Optimization of GM^r (1,1) Model and Its Application in Forecast the Number of Tourist Visits to Quang Ninh Province

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Abstract: **-** Currently, many researchers pay more attention to improving the accuracy of the Grey forecasting model. One of tendency is focused on the modification of the accumulated generating operation. In 2015, some scholars used the *r*-fractional order accumulation to improve the accuracy. However, With the desire of users to have a set of forecasting tools as accurate as possible. This paper based on the flexibility parameter of *r*-accumulated generation operation proposed the systematic approach by optimizing the number of *r* for improving the precision. To verify the performance in advance of the proposed approach, three case examples were used, the simulation results demonstrated that the proposed systematic approach provides very remarkable predictive performance with the accuracy performance of the proposed approach being higher than other models in comparison. Furthermore, the real case in forecasting the number of tourism visits to Quang Ninh was also conducted to compare the performance of models. The empirical results show that the proposed model will get a higher accuracy performance with the lowest MAPE =19.722%. This result offers valuable insights for Quang Ninh policymakers in building and developing policies regarding tourism industry management in the future.

Key-Words: - GM^r (1,1), fractional order accumulation, optimization, systematic approach, accuracy, number of tourists, Quang Ninh province.

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1 Introduction

The grey forecasting model is an important part of the grey system theory. It is a time series forecasting model. It was proposed by Prof. Deng in the early 1980s, [1], [2]. Among the grey models' series, grey model GM (1,1) is the basic model. It is constructed by the first-order differential equation and one variable with the non-negative original sequence. Because of easy simulation and higher accuracy compared with other time series forecasting models in the uncertain information and the limited data, [3], [4], [5], So, the GM (1,1) has been widely and successfully applied to various fields such as tourism, [6], [7], transportation, [8], [9], and, [10], financial and economic, [11], [12], [13], integrated circuit industry, [14], [15], [16], [17], energy industry, [18], [19], [20], and so forth.

Some scholars concentrate on the improvement of the GM (1,1) model with different perspectives. For an instant, Wang and Lin used optimal methodology to improve the background values, [20], [21]. Hsu and Wang used different methods to modify the development coefficient and the grey input coefficient, [17], [22]. Some scholars focused on modifying the residual error to improve the performance of the GM (1,1) model, [7], and, [13]. In addition, some research proposed hybrid models based on the combination of the GM (1,1) and other methods like the grey econometric model, [5], the grey Markov model, [23], [24], the grey fuzzy model, [20], and so on. Despite its improvement in prediction accuracy, the prediction accuracy of these models is not always satisfactory. Especially, in an environment with the data are highly fluctuating, or with lots of noise.

In recent years, Wu developed the grey model with fractional order accumulation (abbreviated as GM^r (1,1)) to deal with fluctuating data, [25], [26]. This model is based on two fundamental principles which are the principle of information differences and the principle of new information priority. Therefore, this model can solve the shortcomings of previous models. For the GM^r (1,1), the changing initial condition affects the simulative value of GM^r (1,1) and the monotonicity and convexity of the simulative value are uncertain when the actual value is nonnegative increased. With the advantages of this model, the GM^r (1,1) has been widely applied in the community, [27], [28]. In addition, with the desire to improve the predictive accuracy, this study proposed a new effective systematic approach based on the mathematical algorithm of GM^r (1,1) to enhance the predictive performance. Based on three case studies in research paper, [25], and the real case in forecasting the number of tourists visit to Quang Ninh province, the stimulation results indicated that the proposed systematic approach can significantly enhance the precision of the GM^r (1,1) model.

This paper is organized as follows. In section 2, the main concept of the GM (1,1) with fractional order accumulation is briefly introduced and proposed a new systematic approach aims to improve the accuracy precision of GM^r (1,1). Section 3 demonstrates the proposed approach has better performances in several numerical examples by compared with GM (1,1), and GM^r (1,1). Section 4 illustrates the application of the proposed approach in forecasting the number of tourism visits to Quang Ninh province, Finally, the conclusions are made in Section 5.

