Simulation Approach for a Two Player Real Options Signaling Game

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Abstract: - Situations where hidden information exists among involved parties can be found in a variety of diverse domains, ranging, for example, from market entry to military operations. Game theory provides valuable tools to model and analyze such complex settings, with signaling games being one of the approaches. Entering an existing market poses several challenges for a new player and can be studied from a variety of viewpoints. One way to approach it, is by a real options signaling game, where in the simplest form an entrant and an incumbent firm are participating and hidden information exists. In this paper we focus on the market entry scenario and approach it by means of a real options signaling game. The work builds on previous work and contributes to the limited literature on the domain. We introduce the basic notations and background and describe the game setting. Next, we present a simulation approach demonstrating the basic steps, according to the theory, and present the results of simulation executions. The work aims to build a generic model for such market games on top of a two player setting, but the concept is not limited to market entry only, but further expanded in relevant domains where hidden information exists.

Key-Words: - Real Options, Signaling Game.

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1 Introduction

Hidden information is always present in business environments, where players avoid to disclose sensitive or private information to their competitors. Competitors perceive opponents' actions as signals of hidden information and they select their strategies, based on this assumption. Except for the business, this type of setting is more than obvious at more critical environments, like military operations, where privacy of information is secured at any cost. In such settings, game theory offers a formal language for analysis and strategy formulation, by the well-studied class of signaling games.

Signaling games are dynamic games of incomplete information. The simplest form of a signaling game comprises of two players who share different degrees of information. Player one is considered as the sender, who selects an action, based on some parameter selected by 'nature'. This action is perceived as a signal from the second player, who then takes an action considering this signal. At the end, players' payoffs are calculated based on their actions and nature's parameter value. More general, this type of games can be part of a strategic setting with players carrying various levels of information and use their actions as signals to their opponents to control their access to hidden information. Research in signaling games is wide enough and spans to generic problems like labor markets [1], online auctions [2] and contracting [3], or more specific areas like real options [4], [5], [6], [7], [8], [9], [21], [22], [23], [24], [25], [26].

Entry into an existing market poses several challenges for a new player and can be studied from a variety of viewpoints. One way to approach it, is by a real options signaling game, where in the simplest form an entrant and an incumbent firm are participating and hidden information exists. Research in real options signaling games is relatively limited, despite its potential [4].

In this paper we present a simulation model for a real options signaling game for the marker entry case. The model and the relevant theory are introduced, and results are presented from the simulation. This work builds upon previous research and contributes in the existing literature by introducing a relative novel approach ([4], [5], [6], [7], [8], [9], [10], [11]; [12], [13], [14], [15], [16], [17], [18], [19]).

2 Background

In game theory, signaling games comprise a special and very interesting category of games with incomplete information, which can be applied to a variety of real world settings. One application domain is real options, where information asymmetry exists between players and also there is a value in real option. For example, a player can apply a delay option for an investment. The complexity of this type of games in real options makes hard to achieve analytical solutions. So, approaches in literature are relatively limited.

One of the key difficulties to derive analytical solutions is the stochastic variables that are included in the model. Infinite future paths can be generated and, as such, it is not possible to solve the model and generate equilibrium results. It is possible that other game parameters may also have asymmetry, like the players for example, but the most challenging issue regards the information asymmetry or imperfect information.

From the literature review, we can see that almost all works present equilibrium results in the form of formulas that provide a kind of entrance threshold for the investment [20]. Very few works present specific equilibrium and corresponding payoffs. They present it under certain assumptions which reduce the infinite values to a finite level. However, despite the complexity, the domain is active and recent research works on the field prove the increasing interest and research importance for the domain.

In general, this type of games, and subsequent models, can be distinguished in continuous time models and models of discrete time. The most significant contributions are coming from Grenadier and Watanabe, who work on real options signaling games in continuous setting, and van de Walle, who uses discrete time setting [13], [18], [19]. Van de Walle uses approximation methods and he models a real option investment game with asymmetric information as a binomial lattice model. In his model, the game setting comprises from an incumbent firm which has the information advantage, and an entrant firm which lacks information about a specific parameter, the investment costs. In this setting, the investment decision depends on a set of parameters which include the present value of the project, the binomial parameters, the market share, the continuous-time discount rate and of course the private information which is the investment costs. So, a player, which represents a firm in the game, will decide to proceed at an investment if the player expects that the payoff will be positive. Also, the player expects that his investment timing is a result of the possible option value of waiting. We can identify some limitations in this approach. The key issue is that the game complexity in the existence of more than three periods is very high. Also the approach is limited to two players setting, and the value function is the present value. However, van de Walle [13] tries to present an approach, which is mostly applied and tries to deal with the complexity of real options signaling games.

