Impact of Tunisian Political and COVID-19 Crisis on Asset Allocation: Traditional Theory of Portfolio Selection Versus Behavioral Theory

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Abstract: - Investing in the financial market is a way to grow wealth. This investment undoubtedly generates a return accompanied by a certain level of risk. In finance, risk occupies a crucial place in the stock market. Indeed, it intervenes in the process of choice and selection of the portfolio. Investment decisions can be tricky from time to time and require further thought. The achievement of judicious investment is based on a knowledge of the financial market evolution, the behavior of investors as well as techniques of portfolio management. Multitudes of strategies have been implemented over time to effectively manage the portfolio. Within this framework, various strategies have been implemented such as modern portfolio theory (MPT) and behavioral portfolio theory (BPT). We concentrate on portfolio optimization for two alternative approaches: the MVT and the BPT. This study aims to compare portfolios generated by these two approaches during political and COVID-19 crisis periods using data from the Tunisian stock market exchange for the period 2009 –2022. The results show that in the case of a higher degree of risk aversion induced by investors' BPT, all the stock is located at the top right of the mean-variance frontier. However, during the crisis, the portfolios selected by rational investors were not systematically selected by irrational investors, even if the optimal portfolio of BPT coincides with the Markowitz efficiency frontier. The results indicate that the crisis induces simultaneously an increase in risk and a sharp decrease in the portfolio return of individuals who follow the mean-variance theory of Markowitz.

Key-Words: - Mean-Variance Theory, Behavioral Portfolio Theory, portfolio optimization, political crisis, COVID-19.

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1 Introduction

Investing in the financial market is a means used to build wealth. Successful investing relies on knowledge of financial industry developments, investor behavior, and portfolio management techniques. Multitudes of strategies have been implemented over time to effectively manage the portfolio. In this context, we cite the mean-variance theory (MVT), developed by [1], which was revolutionary in the world of portfolio management. However, some predictions made were invalidated when confronted with the reality of the market and thus opening the way to new explanations. The main criticism is the risk quantification method since it examines gains and losses similarly, as developed by: [2], [3], [4], [5], [6], [7], [8]. Also, they assume that the distribution of returns is normal as mentioned in [9], [10], [11], [12], [13]. To overcome this shortcoming, [14], was the first researcher to use probability as an alternative risk measure to that proposed by Markowitz which takes into account only negative downside risk deviations from a reference. But [14], does not specify how to allocate the remaining wealth once the level of substance is reached. To fill the gap in Roy's safety-first model, [15], added a criterion for arranging portfolios. This criterion is the expectation of final wealth or the portfolio return denoted. In addition, they defined a probability of satisfactory bankruptcy noted α by [16], as the probability that the subsistence threshold is not reached. Moreover, several, for example, [17], [18], have called into question the hypothesis of the rationality of investors. Therefore, the existence of investors did not behave rationally in the financial market; the utility function became dysfunctional, [19]. To deal with this difficulty, the managers have tried to highlight portfolio selection theories that appeal to investor behaviors. Among these new theories, we find the Behavioral Portfolio Theory (BPT), [20], which was established at the beginning of the nineties, by formulating new more realistic hypotheses for understanding financial behavior. In recent developments, the theory of Markowitz and that of [20], have become the central research hypotheses. This gives rise to a new flow of literature that attempts to compare the asset allocation generated by the BPT model with that generated by the MVT model. There are two opposite ways of literature. The first path is that of [21], [22], [23], [24], [25], [20], [26], who compared the efficiency frontier of [1], with the efficiency frontier of [20], and has shown that generally, these two borders do not occur simultaneously. These researchers have explained this discrepancy for several reasons. The explanation for this discrepancy was that mean-variance investors choose their portfolios, a combination of a market portfolio and risk-free assets, based solely on return and risk. In contrast, BPT investors base their portfolios, a mix of bonds and lottery tickets, on expected wealth, desire for security, and level of aspiration. This was asserted by [26], who proved that the optimal portfolio of a normal investor, who also considers the three dimensions of benefits, is lower than the optimal portfolio of a rational investor who ignores expressive and emotional benefits. Behavioral investors are willing to give up a certain portion of the expected return to gain expressive and emotional benefits. This is precisely why the optimal portfolio and the efficiency frontier of BPT are positioned below that of MPT. While the behavioral portfolio does not produce the highest utility benefits, it is optimal because it produces the highest overall benefits for normal investors. Furthermore, the optimal portfolio in terms of the theory of optimal portfolio diversification varies from one investor to another, depending on the investor's attitude towards risk, while the optimal portfolio in terms of BPT varies from investor to investor not only because of the different levels of risk tolerance but also because of the different wants, needs, biases, habits, preferences, and emotions of these investors.

The second path is that of [27], [28], [29], [30], as well as, [19], who have shown that certain characteristics of the theory of [20], and that, [1], almost coincide their distribution of assets. In addition, [21], [31], as well as, [32], [33], have proved that the efficiency frontiers resulting from BPT and that obtained within the framework of the mean-variance model coincide when the returns on assets are normally distributed. While the normality hypothesis is often accepted in the literature but not verified in real markets as shown in [34], [19], [35], [36]. Whereas, [37], [38], have shown that the optimization of the mean-variance portfolio and the behavioral portfolio, in the absence of probability distortion, produces very similar results in the presence of returns distributions that do not follow the normal law. Consequently, there are varieties of conclusions. Motivated by these studies, this research aims to compare portfolios generated by these two approaches during periods of political and COVID-19 crisis. The interest of this paper lies in the fact that it analyzes the effect of crises on the choice of portfolios in an emerging market such as the Tunisian financial market.

Through an empirical investigation, we use daily stock price data from the Tunisian stock exchange market «TUNINDEX» over a period extending from January 2009 to August 2022. In order, to study the effect of the 2011 political crisis and sanitary crisis, on the asset allocations generated by MVT and BPT, we have divided our study period into four sub-periods, before, during, and after the political crisis and the COVID-19 crisis.

The paper is organized as follows: Section 2 is a literature review of [1], [20] models. Section 3 provides the data and describes our methodology. Section 4 illustrates the results of this empirical study. Section 5 concludes with a summary of our findings.

2 Model

In this paper, we concentrate on portfolio optimization for two alternative approaches: the MVT developed by [1], and the BPT developed by [20].

