Double Moving Average Control Chart for Time Series Data with Poisson INARCH(1)

SUGANYA PHANTU[1], YUPAPORN AREEPONG[2], SAOWANIT SUKPARUNGSEE[2,*]
[1]Faulty of Science, Energy, and Environment,
King's Mongkut University of Technology North Bangkok,
Rayong 21120,
THAILAND

[2]Department of Applied Statistics, Faculty of Applied Science,
King Mongkut's University of Technology, North Bangkok,
Bangkok,
THAILAND

[*]*Corresponding Author*

Abstract: - The objectives of this research are to find the explicit formulas of the average run length (ARL) of a double moving average (DMA) control chart for first-order integer-valued autoregressive conditional heteroscedasticity (INARCH1))) of Poisson count data. In addition, the numerical results obtained from the proposed explicit formulas are compared with those obtained from Monte Carlo simulations (MC) for the Poisson INARCH(1) counting process. An out-of-control ARL (ARL_1) is the criteria for measuring the performance of control charts. The numerical results found that the values of both ARL0 and ARL_1 obtained from explicit formulas agree with the numerical results obtained from the Monte Carlo simulation (MC), but the latter is very time-consuming.

Key-Words: - Poison Process, Average Run Length, Explicit Formulas, Optimal Design, Volatility, Time series.

Received: November 14, 2023. Revised: January 7, 2024. Accepted: January 29, 2024. Published: February 23, 2024.

1 Introduction

Statistical Quality Control (SQC) is critically important in various industries, [1], and business, [2], sectors. It plays a crucial role in ensuring that products and processes meet specified standards and customer expectations. Here are some reasons highlighting the importance of statistical quality control: consistency and uniformity, defect reduction, cost reduction, customer satisfaction, process improvement, and early detection of issues. In summary, Statistical Quality Control is a fundamental aspect of quality management in various industries. It enables organizations to produce high-quality products, reduce costs, meet customer expectations, and stay competitive in the market. The SQC relies on several key assumptions to effectively apply statistical methods and tools to monitor and control processes. Many statistical methods in SQC assume that the data follows a normal distribution. This assumption is particularly important when using control charts and other statistical tools. While normality is not always strictly required, deviations from normal distribution may affect the accuracy and interpretation of results. It's important for practitioners to be aware of these assumptions and to assess whether they hold in a particular context. Deviations from these assumptions may necessitate adjustments to the statistical approach or additional considerations in the interpretation of results.

In real-world applications, time-series data which may be found in a variety of fields including communication engineering, epidemiology, [3], monetary economics [4], financial and insurance [5], environmental science [6], and so on—may be tied to time. The number of incidents and accident rates, multiple crimes, the identification of communication errors, the number of customers using the internet in an hour, the volume of business phone calls, the number of customers from different organizations who have used the service in the previous year, and the amount of time spent waiting for the plane to take off are other scenarios that may give rise to it. Consequently, utilizing the thinning operator concepts described by, [7], [8], [9], several researchers have created a model appropriate for these data, dubbed the first order of Integer-valued Autoregressive (INAR(1)), proposed by, [10], [11]. They took advantage of a discrete distribution time series model created by, [12].

The discrete probability distribution known as the Poisson distribution, which represents the number of occurrences that take place within a specific time or location, is especially well-suited for describing counting operations. A sort of time series model intended for count data that displays conditional heteroscedasticity and autocorrelation is an integer-valued autoregressive conditional heteroscedasticity (INARCH) model with a Poisson process. The term "autoregressive conditional heteroscedasticity" (ARCH) in this context denotes that the conditional variance of the process is not constant but rather fluctuates with time, and "integer-valued" refers to the type of data. It can also happen in other circumstances, like the quantity and rate of accidents, multiple crimes, the identification of communication breakdowns, the number of users accessing the internet in a single hour, the number of users from different organizations who have used the service in the previous year, and the amount of time spent waiting for an aircraft to take off from the airport. Consequently, utilizing the thinning operator concepts described by, [7], [8], [9], several researchers have created a model appropriate for these data, dubbed the first order of Integer-valued Autoregressive (INAR(1)), proposed by, [10], [11]. The first-order numerical correlation Poisson model with unstable variance Poisson Integer-valued Autoregressive Conditional Heteroscedasticity (INARCH(1)) was developed from many research fields such as application use in pharmaceutical science by, [13], infectious rates by, [14], [15], studied the number of insurance claims from insurance companies, [16], and applied to the queueing of internet access claims data, [17].

With its Poisson distribution and constant variance, the Poisson INARCH(1) model bears a resemblance to the AR(1) model. By multiplying the random operator by a numerical random variable to construct an INAR(1) and INARCH(1) by, [18], [19], [20], [21], [22], these use a random operator known as the Binomial Thinning Operator to assist in the creation of an integer value model.

Counting data has been the subject of numerous studies, many of which have taken dependent data characteristics and variable variance into account. To effectively identify the nature of this type of data, such as when it is highly correlated, they have created a control chart. The autocorrelation model for Poisson counting first-order unstable variance (INARCH(1)) is a model that works well with this kind of data. Its control chart performance has been examined by several researchers using various control charts examined the Poisson INAR(1) model's cumulative combined control chart, [23], [24], examined the Poisson INAR(1) model of the counting data's two-sided cumulative combined control chart, and [25], examined the correlated Poisson model's quality control.

The expected number of samples or observations collected from a process before a signal indicating a shift or change in the process mean or variability is detected is referred to as the average run length (ARL) in statistical process control. A statistical process control chart is a graphical tool used to monitor and control a process over time. The ARL evaluates the performance of these charts. It can be broken down into two categories: processes that are under control (in-control processes) are represented by the symbol ARL_0, and processes that are not under control are represented by the symbol ARL_1. In mathematical notation, they can be expressed as follows:

$$ARL_0 \equiv E_\infty(\tau) = T$$

and

$$\text{Minimize } ARL_1 \equiv E_1(\tau - \theta + 1)$$

where $E_\theta(.)$ is the expected time.

τ is the first passage time (stopping times)

T is a constant (usually given T =370)

θ is the change-point time process has changed from $F(x,\alpha_1)$ to $F(x,\alpha_0)$.