This work contributes to existing research, by introducing a simulation approach for a discrete time setting in real options signaling games.

3 Formal Game Definition

In its very basic form, a signaling game is a Bayesian game in extensive form with observable actions, which comprises of the following:

Player 1, called the “Sender” (S).

Player 2, called the “Receiver” (R).

Random variable t, whose support is given by a set T and is called the type of S (player S knows the value of *t* and is considered as private information).

Probability distribution $\pi()$ over T, which comprises the prior beliefs of player R.

Set M of “Sender” (S) actions (called signals or messages $s \in M$).

Set A of “Receiver” (R) actions, with $\alpha \in A$.

Function $u^i: T \times M \times A \rightarrow \square$ (the payoff for player i at the end of the game).

The timing of the game is as follows: Nature selects one type t_i for the Sender (S) from the set $T = \{t_1, t_2, \ldots, t_i\}$ of the feasible types, according to a probability distribution $\pi(t_i)$, where $\pi(t_i) > 0\ \forall i$ and $\sum_i \pi(t_i) = 1$.

The player with the private information moves first. The Sender (S) observes t_i and selects an action (message) $s_j \in M$from her set of feasible actions $M = \{s_1, \ldots, s_j\}$.

Player 2 with no knowledge of player’s 1 private information moves second. The Receiver (R) observes the message s_j (but does not know

Sender's S type:t_i) and selects an action $\alpha_k \in A$ from her set of feasible actions $A = \{a_1, \dots, \alpha_k\}$.

The game ends and players receive their payoffs, calculated by the function $u^S(t_i, s_j, \alpha_k)$ and $u^R(t_i, s_j, \alpha_k)$ accordingly. Player's 2 payoff depends on the type of Player1 (t_i).

4 Structure of the Market Entrance Real Options Signaling Game

In order to simulate the investment threshold in a market, we considered a real options signaling game where one player represents an incumbent firm and the other the entrant firm. The following conditions were set for the game:

A set of players $N = \{S, R\} = \{Inc, Ent\}$ and Nature, where Inc is the incumbent firm (Sender), and Ent is the entrant firm (Receiver). Nature selects the type of the incumbent firm.

A random variable *t*, given by the set $T = \{L, H\}$ (known to *S*). It represents the investment cost level and it can take either the value L (low), or H (high). Nature selects the actual investment cost at the beginning of the game and this is known to the incumbent firm, but not to the entrant. Incumbent form thus, has private information which is not known to the entrant firm.

A set of costs $C^t = \{C^L, C^H\}$ which reflect the actual investment cost per type *t*. The investment cost can take either low (C^L) or high value (C^H) and it remains constant during the game. (In case of investment the two firms face the same cost and they know the actual values). The values of $C^t = \{C^L, C^H\}$ are common knowledge, but the actual investment cost is known only to the incumbent and is selected by nature at the beginning of the game.

A probability distribution *π(t)* over $T = \{L, H\}$ (the prior probability that the incumbent Inc is of type *t*). The values of *π(t)* are common knowledge.

A probability distribution for the incumbent $\pi_s[.|t]$ over the set of messages $s_j \in M$ for every type $t_i \in T$ (the probability for each message that the incumbent will send the specific message conditional on his type $\pi_s[s|t_i]$. These values are common knowledge.

A set of entrant's Ent posterior beliefs. They represent entrant's beliefs about incumbent's type, conditional on the message (can be considered as common knowledge). The entrant firm assigns these probabilities for every incumbent type and on each message. Based on this, when the incumbent sends a message, the entrant firm updates beliefs according to Bayes rule. These are common knowledge as well. The beliefs are updated as follows

$$\mu(t_i, s_j) = \mu(t_i|s_j) = \frac{\pi(t_i)\,\pi(s_j \mid t_i)}{\sum_{t_i \in T} \pi(t_i)\,\pi(s_j \mid t_i)}$$

where $\sum_{t_i \in T} \pi(s_j \mid t_i) = 1$.

A set M of incumbent's actions (signals or messages). They are of type $s_j = \{d_1, \{d_{2,1}, d_{2,2}\}\}$, where $d_1 = \{I, N\}, d_{2,1} = \{I, N\}, d_{2,2} = \{I, N\}$ (subscripts represent nodes). We allow decisions to invest I, or not invest N.