2.1 The Mean-Variance Model

[1], mean-variance model is considered the backbone of the vast majority of portfolio optimization frameworks which continue to be widely applied in practice. Markowitz's model provided the first systematic treatment of the choice investors face: conflicting goals between having high profits and low risk. This optimization model stipulates that investors only use two specific parameters in their decision-making processes. These are the expected return and the standard deviation which is none other than the square root of the variance. [1], assumes that the investor seeks to minimize the risk of his portfolio for a given level of return. So, the formulation of Markowitz's, [1], optimization model is presented as follows:

$$Min\sigma^2(R_p) = \sum_{i=1}^{n}\sum_{j=1}^{n}\omega_i\omega_j\sigma_{ij}$$

$$S.cE(R_p) = \sum_{i=1}^{n} \omega_i E(R_i) \geq \mu_0 \qquad (1)$$

$$\sum_{i=1}^{n} \omega_i = 1 \,;\, \omega_i \geq 0 \; (i = 1, \dots, n)$$

Where $\sigma^2(R_p)$ symbolizes the variance of the portfolio, n represents the number of assets that make up the portfolio, ω_i and ω_j are respectively the weights of asset i and j in the portfolio, σ_{ij} is the covariance between the returns on assets $iandj$, $E(R_p)$ is the expected return on the portfolio, $E(R_i)$ is the expected return on asset i and μ_0 is the minimum return predetermined by the investor.

2.2 The Behavioral Portfolio Model

The perspective theory of [18], [39], is considered to be the basis for the development of Behavioral Portfolio Theory (BPT). BPT takes into account the fact that investors are not rational and assume two contradictory emotions, fear, and hope, which determine their portfolio choice. [20], proposed that there are two versions of portfolio management. The first version called the single mental account version (BPT-SA), applies the safety-first concept to the ideas of [39]. The second version called the multiple mental account version (BPT-MA), introduces another psychological bias known as mental accounting which was introduced by [40].

In our study, we use the BPT-SA since it integrates investor portfolios into a single mental account like mean-variance model investors. In BPT-SA, investors aim to maximize their final expected wealth while respecting their security constraints. Therefore, this optimization program is as follows:

$$\begin{gathered} MaxE_h(W) \\ s.cP(W \leq A) \leq \alpha \end{gathered} \qquad (2)$$

Where E_h is the expected final wealth of the investor, which is calculated by using the probabilities obtained by the transformation h, h is a transformation function of the probabilities, A is the aspiration level and α is the probability of eligible bankruptcy.

3 Data and Methodology

3.1 Data

The data set used in this article is made up of stocks listed on the "TUNINDEX" over the period from January 2009 to August 2022. TUNINDEX is a benchmark on the Tunis stock exchange, representative of the most capitalized and liquid stock market values. The analysis shows the evolution of the Tunisian stock index throughout the study. We note that the returns on the "TUNINDEX", during the period from 2010 to 2022, are more volatile compared to other periods. Indeed, this increased volatility is due to the impact of the crisis on the Tunisian stock market. It is essential to know the effect of the political crisis and the COVID-19 crisis on the return and the risk of the portfolio. To do this, we have divided our study period into four sub-periods, before, during, and after the political crisis and the COVID-19 crisis.

Therefore, we considered the first period from 02 January 2009 to 30 November 2010 as the pre-crisis period (480 observations of daily asset returns). The second period spanned from 1st December 2010 until 29 May 2015 as a crisis period (1107 observations of daily asset returns), the third period from 1st June 2015 to 22 August 2019 as a post-crisis period (1036 observations of daily returns on assets) and the fourth period from July 23, 2019, until August 12, 2022 (786 observations of daily asset returns).

To construct the sample for our study, we first examine the securities that make up the TUNINDEX index since 2009. Subsequently, we eliminate all the assets exiting the index during the period of analysis such as PALM BEACH and STIP. Therefore, our final sample contains 43 shares of 45 companies listed on the Tunis Stock Exchange (BVMT). Among the stocks selected are the largest market capitalizations such as BIAT, BT, POULINA GROUP HOLDING, and SFBT.

We start our empirical part by calculating the daily returns for our entire sample over the period (2009-2022). The daily returns asset i, during period t, was calculated as follows:

$$\mathrm{R_{i,t}} = \frac{\mathrm{P_{i,t}} - \mathrm{P_{i,t-1}} + \mathrm{D_{i,t}}}{\mathrm{P_{i,t-1}}} \qquad (3)$$

where $\mathrm{R_{i,t}}$ denotes the daily return of asset i for day t, and $\mathrm{P_{i,t-1}}$ are the asset prices respectively for day t and t-1 and $\mathrm{D_{i,t}}$ is the dividend paid by asset i during period t.

Then, we summarize in Table 1, for the different sub-periods, the maximum and minimum values of the main descriptive statistics for each security studied.

Table 1. Descriptive statistics for daily returns

	Pre-revolution	
	Return	Standard deviation
Maximum	0.027249 SOTUVER	0.570745 SOTUVER
Minimum	-0.001568 BT	0.009749 UIB
	Revolution	
	Return	Standard deviation
Maximum	0.001014 SFBT	0.029134 MAGASIN GENERAL
Minimum	-0.001548 SOTETEL	0.011601 ATTIJARI BANK
	Post-revolution	
	Return	Standard deviation
Maximum	0.002366 ICF	0.036412 STEQ
Minimum	-0.001319 GIF-FILTER	0.010093 UIB
	Covid-19	
	Return	Standard deviation
Maximum	0.049829 SOTUVER	0.775975 SOTUVER
Minimum	-0.004254 ALKIMIA	0.009718 SFBT

We find that, during the four periods studied, the values of Kurtosis were greater than three. This means that the distributions of returns on securities are sharper than the distribution of the normal distribution. As a result, the distributions of returns on the securities studied are sharper with thick tails on the left and the right. Consequently, the assumption of normality is rejected for Tunisian stock returns. To deal with this difficulty, we need to highlight the portfolio selection theories that appeal to investor behaviors. This trend is associated with what is now called behavioral finance. Behavioral models were subsequently developed. One of the best-known of these alternative models of portfolio management is that of [20]. In addition, the BPT model of [20], is found to be more adequate than the MVT model of [1], to describe the observed behavior in reality. For this, this theory is positioned as a real alternative to that of [1]. This extension has encouraged the emergence of numerous empirical studies. In this article, using an empirical study, we compare the choices of an investor following MVT with those of BPT.

3.2 Methodology

We assume an individual investor who is in a space where there are only 43 stocks. Operationally, the construction of Markowitz's, [1], optimal portfolio does not require complicated calculations and can be performed on Matlab software. This is not the case for an optimal portfolio derived from the BPT model of [20]. The more the number of securities in the portfolio is important, the more construction is heavy. To avoid this operational problem, we are reducing the number of securities making up the final portfolio. In other words, we assume that the number of securities identified by the investor cannot exceed a specific limit, and must be less than 43 securities. Given that the objective of our study was to compare Markowitz's, [1], model with that of [20], it is therefore essential that our portfolio be as diversified as possible. To achieve our goal, we followed the methodology of [41], which consists of going through the following three steps. The first step is to determine the optimal threshold for diversification. The second step is to estimate the annual returns using the Bootstrap method. The third step is to build a generation of 100,000 portfolios.