When comparing the capabilities of various control charts, numerical techniques for calculating the ARLs are frequently employed. There are multiple methods for figuring out this number. Its benefits and drawbacks are as follows: conventional techniques, like Monte Carlo Simulation (MC), are frequently employed. It is frequently used to confirm accuracy in comparison to alternative methods, but processing times may occasionally be lengthy. The inverse of a matrix is found using the Markov chain approach (MCA), [26], [27], but the theory of convergence properties still needs to be backed up. To estimate, the Numerical Integral Equations (NIE) method makes use of sophisticated mathematical computations, [28]. The Martingale Approach is a contemporary technique that is quick, simple to calculate, and convenient, but it could take some time for the simulation method to verify accuracy, [29], [30].

Small changes in the mean or variability can be more easily detected with a control chart that has a short ARL since it is more responsive to process variations. A control chart with a longer ARL, on the other hand, is less sensitive and might be more suitable for processes that naturally fluctuate.

Therefore, the purpose of this research is to use moving average control charts in conjunction with the Poisson INARCH(1) model to design an appropriate control for detecting the process's rapid dynamics. to identify abrupt average shifts in the manufacturing process. By comparing the detection efficiency of the MA control chart with the DMA chart while taking the average run length into account, and by verifying the accuracy of the results obtained from the formula with the simulation results.

2 Research Methodology

The Poisson INARCH(1) model, moving average control charts, double-moving average control charts, and their properties are briefly reviewed in this section.

2.1 Binomial Thinning Operator

The probabilistic operation of binomial thinning is introduced by, [31]. If X is a discrete random variable with range $N_0 = \{0,1,...\}$ and if $\alpha \in [0,1]$, then the random variable $\alpha \circ N := \sum_{i=1}^{N} X_i$ is said to arise from X by binomial thinning, and the X_i are referred to as the counting series. The thinning operation, when operated on by a parameter proven to be an adequate alternative to scalar multiplication, is defined as

$$\alpha \circ N = \sum_{i=1}^{N} X_i$$

where X_i are independent and identically distributed (i.i.d.) Bernoulli random variables with success probability α. The operator is a random operator, and the random variable $\alpha \circ N$ has a binomial distribution with parameters N, α and counts the number of survivors from the count N remaining after thinning.

The variance and expectations of $\alpha \circ N$ can be obtained with ease by using the following well-known rules for a conditional moment

$$E[\alpha \circ N] = \alpha E[N]$$

and

$$V[\alpha \circ N] = \alpha^2 V[N] + \alpha(1-\alpha)E[N].$$

2.2 Poisson INARCH (1) Model

A particular kind of time series model called an INARCH (1) process is used to model the number of events that transpire within a given time interval to explain the behavior of a counting process. Specifically, an integrated autoregressive conditional heteroskedasticity model that assumes the counting process is driven by its past values and a stochastic component is called an INARCH(1) Poisson counting process. The following details relate to the INARCH(1) process:

$$N_t = \alpha \circ N_{t-1} + \varepsilon_t$$

where $\circ$ is a random operator and at time t is independent of (ε_t) and $(N_s)_{S<t}$

ε_t is an innovation-independent counts of $(N_s)_{S<t}$ the distribution $Pois(\beta + \alpha \cdot N_{t-1})$

N_t is counting observations at time t.

If the initial count N_0 is distributed as $Pois(\frac{\beta}{1-\alpha})$ then N_t is stationary and distributed as $Pois(\frac{\beta}{1-\alpha})$. According to the above situation, it can be modeled as a Poisson INARCH(1) model. The expectation and variance of the Poisson INARCH(1) model are:

$$E[N_t] = \frac{\beta}{1-\alpha}$$

and

$$V[N_t] = \frac{\beta}{(1-\alpha)(1-\alpha^2)}.$$

2.3 Moving Average Control Chart for Poisson INARCH(1) Model

A time-varying control chart with unequal weights, known as a moving average control chart (MA chart), [32], was developed to count variables such as the quantity of nonconformities in a product's inspection unit. Assume that discrete observations are obtained from a sequence of identically independent distributions and the Poisson INARCH(1) model. The definition of the width at a time moving average is:

$$MA_t = \begin{cases} \frac{1}{i}\sum_{j=1}^{t} N_j & ; i < k \\ \frac{1}{k}\sum_{j=t-k+1}^{t} N_j & ; i \ge k \end{cases}.$$

The expectation of the MA statistics for the Poisson INARCH(1) model when $i<k$ and $i \ge k$ is:

$$E(MA_t) = \frac{\beta}{1-\alpha}.$$

The variance of the MA statistics for the Poisson INARCH(1) model for both cases of $i<k$ and $i \ge k$ is:

$$Var(MA_i) = \begin{cases} \frac{\beta}{i(1-\alpha)(1-\alpha^2)} & , i \le k \\ \frac{\beta}{k(1-\alpha)(1-\alpha^2)} & , i > k \end{cases}.$$

The upper and lower control limits of the MA statistics are given as follows.

$$= \begin{cases} \frac{\beta}{1-\alpha} \pm H_1\sqrt{\frac{\beta}{i(1-\alpha)(1-\alpha^2)}} & , i \le k \\ \frac{\beta}{1-\alpha} \pm H_1\sqrt{\frac{\beta}{k(1-\alpha)(1-\alpha^2)}} & , i > k \end{cases}$$

where H_1 refers to a coefficient of control limit of the MA chart.

2.4 Double Moving Average Control Chart for Poisson INARCH(1) Model

A double-moving average control chart (DMA chart) was proposed by, [33]. The observations of DMA statistics are the collected double moving average of the MA statistics. The DMA of span k at the time t is defined as:

$$DMA_i = \begin{cases} \frac{MA_i + MA_{i-1} + MA_{i-2} + \ldots}{i} & ; i \le k \\ \frac{MA_i + MA_{i-1} + \ldots + MA_{i-w+1}}{k} & ; k < i < 2k-1 \\ \frac{MA_i + MA_{i-1} + \ldots + MA_{i-w+1}}{k} & ; i \ge 2k-1 \end{cases} \quad (1)$$

where MA_i refers to the statistic of the MA chart. It is a time-weighted moving control chart based on a simple, unweighted moving average. Assume N_1, N_2, ... are obtained from a Poisson INARCH(1) process. The MA statistic of span w at a time i defined as, [34]

$$MA_i = \frac{X_i + X_{i-1} + \ldots + X_{i-k+1}}{k} \quad ; \text{for } i \ge k.$$

