Small Portfolio Construction with Cryptocurrencies

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Abstract: - In this paper, we describe and apply different models of portfolio construction in the selection between a small number of big-cap cryptocurrencies. Our purpose is to select the minimum riskiness between cryptocurrencies, comparing different risk measures and maximum diversification. We build our models without the constraints of the expected returns. Without relying on expected returns, we have the same condition on the comparison between them. Cryptocurrencies are not common stock or other assets indexed in the market but it is interesting to study how diversification can significantly improve investment performance. We first give the methodology to use high-frequency observation data, in the numeral approximation especially in the novel application of the Risk parity models, used with different risk measures we can achieve a very good result, from the position of gaining and variation. Since Risk parity models divide the weights of the asset in equal risk contribution proportion, it is suggested to use a small number of cryptocurrencies, otherwise their performance will be close to the uniform portfolio. To the traditional Mean Variance model, and the alternative, Expected shortfall/Conditional Value at Risk, we use three versions of Risk Parity with two different risk measures and a naive risk parity. The uniform portfolio is used as a benchmark for selection comparison with the other portfolio models. We give the conditions for the Risk Parity with the Expected shortfall/Conditional Value at Risk (CVaR) to guarantee convergence with the numerical approximation. In the end, we study the tradeoff between each model and which is more suitable for a small cryptocurrency portfolio.

Key-Words: - Bitcoin, Cryptocurrency, Asset allocation, Portfolio optimization, Risk diversification, Risk Parity, Markowitz, Marginal Risk Contribution, Robust Optimization, Risk Management.

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1 Introduction

Trading with Cryptocurrencies, as a non-regulated market, has achieved a lot of focus from the point of view of the view of speculation and research. The most famous cryptocurrency, built using blockchain technology, is Bitcoin with a market price of about 16.700 US dollars per coin and a market capitalization of about 320 billion dollars (December 2022), which has decreased from 359 billion USD from the last year.

The Cryptocurrency market capitalization has reached in 2022 above 2 trillion U.S; until 2016 the total market capitalization was below 18 billion U.S. dollars, (Yahoo Finance 2020).

Trading is easily accessible in more than 100 different cryptocurrencies to be used as currency or as financial assets. Thus, cryptocurrencies can be seen as an alternative asset of investment, since they obtained the increasing attention of many investors for very high gains in an observed short time.

The price of Bitcoin ranges between 13.00 USD and about 62.000 USD, thus, some professional investors must not invest in cryptocurrency due to its unpredictable price movements and high volatility, not to mention the fact that there is no reliable way to value a crypto asset. The price is very sensitive to the authority's prohibition in accepting Bitcoin as a currency, but also, the price can go up if any public figure such as Elon Musk, declares positive things about this method of payment. However, several studies show that including Bitcoin in a portfolio has significant benefits, [1], [2]. The crucial problem with Cryptocurrency markets is that they are not under the market classic regulation, thus, we should use models that rely on the minimum risk.

A few cryptocurrencies have large market capitalization and in terms of millions or even billions, while they usually provide lower transaction costs to individuals demonstrating an efficient financial market characteristic that indicates immediate liquidity, [3].

If we consider cryptocurrencies as a new class of assets or as traditional currency, [4], we should first see their statistical characteristics in the distribution of the returns such as the skewness, kurtosis, and heteroscedasticity. Many works on the study of cryptocurrencies found recently are common in financial assets and long-memory, [5]. Moreover, researches on cryptos further demonstrate potential diversification in this emerging market for institutional and retail investors.

There are cryptocurrencies that evolvement is relatively isolated from the others, [6], which may offer diversification benefits for speculators and the variety of cryptocurrencies is still uprising, thus the cryptocurrency market has an increasing place in diversification and portfolio composition, thus, the research on the portfolio diversification of cryptocurrencies has been increased. Today, you may find more than two thousand different cryptocurrencies, but do we trust all of them?