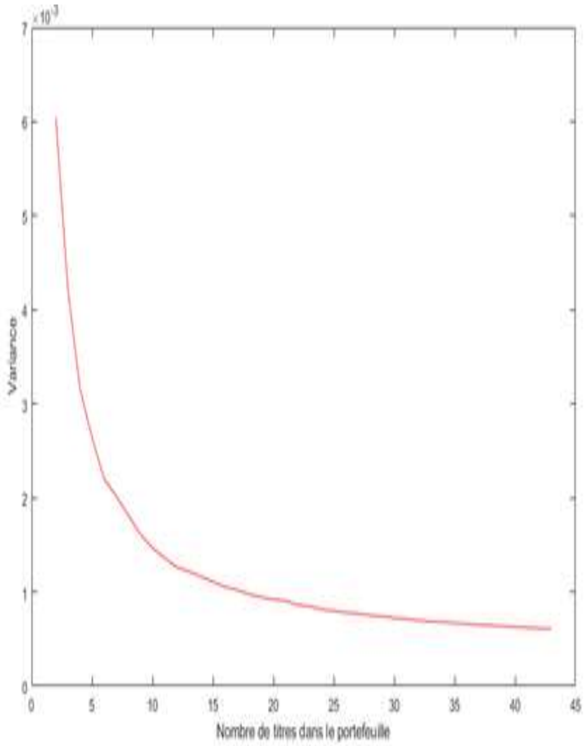
Step 1. Determination of the ideal number of assets

Among the key points of Markowitz's, [1], mean-variance model is portfolio diversification. According to the Markowitz principle of diversification, an optimal portfolio should consist of all the securities traded and available on the market. Several studies have been carried out to determine the ideal number of securities to construct the optimal threshold of diversification in terms of the mean-variance model. For example, [42], defined the most diversified portfolio possible as that which contains at least twenty securities.

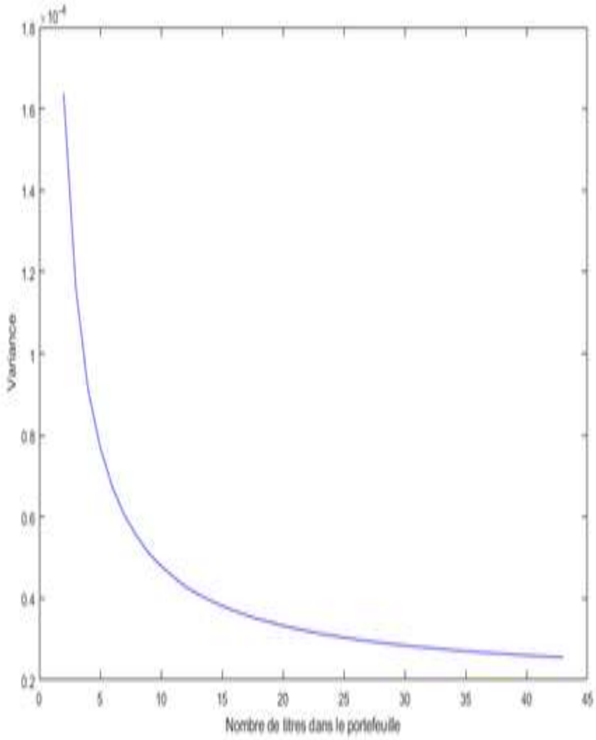
Whereas, [43], proposed that the number of securities in the portfolio is greater than thirty to achieve the optimal level of diversification. However, after 16 years, it has shown that the portfolio is as diversified as possible by one that contains several securities. The results show that their number is greater than 120. [44], have shown by a study carried out on 40,000 investors from 1991 to 1996, that the average number of securities that make up a portfolio is 4. For that, the principle enunciated by [1], is far from being applied in the field. [26], defined a well-diversified portfolio as one that generates at least 90% of the variance reduction.

In our study, we determine the optimal threshold for diversification by following the methodology of [41], which states that optimal diversification can be achieved with a limited number of securities. First, we randomly chose, among the 43 available stocks, the number of assets in each portfolio where n can take 2, 3 up to 43. Subsequently, we calculate for each value of n the average variance of 10,000 randomly constructed portfolios made up of n stocks of equal weights.

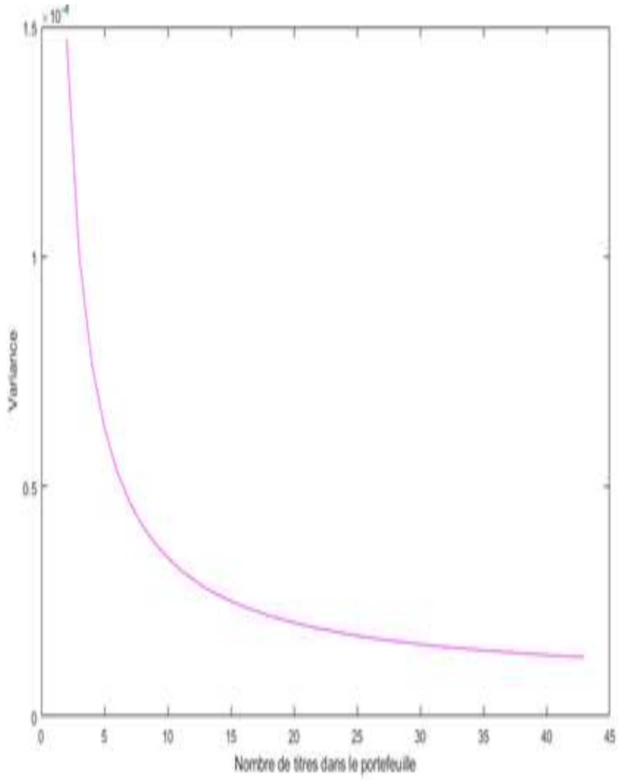
The results of diversification for the four sub-periods are presented in Figure 1.



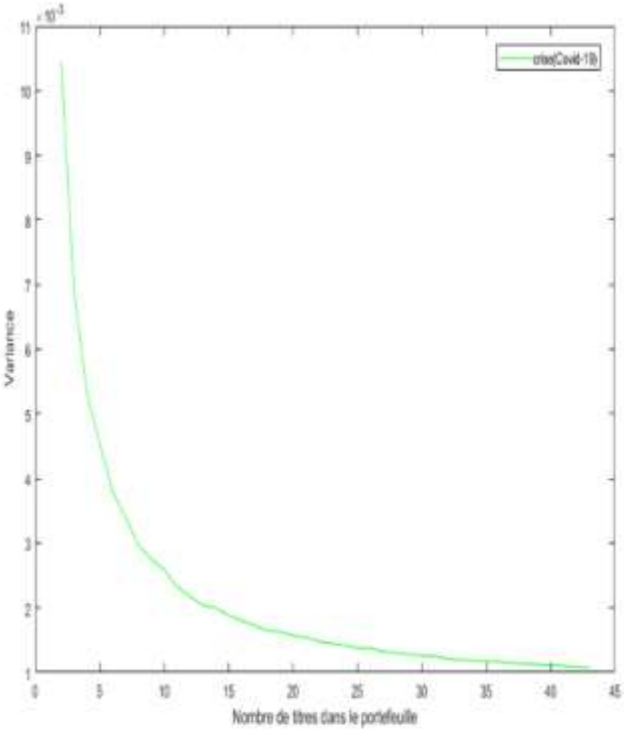
(a) —— Effect of diversification before the political crisis



(b) —— Effect of diversification during the political crisis



(c) —— Effect of diversification after the political crisis



(d) Effect of diversification during the Covid-19

Fig. 1: Effect of diversification

We note that for the three sub-periods the variance of the portfolio which contains all the securities available, the minimum variance, amounts respectively to 6.0680×10^{-4}, 2.5553×10^{-5} and 1.2833×10^{-5} while the variance of the portfolio which consists of only two securities, the maximum variance, amounts to 0.0061, 1.6335×10^{-4} and 1.4731×10^{-4} respectively. We find that the more diversified the portfolio, the lower its risk exposure. This conclusion was affirmed by Table 2.