A set A of entrant's actions (actions). They are of type $\alpha_k = \{d_1, \{d_{2,1}, d_{2,2}\}\}$, where $d_1 = \{I, N\}, d_{2,1} = \{I, N\}, d_{2,2} = \{I, N\}$ (subscripts represent nodes). We allow decisions to invest I or not invest N.

A function $u^i: T \times M \times A \rightarrow$R that is the payoff to player *i* at the end of the game.

Players' payoffs are given by the functions $u^s(t_i, s_j, a_k)$ and $u^R(t_i, s_j, a_k)$ accordingly.

We consider the market as Cournot like and the demand as $p(Q) = a_{i,j} - bQ$ where $a_{i,j}$ is a stochastic variable (approximated by a binomial tree evolving over time).

Incumbent's marginal cost before the investment is c.

For the payoff functions, the overall approach of van de Walle [13] was followed.

The solution concept that is used for the game is the following. We consider an assessment $\{(s(L), s(H)), \alpha(.), q(.)\}$ which is consisted of the incumbent's messages $s(L), s(H)$ for high and low type, the entrant's action $\alpha(.)$ for the incumbent's message and the posterior entrant's belief for the incumbent's message and type [4].

5 Simulation Model for the Signaling Game

In general, an elementary two-period game can be implemented in a straightforward way using relative widely used software, such as spreadsheets for both the analysis and solution [13]. This approach is sufficient for handling data and low level of complexity of a two-period game. However, in order to model a more advanced game, like a multi period or a multi-player model, some advanced approach (for example an object-oriented programming language), should be used in order to achieve better performance and scalability. Especially, for the approach we followed using binomial lattice, the complexity of the game increases as the periods increase and more players participate.

For the present work, our approach was to use an object-oriented language, as it can handle all the issues and provide a fast and accurate solution. The model and simulations were developed in C++ language and the graphs were produced by importing the output data to a spreadsheet program. Although the model was designed to be generic enough for quick scaling up to multi periods or multi players, our initial study was limited to a two-period, two-player setting. Refinements or theoretical enhancements will be the subject of future research. Future work also includes the presentation of the algorithm in a more formal way along with metrics of performance and efficiency.
The solution implementation steps are presented below, following the way they are implemented in the C++ program. For the two-period game the steps are the following:

- Step 1: Definition of parameters and variables.

This is the starting point of the game. In this step the game parameters and variables of the game are defined. For the simulation we set the following initial values to the parameters and derived variables (Table I):

TABLE I. SIMULATION PARAMETERS

	Node n_1		
Parameter/variable	**Value**	**Formula**	**Value**
Risk adjusted discount rate k	1%		1%
Initial market demand $a_{1,1}$	50		50
High cost C^H	40		40
Low cost C^L	20		20
Initial incumbent's marginal cost c	6		6
Reduced incumbent's marginal cost c_1	4		4
Binomial parameters			
Risk free rate r	1%		1%
Time T	4		4
Periods n	1		1
Maturity ΔT		$\Delta T = \frac{T}{n}$	4
Volatility σ	20%		20%
u		$u = e^{\sqrt{\sigma \Delta T}}$	1,492
d		$d = \frac{1}{u}$	0,67
Constant asset payout yield δ		$\delta = \frac{k}{1+k}$	0,038
Risk neutral probabilities			
p_u		$p_u = \frac{(1+r-\delta)-d}{u-d}$	0,401
p_d		$p_d = 1 - p_u$	0,599
Profit share $\delta\varphi$	0,2		0,2
Prior probabilities			
$\pi(s = L)$	0,5		0,5
$\pi(s = H)$	0,5		0,5

- Step 2: Assignment of values to parameters and initiation of variables.

At this step all the parameters are assigned values according to the model to be studied and the variables and other structures (such as tables and arrays) are initialized. All these values comprise actually the public information that is available to the players. Prior probabilities are initialized without a formal procedure. Also, this step can be executed once, in case of a single play, or recurring in case of a simulation scenario, where the values are reassigned partially.

- Step 3: Nature's selection

This is the step where nature selects the incumbent's type. In a realistic scenario the algorithm uses a random number generator process to produce the outcome.

- Step 4: Formulation of players' strategies per node

This is the step where player's strategies are formulated in tables of decisions per node to cover all possible combinations. This is a dynamic process and depends on the number of players and nodes of the game. This is easy to depict in the two-player/two period game, however in the case of a multi model complexity increases and depiction is not always easy.

- Step 5: Entrant's beliefs update

Here, the entrant's posterior probabilities are calculated for all the incumbent's messages according to Bayes formula. Although in the game flow the incumbent selects the message first and the entrant calculates the posterior belief, for the solution we calculate the values beforehand.