2Methodology and Proposed Approach

Following the perspective of improving the GM (1,1) model by modifying the background value, many scholars indicated that the growth trend of the original data sequence has a great influence on the accuracy of prediction because the GM (1,1) model is a kind of homogeneous exponential growth model. If the original data sequence is smooth, the closer to the exponential growth it is, the higher the prediction precision the model can produce. Therefore, Wu and partners proposed the *r*-order accumulated generating operation to reduce the fluctuation of the original data sequence, [25]. All procedures for modifying the *r*-order accumulated generating operation of GM (1,1) are given in section 2.1

2.1 The GM^r (1,1) Model

According to the research paper of Wu and partners, [25], the overall process of the GM^r (1,1) is as follows:

Step1: Preparing the non-negative original data sequence X_0

$$X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(k), \ldots, x^{(0)}(n)\right\} \quad n \geq 4 \tag{1}$$

Where n is the sample size of the data

Step 2: Construct a new series data X^r by using *r*-order accumulated generating operation (*r*-AGO)

$$X^{(r)} = (x^{(r)}(1), x^{(r)}(2), \ldots x^{(r)}(n)) = \left[x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(k), \ldots, x^{(0)}(n)\right] \begin{bmatrix} 1 & C_r^1 & \ldots. & C_{r+n-3}^{n-2} & C_{r+n-2}^{n-1} \\ 0 & 1 & \ldots & C_{r+n-4}^{n-3} & C_{r+n-3}^{n-2} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 & C_r^1 \\ 0 & 0 & \ldots & 0 & 1 \end{bmatrix} \tag{2}$$

Step 3: Building the GM (1,1) model by establishing the *r*-order differential equation with one variable is expressed as:

$$x^r(k) + az^{(r)}(k) = b$$
$$(z^{(r)}(k) = 0.5x^{(r)}((k) + (k-1)), k = 2,3,\ldots,n.) \tag{3}$$

Where $z^{(1)}(k)$ is the average value of consecutive data.

So, whiteness equation is the following:

$$\frac{dx^{(r)}(k)}{d(k)} + az^{(r)}(k) = b \tag{4}$$

The ordinary least square estimate of a and b can be obtained by:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (A^T A)^{-1} A^T Y \tag{5}$$

Where

$$Y = \begin{bmatrix} x^{(r)}(2) - x^{(r)}(1) \\ x^{(r)}(3) - x^{(r)}(2) \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ x^{(r)}(n) - x^{(r)}(n-1) \end{bmatrix} \quad (6)$$

and

$$A = \begin{bmatrix} -z^{(r)}(2) & 1 \\ -z^{(r)}(3) & 1 \\ \cdots\cdots & . \\ \cdots\cdots & . \\ -z^{(r)}(n) & 1 \end{bmatrix} \quad (7)$$

Step 4: The solution of the whitenization equation (4) can be expressed as follows

$$\hat{x}^{(r)}(k) = \left[x^{(r)}(1) - \frac{b}{a} \right] e^{-a(k-1)} (1 - e^{a}) \quad (8)$$

Step 5: The simulations and forecasting value can be obtained by applying the *r*-order inverse accumulated generating operator (*r*-IAGO). Therefore, the fitted and predicted sequence is given $\hat{x}^{(0)}$ as:

$$\hat{X}^{(0)} = (\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots \hat{x}^{(0)}(n)) = \left[\hat{x}^{(r)}(1), \hat{x}^{(r)}(2), \ldots, \hat{x}^{(r)}(k), \ldots, \hat{x}^{(r)}(n) \right] \begin{bmatrix} 1 & -C_r^1 & \ldots & (-1)^{n-1} C_{r+n-2}^{n-1} \\ 0 & 1 & \ldots & (-1)^{n-2} C_{r+n-3}^{n-2} \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & -C_r^1 \\ 0 & 0 & \ldots & 1 \end{bmatrix} \quad (9)$$

2.2 Proposed a Systematic Approach to Improve the Predictive Accuracy of GM^r (1,1) by the *r*-order Accumulation Optimization

Based on the mathematical algorithm of the GM^r (1,1), this study proposed a systematic approach to improve the predictive accuracy of GM^r (1,1) by *r*-order accumulation optimization. The overall procedure of the proposed approach is illustrated in Figure 1:

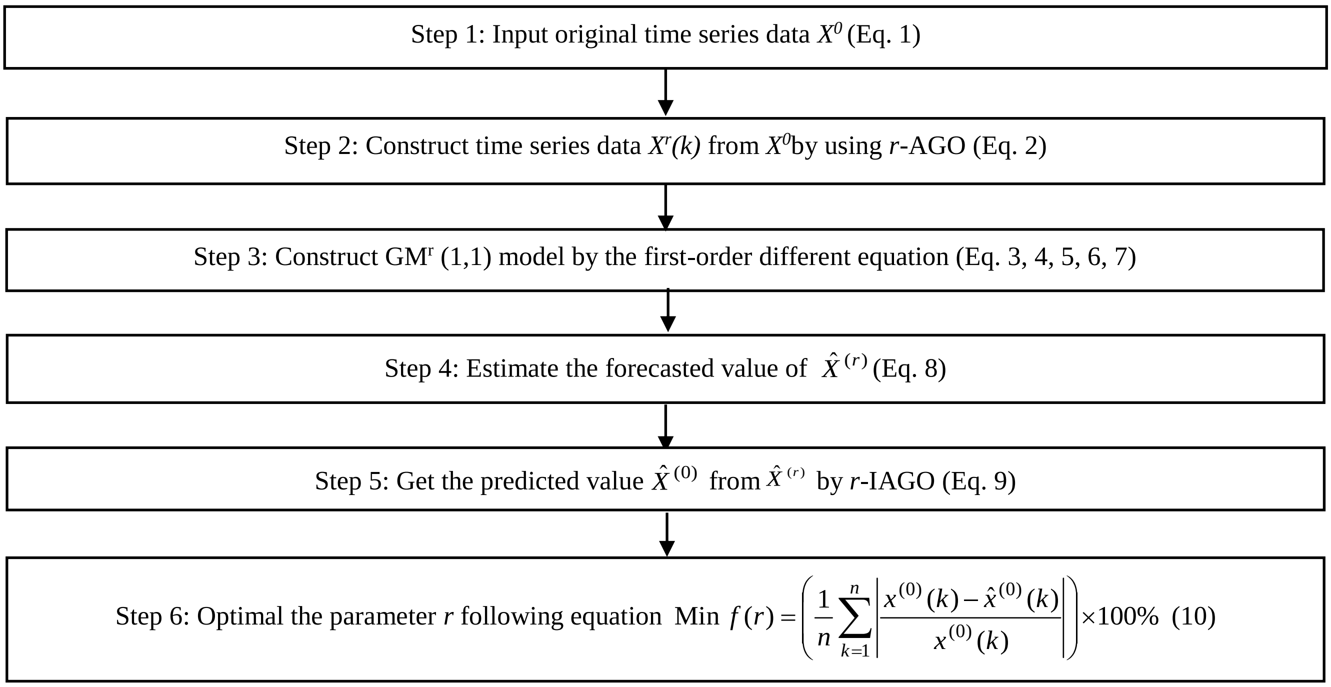

Fig. 1: The procedure of the proposed approach

2.3 Evaluation Indicators Indexes

In order to evaluate forecast results, the present study applied the MAPE (Mean Absolute Percentages Error) performance indicator which is expressed as follows:

$$MAPE = \frac{1}{n}\sum_{k=2}^{n}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\% \tag{11}$$

Where $x^{(0)}(k)$ and $\hat{x}^{(0)}(k)$ are actual and forecasting values at time k, respectively, and n is the total number of predictions.

3 Verification of the Proposed Approach

In order to illustrate the effectiveness of the proposed model, three cases in the research paper, [25], are given below:

3.1 Electricity Consumption Forecasting

The examples from papers, [18], and, [25], provide the sample data. As the same setting situation in Wu's paper, [25], the data from 2000 to 2003 (in-sample data) are utilized to forecast the value of 2004 to 2007, the results are listed in Table 1. For the electricity consumption in Russia, the three compared models yielded lower MAPE in out-of-sample (2004 to 2007). But the proposed approach provides the best among them with a MAPE is 1.42. This implies that the proposed approach has a strong forecasting performance. Furthermore, Figure 2 illustrates the relative between MAPE and the *r*-order accumulated generating operators.