Table 2. Cumulative proportion of variance reduction

	The pre - Crisis	Crisis	The post-crisis	Covid-19
2	0	0	0	0
3	0,33	0,34	0,35	0.38
4	0,53	0,52	0,52	0.55
5	0,62	0,62	0,62	0.63
6	0,70	0,69	0,69	0.71
7	0,74	0,74	0,74	0.75
8	0,77	0,78	0,78	0.80
9	0,815	0,815	0,815	0.82
10	0,84	0,83	0,83	0.84
11	0,861	0,857	0,858	0.87
12	0,879	0,874	0,873	0.87
13	0,887	0,887	0,887	0.88
14	0,897	0,898	0,899	0.9
15	0,908	0,908	0,909	0.9
20	0,942	0,943	0,943	0.94
30	0,978	0,978	0,978	0.98
40	0,996	0,996	0,996	0.99
43	1	1	1	1

Table 2 shows that a large number of stocks in the portfolio results in a large decrease in variance. In addition, the risk is reduced when the wealth of investors is distributed equally over several assets. Furthermore, we find in the three periods, that the variance of the portfolio decreases by more than 90% when the portfolio contains 15 stocks. We assume that this number of securities in the portfolio allows sufficient diversification throughout the study period.

Step2. Estimation of annual returns using the bootstrap method

After having fixed the number of securities in the portfolio, we will move on to determining the method used to choose them. In the field, investor preference from one security to another is influenced by several factors. In order not to favor one factor over another, and to rule out any prejudice on the choice of the 15 titles, we choose them randomly from our database. Subsequently, we create the matrix R_J (J = 1, 2, 3, and 4) that contains the T daily returns of the 15 randomly chosen assets, where N is the number of observations for each period (N= 480, 1107, 1036 and 786). Then, we try to establish a series of annual returns via the Bootstrap method initiated by B. Efron in 1979 from our series of daily returns. R_J is given by

$$R_J = \begin{bmatrix} R_{1.1} & \cdots & R_{1,43} \\ \vdots & \ddots & \vdots \\ R_{N,1} & \cdots & R_{N,43} \end{bmatrix}$$

According to [45], a year is on average made up of 250 trading days. We randomly select a line, denoted i, from our matrix R_J which corresponds to the moment when the investor builds, from his initial wealth, his portfolio. Therefore, the first Bootstrap sample is the 250 randomly sampled daily returns preceding line i. Then, we take the sum of the 250 daily returns to calculate the annual returns for each of the 15 securities selected. We repeat this process 1000 times to obtain the 1000 states of nature of 15 assets.

Finally, we obtain a matrix, denoted R_1^*, of dimension 1000 × 15 of the probable annual returns for the 15 securities previously chosen in the first step. For the other matrices (R_2^*, $R_3^*$and R_4^*) we carried out the same work as that carried out on R_1^*.

$$R_J^* = \begin{bmatrix} r_{1,1} & \cdots & r_{1,15} \\ \vdots & \ddots & \vdots \\ r_{1000,1} & \cdots & r_{1000,15} \end{bmatrix}$$

Step3. Generation of 100000 portfolios

The method of selecting securities and distributing wealth is not always fixed: in some cases, the investor may decide to invest all of his fortunes in just one asset. Or, he may decide to invest in specific securities and not invest in others. In addition, when building their portfolio, it does not necessarily share their wealth between securities equally. Therefore, it is necessary to take these variable criteria into account. According to our study, we have grouped the portfolios according to the number of securities that compose them. As a result, we then have 15 groups: the first group only includes portfolios made up of a single security. The second group contains portfolios made up of two securities. The third group contains portfolios with three titles and so on. Then, we determine the portion invested in each security, we use a step of 1/15. This means that this part can take proportions equal to 0, 1/15, 2/15, 3/15, ..., 14/15, or 1. After taking into account all the possibilities the total number of portfolios amounts to 77,558,760 as shown in Table 3.

Table 3. Distribution of portfolios in group

Groupe g	$\binom{15}{g} \times m$	**Portfolio number**
1	15 x 1	15
2	105 x 14	1470
3	455x91	41405
4	1365x364	496860
5	3003x1001	3006003
6	5005x2002	10020010
7	6435x3003	19324305
8	6435x3432	22084920
9	5005x3003	15030015
10	3003x2002	6012006
11	1365x1001	1366365
12	455x364	165620
13	105x91	9555
14	15 x14	210
15	1 x1	1
	Total	77558760

Due to technical reasons, we randomly selected 100,000 portfolios among the 77,558,760 possible portfolios. As a result, we get 100,000 different portfolio proposals with different numbers of assets and different weight distributions. In this case, we obtain a P matrix of dimension 100,000 × 15.

Step 4: Construction of the optimal portfolio of Shefrin and Statman

[20], stipulate that the individual following the BPT model seeks to maximize the expected return of his portfolio while respecting his security constraint.

The maximization program is as follows:

$$\begin{cases} Max\ E_h(\tilde{r}) \\ s.c\ P(\tilde{r} < r^*) \leq \alpha \end{cases}$$

where $\tilde{r}$ is a random variable that designates the portfolio's profitability, r^* corresponds to the minimum profitability below which the investor does not wish to fall, and α qualifies the admissible failure threshold. In this study, we begin by calculating the return of the portfolio, $\tilde{r}$, which corresponds to the linear combination of the returns of the 15 randomly chosen securities. Therefore, it is determined by the matrix R_j^* and the weights invested in each security. Since each investor defines a different security constraint than other investors, we consider several configurations for α and r^*. To solve this problem, we consider 12 different specifications with r^*={0; 0.05; 0.1} and α = {0; 0.1; 0.2; 0.3}. Subsequently, for each of the 100,000 wallets, we examine the security constraint.

4 Empirical Results and Analysis

4.1 Impact of the Political and COVID-19 Crisis on the Optimization of Markowitz Portfolios

To study the effect of the Tunisian revolution and COVID-19 crisis on the portfolios of investors according to the mean-variance model of [1], we calculate for each of 100000 portfolios the expected return and the standard deviation for the four sub-periods (before, during, after the political crisis, and covid-19 crisis). In Figure 2, we present the 100000 portfolios for each period in the return standard deviation space.

