Quantitative Management of Business Disbursements by Portfolio Optimization

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Abstract: - The paper aims to solve the problem of reallocating financial payments between disbursements and outflows by increasing the efficiency of working capital. The reallocation of financial payments is a different case from the reallocation of investments. The disbursements can be regarded as a set of assets, which generate negative returns in comparison with the assets in the portfolio theory. The purpose of this study is to derive a formal model, which gives quantitative solutions for the reallocation of resources between disbursements. Thus, the disbursements can indirectly influence positively the business profit of an economic entity. The reallocation of payments between a set of disbursements can improve the financial outcome of a business entity. Such redistribution plays an important role in the business management of the manufacturing units. The paper derives a quantitative model for the assessment and decision-making payment redistribution between payments. The quantitative solution is based on the application of a portfolio model. The latter is modified by minimizing disbursements in the portfolio problem. The empirical application of this model is applied to dairy farm payment management cases. Comparisons applied to the model with the actual set of payments show that the derived model is better at reducing the total values of the disbursements.

Key-Words: - management of cash flows, disbursement minimization, portfolio theory, decision-making, maximal return, inventory costs

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1 Introduction

A disbursement is a payment event for a specific resource and/or service. For a business and production entity, it is mandatory to maintain and condition the normal rules for production results. Disbursements are an important part of cash outflows that can be claimed over a short time horizon. This makes it important for business management to plan and consider optimally the allocation of inflows or available cash to disbursement requests. Disbursements are considered negative flows for the financial balance of the business. Negative complementarity of payments added to financial flows can lead to losses, cash shortages, and a lack of profit. Thus, payments are seen as an important prerequisite for effective business management. Disbursement management is considered a prerequisite for maintaining the liquidity of small and medium-sized enterprises, [1]. Managing financial flows is not easy, because the production and sale of products do not give immediate cash flows, [2]. Made disbursements are critical for supply chain management, [3].

Thus, disbursements to maintain a continuous supply chain are not always eligible due to the difference between the timing of cash inflows and outflows. This delay can reduce the working capital. Payment scheduling solutions are one possible way to increase working capital productivity. An alternative solution to the scheduling is the reallocation of financial resources during the year, [4]. In, [5], are considered cases that deal with the reallocation of resources in business entities. The reallocation of financial resources is frequently applied for investment by means to achieve high returns and low risk following the dynamic changes in market behavior, [6]. This means that the target is to obtain positive profit from the reallocation.

The case of disbursements and their reallocation not only directly influence the business profit. They are related to the production program of the business entity and they cannot be overcome or ignored, [7]. Thus, payment and disbursements have to be considered as a "necessary evil" for the normal production operation. The management of the disbursement has an important role in business decision-making.

Payment and disbursement optimization is targeted by developing quantitative and optimization models. The need for optimal management of financial disbursements insists definition of optimization problems for cost accounting, [8]. The reasons for the usage of quantitative decision-making are motivated in [9]. In, [10], the reduction and reallocation of payment resources are based on the definition and solution of an optimization problem. The latter is based on formal modeling from network graph theories. The issue allows for an increase in payment transactions and, in general, this improves the competitiveness, well-being, and production activities of the economic entity. The problem of covering payout payments in case of resource constraints is solved by applying a simulated annealing algorithm. Backward scheduling is implemented to complete payments and maximize payment flow.

This study applies the redistribution of financial resources between material resources and services that are carried out for the production policy of the economic entity. The added value of this research is that the redistribution is performed not for financial investments but for disbursements, which can be regarded as inverse tools, which decrease the business profit. Redistribution is carried out in the absence of additional resources other than those of investments, which gives benefits to the business management. The redistribution is carried out with means to preserve the production profit. Redistribution is quantified by defining and solving an optimization problem. The problem is based on the portfolio theory, but the problem is a modification of the portfolio problems. The modification is necessary because portfolio theory reallocates investment resources to obtain maximum returns. In our case, we seek to redistribute payments by reducing their total value, but maintaining a high level of production profit. The problem gives optimal values of resource allocation. The derived problem is applied to the case of animal husbandry with the production of milk and milk products.