- Step 6: Entrant's expected costs

Here, the expected costs are calculated for all the incumbent's messages. These are the expected costs that the entrant assigns to each incumbent's message.

- Step 7: Entrant's expected payoffs per node

Here, the expected payoffs are calculated for all the incumbent's messages per node. These are the expected payoffs that the entrant calculates for each incumbent's message per node.

• Step 7: Entrant's best expected payoff per message

Here, the best responses are calculated for all the incumbent's messages. These are the maximum expected payoffs that the entrant calculates for each incumbent's message.

• Step 8: Incumbent's payoffs per node

Here, the payoffs are calculated for all the entrant's actions given the incumbent's messages per node. These are the payoffs that the incumbent calculates for each entrant's action per node.

• Step 9: Incumbent's best payoff per action

Here, the best responses are calculated for the entire entrant's actions. These are the maximum payoffs that the incumbent calculates for each entrant's action.

• Step 10: Entrant's best payoff

Here, the best entrant's payoff is calculated from the best payoffs per action, given the incumbent's messages. The action that is selected is the entrant's best response.

• Step 11: Incumbent's best payoff

Here, the best incumbent's payoff is calculated from the best payoffs per message, given the entrant's best responses. The message that is selected is the incumbent's best response.

• Step 12: Solution

The message-action combination is the Bayes equilibrium with the final payoffs for both companies.

In case of simulation the above steps are repeated with modifications of the parameters, which however do not affect the overall flow and calculations.

Multi period or multi player setting requires the repetition of the previous steps, not necessarily in the same form or sequence, as in this case another algorithm may be more efficient. However, as said, it is not within the scope of present work to work on more complex settings. In addition, the specific flow has been implemented without any consideration to performance optimization.

6 Simulation Results for the Entrance/Investment Threshold

Entrance or investment threshold in a market definition varies in relevant studies. However, a common approach is to define it as the demand level which is appropriate for a firm to enter a market in order to acquire profits. In the case of continuous time models the model is solved and it provides the demand level is a function of a number of model's variables and offers a decision rule to the firm as a threshold to invest. As in continuous time models equilibrium analysis cannot be provided due to the infinity of combinations the analytical results are often limited to the investment thresholds.

In the present study as we follow a discrete framework, we cannot provide analytical formula for the threshold however we can simulate the threshold for various demand values. The threshold is defined as the demand level for which the payoff of the firm is larger than zero. As the cost is included in the calculation of the payoff the threshold definition is sufficient in order to provide for an investment decision rule.

So, in a more formal way we can define the investment threshold as the demand level for which the payoff is positive (larger than zero). However, the decision to invest or no, given that the threshold is reached, depends on the real option value as well. So, if the payoff is less than the option value investment can be postponed till the payoff equals the real option value. In case the payoff is larger than zero and larger than the option value investment can be done.

According to this definition we calculate the investment threshold as the demand value a where the equilibrium payoff of the firms is larger than zero. In order to identify that value, it is necessary to calculate the equilibrium payoffs till the value of payoff turns to a positive value.

So, for the incumbent the demand threshold is defined as

$$a_{T\square r}^{INC} = \left\{ \begin{matrix} a{:}\,for\ \ \text{a=0 to n step 1,} \\ max\left(u^{INC}\left(s^*(t), \alpha^*\left(s^*(t)\right), q(s)\right), 0\right) \\ > 0 \end{matrix} \right\}$$

As the levels of demand and relevant payoffs are subject to volatility changes, we run a set of simulations on the way the investment level changes when volatility is modified between zero and one (0%-100%). In the following diagrams we depict the results of the simulation for various scenarios.

- In Fig. 1, we see the decrease of investment thresholds for both incumbent and entrant for volatility increase. The threshold is the demand value where the above condition is true.
- In Fig. 2 we see the threshold I for incumbent in addition to the payoff and real option value.

- In Fig. 3 we see the threshold II for incumbent in addition to the payoff and real option value.
- In Fig. 4 we see the threshold for the entrant in addition to the payoff and real option value.