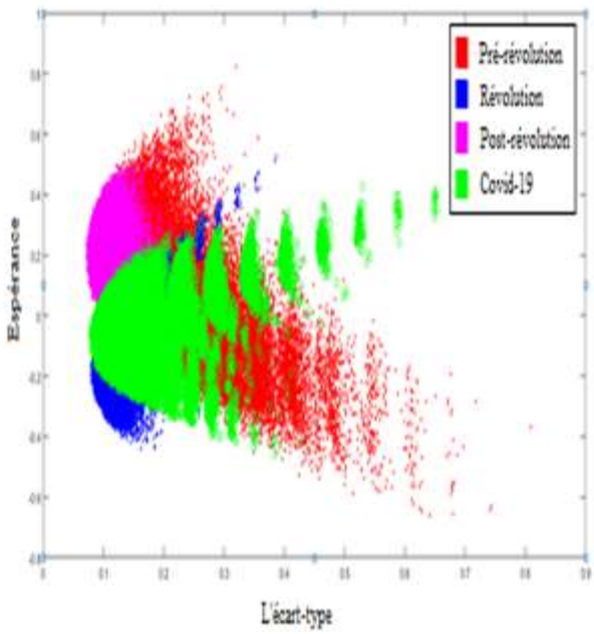


Fig. 2: Impact of the Revolution and the COVID-19 Pandemic on Markowitz's portfolio optimization

During the period of the revolution and Covid-19, we find that these periods of disruption induce both an increase in risk and a decrease in the expected return on the 100,000 portfolios of [1]. Thus, the lowest negative return of the portfolios can be observed in times of crisis. We also observe that the efficiency frontier for the two crisis periods is below all the other efficiency frontiers. Moreover, the expected returns of efficient portfolios during the political crisis period are lower for a given level of risk, compared to efficient portfolios before and after the political crisis. These results indicate that the political crisis and the health crisis lead to sharp declines in the market values of efficient portfolios. Therefore, these periods are characterized by their negative effect on optimal portfolio selection for Tunisian investors.

4.2 Impact of the Political Crisis on the Shefrin and Statman Portfolios Optimization

To analyze the period effect of stress, which affects the Tunisian market, on the BPT portfolios, we start by calculating the portfolio's profitability, $\tilde{r}$, which corresponds to the linear combination of the returns of the 15 securities chosen randomly. Therefore, it is determined by the matrix R_j^* and the weightings invested in each security. Since each investor defines a different security constraint than other investors, we consider several configurations for α and r^*. To solve this problem, we consider 12 different specifications with r^*= {0; 0.05; 0.1} and α = {0; 0.1; 0.2; 0.3}. Next, for each of the 100,000 portfolios, we look at the security constraint. The proportion of the BPT portfolio respecting the safety constraint for the various parameters α and r^* is shown in Table 4.

Table 4. Proportions of Shefrin and Statman portfolios

	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	**Secure portfolio number**
Pre-revolution					
$r^* = 0$	68.76 %	69.16 %	88.28%	91.52 %	814970
$r^* = 0.05$	47.85 %	57.98 %	78.14%	88.71 %	
$r^* = 0.1$	45%	56.57 %	57.86%	65.14 %	
Revolution					
$r^* = 0$	1%	32.86 %	56.29%	68.29 %	320330
$r^* = 0.05$	0%	10.71 %	33.19%	44.57 %	
$r^* = 0.1$	0%	9.43 %	28.5 %	35.49 %	
Post-revolution					
$r^* = 0$	55%	67.14 %	72.86%	83.57 %	701680
$r^* = 0.05$	38.57 %	55.43 %	61.29%	71.25 %	
$r^* = 0.1$	27.86 %	47.29 %	53.85%	67.57 %	
Pendant Covid-19					
$r^* = 0$	0%	1.29 %	5.89 %	19.47 %	38971
$r^* = 0.05$	0%	0.98 %	1.52 %	9.56 %	
$r^* = 0.1$	0%	0.042 %	0.077%	0.142 %	

According to Table 3 and Table 4 the number of the optimal portfolio constructed according to the model of Shefrin and Statman (2000) during the four sub-periods, from 1,200,000 draws made, is respectively equal to 814970, 320330, and 701680 for the period pre-revolution, revolution post-revolution. We find that the number of secure portfolios decreases during the period of crisis compared to other periods of economic stability. The same result is noted for the period of the COVID crisis.

We also note, for all the sub-periods, that the number of portfolios corresponding to α = 0, for a given level of profitability, are the least numerous. And as a result, their set of secure wallets is hugely restricted. In addition, we find, for example, during the pre-revolution period where α is equal to 0.3 and the suction level is set to zero, the proportion of the portfolio that satisfies the constraint is 91.52%. By choosing α equal to 0 and at the same suction level, this proportion decreases by 22.76%. This result indicates that the investor's expectation decreases with α. Therefore, we show that the higher the probability of admissible failure, α, the greater the set of BPT portfolios meeting the security constraint. This result seems quite natural since the BPT agent wants to secure more (fewer) states of nature when α decreases (increases) and therefore it is qualified as the most (less) demanding agent in terms of security.

4.3 Evolution Secure Portfolio Construction for the Different Levels of Defeat and Aspiration during the Tunisian Crisis

Figure 3 illustrates the evolvement of the BPT portfolio under different admissible probabilities of failure during the Tunisian crisis.

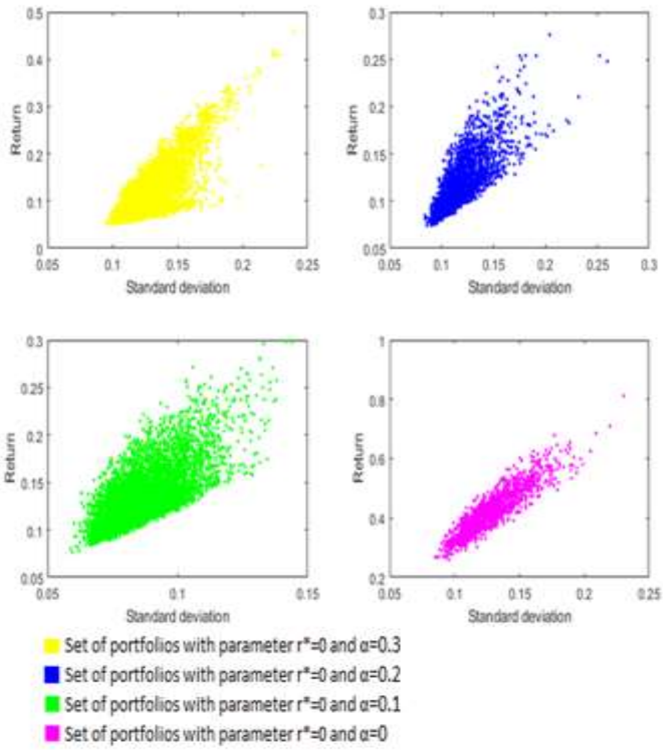


Fig. 3: Evolution of the BPT portfolio for different levels of permissible default probability during political and covid-19 crisis

In this section, we first study how secure portfolios evolve following the increase in the probability of admissible failure and the levels of aspirations.