Table 1. The actual, predicted values and MAPE index of different grey models

Year	Actual value	GM (1,1)	$GM^{0.01}(1,1)$	Proposed approach with *r=0.009*
2004	55,516	54,800	55,294	55,329
2005	55,898	55,431	56,278	56,379
2006	58,600	56,069	57,308	57,510
2007	60,281	56,714	58,386	58,726
MAPE (%)		3.10	1.61	**1.42**

For the case of forecasting the electricity consumption in Vietnam, as can be seen from Table 2, from a forecasting viewpoint, the proposed approach model is the best forecasting model compared the GM (1,1) and $GM^{0.05}$ (1,1). The average MAPE from the years 2004 to 2007 of the proposed approach is 0.6. This means that the proposed approach can significantly enhance the precision of the grey forecasting model. More specific are shown in Figure 3.

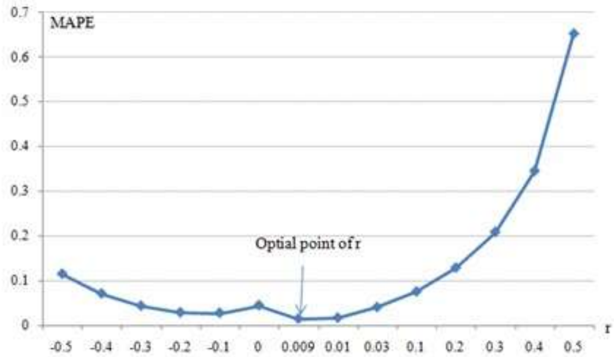

Fig. 2: The effectiveness between MAPE and *r-order*

Table 2. The actual, predicted values and three error indicators of different grey models

Year	Actual value	GM (1,1)	$GM^{0.05}(1,1)$	Proposed approach with *r=0.037*
2004	3,437	3,477	3,457	3,464
2005	3,967	4,042	3,971	3,994
2006	4,630	4,699	4,537	4,587
2007	5,256	5,462	5,163	5,255
MAPE (%)		2.16	1.13	**0.6**

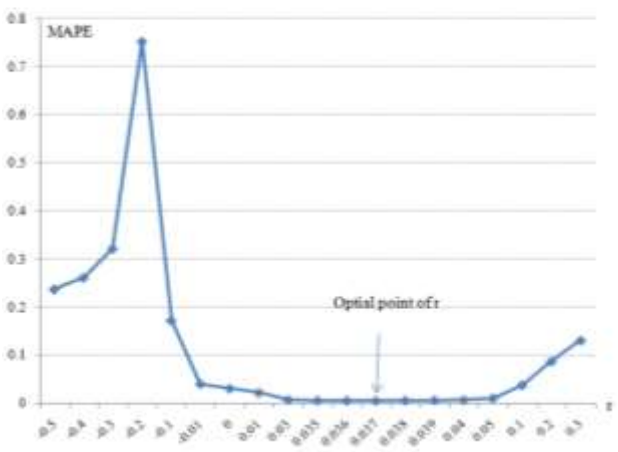

Fig. 3: The impact of *r-order* to MAPE

3.2 The Power Load of Hubei Province in China Forecasting

The sample from the paper, [25], is applied here to compare the precision, the history load data of the Hubei electric power network in China from the years 1996 to 2007 are used to verify the forecasting results. The power load for the year 2006 and 2007 are predicted based on the data before the year 2006. The performance of three compared models is shown in Table 3. As can be seen from Table 3, the MAPE of the proposed approach and the $GM^{0.01}$

(1,1) are the same. It means that the r = 0.01 in the GM^r (1,1) model is the optimal solution of the proposed approach (Figure 4).