The paper contains five sections. The introduction tells the negative role of the disbursements on the financial balance. The second section motivates the opportunity for usage of the formal portfolio problem to disbursement management. Correspondence between the portfolio modeling and disbursement management formalized inappropriate optimization problems. In section three the defined problem for disbursement reallocation is applied to the case of animal husbandry, in which accounting data are taken from their financial records. Section four derives a business management policy for the reallocation of disbursements. This policy is empirically applied to the accounting data of husbandry. The final section makes a comparison and conclusion that the derived formal problem for disbursement reallocation gives benefit for the business profit of animal husbandry. Further trends of improvements are mentioned for the integration of the disbursements with the production of the husbandry.

2 Portfolio Problem for Disbursement Reallocations

Portfolio theory is a powerful basis for the optimal reallocation of financial resources for investment. The practical problem at which portfolio optimization is directed is to estimate the shares of the investment that should be invested in various assets to achieve future maximization of returns and to maintain low or minimal risk to the investment. Portfolio theory has its achievements and proper applications, [11]. Trends and improvements are discussed in, [12]. An overview of applied optimization technologies can be found in, [13]. Portfolio theory raises two important criteria for optimizing investments: risk and return. Risk is defined as a range of real portfolio return values that are close to the portfolio's average return. This range is quantified by the standard deviation of the portfolio's real returns that can occur around the mean. The risk and return of a portfolio are functions of the respective risks and returns of the assets participating in the portfolio. The inputs to the portfolio problem are the historically obtained returns on the assets that are commonly traded on the exchange

$$\mathbf{R_i} = (R_i^{(1)}, R_i^{(2)}, \dots, R_i^{(j)}), i = 1, \dots, N; j = 1, \dots, n\,, \tag{1}$$

where N is the number of assets in the portfolio,

n is the number of historical returns,

$R_i^{(j)}$ is the real return on asset i at time j.

The average return $\mathbf{R_i}$ of asset, i is the average sum

$$E_i = \frac{1}{n}\sum_{k=1}^{n} R_i^{(k)}, \quad \mathrm{E}=(E_1, \dots, E_{N)}. \tag{2}$$

The risk of the asset t is quantified by the standard deviation σ_i of the series of values $\mathbf{R_i}$ or

$$\sigma_i^2 = \frac{1}{n}\sum_{k=1}^{n} \left(E_i - R_i^{(k)}\right)^2, i = 1, \dots N. \quad (3)$$

Formally, the portfolio return E_p is the weighted sum of asset returns

$$E_p = \sum_{i=1}^{N} w_i E_i \text{ ι=1,...,N, } \omega = (w_1, \dots, w_N), \quad (4)$$

where $w_i, i = 1, \dots, N$ is the relative amount of the investment that is allocated to purchase asset *i*.

Portfolio returns are a bit more complicated to estimate. It depends not only on the risks of the individual assets σ_i , but takes into account the influence of the covariance between the time series of each pair of assets

$$\sigma_{ij} = \frac{1}{n-1}\sum_{m=1}^{n} \left(R_i^{(m)} - E_i\right)\left(R_j^{(m)} - E_j\right) \quad \forall i, j = 1, \dots, N. \quad (5)$$

These values can be positive and negative depending on whether the time series of asset returns is increasing or decreasing simultaneously for a positive covariance or vice versa for a negative one. The set of covariance coefficients forms the covariance matrix $\boldsymbol{\Sigma}$ **,** which is symmetric, $\sigma_{ij} = \sigma_{ji}$ and its diagonal contains the variances σ_i^2 of the asset returns

$$\boldsymbol{\Sigma} = \begin{vmatrix} \sigma_{11} = \sigma_1^2 \dots & \sigma_{1N} \\ \dots & \dots \\ \sigma_{N1} \dots & \sigma_{NN} = \sigma_N^2 \end{vmatrix} \quad (6)$$

The portfolio risk σ_p is estimated as a quadratic form

$$\sigma_p^2 = w^T \boldsymbol{\Sigma} w \quad (7)$$

Relations (4) and (7) quantify the values of portfolio risk and return depending on the same parameters for the assets and their relative participation in the portfolio.