We notice that, for a lower admissible default probability level, it is difficult to recover the portfolios satisfying the security constraint. We also find that the safety parameters (r^*, α) characterize the behavioral risk function of the investor and determine the investment strategy. The decision to invest is made on both fear and feelings of hope. On the one hand, the more the BPT investor is driven by fear, the more he needs to secure his wealth when he is more risk-averse. On the other hand, BPT investors are willing to take more risks to have the opportunity to increase their potential gains when they are less risk-averse.

From Figure 4, we found that increasing the investor's desired profitability or decreasing the permissible probability of failure brings us to the same previous findings of changing the safety package. In addition, we also found that the higher the aspiration level, r^*, the greater the expected profitability of the portfolio. This finding is adequate with that of [31], [46].

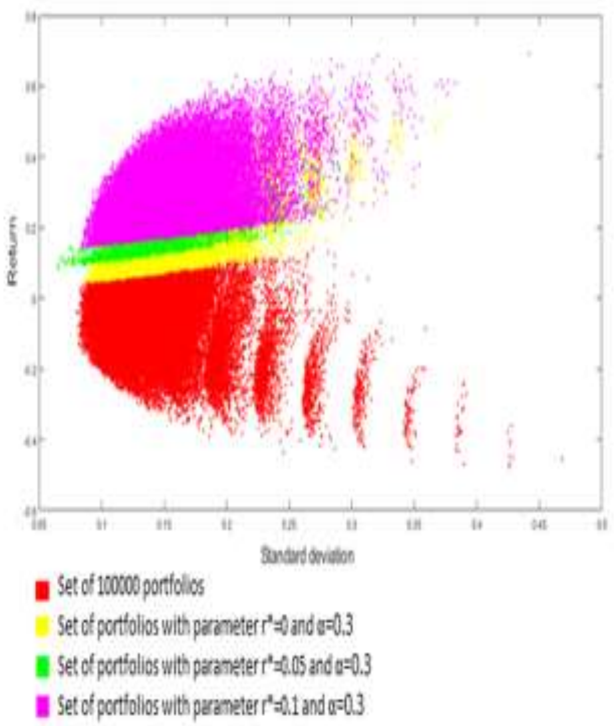


Fig. 4: Evolution of the choice of the BPT investor for the different levels of aspirations

4.4 Comparison of Investors Conforming to the Behavioral Model with Investors in Consonance with the Mean-Variance Model

The objective of our study was to compare the portfolios constructed conforming, [20], intuition with those of the mean-variance Markowitz theory, [1], during the political crisis.

We note P_{ss} is the optimal portfolio of the investor following the BPT model which has already been built in the previous step. First, we calculate the expectation and standard deviation for each of the 100,000 portfolios chosen previously. Subsequently, we check whether there are portfolios with stronger returns while keeping a lower risk than P_{ss} during the stress period.

Table 5 and Figure 5 illustrate the comparison made between the portfolios selected by the agents who follow the mean-variance model with those of BPT during the crisis.

Table 5. Characteristics of optimal portfolios during the Crisis

Portfolio	Expected return	Standard deviation
P_{ss1}	1.7386	0.3772
P_1	0.7421	0.1406
P_2	0.7468	0.1416

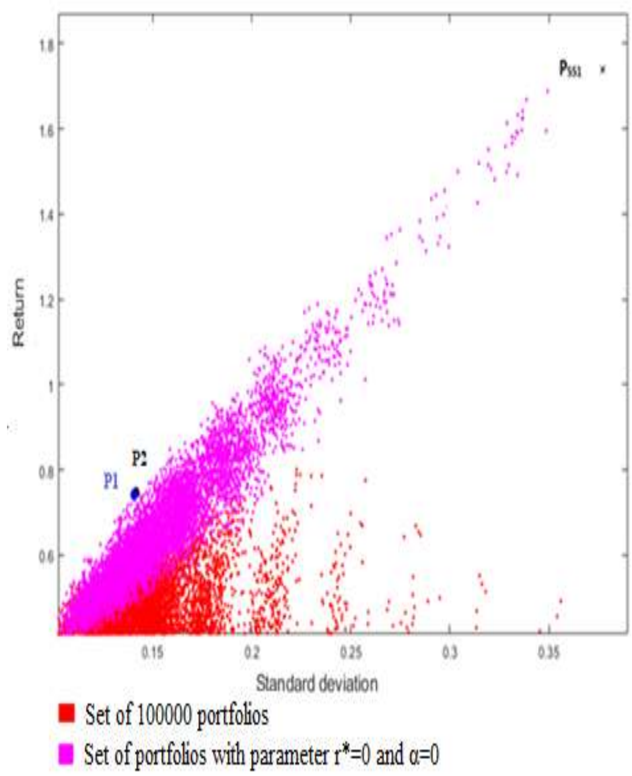


Fig. 5: Choice of Tunisian Investors during the Crisis

Indeed, we note, during the political and COVID-19 crisis, that the Tunisian investor of the BPT type chooses the portfolio which has the highest expected yield and the highest risk compared to other portfolios located on the Markowitz efficiency frontier, [1]. This choice is explained by the fact that, during the crisis, the appointment of a government was able to moderate the fears of investors and offered new hope to Tunisian investors. However, for the same level of return, the Tunisian MVT-type investor chooses a portfolio located on the efficiency frontier with a lower level of risk. More precisely, we also find that Tunisian investors of the MVT type, during the period of stress, select portfolios located to the left of the efficiency frontier. This result confirms that the choice of the BPT investor's portfolio does not necessarily lead to the same choice of the portfolio generated by the mean-variance model of Markowitz, [1], even if the asset allocation of the two approaches coincides. This result implies that even if the optimal BPT portfolio is often located on the efficiency frontier of Markowitz, [1], [47], it will not be chosen by investors following the mean-variance model because it is associated with a degree of aversion to extremely low risk.

Indeed, we find, during the revolution, that the Tunisian investor of the BPT type chooses the portfolio which has the highest expected yield and the highest risk compared to the other portfolios located on the Markowitz efficiency frontier, [1]. This choice is explained by the fact that, during the revolution, the appointment of a new government was able to moderate the fears of investors and offered new hope to Tunisian investors. However, for the same level of return, the Tunisian MPT-type investor chooses a portfolio located on the efficiency frontier with a lower level of risk. More specifically, we also find that Tunisian MPT investors, during the stress period, select portfolios located to the left of the efficiency frontier. This result confirms that the portfolio choice of the BPT investor does not necessarily lead to the same portfolio choice generated by the mean-variance model of Markowitz, [1], even if the asset allocation of the two approaches coincides. This result implies that even if the optimal BPT portfolio often lies on the efficiency frontier of Markowitz, [1], isn't the same one selected by investors following the mean-variance model because it is associated with a degree of aversion to the extremely low risk.