For the period $i<k$ we do not have k measurements to compute a moving average of span k. For these periods, the average of all measurements up to periods i defines the MA. The mean based on an in-control process of the DMA chart are:

$$E(DMA_i) = \frac{\beta}{1-\alpha} \quad (2)$$

and variance based on an in-control process of the DMA chart are:

$$Var(DMA_i) = \begin{cases} \sum_{j=1}^{i}\frac{1}{j}\frac{\beta}{i^2(1-\alpha)(1-\alpha^2)} & ; i \le k \\ \sum_{j=i-k+1}^{k-1}\frac{1}{j} + (j-k+1)\left(\frac{1}{k}\right)\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)} & ; k < i < 2k-1 \\ \frac{\beta}{k^2(1-\alpha)(1-\alpha^2)} & ; i \ge 2k-1. \end{cases} \quad (3)$$

From Eq. (2) and (3), the upper and lower control limit of the DMA chart can be established as follows

$$
=\begin{cases} \dfrac{\beta}{1-\alpha} \pm H_2 \sqrt{\displaystyle\sum_{j=1}^{i} \frac{1}{j} \frac{\beta}{i^2(1-\alpha)(1-\alpha^2)}} & ; i \le k \\ \dfrac{\beta}{1-\alpha} \pm H_2 \sqrt{\displaystyle\sum_{j=i-k+1}^{k-1} \frac{1}{j} + (j-k+1)\left(\frac{1}{k}\right) \frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}} & ; k < i < 2k-1 \\ \dfrac{\beta}{1-\alpha} \pm H_2 \sqrt{\dfrac{\beta}{k^2(1-\alpha)(1-\alpha^2)}} & ; i \ge 2k-1 \end{cases} \tag{4}
$$

when H_2 is a coefficient of control limit based on a desired in-control ARL$_0$. The Poisson INARCH(1) model of the DMA chart will signal the out-of-control situation when $DMA_i < LCL$ or $DMA_i > UCL.$

3 An Explicit Formulas for Average Run Length of DMA Control Chart

In quality control and process monitoring, Average Run Length (ARL) is frequently used to assess how well a control chart detects shifts or modifications in a process. Assuming the process is under control, the ARL value indicates the anticipated number of observations before a control chart indicating a process shift. Explicit formulas, integral equations, Markov chain analysis, simulation, and other mathematical techniques can all be used to calculate the ARL value. The results of these techniques can then be compared with Monte Carlo simulation results. The latter, which is employed in situations where ARL's explicit formulas or closed-form formulas are unavailable, takes a very long time.

Based on the central limit theorem (CLT), this section presents the derivative analytical ARL of the DMA chart for the Poisson INARCH(1) observations. The following formula can be used to determine the DMA chart's average run length:

Let $ARL = n,$ then

$$
\begin{aligned}
\frac{1}{ARL} &\cong \frac{1}{n} P(\text{out of control signal at time } i \le k) \\
&+ \frac{1}{n} P(\text{out of control signal at time } k < i < 2k-1) \\
&+ \frac{n-(2k-2)}{n} P(\text{out of control signal at time } i \ge 2k-1).
\end{aligned} \tag{5}
$$

According to Eq. (5), the DMA statistics in terms of out-of-control signals at time i state are replaced as follows:

$$
\begin{aligned}
&\cong \frac{1}{n} \sum_{i=1}^{k} \left(P\left(M_i > UCL_{i \le k}\right) + P\left(M_i < LCL_{i \le k}\right)\right) \\
&+ \frac{1}{n} \sum_{j=i-k+1}^{2k-2} \left(P\left(M_j > UCL_{k<i<2k-1}\right) + P\left(M_j < LCL_{k<i<2k-1}\right)\right) \\
&+ \frac{n-(2k-2)}{n} \left(P\left(M_i > UCL_{i \ge 2k-1}\right) + P\left(M_i < LCL_{i \ge 2k-1}\right)\right).
\end{aligned} \tag{6}
$$

Then, substitute the upper and lower control limits of DMA statistics from Eq. (1) into Eq. (6), which can be rewritten.

$$
\begin{aligned}
&\cong \frac{1}{n} \sum_{i=1}^{k} \left(P\left(\frac{\sum_{j=1}^{i} MA_j}{i} > \frac{\beta}{1-\alpha} + H_2 \sqrt{\frac{\beta}{i^2(1-\alpha)(1-\alpha^2)} \sum_{j=1}^{i} \frac{1}{j}} \right) \right. \\
&\left. + P\left(\frac{\sum_{j=1}^{i} MA_j}{i} < \frac{\beta}{1-\alpha} - H_2 \sqrt{\frac{\beta}{i^2(1-\alpha)(1-\alpha^2)} \sum_{j=1}^{i} \frac{1}{j}} \right) \right) \\
&+ \frac{1}{n} \sum_{j=i-k+1}^{2k-2} \left(P\left(\frac{\sum_{j=i-k+1}^{i} MA_j}{k} > \frac{\beta}{1-\alpha} + H_2 \sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)} \sum_{j=i-k+1}^{k-1} \frac{1}{j} + (j-k+1)\left(\frac{1}{k}\right)} \right) \right. \\
&\left. + P\left(\frac{\sum_{j=i-k+1}^{i} MA_j}{k} < \frac{\beta}{1-\alpha} - H \sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)} \sum_{j=i-k+1}^{k-1} \frac{1}{j} + (j-k+1)\left(\frac{1}{k}\right)} \right) \right) \\
&+ \frac{n-(2k-2)}{n} \left(P\left(\frac{\sum_{j=i-k+1}^{i} MA_j}{k} > \frac{\beta}{1-\alpha} + H_2 \sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}} \right) \right. \\
&\left. + P\left(\frac{\sum_{j=i-k+1}^{i} MA_j}{k} > \frac{\beta}{1-\alpha} - H_2 \sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}} \right) \right).
\end{aligned}
$$

(7)