The portfolio problem is defined as minimizing portfolio risk and maximizing portfolio return, [14].

$$\min_{w} \{\tfrac{1}{2}\lambda\, w^T \boldsymbol{\Sigma}\, w - (1-\lambda)\mathbf{E}^T w\} \quad (8)$$

$$\sum_{i=1}^{N} w_i = 1, \qquad w_i \geq 0, i{=}1,N,$$

where the coefficient $0 \leq \lambda \leq 1$ gives the weighting preferences in the risk or return objective function of the portfolio. The coefficient λ must be determined by the investor, depending on his intention to take or not risk. To overcome this subjective choice, the broad practice is to choose such a value of λ^* that gives the minimum relation.

$$\textbf{Sharpe ration} = \min_{\lambda} \frac{\mathbf{w^T}(\lambda^*)\boldsymbol{\Sigma}\mathbf{w}(\lambda^*)}{\mathbf{E^T}\mathbf{w}(\lambda^*)} \quad (9)$$

This research applies the methodological formalization of the portfolio theory but for a different set of assets. This set is constituted of disbursements, which play an opposite role in the portfolio return. The innovativeness of this approach is that it can formalize an optimization problem in the reallocation of the disbursements. A particular case is that this reallocation can be provided without additional financial resources. Thus quantitative solutions in an optimal manner can be evaluated for the reallocation of the disbursement. The formal background for the definition of the optimization problem is motivated below, based on the portfolio theory.

These general relations (1)-(9) of portfolio theory apply to the case of redistribution of payments. Historical data is assumed to be available for the various payment categories

$$\mathbf{D_i} = (D_i^{(1)}, D_i^{(2)}, \dots, D), i = 1, \dots, N; j = 1, \dots, n \quad , \quad (10)$$

where N is the different payout category and n is historical data available for them. The corresponding mean and covariance matrix are calculated according to (4-6).

$$DE_i = \frac{1}{n}\sum_{k=1}^{n} D_i^{(k)}, \quad \mathbf{\Delta E} = (DE_1, \dots, DE_N) \quad (11)$$

$$d\sigma_{ij} = \frac{1}{n-1}\sum_{m=1}^{n} \left(D_i^{(m)} - DE_i\right)\left(RD_j^{(m)} - DE_j\right), \quad \forall i, j = 1, \dots, N.$$

$$\boldsymbol{D\Sigma} = \begin{vmatrix} d\sigma_{11} = d\sigma_1^2 \dots & d\sigma_{1N} \\ \dots & \dots \\ d\sigma_{N1} \dots & d\sigma_{NN} = d\sigma_N^2 \end{vmatrix}.$$

The payment redistribution portfolio problem is of the form (8) but modified to minimize the two components of the mean and risk of the payments. For simplicity, we keep the notation for the weights by $\mathbf{w}$

$$\min_{w} \{\tfrac{1}{2}\lambda\, \mathbf{w^T D\Sigma\, w} + (1-\lambda)\mathbf{DE}^T\, \mathbf{w}\} \tag{12}$$

$$\sum_{i=1}^{N} w_i = 1\,, \qquad w_i \geq 0,\ i=1,N.$$

Problem (12) generates many solutions for different values of λ. To choose the optimal one, we define a criterion that corresponds to (9), but the point is to reduce both the portfolio payoff values $\mathbf{DE}^T\, w$ and the corresponding risk $w^T\mathbf{D\Sigma}\, w$. The new distribution criterion **REL** is defined as the minimum value of the sum

$$\min_{\boldsymbol{\lambda}} \{\mathbf{REL} = \mathbf{w(\lambda)^T D\Sigma w(\lambda)} + \mathbf{DE}^T\, \mathbf{w(\lambda)}\} \tag{13}$$

The solutions of (12) $\mathbf{w}^{opt}$ will give the relative mode of payments according to their total amount. For the case when
w_i≤0.5, payoff i should be increased by the value w_i. In the opposite case, when w_i≥ 0.5, the payoff i must decrease to w_i fraction of its current value.