Speaking of optimization models, many other portfolio optimization models, such as $VaR_\alpha(x)$, as a maximum potential loss of a portfolio in an interval of time, have been proposed in the literature after the Nobel prize H. Markowitz, [7], with his first step in modern portfolio theory. Numerous studies on a similar risk measure, the Conditional Value at Risk $CVaR_\alpha(x)$, [8], demonstrate why it is preferred to Value at Risk $VaR_\alpha(x)$ because the later does not allow diversification. The most crucial characteristics are that $CVaR_\alpha(x)$ is a convex and coherent risk measure demonstrated in the model function, [9], a model that supports diversification.

All of these models rely on the estimated expected return of the assets as an input, which causes them to concentrate heavily on a restricted set of assets and perform badly outside of the sample, [10]. Additionally, these models generate extremely high weights and demonstrate large fluctuations over time. So, comparable to within a Mean Variance portfolio, a major adjustment in the input parameters can affect the portfolio's composition significantly.

The Risk Parity approach's ability to avoid requiring the estimation of expected returns is one of its main advantages. The Risk Parity methodologies divide the entire risk of the portfolio into the risk contributions of each asset in the same proportion

Using the Euler breakdown for the first order homogeneous function, we will be able to apply the Risk Parity technique to the Expected shortfall or more common $CVaR_\alpha(x)$.

By observation, we know that the idea that the returns are a normal multivariate distribution is less credible due to the lack of reality. Other authors, [11], use a Mixed Tempered stable distributed for the source of risk in the Risk Parity models. An alternative approach, called Equal Risk Bounding (ERB), requires the solution of a nonconvex quadratically constrained optimization problem. The ERB approach, while starting from different requirements, turns out to be firmly connected to the RP approach, [12]. In this paper, we will treat cryptocurrencies as usual stocks or bonds, with a purpose of studying how the novices' digital currency, which operates without a financial system or government authorities, will behave in these conditions. We first describe the selected small crypto portfolio, justifying our selection on these, to analyse the performance in a out of sample period with the use of a rolling window. Another important step is the analysis of riskiness, portfolio turnover, and diversification.

2 Cryptocurrency Datasets and Models Used

The cryptocurrency dataset selected, includes the period from 1/1/2018 to 31/01/2021 with 1123 trading days in total (remember that you can trade each day of the year 24/7). We chose this time span because it does not included, the moment in which Bitcoin reached its highest peak in November 2021, avoiding the unusual distortion of the data.

We collect ten cryptocurrencies with a market capitalization larger than half a billion. To avoid any currency fluctuation, all the prices are in dollars as they are listed on Yahoo Finance.

Table 1. Data of the distribution of the daily returns (in %)

Cryptocurrency	Mean	Median	Range	Skewness	Kurtosis
Cardano USD (ADA-USD)	-0.07	-0.01	82.54	-21.40	924.72
Ethereum Classic USD (ETC-USD)	0.24	0.06	81.99	78.10	1436.37
Binance Coin USD (BNB-USD)	0.15	0.07	102.4	-13.64	2083
Bitcoin USD (BTC-USD)	0.08	0.15	63.18	-147.85	2081.29
Dogecoin USD (DOGE-USD)	0.13	-0.10	183.8	592.56	10552
Chainlink USD (LINK-USD)	0.31	-0.10	109.5	2.31	1097.80
Litecoin USD (LTC-USD)	-0.05	-0.07	73.97	-30.90	1021.65
Tether USD (USDT-USD)	0.00	-0.01	10.60	27.23	2885.30
Stellar USD (XLM-USD)	-0.04	-0.18	96.91	112.39	1671.9
Monero USD (XMR-USD)	-0.09	0.07	72.11	-99.16	1153.8

Source: Authors' calculation

As we see in Table 1, except for Dogecoin, the other daily returns are almost normally distributed, even if we compare the skewness and the kurtosis. We decided to also include Dogecoin to see the difference between the cryptocurrencies.