The central limit theorem is used to derive the explicit formulas. Then, Eq. (7) can be rewritten as:

$$
\frac{1}{ARL} = \frac{1}{n} \left\{ \sum_{t=1}^{k} \left[P\left(Z_A > \frac{UCL_{t \le k} - \dfrac{\beta}{1-\alpha}}{\sqrt{\displaystyle\sum_{j=1}^{i} \frac{1}{j} \frac{\beta}{i^2(1-\alpha)(1-\alpha^2)}}} \right) \right. \right.
$$

$$+P\left(Z_A < \frac{LCL_{t\leq k} - \frac{\beta}{1-\alpha}}{\sqrt{\sum_{j=1}^{i}\frac{1}{j}\frac{\beta}{i^2(1-\alpha)(1-\alpha^2)}}}\right)\Bigg]\Bigg]\Bigg\}$$

$$+\frac{1}{n}\sum_{j=i-k+1}^{2k-2}\left[P\left(Z_B > \frac{UCL_{k<i<2k-1} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}\sum_{j=i-k+1}^{k-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}}\right)\right.$$

$$\left.+P\left(Z_B < \frac{LCL_{k<i<2k-1} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}\sum_{j=i-k+1}^{k-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}}\right)\right]$$

$$+\left[\frac{n-(2k-2)}{n}\right]\left\{P\left(Z_C > \frac{UCL_{i\geq 2k-1} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}}}\right)\right.$$

$$\left.+P\left(Z_C < \frac{LCL_{i\geq 2k-1} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}}}\right)\right\}. \tag{8}$$

Next, the statistics of the DMA chart in Eq. (8) are transformed to be standardized, then assume that.

$$Z_A = \frac{\frac{\sum_{j=1}^{i} MA_j}{i} - \frac{\beta}{1-\alpha}}{\sqrt{\sum_{j=1}^{i}\frac{1}{j}\frac{\beta}{i^2(1-\alpha)(1-\alpha^2)}}},$$

$$Z_B = \frac{\frac{\sum_{j=1}^{i} MA_j}{k} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}}$$

and $Z_C = \dfrac{\frac{\sum_{j=1}^{i} MA_j}{k} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}}}.$

According to Eq. (8), let

$$A = \sum_{t=1}^{k}\left[P\left(Z_A > \frac{UCL_{i\leq k} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{i^2(1-\alpha)(1-\alpha^2)}}}\right) + P\left(Z_A < \frac{LCL_{i\leq k} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{i^2(1-\alpha)(1-\alpha^2)}}}\right)\right]$$

$$B = \frac{1}{n}\sum_{j=i-k+1}^{2k-2}\left(P\left(Z_B > \frac{UCL_{k<i<2k-1} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}\sum_{j=i-k+1}^{k-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}}\right) + P\left(Z_B < \frac{LCL_{k<i<2k-1} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}\sum_{j=i-k+1}^{k-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}}\right)\right)$$

and

$$C = P\left(Z_C > \frac{UCL_{i\geq 2k-1} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}}}\right) + P\left(Z_C < \frac{LCL_{i\geq 2k-1} - \frac{\beta}{1-\alpha}}{\sqrt{\frac{\beta}{k^2(1-\alpha)(1-\alpha^2)}}}\right).$$

Then, the explicit formulas of ARL_0 and ARL_1 for the DMA chart are rewritten by substituting *A*, *B*, and *C* into Eq. (8).

$$\frac{1}{n} \cong \frac{1}{n}(A) + \frac{1}{n}(B) + \frac{n-(2k-2)}{n}(C)$$

$$n \cong \frac{(1-A-B)}{C} + (2k-2).$$

As we give $ARL = n$, then

$$ARL \cong [(1-A)-B]C^{-1} + (2k-2). \tag{9}$$

From Eq. (9), proving the explicit formula of the DMA chart can be divided into two cases:

Proposition I: Explicit formulas of ARL_0 for the DMA chart.

$$ARL_0 \cong \left\{ \begin{array}{l} 1-\sum_{i=1}^{w}\left(P\left(Z_A > \dfrac{\frac{\beta_0}{1-\alpha_0}+H_2\sqrt{\sum_{j=1}^{i}\frac{1}{j}\frac{\beta_0}{i^2(1-\alpha_0)(1-\alpha_0^2)}}-\frac{\beta_0}{1-\alpha_0}}{\sqrt{\sum_{j=1}^{i}\frac{1}{j}\frac{\beta_0}{i^2(1-\alpha_0)(1-\alpha_0^2)}}} \right) \right. \\ \left. +P\left(Z_A < \dfrac{\frac{\beta_0}{1-\alpha_0}-H_2\sqrt{\sum_{j=1}^{i}\frac{1}{j}\frac{\beta_0}{i^2(1-\alpha_0)(1-\alpha_0^2)}}-\frac{\beta_0}{1-\alpha_0}}{\sqrt{\sum_{j=1}^{i}\frac{1}{j}\frac{\beta_0}{i^2(1-\alpha_0)(1-\alpha_0^2)}}} \right) \right) \\ -\sum_{j=i-w+1}^{2w-2}\left(P\left(Z > \dfrac{\frac{\beta_0}{1-\alpha_0}+H_2\sqrt{\frac{\beta_2}{k^2(1-\alpha_0)(1-\alpha_0^2)}\sum_{j=i-k+1}^{k-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}-\frac{\beta_0}{1-\alpha_0}}{\sqrt{\frac{\beta_0}{k^2(1-\alpha_0)(1-\alpha_0^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}} \right) \right. \\ \left. +P\left(Z < \dfrac{\frac{\beta_0}{1-\alpha_0}-H_2\sqrt{\frac{\beta_0}{k^2(1-\alpha_0)(1-\alpha_0^2)}\sum_{j=i-k+1}^{k-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}-\frac{\beta_0}{1-\alpha_0}}{\sqrt{\frac{\beta_0}{k^2(1-\alpha_0)(1-\alpha_0^2)}\sum_{j=i-k+1}^{k-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}} \right) \right) \end{array} \right\}$$

$$\times \left\{ \begin{array}{l} P\left(Z_C > \dfrac{\frac{\beta_0}{1-\alpha_0}+H_2\sqrt{\frac{\beta_0}{k^2(1-\alpha_0)(1-\alpha_0^2)}}-\frac{\beta_0}{1-\alpha_0}}{\sqrt{\frac{\beta_0}{k^2(1-\alpha_0)(1-\alpha_0^2)}}} \right) \\ +P\left(Z_C < \dfrac{\frac{\beta_0}{1-\alpha_0}-H_2\sqrt{\frac{\beta_0}{k^2(1-\alpha_0)(1-\alpha_0^2)}}-\frac{\beta_0}{1-\alpha_0}}{\sqrt{\frac{\beta_0}{k^2(1-\alpha_0)(1-\alpha_0^2)}}} \right) \end{array} \right\}^{-1} +(2w-2) \quad (10)$$

Proposition II: Explicit formulas of ARL_1 for the DMA chart.

