of this resource arising from the production technology. The graphical representation of the delivery process is presented in Figure 1 and y is the level of the resource that is stored in the inventory. The level of y decreases, coming from the production process. The rate of decrease in the level y depends on the value of demand D. For this ideal configuration, the new request is executed when $y=0$ and the delivery y arrives without delay.

The period T_0, for repeating the delivery of volume y is:

$$T_0 = D/y\,. \tag{1}$$

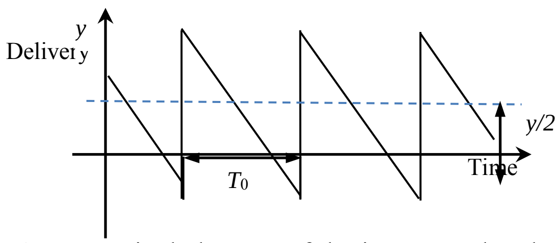

Fig. 1: Dynamical changes of the inventory level y

This formal inventory model is called the static EOQ (Economic-Order-Quantity) model, [11]. The materials holding cost for one delivery period is the product ($h \times y/2$). Here, the average value of the inventory resource is assumed to be $y/2$, Figure 1. Therefore, the costs $RC(y)$ of making one delivery in one cycle T_0 is:

$$RC(x) = \frac{K + h\left(\frac{y}{2}\right)T_0}{T_0}\,. \tag{2}$$

By substituting (1) into (2), the costs for one inventory cycle are:

$$RC(y) = \frac{K\,D}{y} + h\frac{y}{2}\,. \tag{3}$$

This relationship is used to estimate $\boldsymbol{y}^{opt}$ that minimizes $RC(y)$ or

$$y^{opt} = arg\left\{\frac{dRC(y)}{dy} = -\frac{KD}{y^2} + \frac{h}{2}\right\} \tag{4}$$

or
$$y^{opt} = \sqrt{\frac{2\,KD}{h}} \tag{5}$$

Relation (5) gives the overall result of the EOQ model for inventory management. It gives the optimal value of the delivery $\boldsymbol{y}^{opt}$ to be made in period T_0.

For the case of several resources y_j , j=1,..,m, problem (4) decomposes into several independent equations. From a practical consideration, the capacity of the inventory location is limited, which is formalized by an additional inequality, and the inventory problem becomes a nonlinear optimization one:

$$\min_{\boldsymbol{y}} \left\{RC(y) = \sum_{j=1}^{m}\left(\frac{K_jD_j}{y_j} + h_j\ \frac{x_y}{2}\right)\right\} \tag{6}$$
$$\sum_{j=1}^{m} a_j y_j \ \le d\ ,$$

where a_j is the relative space that resource y_i needs for total inventory stock d. The solution of (6) must be performed with nonlinear constraint optimization algorithms.

This research applies integration between the supply of resources and the production of goods with those resources. Each commodity needs a set of resources. The goal of the problem is to find those optimal delivery quantities for a set of resources that maximize the production of goods for predetermined requirements. The formal problem that is defined is bi-level optimization. The low-level problem maximizes the production. The upper level evaluates the optimal values of deliveries for the different resources, used on the production stage, by minimizing the inventory costs.

3 Definition of the Bi-level Optimization Problem

The production problem is included as an additional constraint in the inventory problem (6). This inclusion leads to the definition in a new optimization problem of bi-level form. The bi-level formalization allows the optimization content to simultaneously optimize two objective functions in a hierarchical order and consider constraints that are given as additional optimization problems, [14].