Table 2. The correlation matrix of the returns of the cryptocurrencies

1.00	0.73	0.61	0.73	0.45	0.59	0.76	-0.06	0.79	0.72
0.73	1.00	0.62	0.79	0.44	0.54	0.81	-0.05	0.63	0.74
0.61	0.62	1.00	0.68	0.39	0.50	0.67	-0.06	0.55	0.65
0.73	0.79	0.68	1.00	0.49	0.56	0.82	-0.03	0.64	0.79
0.45	0.44	0.39	0.49	1.00	0.34	0.48	-0.02	0.45	0.45
0.59	0.54	0.50	0.56	0.34	1.00	0.55	-0.02	0.51	0.53
0.76	0.81	0.67	0.82	0.48	0.55	1.00	-0.06	0.65	0.77
-0.06	-0.05	-0.06	-0.03	-0.02	-0.02	-0.06	1.00	-0.03	-0.03
0.79	0.63	0.55	0.64	0.45	0.51	0.65	-0.03	1.00	0.64
0.72	0.74	0.65	0.79	0.45	0.53	0.77	-0.03	0.64	1.00

Source: Authors' calculation

In Table 2 we notice that Tether (row/column 8) is the only crypto with returns negatively correlated with the other nine cryptocurrencies. The mean of returns is centered around zero and close to the median which is a good condition. The next step is to measure the performance of the portfolio.

The correlation matrix is important to understand the ongoing market, and how the behavior of the models is based on the variance of the portfolio.

The portfolio models considered in this article are:

- 1/N equal weighted rule (Naive Portfolio),
- minimum variance (MV),
- minimum CVaR,
- Risk Parity with standard deviation as risk measure (RP-std),
- Risk Parity with conditional value at risk measure CVaR (RP-CVaR),
- Naive Risk Parity CVaR (RP-CVaR naive).

The last one is a special case in which we have the worst-case scenario (highest CVaR, useful as an upper bound, [13].

In all these methods, we do not use expected returns, so at the minimum variance i.e. we don't have the constraints of the return of the portfolio, to have the smallest possible variance.

In all the cases we do not allow short selling, so the weights allocated at each cryptocurrency can assume only positive values.

We will try different rolling windows to see how do they perform by measuring the cumulated return of the portfolio.

The first indicator of performance is the following:

$$\mu_k^c(R_P) = \prod_{j=1}^{k}\left(1 + r_{pj}\right) - 1$$

so that $\mu_T^c(R_P)$ is the total compounded return for the whole period.

All portfolios will have n assets, for weight x_i assigned and $\mathcal{R}(x)$ as a measure of risk for the portfolio $x = (x_1, x_2, \ldots, x_n)$.

In other works, [14], the more common use of Risk Parity is as a risk measure of the standard deviation. Considering the weights $x = (x_1, x_2, \ldots, x_n)$ assigned to n assets, the risk measure, in this case, standard deviation is given by:

$$\mathcal{R}(x) = \sigma_P(x) = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij}} = \sqrt{x'\mathrm{V}x}$$

whereV is the covariance matrix. For the i asset, the marginal risk contribution is:

$$MRC_i(x) = \frac{\partial \sigma_P(x)}{\partial x_i} = \frac{\partial \sigma_i^2 + \sum_{j=1}^{n} x_i \sigma_{ij}}{\sigma_P(x)} = \frac{(\mathrm{V}x)_i}{\sqrt{x'\mathrm{V}x}}$$

and the total risk contribution:

$$TRC_i(x) = x_i \frac{\partial \sigma_P(x)}{\partial x_i} = x_i \frac{\partial \sigma_i^2 + \sum_{j=1}^{n} x_i \sigma_{ij}}{\sigma_P(x)} = x_i \frac{(\mathrm{V}x)_i}{\sqrt{x'\mathrm{V}x}}$$

The following optimization problem can be used to represent the Risk Parity model:

$$x^* = \arg\min \sum_{i=1}^{n} \sum_{j=1}^{n} \left(TRC_i(x) - TRC_j(x)\right)^2$$
$$\sum_{i=1}^{n} x_i = 1$$
$$x \geq 0$$

Remind that the Mean Variance Markowitz model equalizes the marginal Risk Contribution.

For the numerical approximation in the proof of the partial derivatives of $CVaR_\alpha(x)$, some conditions are needed on the distribution of the return vector $R = (r_1, r_2, \ldots, r_n)$.

The denote with $X = R'x = \sum_{i=1}^{n} x_i r_i$ the portfolio return, which must be differentiable to the weights x_i to apply the Euler decomposition.