$$ARL_1 \cong \left\{ \begin{array}{l} 1-\sum_{i=1}^{k}\left(P\left(Z_A > \dfrac{\frac{\beta_0}{1-\alpha_0}+H_2\sqrt{\frac{\beta_0}{i^2(1-\alpha_0)(1-\alpha_0^2)}}-\frac{\beta_1}{1-\alpha_1}}{\sqrt{\frac{\beta_1}{i^2(1-\alpha_1)(1-\alpha_1^2)}}} \right) \right. \\ \left. +P\left(Z_A < \dfrac{\frac{\beta_0}{1-\alpha_0}-H_2\sqrt{\frac{\beta_0}{i^2(1-\alpha_0)(1-\alpha_0^2)}}-\frac{\beta_1}{1-\alpha_1}}{\sqrt{\frac{\beta_1}{t^2(1-\alpha_1)(1-\alpha_1^2)}}} \right) \right) \\ -\sum_{j=i-k+1}^{2k-2}\left(P\left(Z_B > \dfrac{\frac{\beta_0}{1-\alpha_0}+H_2\sqrt{\frac{\beta_0}{w^2(1-\alpha_0)(1-\alpha_0^2)}\sum_{j=i-k+1}^{k-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}-\frac{\beta_1}{1-\alpha_1}}{\sqrt{\frac{\beta_1}{w^2(1-\alpha_1)(1-\alpha_1^2)}\sum_{j=i-k+1}^{k-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}} \right) \right. \\ \left. +P\left(Z_B < \dfrac{\frac{\beta_0}{1-\alpha_0}-H_2\sqrt{\frac{\beta_0}{w^2(1-\alpha_0)(1-\alpha_0^2)}\sum_{j=i-k+1}^{k-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}-\frac{\beta_1}{1-\alpha_1}}{\sqrt{\frac{\beta_1}{w^2(1-\alpha_1)(1-\alpha_1^2)}\sum_{j=i-k+1}^{k-1}\frac{1}{j}+(j-k+1)\left(\frac{1}{k}\right)}} \right) \right) \end{array} \right\}$$

$$\times \left\{ \begin{array}{l} P\left(Z_C > \dfrac{\frac{\beta_0}{1-\alpha_0}+H_2\sqrt{\frac{\beta_0}{k^2(1-\alpha_0)(1-\alpha_0^2)}}-\frac{\beta_1}{1-\alpha_1}}{\sqrt{\frac{\beta_1}{k^2(1-\alpha_1)(1-\alpha_1^2)}}} \right) \\ +P\left(Z_C < \dfrac{\frac{\beta_0}{1-\alpha_0}-H_2\sqrt{\frac{\beta_0}{k^2(1-\alpha_0)(1-\alpha_0^2)}}-\frac{\beta_1}{1-\alpha_1}}{\sqrt{\frac{\beta_1}{k^2(1-\alpha_1)(1-\alpha_1^2)}}} \right) \end{array} \right\}^{-1} +(2k-2) \quad (11)$$

4 Numerical Results

This section presents the performance results of the DMA chart for the INARCH(1) model. It is divided into two parts: Part 1 presents the average run length determination of the DMA chart, and Part 2 presents the performance comparison control chart for actual data.

4.1 Average Run Length of DMA Chart

The study compares the accuracy and precision of the results calculated from the explicit formula from Eqs. (10) and (11). This research presents the results obtained from the Monte Carlo simulation method. The in-control parameter was given $\beta_0 = 1,\ \alpha_0 = 0.1$, and ARL_0 = 370. The magnitude of changes of β_0 is δ_1 which set value equal to 0.1, 0.2,…,1. The magnitude of the changes of α_0 is δ_2 which set value equal to 0.1, 0.2, …,1. The period for finding the moving average (k) equals 2, 5, 10, 15, and 20; for the MC method, the number of iteration cycles is 50,000 replications. The numerical results are shown in Table 1 and Table 2 for the given β change and Table 3 and Table 4 for the given α change as follows.

Table 1. Average run length of DMA chart for the INARCH(1) model where $\beta_0 = 1,\ \alpha_0 = 0.1$ and given β change

δ_1	k				
	2	5	10	15	20
0.0	370.398* 371.560** (1.455)	370.398 370.589 (1.513)	370.398 370.677 (1.643)	370.398 371.294 (1.545)	370.398 370.499 (1.451)
0.1	318.871 318.566 (0.956)	179.137 179.556 (0.942)	67.596 67.659 (0.933)	42.582 42.355 (0.956)	*41.641* *41.328* (0.973)
0.2	236.985 236.454 (0.853)	71.536 71.298 (0.835)	*25.468* *25.436* (0.867)	27.526 27.483 (0.862)	35.219 35.748 (0.845)
0.3	173.99 173.452 (0.760)	36.633 36.806 (0.743)	*18.907* *18.706* (0.732)	25.343 25.398 (0.763)	32.345 32.472 (0.739)
0.4	131.593 131.839 (0.678)	23.070 23.547 (0.665)	*17.016* *17.430* (0.645)	23.726 23.094 (0.654)	28.357 28.445 (0.637)
0.5	103.213 103.454 (0.565)	16.822 16.745 (0.549)	*16.090* *16.375* (0.548)	21.841 21.427 (0.561)	24.151 24.964 (0.535)
0.6	83.696 83.409 (0.453)	*13.541* *13.528* (0.467)	15.408 15.398 (0.403)	19.820 19.406 (0.438)	20.464 20.673 (0.468)
0.7	69.822 69.328 (0.346)	*11.636* *11.545* (0.369)	14.787 14.981 (0.347)	17.888 17.059 (0.317)	17.510 17.451 (0.336)
0.8	59.643 59.462 (0.288)	*10.439* *10.506* (0.243)	14.180 14.227 (0.231)	16.171 16.548 (0.243)	15.249 15.493 (0.269)
0.9	51.961 51.398 (0.185)	*9.636* *9.782* (0.187)	13.585 13.679 (0.168)	14.703 14.093 (0.189)	13.554 13.603 (0.172)
1.0	46.019 46.475 (0.098)	*9.070* *9.113* (0.096)	13.008 13.241 (0.093)	13.473 13.326 (0.087)	12.286 12.096 (0.097)

**results from explicit formulas, **results from simulation, the Italic number is minimum ARL_1*
The number in parentheses is the standard deviation of run length.