The production problem concerns the maximization of the objective function $F(\boldsymbol{x})$, where $\boldsymbol{x}$ is a vector with n components for the number of products $x = (x_1, \dots, x_n)$. The function $F()$ estimates the profit from producing $\boldsymbol{x}$ amount of goods. In the case of minimizing the computational load, the profit value $F()$ is used in the linear form $F=\boldsymbol{c}^T\boldsymbol{x}$ **,** where the coefficients $\boldsymbol{c}=(c_1, \dots, c_n)$ give the relative profit value for commodity x_i. For the production process, a set of inequalities is used, giving relationships between the relative level of resources $b_{j,I}$, which is necessary for the production of the commodity x_i.

$$\sum_{j=1}^{m} b_{j,i}y_j \ \ge\ x_i \ \ j = 1, \dots, m; i = 1, \dots, n \tag{7}$$

Similarly, additional inequalities can be added to the production problem, which formalizes the storage constraints of goods x_i,

$$\sum_{i=1}^{n} g_i x_i \ \le f\ \ , \tag{8}$$

where g_i is the relative space that commodity x_i occupies and f is the total space of the warehouse.

Therefore, the production problem takes the form

$$\max_{\boldsymbol{x}} \{F(\boldsymbol{x}(\boldsymbol{y}))\} \tag{9}$$

with additional constraints (7) and (8). This additional optimization problem explicitly contains in its objective function F($\boldsymbol{x}(\boldsymbol{y})$) the solutions $\boldsymbol{y}$ of the inventory optimization. The integration of the production problem with the objective function (9) and constraints (8) is performed in a bi-level optimization form. The bi-level optimization with simultaneous delivery and production solutions gives solutions to both problems (6) and (9). These problems have a common set of inequalities (7) and their arguments are denoted by $\boldsymbol{x}$ and $\boldsymbol{y}$. The analytical description of the bi-level problem is:

$$\min_{\boldsymbol{y}} \left\{RC(\boldsymbol{y}(\boldsymbol{x})) = \sum_{j=1}^{m}\left(\frac{K_jD_j}{y_j} + h_j\ \frac{y_j}{2}\right)\right\} \tag{10}$$
$$\sum_{j=1}^{m} a_j y \ \le d$$
$$\sum_{j=1}^{m} b_{j,i}x_j \ \ge\ y_i \ \ j = 1, \dots, m; i = 1, \dots, n$$

subject to
$$\max_{\boldsymbol{x}} \{F(\boldsymbol{x}(\boldsymbol{y}))\}$$
$$\sum_{i=1}^{n} g_i y_i \ \le f$$
$$\sum_{j=1}^{m} b_{j,i}x \ge\ y_i \ \ j = 1, \dots, m; i = 1, \dots, n.$$

Problem (10) gives both the optimal values of the inventory volumes $\boldsymbol{y}$, which are used for the production of goods $\boldsymbol{x}$. The latter, in turn, will give

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maximum profit from production. Therefore, bi-level optimization allows to simultaneously reduce the costs of inventory $\boldsymbol{y}$ and maximize the return from the production of a volume of goods $\boldsymbol{x}$.

4 Empirical Solution and Comparisons of Bi-level Results

Bi-level optimization is applied in the case of drawing up food standards for animals. The problem is to find the optimal content in the food of the nutritional components of protein and fiber. These components are contained in the agricultural products of corn and soybean meal, which have a percentage of nutrients. The problem of composition of the nutritional diet is defined according to the data from Table 1.

Table 1. Content of the Agricultural Products

Content	Proteins, x_1	Fibers, x_2
Corn, y_1	0.9	0.03
Soybean meal, y_2	0.01	0.8

To prepare the food for the animals, it is necessary to prepare a food mixture with the corresponding protein and fiber content $\boldsymbol{x}=(x_1, x_2)$. They should be taken from the corn and soybean meal products $\boldsymbol{y}=(y_1, y_2)$. Appropriate quantities of $\boldsymbol{y}$ are accordingly required. The production-delivery problem is that the quantities of corn and soybean meal are delivered with minimum inventory costs and that they satisfy the required nutrient content ratios. Following the form of problem (10), it is necessary to estimate the initial delivery cost $\boldsymbol{K}$, the holding costs $\boldsymbol{h}$ and the demand value $\boldsymbol{D}$. The commonly used inequalities (7) for the arguments $\boldsymbol{y}$ and $\boldsymbol{x}$ using the data in Table 1 will be of the form:

$$\boldsymbol{A}\,\boldsymbol{y} \geq \boldsymbol{x}, \tag{11}$$

where $\boldsymbol{A}=[0.9\ 0.03;\ 0.01\ 0.8]$ from Table 1.