The return r_i from t to time $t+1$ is measured as follow:

$$r_{i,t+1} = \frac{P_{i,t+1} - P_{i,t}}{P_{i,t}}$$

We can compute the partial derivatives of the $CVaR_\alpha(x)$ from partial derivatives for the Value at Risk. Starting from the definition of $CVaR_\alpha(x)$, [8], we have

$$CVaR_\alpha(x) = \frac{1}{\alpha} \int_0^\alpha VaR_v(x) \mathrm{d}v$$

Thus, using the needed assumptions, [9], from the mathematical point of view, we can proceed with the differentiation to the variable x_i weights.

$$\frac{\partial CVaR_\alpha(x)}{\partial x_i} = \frac{1}{\alpha} \int_0^\alpha \frac{\partial CVaR_\alpha(x)}{\partial x_i} \mathrm{d}v = -\frac{1}{\alpha} \int_0^\alpha E[r_i| -R'x = VaR_\alpha(x)] \mathrm{d}v = -\frac{1}{\alpha} \int_0^\alpha E[r_i | X = q_\alpha(X)] \mathrm{d}v = -\ E[r_i | X \leq -VaR_\alpha(x)]$$

The Total Risk is given from the following expression for the asset i:

$$TRC_i^{CVaR_\alpha(x)}(x) = x_i \frac{\partial CVaR_\alpha(x)}{\partial x_i}$$

In case of continuous returns distribution, we can pass to the following presentation:

$$TRC_i^{CVaR_\alpha(x)}(x) = -x_i E[r_i | X \leq -VaR_\alpha(x)]$$
$$CVaR_\alpha(x) = \sum_{i=1}^{n} TRC_i^{CVaR_\alpha(x)}(x) = -\sum_{i=1}^{n} x_i E[r_i | X \leq -VaR_\alpha(x)]$$

For the discrete variables in the numerical finding for $\boldsymbol{VaR_\alpha(x)}$ and $\boldsymbol{CVaR_\alpha(x)}$ Risk Parity using times series observations we have to do the following assumption. Assuming that the i-th asset return r_{ji} with i=1,.....,n and j=1,...,T where n is the quantity of elements considered at our portfolio and T the laps of time of observation. The vector of the created portfolio returns is $R_P = (r_{p1}, \ldots\ldots, r_{pT})$ in which:

$r_{pj} = x'r^j$ with j=1,....,T where $r^j = (r_{j1}, \ldots\ldots, r_{jT})$.

Using the central limit theorem for the number of observation large enough, the approximation of the empirical distribution of the observed portfolio returns:

$$P(R_P \leq y) \approx \frac{\#\left(j = 1, \ldots., T | r_{p1} \leq y\right)}{T}$$

So, approximation for the $VaR_\alpha(x)$ and $CVaR_\alpha(x)$ of portfolio returns will be as follows:

$$VaR_\alpha(x) \approx -r_{\mathrm{p}\lfloor\alpha\mathrm{T}\rfloor}^{\mathrm{sorted}}$$
$$CVaR_\alpha(x) \approx -\frac{1}{\alpha\mathrm{T}} \sum_{\mathrm{j}=1}^{\lfloor\alpha\mathrm{T}\rfloor} r_{\mathrm{p\,j}}^{\mathrm{sorted}}$$

where α is the significance level and $\mathrm{r}_{\mathrm{p\,j}}^{\mathrm{sorted}}$ are the sorted portfolio returns that must satisfy

$$r_{p\,1}^{sorted} \leq r_{p\,2}^{sorted} \leq \cdots r_{p\,j}^{sorted} \leq \cdots \leq r_{p\,j}^{sorted}$$

For more details, [15].