Table 2. Average run length of DMA chart for the INARCH(1) model where $\beta_0 = 5,\ \alpha_0 = 0.1$ and given β change

δ_1	k				
	2	5	10	15	20
0.0	370.398 370.623 (1.451)	370.398 371.558 (1.565)	370.398 370.699 (1.630)	370.398 370.664 (1.597)	370.398 371.745 (1.468)
0.1	318.871 318.452 (0.988)	179.137 179.564 (0.965)	67.596 67.493 (0.934)	42.582 42.389 (0.974)	*41.641* *41.657* (0.982)
0.2	236.985 236.548 (0.865)	71.536 71.478 (0.837)	*25.468* *25.489* (0.863)	27.526 27.438 (0.844)	35.219 35.462 (0.847)
0.3	173.99 173.495 (0.744)	36.633 36.291 (0.732)	*18.907* *18.539* (0.752)	25.343 25.493 (0.716)	32.345 32.367 (0.759)
0.4	131.593 131.522 (0.676)	23.070 23.541 (0.629)	*17.016* *17.698* (0.645)	23.726 23.578 (0.653)	28.357 28.433 (0.611)
0.5	103.213 104.540 (0.577)	16.822 16.483 (0.509)	*16.090* *16.493* (0.521)	21.841 21.437 (0.547)	24.151 24.368 (0.508)
0.6	83.696 83.533 (0.409)	*13.541* *13.276* (0.425)	15.408 15.463 (0.487)	19.820 19.439 (0.489)	20.464 20.433 (0.465)
0.7	69.822 69.789 (0.354)	*11.636* *11.478* (0.328)	14.787 14.728 (0.369)	17.888 17.439 (0.376)	17.510 17.213 (0.361)
0.8	59.643 59.675 (0.265)	*10.439* *10.438* (0.230)	14.180 14.287 (0.261)	16.171 16.585 (0.255)	15.249 15.309 (0.254)
0.9	51.961 51.433 (0.196)	*9.636* *9.456* (0.126)	13.585 13.269 (0.134)	14.703 14.979 (0.183)	13.554 13.271 (0.168)
1.0	46.019 46.767 (0.093)	*9.070* *9.327* (0.096)	13.009 13.726 (0.089)	13.473 13.896 (0.097)	12.286 12.078 (0.098)

**results from explicit formulas, **results from simulation, the Italic number is minimum ARL_1*
The number in parentheses is the standard deviation of run length.

The results obtained from the proposed explicit formulas are compared with those obtained from Monte Carlo simulations. The results showed that the efficiency of explicit formulas is the same as those obtained from the Monte Carlo simulation; however, the former is less time-consuming. When parameter values were changed, for example, when the process was under control $\beta_0 = 1,\ \alpha_0 = 0.1,$ given in Table 1, it was found that when the parameter value changed magnitude, $\delta_1 \leq 0.1,$ that $k = 20$ value would minimize ARL_1 when $0.2 \leq \delta_1 \leq 0.5$ that $k = 15$ value makes ARL_1 minimum, and when $\delta_1 \geq 0.6$ that $k = 10$ value makes ARL_1, which yields the same findings as $\beta_0 = 5,\ \alpha_0 = 0.1$ shown on Table 2.

For the case of changing the parameters α and the control parameter is given $\beta_0 = 1,$ and $\alpha_0 = 0.1$ the numerical results showed that the performance of explicit formulas for ARL of the DMA chart was excellent when compared with the Monte Carlo simulation method. Unfortunately, the latter is very time-consuming. In Table 3 and Table 4, it was found that when the magnitude of the parameter change $\delta_2 \geq 0.1,$ that value $k = 20$ caused the lowest ARL_1 value.

Table 3. Average run length of DMA chart for the INARCH(1)model where $\beta_0 = 1,\ \alpha_0 = 0.1$ and given α change

δ_2	k				
	2	5	10	15	20
0.0	370.398 371.799(1.406)	370.398 370.573(1.476)	370.398 370.599 (1.465)	370.398 371.981(1.462)	370.398 370.568(1.532)
0.1	370.388 370.572(0.978)	370.337 370.452(0.957)	370.159 370.376 (0.954)	369.874 369.756(0.976)	*369.491* *369.769(* *0.966)*
0.2	370.359 370.465(0.864)	370.154 370.306(0.837)	369.443 369.392 (0.834)	368.307 368.776(0.879)	*366.793* *366.542(* *0.854)*
0.3	370.309 370.284(0.776)	369.848 369.243(0.767)	368.253 368.583 (0.766)	365.72 365.548(0.716)	*362.373* *362.452(* *0.754)*
0.4	370.24 370.199(0.649)	369.419 369.292(0.608)	366.569 366.592 (0.628)	362.151 362.434(0.645)	*356.35* *356.539(* *0.605)*
0.5	370.15 370.173(0.506)	368.869 368.658(0.536)	364.483 364.591 (0.547)	357.652 357.563(0.563)	*348.878* *348.563(* *0.564)*
0.6	370.041 370.067(0.452)	368.197 368.549(0.433)	361.927 361.973 (0.463)	352.289 352.479(0.462)	*340.141* *340.568(* *0.438)*
0.7	369.911 369.985(0.377)	367.405 367.291(0.342)	358.945 358.607 (0.328)	346.136 346.511(0.342)	*330.34* *330.522(* *0.387)*
0.8	369.762 369.675(0.265)	366.494 366.485(0.299)	355.555 355.376 (0.265)	339.277 339.678(0.257)	*319.686* *319.382(* *0.203)*
0.9	369.592 369.489(0.183)	365.464 365.752 (0.124)	351.779 (351.288(0.189)	331.799 331.575 (0.176)	*308.385* *308.531(* *0.183)*
1.0	369.403 369.254(0.096)	364.318 364.589(0.074)	347.64 347.547 (0.087)	323.7953 23.609(0. 096)	*296.635* *296.546(* *0.092)*

**results from explicit formulas, **results from simulation, , Italic number is minimum ARL_1*
The number in parentheses is the standard deviation of run length

Table 4. Average run length of DMA chart for the INARCH(1)model where $\beta_0 = 5$, $\alpha_0 = 0.1$ and given α change