For the storage of agricultural produce, the warehouse has limitations following the ratio (8) or

$$\boldsymbol{A1}\boldsymbol{y} \leq b(1)$$

where $\boldsymbol{A1}=[0.4;\ 0.6]$, $b(1)=2$.

The coefficients of the problem $\boldsymbol{K}=[50;\ 10]$, $\boldsymbol{h}=[25;\ 1]$, which corresponds to the delivery and holding of corn have greater costs. The demand value $\boldsymbol{D}$ results from the product-nutrition relationships given in Table 1. Corn demand is estimated by the coefficients for protein and fiber as a column sum or

$$D(1) = 0.9 + 0.01 = 0.91\ .$$

Accordingly, the demand for soybean meal is $D(2) = 0.03 + 0.8 = 0.83$ or a total $\boldsymbol{D}=[0.93;\ 0.83]$.

The bi-level optimization problem is aimed at maximizing the protein and fiber content $\boldsymbol{x}$ and minimizing the delivery costs of corn and soybean meal. To reduce the computational workload, the objective function of the production problem is defined in quadratic form as:

$$\min_{\boldsymbol{x}} \{ -\boldsymbol{x}^T\, \boldsymbol{Q2}\, \boldsymbol{x} + \boldsymbol{y}^T\, \boldsymbol{Q1}\, \boldsymbol{y}) \,.$$

The matrices $\boldsymbol{Q1}$ and $\boldsymbol{Q2}$ are taken from considerations regarding the relative weights of the various components $\boldsymbol{x}$ and $\boldsymbol{y}$ for the production and delivery processes. For the present case, the numerical values of the matrices are:

$$\boldsymbol{Q1}=[1\ 0;\ 0\ 1];\quad \boldsymbol{Q2}=[19\ 0;\ 0\ 40].$$

The food content constraint is given as:

$$\boldsymbol{A2}\,\boldsymbol{x} \leq b(2),$$

where $\boldsymbol{A2}=[0.8;\ 0.1]$, $b(2) = 2$.

For practical reasons, additional constraints are added for the upper and lower bounds of agricultural products $\boldsymbol{x}$, $\boldsymbol{x} \geq \boldsymbol{LB}$. $\boldsymbol{LB}$ values were applied to check the sensitivity of the solutions to the amount of nutrient content for different possible diets for the animals. The numerical form of the bi-level problem becomes:

$$\min_{\boldsymbol{y}} \left\{ \frac{K(1)D(1)}{y(1)} + h(1)\,\frac{y(1)}{2} + (\frac{K(2)D(1)}{y(2)} + h(2)\,\frac{y(2)}{2} \right\}$$

$$\begin{aligned} &\boldsymbol{Ay} \geq \boldsymbol{x} \\ &\boldsymbol{A1}\,\boldsymbol{y} \leq b(1)\,, \end{aligned}$$

where (12)

$$\begin{aligned} &\min_{\boldsymbol{x}} \{ -\boldsymbol{x}^T\, \boldsymbol{Q2}\, \boldsymbol{x} + \boldsymbol{y}^T\, \boldsymbol{Q1}\, \boldsymbol{y}) \\ &\boldsymbol{Ay} \geq \boldsymbol{x} \\ &\boldsymbol{A2}\,\boldsymbol{x} \leq b(2) \\ &x \geq \boldsymbol{LB}\,. \end{aligned}$$

The solution of (12) is evaluated with $\boldsymbol{LB}=[0.14;0.14]$ for the low values for protein and fiber and the storage capacity constraints as $\boldsymbol{b}=[4;4]$ are:

$$\boldsymbol{y}^{\text{opt}}=[1.45;\ 3.06]\ ,\ \boldsymbol{x}^{\boldsymbol{opt}}=[1.4;\ 2.46].$$