Problem (12) gives a set of solutions for the disbursement reallocations, which are sensitive to the value of the parameter λ. The last gives a reference for the manager to choose this reallocation **w**, which minimizes the loss $\mathbf{DE}^T\,\mathbf{w}$ or the risk from the redistribution $\mathbf{w^T D\Sigma}$. The criteria (13) give unique solutions, which minimize both components of the goal function (12).

Ratios (12) and (13) are applied to estimate the costs of animal husbandry from the central region of Bulgaria.

3 Assessment of Disbursements in Animal Husbandry

The input data are taken from the 2021 ledger of a livestock farm. Three categories of payments are considered that have a relatively large impact on payments: electricity, staff wages, and fleet fuel. Payments are recorded for each month of the year and their values are given in Table 1.

Table 1. monthly disbursements per category

CATEGORY [BGN}	ELECTRICITY, D_1	SALARIES, D_2	FUEL, D_3	TOTAL
JAN	5940	1530	7354	14824
FEB	5290	1596	8354	15240
MARCH	6089	1558	8433	16080
APR	5549	1463	7362	14374
MAY	4858	1498	6602	12958
JUNE	5931	1447	7521	14899
JULY	8440	1395	8087	17922
AUG	7455	1153	7911	16519
SEPT	7302	1153	8905	17360
OCT	10617	1275	7688	19580
NOV	11643	1280	7049	19972
DEC	13688	1312	6996	21996

The means **DE** and covariance matrix of payments **DΣ** have values

DE = [0.0773 0.0139 0.0769]x10^5

DE = [8.0002 -0.2420 -0.4585;
-0.2420 0.0231 -0.0136;
-0.4585 -0.0136 0.4546] x10^6.

The solution of the modified portfolio problem (12) is solved for the parameter set $0 \leq \lambda \leq 1$. These solutions are represented graphically in Figure 1, where on the horizontal axis are the risk estimates $\mathbf{w^T D\Sigma\, w}$ and on the vertical axis are the payoffs $\mathbf{DE}^T\ \mathbf{w}$.

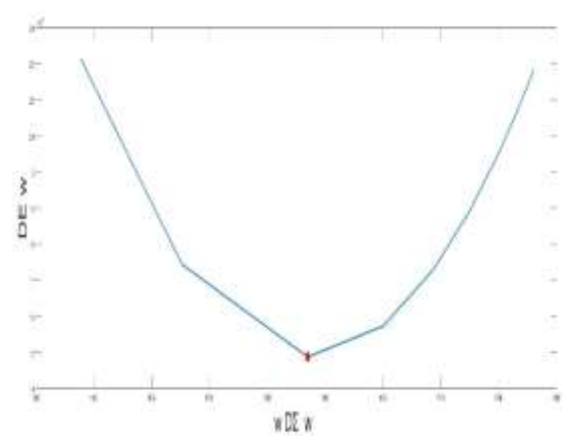

Fig. 1: Graphical representation of the results of the modified portfolio problem

Applying (13) to the choice of the minimum relation **REL**, the values are obtained:

$\mathbf{DE}^{min} = 1585$; $\mathbf{D\Sigma}^{min} = 14873$; $\mathbf{w}^{min} = [0.03; 0.97; 0]$.

The solution $\mathbf{w}^{min}$ says that payments D_1 should increase by 3%, D_2 should decrease to 97% of their current value, and D_3 should remain at their current value.

To verify the effectiveness of this payment adaptation and redistribution policy, we will apply a predictive control model approach and compare the results of applying the modified portfolio problem (12) and the evaluation criteria (13).

4 Implementing the Modified Portfolio Solution in Sliding Mode

The portfolio model is applied and evaluated with a sliding mode approach to evaluate portfolio decisions. The reallocation of the disbursements is made with the accounting data of previous months. Then, this reallocation is applied for the current month, for which the accounting data are not yet available. At the end of the current month, a comparison is made for the real obtained profit from the accounting records and the protentional profit, which the applied reallocation of disbursement would be obtained. The comparison assesses whether the results from the derived problem give benefits according to the real recorded state of payments. Hence this management policy performs adaptation and reallocation of disbursement funds. It is an example of the usage of problems (12) and (13) for the real management of the disbursements. Figure 2 illustrates the management approach.