δ_2	k				
	2	5	10	15	20
0.0	370.398 370.571 (1.406)	370.398 371.292 (1.392)	370.398 371.746 (1.434)	370.398 370.579 (1.419)	370.398 371.856 (1.333)
0.1	370.349 370.426 (0.968)	370.093 370.569 (0.924)	369.206 369.760 (0.966)	367.791 367.309 (0.933)	*365.906* *365.940* (0.954)
0.2	370.2 379.765 (0.843)	369.178 369.675 (0.846)	365.666 365.478 (0.862)	360.164 361.430 (0.873)	*353.034* *353.403* (0.879)
0.3	369.953 369.675 (0.784)	367.659 367.565 (0.709)	359.897 359.420 (0.774)	348.087 348.466 (0.761)	*333.424* *333.219* (0.791)
0.4	369.606 369.496 (0.655)	365.549 365.354 (0.611)	352.088 352.218 (0.647)	332.404 332.085 (0.679)	*309.286* *309.739* (0.675)
0.5	369.161 369.217 (0.579)	362.864 362.948 (0.554)	342.486 342.392 (0.561)	314.096 314.204 (0.566)	*282.862* *(282.571* (0.546)
0.6	368.617 368.452 (0.463)	359.627 359.650 (0.483)	331.376 331.084 (0.435)	294.157 295.641 (0.463)	*256.039* *256.659* (0.441)
0.7	367.975 367.290 (0.354)	355.599 355.642 (0.343)	319.064 319.302 (0.372)	273.482 273.439 (0.395)	*230.184* *230.798* (0.365)
0.8	367.236 367.928 (0.275)	351.599 351.203 (0.265)	305.855 306.549 (0.228)	252.813 253.074 (0.245)	*206.151* *206.454* (0.277)
0.9	366.4 366.097 (0.187)	346.872 346.087 (0.125)	292.043 292.438 (0.167)	232.713 232.669 (0.173)	*184.372* *185.078* (0.169)
1.0	365.468 365.289 (0.092)	341.716 341.685 (0.096)	277.894 277.439 (0.085)	213.577 213.094 (0.091)	*164.984* *165.938* (0.087)

**results from explicit formulas, **results from simulation, the Italic number is minimum ARL_1*
The number in parentheses is the standard deviation of run length.

4.2 Real Application

This section presents the performance of the DMA chart with the Shewhart and MA chart. The data consists of n = 108 monthly work stoppage count displayed in Figure 1, originally published in, [35]. The mean of INARCH(1) model is 1.173, and the variance of INARCH(1) model is 0.766. The result shows that the first sample outside the control limit is no. 4 for the Shewhart, MA, and DMA charts in Figure 2, Figure 3 and Figure 4, respectively. It can be concluded that all three charts are equally effective in detecting such data change.

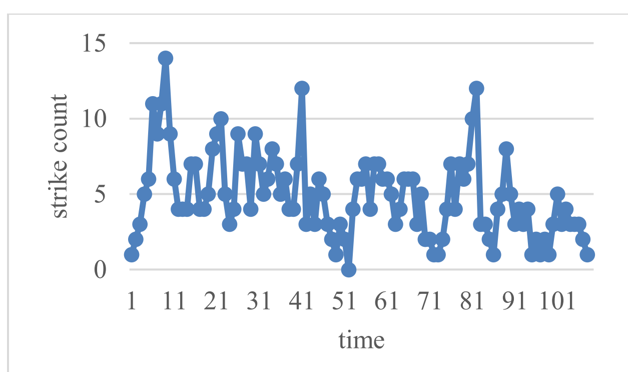

Fig. 1: Strikes counts data

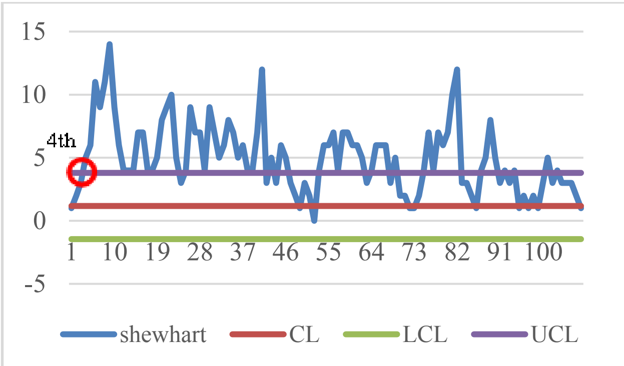

Fig. 2: Shewhart chart of strike count data

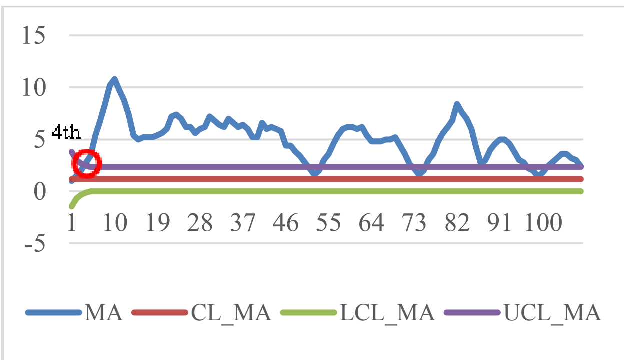

Fig. 3: MA chart (k=5) of strike count data

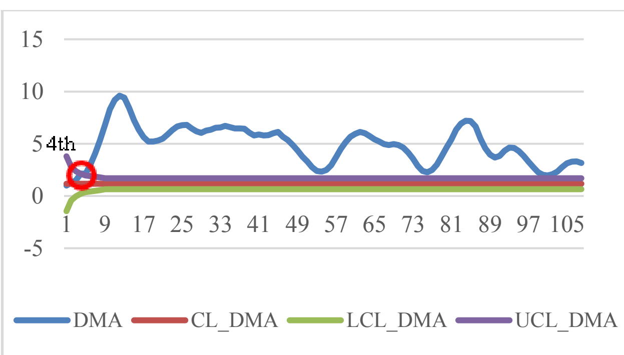

Fig. 4: DMA chart (k=5) of strike count data

5 Conclusion and Future Work

The derivative proof of explicit ARL formulas of the DMA control chart for a Poisson INARCH(1) process was presented. The numerical results were obtained from the explicit formulas and compared with the Monte Carlo simulation. They show that the explicit formulas' accuracy is in excellent agreement with the MC. Furthermore, the results found that when a parameter both β and α increases, the DMA chart will perform better as the value of k decreases for all case studies. In addition, these explicit formulas are simple and easy to implement with reduced computation times. For future work, this work can extend to other time series models and use for the other control charts that practitioners may apply in several fields.

Acknowledgment:
The authors would like to express their sincere gratitude to the Department of Applied Statistics for allowance and financial support to the Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Thailand for supporting research grants with contract no. 672158.

References:

[1] H. Kim, Y. Lin, and T.-L.B. Tseng, “A review on quality control in additive manufacturing,” Rapid Prototyping Journal 24, 645-669 (2018).