Table 3. The fitted values and three error indicators of different models

Year	Actual value (10^4 kW)	GM (1,1)	GM$^{0.01}$(1,1)	Proposed approach with r=*0.01*
1996	425.38	425.38	425.38	425.38
1997	440.26	402.20	433.94	433.94
1998	457.24	434.89	448.05	448.05
1999	457.38	470.23	467.27	467.27
2000	503.02	508.44	492.49	492.49
2001	526.02	549.77	525.11	525.11
2002	561.96	594.44	567.06	567.06
2003	629.20	642.75	620.83	620.83
2004	700.21	694.99	689.63	689.63
2005	788.15	751.47	777.57	777.57
MAPE (%)		3.53	**1.29**	**1.29**
2006	876.76	812.54	889.88	889.88
2007	989.23	878.58	1033.26	1033.26
MAPE (%)		9.26	**2.97**	**2.97**

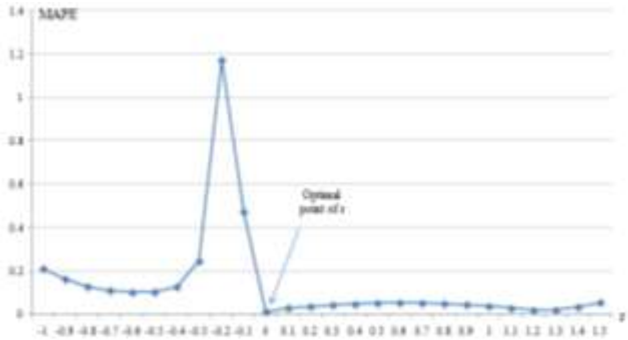

Fig. 4: The effectiveness between MAPE and *r-order*

3.3 Computer Industry Output Value Forecasting in Hsinchu Science Park

The example from a reference in, [25], provides the sample data, the data from the years 2001 to 2007 are used to construct three forecasting models. Actual values and simulative values of these compared models are presented in Table 4. As can be seen from Table 4, the proposed approach with r=0.75 yielded the lowest MAPE in sample data. This indicates that the proposed approach can improve the fitted error of the GM (1,1) with fractional order accumulation. More visualization of r- order accumulation optimization was shown in Figure 5.

Table 4. The fitted values and three error indicators of different models.

Year	Actual value	GM (1,1)	GM$^{0.8}$(1,1)	Proposed approach with r=*0.75*
2001	1,610.71	1,610.71	1,610.71	1,610.71
2002	1,245.28	1,363.88	1,342.76	1,340.37
2003	1,347.71	1,274.91	1,280.32	1,279.60
2004	1,382.45	1,191.73	1,204.58	1,205.14
2005	1,018.45	1,113.99	1,122.64	1,123.34
2006	1,014.96	1,041.31	1,040.18	1,040.47
2007	949.46	973.38	960.34	960.12
MAPE (%)		6.17	5.65	**5.64**

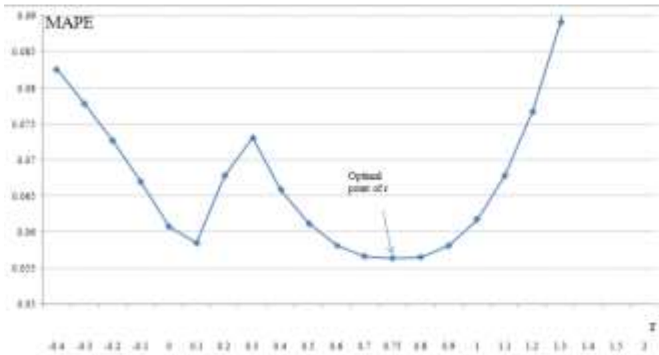

Fig. 5: The relative between MAPE and *r-order*

4 Application of Improved GM^r (1,1) in Forecast the Number of Tourist Visits to Quang Ninh Province

In order to help the policymaker have more efficient tools in formulating sustainable tourism industry development, this paper uses three models which are GM (1,1), GM^r (1,1) and optimization of GM^r (1,1) model to forecast the number of tourism visits to Quang Ninh province. Then we compare the accuracy performance of these models to find out the best model to suggest for forecasting in this case.

4.1 Data Collection

The number of tourism visited to Quang Ninh province from 2011- 2021 were obtained from the report of Department of Tourism, Quang Ninh province, [29], and are shown in Table 5.