Using times series observations, the discrete values of the partial derivatives $CVaR_\alpha(x)$ for each asst i becomes:

$$\frac{\partial CVaR_\alpha(x)}{\partial x_i} \approx -\frac{1}{\alpha\mathrm{T}} \sum_{\mathrm{k}=1}^{\lfloor\alpha\mathrm{T}\rfloor} r_{\mathrm{k\,i}}^{\mathrm{sorted}}\ \forall\ i=1,\ldots,n$$

and then the total risk contribution of asset i is

$$TRC_i^{CVaR_\alpha(x)}(x) = x_i \frac{\partial CVaR_\alpha(x)}{\partial x_i} \approx -\frac{1}{\lfloor \alpha T \rfloor} x_i \sum_{k=1}^{\lfloor \alpha T \rfloor} r_{k\,i}^{\mathrm{sorted}}$$

where $r_{k\,i}^{\mathrm{sorted}}$ are the related portfolio returns to the ordered from the smallest to the largest value.

We consider three diversification measures, to control, if the portfolios are well diversified. Consider a portfolio x = (x_1, x_2,.....,x_n) satisfying the budget constraint $\sum_{i=1}^{n} x_i = 1$ with no short selling ($x_i \geq 0$) .

The Bera and Park D_{BP} measure, [16], is very important to understand the entropy of the portfolio, for strategies no short selling strategies.

$$D_{BP} = -\sum_{i=1}^{n} w_i \log(w_i) = \sum_{i=1}^{n} w_i \log(\frac{1}{w_i})$$

The D_{BP} assumes values between 0 (all in one) and $\log(n)$ for uniform allocation: the diversification measures only accurately reflect diversity in terms of weights allocated and do not consider the fact that different types of assets will have changing effects on the high change of the entire portfolio.

The turnover of the portfolio is another helpful amount for determining transaction costs:

$$TO = \sum_{i=1}^{n} |w_i^{t+1} - w_i^t|,$$

where w_i^t is the amount assigned in i the observation t.

For the computation, we use MATLAB software running on a Laptop with Windows 10 Home operation system, Intel(R) Core(TM) i7-7500U CPU @ 2.70GHz 2.90 GHz, 12 RAM and NVIDIA GeForce 930MX graphic card. The timing of optimization is very short compared to the other.

3 Results in Performance out of Sample and Diversification

To succeed in the numerical approximation, we create a rolling window using the data of the past 2 years (from 1/1/2018 to 1/1/2020 so L=365*2=730 observation) for the estimation of the weights of each of the portfolio models and move the rolling window in the period from 1/1/2020 to 31/1/2021 to measure the out of sample for the next 7 days (Holding period). This will take 56 iterations for the calculation of the optimal portfolios. Remembering that short selling is not allowed. The expected returns constraint is eliminated, for the Minimum Variance and Conditional Value at Risk model: the minimum risk is achieved for each risk measure.
Let us see the weights only for the first optimization (L=730 days).

Table 3. Weight allocation of the first optimization (in %)

	Portfolio Model					
Crypto.	Uniform	R. P. S. D.	R. P. CVaR	R. P. CVaR N.	Min-CVaR	Min-Var
Cardano	10	2.97	4.38	2.07	0.00	0.07
Ethereum	10	2.76	4.07	1.98	0.00	0.03
Binance	10	3.48	4.94	2.70	0.00	0.06
Bitcoin	10	4.26	6.32	2.96	0.00	0.03
Dogecoin	10	3.62	4.96	2.78	0.00	0.07
Chainlink	10	2.99	3.96	2.13	0.00	0.01
Litecoin	10	3.28	5.13	2.47	0.00	0.05
Tether	10	70.3	57.1	78.5	100	100
Stellar	10	3.18	4.60	2.28	0.00	0.02
Monero	10	3.20	4.56	2.22	0.00	0.02
TOTAL	100	100	100	100	100	100

Source: Authors' calculation

As it is clear from Table 3, most of the portfolios focus on the Tether USD, in which, Minimum variance and CVaR are fully allocated. The Risk parity models a range between 57% to 78% in the same cryptocurrency. This is very interesting if we compare it with the matrix of covariance, Tether is the only one negatively correlated, as we know, it will have a smaller volatility. Thus, most of the weights are higher for this cryptocurrency.

The better way to show the compound return rate is from the graph below.