[2] V. Golosnoy and W. Schmid, “EWMA control charts for monitoring optimal portfolio weights,” Sequential Analysis 26, 195-224 (2006).

[3] V. Golosnoy and W. Schmid, “EWMA control charts for monitoring optimal portfolio weights,” Sequential Analysis 26, 195-224 (2006).

[4] M. Frisén, “Evaluations of methods for statistical surveillance,” Statistics in Medicine 11, 1489-1502 (1992).

[5] C. C. Torng, P. H. Lee and N. Y. Liao, “An economic-statistical design of double sampling $\bar{X}$ control chart,” International Journal of Production Economics 120, 495-500 (2009).

[6] M. Basseville and I. V. Nikiforov, “Detection of Abrupt Changes: Theory and Application”, Englewood Cliffs: Prentice Hall (1993).

[7] M. A. Al.Osh and A. A. Alzaid, “An integer-valued pth-order autoregressive structure (INAR(p)) process,” Journal of Applied Probability 27, 314-324 (1990).

[8] F. W. Steuel and K. Van Harn, “Discrete analogues of self-decomposability and stability,” Annals of Probability 7, 893-899 (1979).

[9] E. McKenzie, “Some simple models for discrete variate time series,” Water Resources Bulletin 21, 645-650 (1985).

[10] E. McKenzie, “Discrete Variate Time Series in Handbook of Statistics,” Elsevier, Amsterdam, 2003.
[11] M. A. Al.Osh and A. A. Alzaid, “An integer-valued pth-order autoregressive structure (INAR(p)) process,” Journal of Applied Probability 27, 314-324 (1990).
[12] F. W. Steuel and K. Van Harn, “Discrete analogues of self-decomposability and stability,” Annals of Probability 7, 893-899 (1979).
[13] D. Jin-Guan and L. Yuan, “The integer-valued autoregressive (INAR(p)) model,” Journal of Time Series Analysis 12, 129-142 (1991).
[14] J. Franke and T. H. Seligmann, “Conditional maximum likelihood estimates for INAR (1) processes and their application to modeling epileptic seizure counts,” Developments in time series analysis, 310-330 (1993).
[15] M. Cardinal, R. Roy, and J. Lambert, “On the application of integer- valued time series models for the analysis of disease incidence,” Statistics in Medicine 18, 2025-2039 (1999).
[16] C. Gourieroux, and J. Jasiak, “Heterogeneous INAR (1) model with application to car insurance,” Insurance: Mathematics and Economics 34, 177-192 (2004).
[17] K. K. Jose and B. Abraham, “Analysis of DAR (1)/D/s queue with Quasi-negative binomial-II as marginal distribution,” Applied Mathematics 2, 1159 (2011).
[18] C. H. Weiß and M. C. Testik, “CUSUM monitoring of first-order integer-valued autoregressive processes of Poisson counts,” Journal of Quality Technology 41, 389-400 (2009).
[19] C. H. Weiß and S. Schweer, “Detecting overdispersion in INARCH(1) processes,” Statistica Neerlandica 69, 281-297 (2015).
[20] C. H. Weiß, “Thinning operations for modeling time series of counts – a survey,” AStA Advances in Statistical Analysis 92, 319-341 (2008).
[21] S. Phantu, S. Sukparungsee, and Y. Areepong, “DMA chart monitoring of the first integer-valued autoregressive process of Poisson counts,” Advances and Applications in Statistics 52, 97-119 (2017).
[22] C. H. Weiß and M. C. Testik, “The Poisson INAR(1) CUSUM chart under overdispersion and estimation error,” IIE Transactions 43, 805-818 (2011).
[23] P. A. Yontay, “Two-Sided CUSUM for First-order integer-valued autoregressive process of Poisson counts, Master thesis,” Middle East Technical University, Ankara (2011).
[24] C. H. Weiß, “Controlling correlated processes of Poisson counts,” Quality and Reliability Engineering International 23, 741-754 (2007).
[25] D. Brook and D. A. Evans, “An approach to the probability distribution of CUSUM run length,” Biometrika 59, 539-548 (1972).
[26] C. W. Champ and S. E. Rigdon, “A comparison of the Markov chain and the integral equation approaches for evaluating the run length distribution of quality control charts,” Communications in Statistics: Simulation and Computation 20, 191-204 (1991).
[27] M. S. Srivastava and Y. Wu, “Evaluation of optimum weights and average run lengths in EWMA control schemes,” Communications in Statistics: Theory and Methods 26, 1253 -1267 (1997).
[28] Y. Areepong and S. Sukparungsee, “Explicit expression for the average run length of double moving average scheme for zero-inflated binomial process,” International Journal of Applied Mathematics and Statistics 53, 34-43 (2015).
[29] S. Sukparungsee and Y. Areepong, “A study of the performance of EWMA chart with transformed Weibull observation,” Thailand Statistician 7, 179-191 (2009).
[30] C. Petru, “Statistical Tool to Estimate and Optimize the Intensity of the Dependence Between the Parameters of a Dynamic System,” WSEAS Transactions on Computers 21, 165-170 (2022).
[31] F. W. Steuel and K. Van Harn, Discrete analogues of self-decomposability and stability. Annals of Probability, 7, 893-899 (1979).
[32] M. B. C. Khoo, “A moving average control chart for monitoring the fraction non-conforming,” Journal of Quality and Reliability Engineering International, 20, 617-635 (2004).
[33] M. B. C. Khoo and V. H. Wong, “A double moving average control chart,” Communications in Statistics-Simulation and Computation, 37, 1696-1708 (2008).

[34] D. C. Montgomery, "Introduction to Statistical Quality Control," John Wiley & Sons (2009).

[35] U.S. Bureau of Labor Statistics, (data of in Jan/1996 to Dec/2004), [Online]. http://www.bls.gov/wsp/ (Accessed Date: January 18, 2024).

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Suganya Phantu: writing an original draft, software, data analysis, data curation, prove and validate.
- Yupaporn Areepong: investigation, methodology, validate.
- Saowanit Sukparungsee: conceptualization, investigation, writing-review and editing, funding acquisition, project administration, reviewing and editing.

All authors have read and agreed to the published version of the manuscript.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

The authors would like to express their sincere gratitude to the Department of Applied Statistics for allowance and financial support to the Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Thailand for supporting research grants with contract no. 672158.

Conflict of Interest

The authors have no conflict of interest to declare.