Historical accounting data for three initial months are used for the evaluation of the reallocation of the disbursements. For this historical period, the parameters of the portfolio problem (12) are calculated for the average values of total payments **DE(**t_1**)** and the corresponding covariance matrix $\mathbf{D\Sigma}(t_1)$**.** The definition and solution of the portfolio problem and by applying the **REL** criteria of (13) give the minimal solution $\mathbf{DE}^{min}(t_1)$, $\mathbf{D\Sigma}^{min}(t_1)$, $\mathbf{w}^{min}(t_1)$. Since our main interest concerns the total payments for this period $\mathbf{DE}^{min}(t_1)$, this value is compared to the actual total payments **TOTAL**(t_1) for this period. Thus, through a comparison between the recommendations of the problem (12) and the real one, it will identify the benefit of the proposed management procedure.

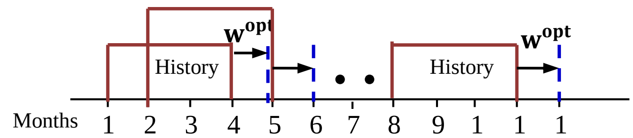

Fig. 2: A sliding mode procedure for evaluating modified portfolio solutions

The sliding procedure is applied from April until the end of the year, which gives nine values for comparison, $t = t_1,\dots,t_9$. The results of this comparison are presented graphically in Figure 3.

Current levels of total payments are higher on the schedules. The values recommended by the portfolio problem are lower, giving advantages to the derivative problem. The results of the portfolio problem are relatively constant. This means that the farm can plan approximately equal total resources for all its payments. Current payment behavior is incremental, requiring additional financial resources to always be planned to cover payments.

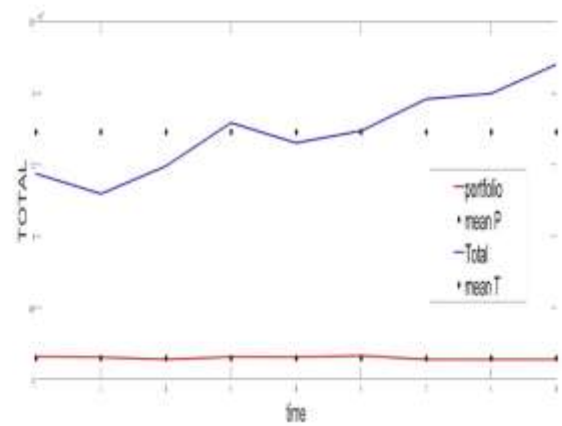

Fig. 3: Comparison between classical and optimization model payoff

5 Conclusions

This study derived a quantitative model for the redistribution of payments that a business entity must cover for its normal operations. Especially for the production case, the level and consistency of payments play an important role in maintaining the liquidity of the enterprise. A possible way to support liquidity is to synchronize income flows with outgoing financial flows. This means appropriate rejection of payments. Our approach was to redistribute payouts by timing and payout category. This is done by defining and solving a modified portfolio problem, which meaning is a reallocation of the financial resources between the disbursements. Such reallocation is performed without the need for additional financial resources for covering the disbursements. Thus, the disbursements indirectly influence the business return in its increase. An optimization problem was defined, in which formal relations are based on the portfolio theory. The problem applies new optimization criteria for the selection of a unique solution to the portfolio problem. It is aimed at minimizing the sum of the average value of the total payments as their potential risk. This risk indicates a relationship for an increase or decrease in a mutually applied payment. Through the defined problem, the reallocation of payments between the different categories of payments is evaluated numerically. The problem is defined and solved with real data from the accounting documentation of the animal breeding activity with the production of milk and milk products. Empirical comparisons between the results of the defined problem and the real records of livestock activities give preference to the derived model. A promising trend and improvement of this quantitative expression can be found by integrating both payment tasks and livestock production. In this way, payments can be closely linked to production volumes and financial flows.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed to the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The authors have no conflicts of interest to declaree.