Table 5 shows that the number of tourist visits to Quang Ninh province significantly dropped from 14,005,090 people in 2019 to 8,843,000 people in 2020 and continues to fall to 4,830,000 people in 2021. The main cause of this decrease is due to the COVID-19 pandemic. Therefore, the number of tourists coming to Quang Ninh has decreased.

Table 5. The total number of tourists visit to Quang Ninh province during period time 2011 to 2022.

Year	Total number of tourists
2011	6,459,000
2012	7,000,000
2013	7,500,000
2014	7,500,000
2015	8,600,000
2016	8,350,000
2017	9,800,000
2018	12,246,000
2019	14,005,090
2020	8,843,000
2021	4,830,000
2022	11,589,000

4.2 Forecasting Models: Testing and Results

In this case, three forecasting models which are traditional GM (1,1), GM^r (1,1) and the proposed approach were used to forecast the number of tourism visits to Quang Ninh province. Then we compare these models to find the best model based on the MAPE index. For the best model strongly suggest applying in this case.

4.2.1 The GM (1,1) Model

The GM (1,1) model is the basic grey forecasting model. It is constructed by the first-order differential model with one input variable. The overall modeling algorithm of the GM (1,1) forecasting model was shown in [21]. Based on the mathematical algorithm of GM (1,1) and the historical data in this case, the parameters were identified as a = -0.02834 and b = 7,590,603.794. The function of GM (1,1) for the number of tourists forecasting is as follows:

$$\hat{x}^{(1)}(k)=\left\lfloor\left(6459000+\frac{7590603.794}{0.02834}\right)e^{0.02834 k}-\frac{7590603.794}{0.02834}\right\rfloor$$

with k=1,2,.....n

All forecasted values and the average MAPE index of the GM (1,1) model are recorded in Table 6. Table 6 shows the performance accuracy of the forecasted value of the number of tourists coming to Quang Ninh by using GM (1,1) with an average of MAPE = 20.427 %.

Table 6. Actual and forecasted value of forecasting models

Actual value	Forecasted value		
	GM (1,1)	GM$^{1.01}$ (1,1)	GM$^{1.029}$ (1,1)
6,459,000	6,459,000	6,459,000	6,459,000
7,000,000	7,699,208.566	7,646,065.689	7,542,921.217
7,500,000	7,920,571.674	7,817,696.142	7,620,177.084
7,500,000	8,148,299.283	8,010,307.708	7,747,608.097
8,600,000	8,382,574.382	8,215,457.149	7,899,337.94
8,350,000	8,623,585.221	8,430,446.6	8,066,989.065
9,800,000	8,871,525.462	8,654,122.258	8,246,792.598
12,246,000	9,126,594.334	8,885,938.281	8,436,771.447
14,005,090	9,388,996.796	9,125,640.063	8,635,795.999
8,843,000	9,658,943.699	9,373,131.559	8,843,190.817
4,830,000	9,936,651.956	9,628,411.584	9,058,546.502
11,589,000	10,222,344.72	9,891,540.14	9,281,620.43
MAPE (%)	20.427	19.969	19.722

4.2.2 The GM^r (1,1) Model

Follow by the mathematical algorithm of GM^r (1,1) was detailed in section 2 and the real data of the tourist number visits to Quang Ninh province. The r-order accumulated generating operation, coefficient parameters a and b are defined as: r= 1.01, a= -0.0276, b= 7,604,573.817. The function of GM^r (1,1) for the number of tourists forecasting is as follows:

$$\hat{x}^{(1.01)}(k)=\left\lfloor\left(6459000^{(1.01)}+\frac{7604573.871}{0.0276}\right)e^{0.0276 k}-\frac{7604573.871}{0.0276}\right\rfloor$$

with k=1,2,.....n

All forecasted values and the average MAPE index of the GM$^{1.01}$ (1,1) model are recorded in Table 6. Table 6 showed the accuracy performance of GM$^{1.01}$ (1,1) is higher than GM (1,1) with the MAPE index decreasing from 20.437 % to 19.969 %.

4.2.3 Optimization of GMr (1,1) Model

As the same calculation of parameter of GM^r (1,1) model and based on the *Eq.(10)* we can get the value of parameters are r= 1.029, a= -0.0263, b= 7,630,772.821.