During the Covid-19 pandemic, we find that portfolios complying with security constraints, (r^*=0 and α =0.3); (r^*=0 and α =0.2) as well as ($r^* = 0$ and α =0.1), have expectations greater than 0.08988, 0.2488, and 0.2739 respectively, independently of the standard deviation. On the other hand, we notice the absence of a BPT portfolio respecting the security constraint (r^*=0 and α =0) this result is shown in Figure 6.

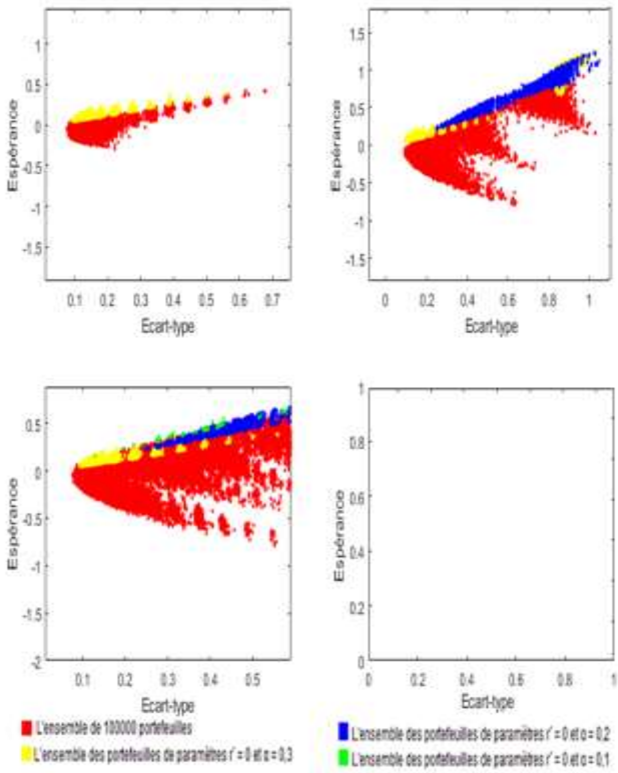


Fig. 6: Evolution of the choice of BPT-type investor during the Covid-19 pandemic for admissible probabilities of defeat equal to 30%, 20%, 10%, and 0%

From Table 3, Table 4, and Figure 6, we see the absence, during the Covid-19 pandemic, of the optimal BPT portfolios for the scenarios r^*= 0, r^*= 0.05, and r^* = 0.1 for a threshold α fixed at 0. Indeed, no portfolio meets the security constraint since this period is characterized by the lowest expected returns. Thus, the potential losses are too high during the health crisis, which leads BPT investors to refrain from choosing a portfolio and not investing in the Tunis stock exchange. This result shows that Tunisian investors of the BPT type are characterized by emotions of fear and security. Consequently, they become very risk-averse and want to secure their assets.

In general, the higher the risk aversion of BPT-type agents, the more their secure portfolios are to the left of the Markowitz efficiency frontier. Moreover, we find that the higher α is, the more the optimal portfolios of BPT are located at the extreme right of the Markowitz efficiency frontier. This implies that these portfolios are characterized by a very high level of expected return and high risk.

In both approaches, we note that the less risk-averse the investor is, the riskier his optimal portfolio will be. Indeed, this investor systematically selects a portfolio located at the efficiency frontier of Markowitz.

5 Conclusion

In this study, we carried out empirical work making it possible to examine the two portfolio management models of MVT developed by Markowitz, [1], and the BPT by [20], on real data from the Tunisian financial market. To do this, we used the daily returns of the 43 securities belonging to the TUNINDEX index over the period from January 2009 to August 2022. To examine the Tunisian revolution and COVID-19 repercussions on asset allocation under MVT and BPT, we have divided the study period into four sub-periods (before, during, and after the revolution and the Covid-19 crisis). The results indicate that the crises cause simultaneously a rise in risk and a sharp decline in portfolio returns constructed under the mean-variance theory of Markowitz, [1]. Furthermore, the results show a remarkable drop in the number of BPT secure portfolios selected by Tunisian investors during the period of disruption. Subsequently, we determined BPT's secure portfolios for the different levels of allowable failure and aspiration. We have found that changing the security setting is consistent with how BPT investors perceive risk. In addition, we have noticed that the more the investor is demanding in terms of security, the more he is driven by the fear of securing his assets. As a result, he becomes less inquire in terms of security and willing to take risks to increase his potential earnings. Comparing the asset allocation constructed by MVT and BPT, we found during the revolution that the optimal portfolio of Shefrin and Statman was located on the Markowitz efficiency frontier. However, we emphasize empirical evidence which stipulates that the optimal portfolio selected by BPT-type investors was located at the top right of the Markowitz efficiency frontier while the optimal portfolios of MVT-type investors were in the top left of the Markowitz efficiency frontier. Therefore, we have shown that the portfolios selected by MVT investors aren't automatically chosen by BPT investors even if the optimal BPT portfolio coincides with the Markowitz efficiency frontier. In terms of perspectives, the presented study could explore the application of a new model which allows the creation of a synergy between the models of classical finance with those of behavioral finance to predict the behavior of stock returns in the future and improve the foresight of investors.

References:

[1] Markowitz, H. Portfolio selection. *Journal of Finance*, Vol 7, 1952. P 77–91.

[2] Bi, J., Jin, H., & Meng, Q. Behavioral mean-variance portfolio selection. *European Journal of Operational Research*, Vol 271**,** No 2, 2018. P 644-663.

[3] Bielecki, T. R., Jin, H., Pliska, S. R., & Zhou, X. Y. (2005). Continuous- time mean- variance portfolio selection with bankruptcy prohibition. *Mathematical Finance: An International Journal of Mathematics, Statistics, and Financial Economics*, Vol 15, No 2, 2005. P 213-244.

[4] Cui, X., Gao, J., Li, X., & Li, D. Optimal mean-variance policy under no-shorting constraint. *European Journal of Operational Research*, Vol 234, No 2, 2014. P 459-468.

[5] Li, X., Zhou, X. Y., & Lim, A. E. Dynamic mean-variance portfolio selection with no-shorting constraints. *SIAM Journal on Control and Optimization*, Vol40, No 5, 2002. P 1540-1555.

[6] Markowitz, Mean-variance approximations to expected utility. *European Journal of Operational Research*, Vol 234, No 2, 2014. P 346-355.

[7] Oliva, I., & Renò, R. Optimal portfolio allocation with volatility and co-jump risk that Markowitz would like. *Journal of Economic Dynamics and Control*, Vol 94, 2018. P 242-256.

[8] Ray, P., & Jenamani, M. Mean-variance analysis of sourcing decision under disruption risk. *European Journal of Operational Research*, Vol 250, No 2, 2016. P 679-689.

[9] Blattberg, R. C., & Gonedes, N. J. A comparison of the stable and student distributions as statistical models for stock prices. *The Journal of Business*, Vol 47, No 2, 1974. P 244-280.