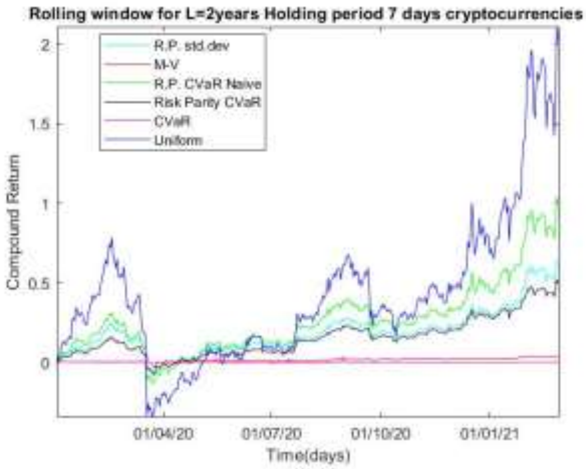

Fig. 1: The compound return rate
Source: Authors' calculation

In Figure 1, we observe that the uniform portfolio (naïve portfolio in blue color) and the Risk Parity with CVaR Naïve, which do not consider the risk in the first place, outperform compared to the other risk measures but on the ongoing have also a higher drawdown (April 2020). The uniform at the end of the observation has doubled his compound return rate.

The Mean Variance Model and the CVaR, have almost the same performance by having the minimum risk possible but a very small value in compound return.

In between, we have the Risk Parity with CVaR and Risk Parity with standard deviation as a risk measure. As we explained, these come for the better diversification of the risk (Figure 1).

If we measure the riskiness of each portfolio using the standard deviation, we will notice that the uniform and the Risk Parity with naïve CVaR will have a very high risk.

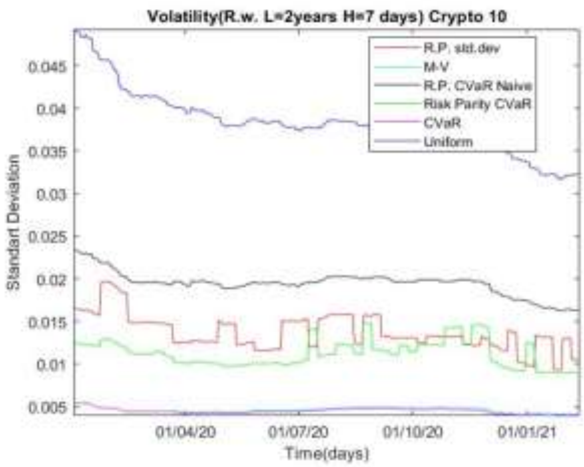

Fig. 2: The standard deviation of portfolio selections
Source: Authors' calculation

The Mean Variance and CVaR models have almost the same results (Figure 2), as we notice in the lower part of the graph. We will have similar results if the holding period changes by one or two days.

In a recent paper, [17], they created portfolios comparing cases with and without Bitcoin only as one asset in portfolios with usual assets such as gold and other indexes. They use the mean variance model and the risk parity only with the standard deviation as a risk measure. They are pointing out that the allocation to Bitcoin in most of the unconstrained or semi-constrained frameworks was minimal. They insist since Bitcoin observers have substantial value variations, stakeholders must exercise attentiveness and limit their acquaintance to Bitcoin, a superfluous exposure to Bitcoin may not principally lead to development in portfolio performance qualities, [16].

To have a close estimation of what happens in case we consider the transaction costs, we can see the portfolio turnover (Figure 3).

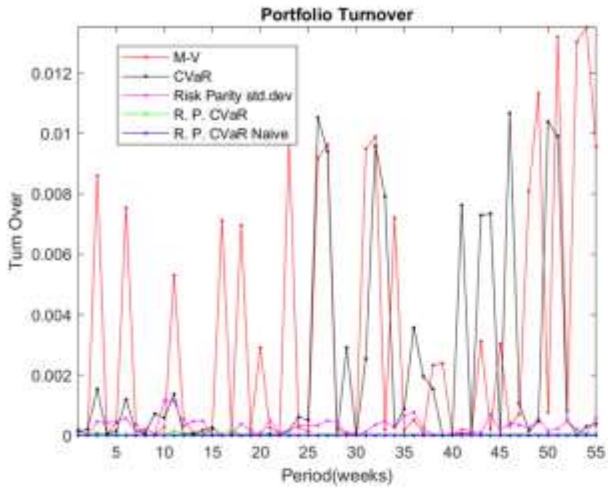

Fig. 3: Portfolio weekly turnover
Source: Authors' calculation

Knowing that the number of cryptocurrencies (ten) is a small one, Mean Variance and CVaR are focused on a smaller number of cryptocurrencies, thus the portfolio turnover will be higher (Figure 3). Considering that cryptocurrencies are easily accessible by different competitive platforms, we still may face different costs, for instance, subscriptions costs and small commissions.