The function of optimization of GM^r (1,1) for the number of tourist forecasting is as follows:

$$\hat{x}^{(1.029)}(k)=\left\lfloor\left(6459000^{(1.029)}+\frac{7630772.821}{0.0263}\right)e^{0.0263 k}-\frac{7630772.821}{0.0263}\right\rfloor$$

with k=1,2,.....n

All forecasted values and the average MAPE index of the optimization of GM^r (1,1) model with r=0.1029 are recorded in Table 6. Table 6 shows the performance accuracy of the forecasted value of the number of tourists coming to Quang Ninh using $GM^{1.029}$ (1,1) with the MAPE =19.722 %.

4.3 Implication

Based on the empirical analysis, Table 6 represents that the optimization of $GM^{1.029}$ (1,1) is better than the other compared models with the lowest MAPE index. Therefore, the optimization of $GM^{1.029}$ (1,1) was strongly suggested for estimation in this case. The forecasted value of tourists coming to Quang Ninh province updated to the years 2023 and 2024 is shown in Table 7.

Table 7. Forecasted value of the number of tourists coming to Quang Ninh by optimization of $GM^{1.029}$ (1,1)

Year	Forecasted
2023	9,512,280.386
2024	9,750,470.664

Table 7 shows that the forecasted values in 2023 and 2024 will be over 9,512,000 and 9,750,000 people, respectively. This figure indicates that the number of tourist visits to Quang Ninh will slightly decrease compared to the year 2022, but will significantly increase in the year 2024. This empirical result will provide a basic scenario for the policymaker to make a good decision in the future regarding tourism industry management and building the developing strategy.

5 Conclusions

Nowadays, the accuracy of forecasting toolkits is becoming more and more important in supporting policymakers in making the right decisions in the future. To fill in this gap, this paper provides a new systematic approach based on the optimization of *r*-accumulated generation operation in *(Eq. 10)* aiming to improve the predictive accuracy of GM^r (1,1). Via simulation in three cases and the real case in forecasting the number of tourists visiting Quang Ninh, the empirical results demonstrate that the proposed systematic approach can significantly improve the precision of the GM^r (1,1) in all cases. In the future direction, the proposed model can be applied to forecast the performance of other industries.

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References:

[1] Deng J. L. (1986). Grey prediction and decision. *Huazhong University of Science and Technology*, Wuhan, China.

[2] Deng J. L. (2002). Solution of grey differential equation for GM (1, 1| τ, r) in matrix train. *Journal of Grey System*, vol. 13, pp. 105-110.

[3] Deng J. L. (1982). Control problems of grey systems. *Systems & Control Letters*, vol. 1, no. 5, pp. 288-294.

[4] Lin Y and Liu S. (2004). A historical introduction to grey systems theory. *2004 IEEE International Conference on Systems*, Man and Cybernetics, vol. 3, pp. 2403-2408.

[5] Liu S., Jeffrey F and Yang Y. (2012). A brief introduction to grey systems theory. *Grey Systems: Theory and Application*, no. 2, pp. 89-104.

[6] Huang Y. L and Lee Y. H. (2011). Accurately forecasting model for the stochastic volatility data in tourism demand. *Modern Economy*, vol. 2, no. 5, pp. 823-829.

[7] Phan P. V. (2015). Enhanced Accuracy of Grey Forecasting Model Case by tourism industry in Vietnam. *Hue University Journal of Science: Economics and Development*, vol. 113, no. 14, pp.157-167.

[8] Jiang F and Lei K. (2009). Grey prediction of port cargo throughput based on GM (1,1, a) model. *Logistics Technology*, vol. 9, pp. 68-70.

[9] Guo, Z. J., Song X. Q and Ye J. (2005). A Verhulst model on time series error corrected for port throughput forecasting. *Journal of the Eastern Asia Society for Transportation Studies*, vol. 6, pp. 881-891.

[10] Lu I. J., Lewis C and Lin S. J. (2009). The forecast of motor vehicle, energy demand and CO_2 emission from Taiwan's road transportation sector. *Energy Policy*, vol. 37, no. 8, pp. 2952-2961.