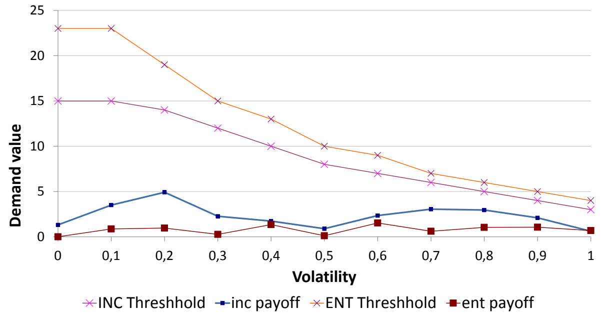

Fig. 1. Investment thresholds vs volatility

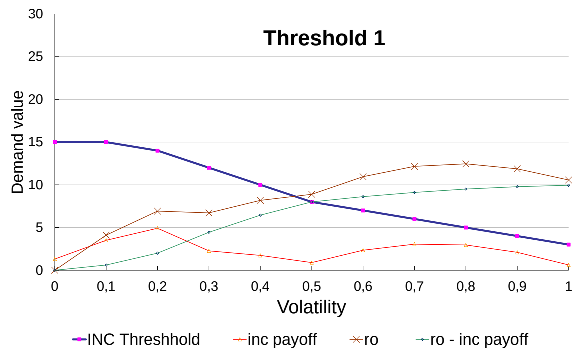

Fig. 2. Incumbent's investment threshold I

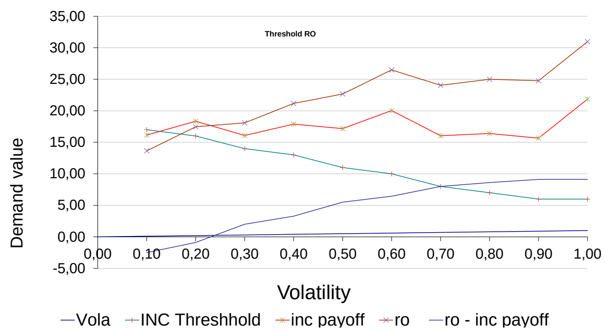

Fig. 3. Incumbent's investment threshold II

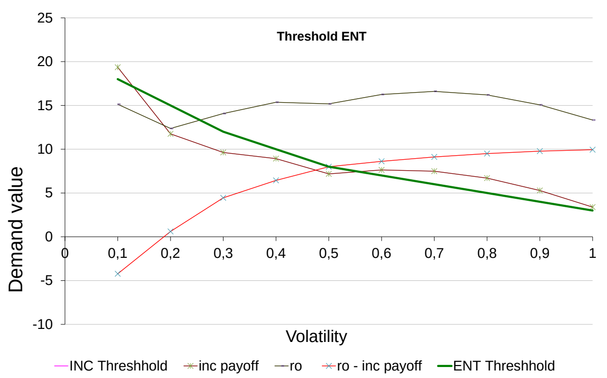

Fig. 4. Entrant's investment threshold

From the above results we see that as volatility increases the thresholds move towards lower demand values, which is in accordance to the previous results as well.

The key question of the specific model is when a firm should invest, or find the optimum strategy, given the other player's strategies and beliefs. In the specific model the payoff is impacted positively by demand increase, especially for high values of market demand. It can be argued, thus, that the higher the payoff the sooner the investment decision is. However, in the case of competition and strategic decision making the decision is not based on the payoff level only and the exact relationship between payoff and demand depends on the level of the cost as well it affects the payoff value. In a real-world scenario, or a richer model, the relationship we found may not be always the case, or we may discover fluctuations. Moreover, if we consider both player's strategies in the two player/two period game the relationship between payoff and demand leads to the presence of three investment zones, the no-invest/no-invest, the invest/no-invest and the invest/invest, where the thresholds are variable according to the values of the rest parameters. In each zone each firm behaves according to the equilibrium. Investment decisions thus seem to be related to investment thresholds which depend on all game's parameters values.

Although the present model is not directly comparable to models presented in literature, however, partial qualitative comparison may be done. Grenadier and Watanabe [18], [19], [20] present continuous time models and they provide analytical solutions for the threshold values for each strategy. They do not solve the model but provide with some numeric examples. The work of van de Walle [13] is closer to our work and it is a discrete model with results similar to ours, in terms of investment strategies and simulation of various parameters. Zhu [10] on the other hand although proceeds to analysis without Bayesian update, he also identifies three regions of equilibrium, which are close to the zones we identified.

7 Conclusion

In this paper we presented a two period real options signaling game simulation. The game setting was introduced along with its parameters and some results from the simulation were also depicted. From the model and the simulations, it is evident that the payoff is impacted positively by increase in demand, especially when market demand remains high. So, we can infer that the higher the payoff the earlier the investment decision is. However, when we have competition and strategic decision-making decision is not based only on the payoff level. The relationship between payoff and demand depends on the level of the cost as well it affects the payoff value. For real world situations, where complexity is greater, we may meet some fluctuations and deviations from the above approach. However, the overall approach seems promising for modeling some simple scenarios and will be further developed in future research.

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