[10] Campbell, J. Y., Campbell, J. J., Campbell, J. W., Lo, A. W., Lo, A. W., & MacKinlay, A. C. The econometrics of financial markets. *Princeton University Press*, 1997.

[11] Harvey, C. R., & Siddique, A. Time-varying conditional skewness and the market risk premium. *Research in Banking and Finance*, Vol 1, No 1, 2000. P 27-60.

[12] Myer, F. C. N., & Webb, J. R. Estimating allocations for mixed-asset portfolios using the bootstrap technique. *In American Real Estate Society Meeting, Sarasota, Florida*. 1991.

[13] Premaratne, G., & Bera, A. K. Modeling asymmetry and excess kurtosis in stock return data. *Illinois Research & Reference Working Paper*, 2000.

[14] Roy, A. D. Safety first and the holding of assets. *Econometrica: Journal of the econometric society*, 1952. P 431-449.

[15] Arzac, E. R., & Bawa, V. S. Portfolio choice and equilibrium in capital markets with safety-first investors. *Journal of Financial Economics*, Vol 4, No 3, 1977. P 277-288.

[16] Telser, L. G. Safety First and Hedging. *The Review of Economic Studies*, Vol 23, No1, 1955. P 1-16.

[17] Baule, R., Korn, O., & Kuntz, L. C. Markowitz with regret. *Journal of Economic Dynamics and Control*, Vol 103, 2019. P 1-24.

[18] Kahneman, D., &. Tversky, A. Prospect theory: An analysis of decision under risk. *Econometrica*, Vol 47, 1979. P 263–291.

[19] Pfiffelmann, M., Roger, T., & Bourachnikova, O. When behavioral portfolio theory meets Markowitz theory. *Economic Modelling*, Vol 53, 2016. P 419-435.

[20] Shefrin, H., & Statman, M. Behavioral portfolio theory. *Journal of Financial and quantitative analysis*, 2000. P 127-151.

[21] Alexander, G. J., & Baptista, A. M. Portfolio selection with mental accounts and delegation. *Journal of Banking & Finance*, Vol 35, No 10, 2011. P 2637-2656.

[22] Baptista, A. M. Portfolio selection with mental accounts and background risk. *Journal of Banking & Finance*, Vol 36, No 4, 2012. P 968-980.

[23] Howard, C. T. Behavioral portfolio management. *Journal of Behavioral Finance & Economics*, Forthcoming. 2014.

[24] Jiang, C., Ma, Y., & An, Y. International portfolio selection with exchange rate risk: A behavioral portfolio theory perspective. *Journal of Banking & Finance*, Vol 37, No 2, 2013. P 648-659.

[25] Leković, M. Behavioral portfolio theory and behavioral asset pricing model as an alternative to standard finance concepts. *Ekonomski horizonti*, Vol 21, No 3, 2019. P 255-279.

[26] Statman, M. Finance for normal people: how investors and markets behave. *Oxford University Press*. 2017.

[27] Alexander, G. J., Baptista, A. M., & Yan, S. Portfolio selection with mental accounts and estimation risk. *Journal of Empirical Finance*, Vol 41, 2017. P 161-186.

[28] Boutti, R., & Hnaka, H. Behavioral theory of choice of portfolio: comparison between asset choices in Morocco by the classical approach and the behavioral approach. State of research. *Revue de Recherches en Economie et en Management Africain*, Vol 4, No 4, 2016.

[29] Kuzmanovic, M., Makajic-Nikolic, D., & Nikolic, N. Preference Based Portfolio for Private Investors: Discrete Choice Analysis Approach. *Mathematics*, Vol 8, No 1, 2020. P 30.

[30] Levy, H., De Giorgi, E. G., & Hens, T. Two paradigms and Nobel prizes in economics: a contradiction or coexistence? *European Financial Management*, Vol 18, No 2, 2012. P 163-182.

[31] Broihanne, M. H., Merli, M., & Roger, P. Behavioral Portfolio Theory. *Revue économique*, Vol 57, No 2, 2006. P 297-314.

[32] Hens, T., & Mayer, J. Cumulative prospect theory and mean-variance analysis: a rigorous comparison. *Swiss Finance Institute Research Paper*, 2014. P 14-23.

[33] Sabri Burak Arzova & Caner Özdurak. Optimal Cryptocurrency and BIST 30 Portfolios with the Perspective of Markowitz Portfolio Theory. *Journal of Finance and Economics*, Vol. 9, No. 4, 2021. P 146-154.

[34] Brenner, M. On the stability of the distribution of the market component in stock price changes. *Journal of Financial and Quantitative Analysis*, 1974. P 945-961.

[35] Walter, C. A. The efficient market hypothesis, the Gaussian assumption, and the investment management industry. *In EFMA Lugano Meetings*, 2001.

[36] Young, M. S., Lee, S. L., & Devaney, S. P. Non-normal real estate returns distributions by property type in the UK. *Journal of Property Research*, Vol 23, No 2, 2006. P 109-133.

[37] Bourachnikova, O., & Burger-Helmchen, T. Investor's behavior and the relevance of asymmetric risk measures. *Banks & bank systems*, Vol 7, No 2, 2012. P 87-94.

[38] Yeny E.Rodríguez, Juan M.Gómez, & Javier Contreras,. Diversified behavioral portfolio as an alternative to Modern Portfolio Theory. *The North American Journal of Economics and Finance*. Vol 58, 2021.

[39] Lopes, L. L. Between hope and fear: The psychology of risk. *Advances in experimental*

social psychology, Vol 20, No 3, 1987. P 255-295.

[40] Thaler, R. Mental accounting and consumer choice. *Marketing Science*, Vol 4, No 3, 1985. P 199-214.

[41] Campbell, J. Y., Lettau, M., Malkiel, B. G., & Xu, Y. Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk. *The Journal of Finance*, Vol 56, No 1, 2001. P 1-43.

[42] Bloomfield, T., Leftwich, R., & Long Jr, J. B. Portfolio strategies and performance. *Journal of Financial Economics*, Vol 5, No 2, 1977. P 201-218.

[43] Statman, M. How many stocks make a diversified portfolio? *Journal of Financial and quantitative analysis*, 1987. P 353-363.

[44] Kumar, A., & Goetzmann, W. N. Diversification decisions of individual investors and asset prices. *Yale School of Management*. 2003.

[45] Hull, J. Options, futures, and other derivatives. *Upper Saddle River, NJ: Prentice Hall*, 2009.

[46] Murat Akkaya. Behavioral Portfolio Theory. *Applying Particle Swarm Optimization*., Vol 306, 2021. P 29-48.

[47] Das, S., Markowitz, H., Scheid, J., & Statman, M. Portfolios for Investors Who Want to Reach TheirGoals While Staying on the Mean-Variance Efficient Frontier. *The Journal of Wealth Management*, Vol 14, No 2, 2011. P 25-31.

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Conflict of Interest

The author declares that no conflict of interest related to this study.

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