[11] Kayacan E., Ulutas B and Kaynak O.(2010). Grey system theory-based models in time series prediction. *Expert Systems with Applications*, vol. 37, pp. 1784-1789.

[12] Askari M and Askari H. (2011). Time series grey system prediction-based models: gold price forecasting. *Trends in Applied Sciences Research*, vol. 6, pp. 1287-1292.

[13] Wang C. N and Phan V. T (2014). Enhancing the accurate of grey prediction for GDP growth rate in Vietnam. *2014 IEEE International Symposium on Computer, Consumer and Control*, pp. 1137-1139.

[14] Tsai L.C and Yu Y. S. (2004). Forecast of the output value of Taiwan's IC industry using the grey forecasting model. *International Journal of Computer Applications in Technology*, vol. 19, no. 1, pp. 23- 27.
[15] Hsu L. C. (2003). Applying the grey prediction model to the global integrated circuit industry. *Technological Forecasting & Social Change*, vol. 70, pp. 563-574.
[16] Hsu L. C., (2010). A genetic algorithm based nonlinear grey Bernoulli model for output forecasting in integrated circuit industry. *Expert Systems with Applications*, vol. 37, no. 6, pp. 4318-4323.
[17] Hsu L. C. (2011). Using improved grey forecasting models to forecast the output of opto-electronics industry. *Expert Systems with Applications*, vol. 38, no. 11, pp. 13879-13885.
[18] Li D. C., Chang C. J., Chen C. C and Chen W. C. (2012). Forecasting short-term electricity consumption using the adaptive grey-based approach-an Asian case. *Omega*, vol. 40, pp. 767-773.
[19] Kang J and Zhao H. (2012). Application of improved grey model in long-term load forecasting of power engineering. *Systems Engineering Procedia*, vol. 3, pp. 85-91.
[20] Lin Y. H., Chiu C. C., Lee P. C and Lin Y. J. (2012). Applying fuzzy grey modification model on inflow forecasting. *Engineering Applications of Artificial Intelligence*, vol. 25, no. 4, pp. 734-743.
[21] Wang Z. X., Dang Y. G and Liu S. F. (2007). The optimization of background value in GM (1,1) model. *Journal of Grey System*, vol. 10, no. 2, pp. 69-74.
[22] Wang C. H and Hsu L. C. (2008). Using genetic algorithms grey theory to forecast high technology industrial output. *Applied Mathematics and Computation*, vol. 195, pp. 256-263.
[23] Dong S., Chi K., Zhang Q. Y and Zhang X. D (2012). The application of a grey Markov model to forecasting annual maximum water levels at hydrological stations. *Journal of Ocean University of China*, vol. 11, no. 1, pp. 13-17.
[24] Hsu Y. T., Liu M. C., Yeh J and Hung H. F. (2009). Forecasting the turning time of stock market based on Markov-Fourier grey model. *Expert Systems with Applications*, vol. 36, no. 4, pp. 8597-8603.
[25] Wu L., Liu S., Fang Z and Xu H. (2015). Properties of the GM (1,1) with fractional order accumulation. *Applied Mathematics and Computation*, vol. 252, pp. 287-293.
[26] Wu L., Liu S., Yao L.,Yan S and Liu D. (2013). Grey system model with the fractional order accumulation, *Communications in Nonlinear Science and Numerical Simulation*, vol. 18, no. 7, pp. 1775-1785.
[27] Wu L. (2016). Using fractional GM (1,1) model to predict the life of complex equipment. *Grey Systems: Theory and Application*, vol. 6, no. 1, pp. 32-40.
[28] Shili F., Wu L., Yan L and Zhigeng F. (2013). Using fractional GM (1,1) model to predict maintenance cost of weapon system, In *Proceedings of IEEE International Conference on Grey systems and Intelligent Services (GSIS)*, pp. 177-181.
[29] The report of Department of Tourism, Quang Ninh province 2023, [Online]. https://bvhttdl.gov.vn/quang-ninh-phuc-hoi-va-phat-trien-ben-vung-nganh-du-lich-20230811100611591.htm (Accessed Date: 20, August, 2023).

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Conflict of Interest

The authors declare no conflicts of interest.