The delivery costs of corn and soybean meal estimated by the objective function of (10) are

$$F(\boldsymbol{y}^{opt}(\boldsymbol{x}^{opt})) = 73.41.$$

The gain from protein and fiber utilization estimated by the objective function (9) is:

$$f(\boldsymbol{x}^{opt}(\boldsymbol{y}^{opt}))=\boldsymbol{x}^{Topt} \quad \boldsymbol{Q2}\ \boldsymbol{x}^{opt}=280.29.$$

The total cost benefit $P(\boldsymbol{y}^{opt}(\boldsymbol{x}^{opt}))$ of integrating delivery and production is:

$$P(\boldsymbol{y}^{opt}(\boldsymbol{x}^{opt}))= f(\boldsymbol{x}^{opt}(\boldsymbol{y}^{opt})) - F(\boldsymbol{y}^{opt}(\boldsymbol{x}^{opt}))=206.88.$$

The solutions of the bi-level problem are compared with the case of problem (6), which gives optimal solutions only for the delivery case without considering integration with production. The delivery decision is indicated by:

$$\boldsymbol{yn}^{opt}= [1.34 \ \ 2.88] .$$

With this agricultural resource for corn and soybean meal, the corresponding amounts of protein and fiber $\boldsymbol{yin}$ can be calculated according to the relationships between $\boldsymbol{x}$ and $\boldsymbol{y}$, defined by constraints (12). This gives:

$$\boldsymbol{xin}^{opt} = \boldsymbol{A}\ \boldsymbol{yn}^{opt} = [1.30; \ 2.31].$$

The delivery cost is calculated by the objective function of (6) $Fn(\boldsymbol{yn}^{opt}(\boldsymbol{xin}^{opt})) = 73.21$.

The total profit is calculated using the estimates for $\boldsymbol{xin}$ or

$$fn(\boldsymbol{xin}^{opt}(\boldsymbol{yn}^{opt}))= \boldsymbol{xin}^{Topt}\ \boldsymbol{Q2}\ \boldsymbol{xin}^{opt}=247.11.$$

Finally, the overall benefit of this delivery model by using the virtual integration of delivery and production gives:

$$Pn\ (\boldsymbol{yn}^{opt}(\boldsymbol{xin}^{opt}))=$$
$$=fn(\boldsymbol{xin}^{opt}(\boldsymbol{yn}^{opt})) - Fn(\boldsymbol{yn}^{opt}(\boldsymbol{xin}^{opt}))=173.89.$$

Obviously, the bi-level problem (12) and the classical inventory EOQ problem (6) give close results for deliveries. However the gain in nutrient utilization in food is significantly different with an advantage over the bi-level problem. Accordingly, the total benefit, compared to the bi-level and classical EOQ problem, gives superiority to the derived bi-level problem with integration of delivery and production requirements.

5 Conclusion

This study formalizes the process of integrating the delivery of resources and their use in the production chain as a bi-level optimization problem. The inventory problem contains all the requirements that are necessary for optimal inventory management. Additional constraints are added in the inventory problem, which means maximizing production returns. Therefore, the inventory problem is complicated by including in the set of constraints an additional optimization problem. This formalization takes the form of a bi-level problem. The benefit of such a formalization is that it takes into account the two objective functions of minimizing inventory costs and maximizing production returns. This new interpretation of the inventory problem has advantages over the single inventory problem. The bi-level problem allows for reducing the balance of raw materials from the inventory. The article defines a production problem in the case of preparing a portion of animal feed with the required content of nutritional components. Production problems have the meaning of correctly defining diets according to food content. Both inventory and production problems are integrated according to the relationship of the content of nutritional components in agricultural foods. The classic EOQ delivery problem does not take into account requirements regarding food components. Thus, its results do not further influence the overall benefit of optimizing animal nutrition.

The bi-level problem integrates both inventory and production tasks. Its decisions have demonstrated significant positive results from an economic point of view. The latter is quantified by the difference between the profit from production and the cost of inventories.

The bi-level problem is solved with real data taken from the management of a dairy farm.

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- Todor Stoilov: conceptualized the study; methodology development; creating models.
- Krasimira Stoilova: formal data analysis, software development.
- Ludmil Iliev: technological assessment.

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Conflict of Interest

The authors have no conflicts of interest to declare.