The last point to discuss is diversification by measuring Bera Park, remember that the higher is the value, the better are diversified the portfolios.

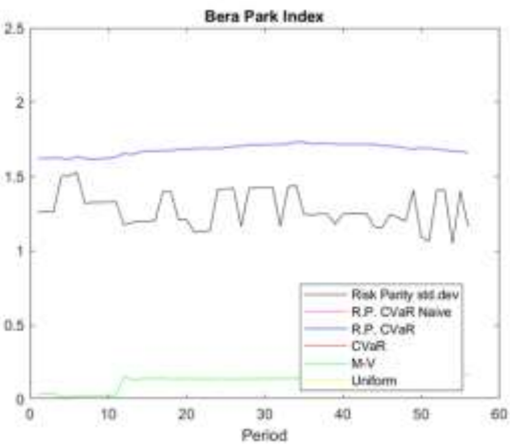

Fig. 4: Diversification by Bera Park.
Source: Authors' calculation

As we notice from the Figure 4, both measures have similar results. Focusing on a smaller number of crypto, M-V, and CVaR are concentrated compared to the models.

The values of the diversification are better with the Risk parity methods, especially with the Conditional Value at Risk as a Risk measure (Figure 4).

4 Conclusion

The novelty of this paper consists in comparing different methods of portfolio optimization in the cryptocurrency market. Without using the expected returns all the models for portfolio selection are in the same condition, The Mean Variance and CVaR are at the minimum risk, as the other models without the use of the expected returns constraint. We have chosen ten cryptocurrencies with the highest capitalization and a lap of time for the observed data large enough to give a significant conclusion. We described the observed data, to make sure that they are suitable for the conditions of the Mean Variance and other models that have conditions on the distribution of the returns and the correlation matrix. After we described the methodology for the numerical approximation, we passed from the continuous case to the discrete observation with high-frequency data.

Considering cryptocurrencies as an asset class we faced an issue of increased volatility, and were very sensitive to the market information. For that, there is a necessity to develop particular models for asset allocation in cryptocurrencies. Traditional approaches, like the Markowitz model, solely concentrate on assets that carry an absolute minimal amount of risk. Therefore, if the investor tries to rebalance the portfolio, this high concentration will likewise have significant transaction costs. Additionally, relying on predicted returns during a downturn in the economy would result in an unrealistic and pessimistic asset allocation. Some investors may have a large collection of cryptocurrencies in their financial holdings for a variety of reasons, most notably for speculation.

When associated with CVaR and Mean Variance, the cryptocurrency portfolio built using the Risk Parity criteria showed higher diversity and less focus on high weights. This results in lower costs for recalibrating the portfolio due to the low turnover.

From the perspectives of performance and volatility, the Risk Parity techniques in each of these situations represent a good compromise between the CVaR, Mean Variance, and the uniform portfolio. The importance of the cryptocurrency has been seen lately as Bitcoin is a novice to the world of exchange-traded funds. Bitcoin ETFs allow investors to get exposure to the tempting potential of BTC without having to directly own it or safely store it, [18]. Some investors may feel safer getting exposure to Bitcoin in their portfolios by purchasing a professionally managed ETF than they do owning an actual BTC. This is to show the importance of cryptocurrencies in the post COVID 19 market and more studies need to be done by including these assets in the investments.

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- Denis Veliu has worked on the theoretical background and literature review of the paper and the overview of the model developments.
- Marin Aranitasi has conducted the quantitative data analysis and the optimization of the problems, by using MATLAB optimization.

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Conflict of Interest

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