A Note on Number Theoretic Approach to Credit Creation in Banking and Other Financial Intermediaries

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Abstract:- Credit is the currency of both banking and non-banking financial intermediaries. The creation of credit has always been seen as one of the key determinants of a healthy financial industry. In this paper, our objective is to use a number theoretic approach to the specification of a credit creation multiplier for interactions of banking system with non-bank financial intermediaries to produce a realistic formula. The research is further extended into more general sorting processes so that coefficients of the powers of the liquidity ratios can be selected at any stage of the process. The proposed new model yields a more realistic approach to the credit creation multiplier than traditional models. This leads in the final section to consideration of the estimation of liquidity ratios, which are one actuarial factor (among many) in superannuation calculations. Used correctly in the financial sector, the new model can provide regulators and stakeholders a better view of the credit creation scenario with increased insight into the micro factors of credit creation.

Key Words: Credit creation, financial intermediaries and number theoretic approach.

1. Introduction

Aftermath of the global financial crisis of 2008-09 the financial community has shown an increased interest in the credit mechanism of banking sector (Abbassi, Iyer, Peydró, & Tous, 2016; Papanikolaou, 2018). Recently in Australia, The Royal Commission of Misconduct in the Banking, Superannuation and Financial Services Industry has raised many issues surrounding the relationship between the Banks' and other Non-Bank Financial Intermediaries' (NBFIs) credit creation practice (Danckert, 2018) even when they have used widely accepted and practiced process of credit creation (Berger, Bouwman, Kick, & Schaeck, 2016). Past authors have argued that credit creation is much more complex then creation of deposits (Bourva, 1992; McLeay, Radia, & Thomas, 2014) but created upon the financial intermediation theory of banking (Allen & Santomero, 1997; Scholtens & Van Wensveen, 2000) with a historical root in fractional reserve theory (De Soto, 1995; Werner, 2015). In this paper, we targeted to create a new model of mathematical structure for the credit creation in banking and other financial intermediaries. In this regard, we consider the elements of this process utilizing mathematical structure as understood in discrete mathematics (Room & Mack, 1966) in which the discussion proceeds on a path from an observable property of an easily visualized collection of objects to the mathematical concept.

2. Mathematical modelling of Credit creation

Suppose we consider a closed banking system with a requisite liquidity ratio of r *(Van Den End & Kruidhof, 2013)*, $0<r<1$, governed by legislation, convention or commercial prudence. If there is an extraneous injection of liquids, D, into the system then deposits rise initially by D, but the ultimate increase in deposits is

$$D + D\left(\frac{1-r}{r}\right) = \frac{D}{r}, \tag{2.1}$$

So that the amount of credit created is $D(1-r)/r$.

Assume now that the banking system is in competition with other deposit taking NBFIs. If the NBFIs hold their liquid assets in the form of bank deposits, then there is no real competition because the process of credit creation in the NBFIs will have no impact on the process of credit creation in the banking system. However, whatever the form of the liquid assets of the NBFIs, the process of credit creation in the banking sector will have an impact on the NBFIs.

If these NBFIs have required liquidity ratios and they do not hold these liquids in the form of bank deposits, there will be leakages from the banking system to the NBFIs. Then make the unrealistic assumption that no part of a deposit lost to the banking system can return, so that any loans made by an NFBI out of leakages of deposits from the banking system will return exclusively to the NBFIs. In this case the ultimate increase in deposits in the banking system is theoretically

$$D\left(1+s(1-r)+s^2(1-r)^2+\ldots\right)=D\sum_{n=0}^{\infty}\left(s(1-r)\right)^n$$
$$=\frac{Ds(1-r)}{1-s(1-r)}$$

so that

$$D+\frac{Ds(1-r)}{1-s(1-r)}=\frac{D}{1-s(1-r)}. \tag{2.2}$$

When s = 1, this reduces to (2.1). Consider a simplified example. Assume that there are balance sheet positions as follows:

Banking System (r_2 = 0.3)				NBFIs (r_1 = 0.1)			
Liabilities		Assets		Liabilities		Assets	
Deposits	1,000	Liquids	300	Deposits	500	Liquids	50
		Advances	700			Advances	450
	1,000		1,000		500		500

Table 1: Simplified comparison

We shall return to this example later in Table 5. We now consider that part of an advance lost to the banking system can return, so that any creation of deposits, whether by the banking system or the NBFIs, will be distributed between the Banks and the NBFIs in the proportions s and (1-s) respectively.

$$\begin{array}{ccc} \boxed{\text{BANKS}} & \overset{\Rightarrow}{\boldsymbol{s}\ \Rightarrow} & \boxed{\text{NBFIs}} \\ \Uparrow & & \Downarrow \\ \Leftarrow & \Leftarrow & \Leftarrow \\ & \mathbf{(1\text{-}s)} & \\ & \Leftarrow & \end{array}$$

This, of course, can continue as in Table 4 until an economic satisfactory result is achieved. To explore what are, in effect, directed graphs, a framework of Generalized Nets (Atanassov, 2016) can be constructed with intuitionistic fuzzy decision processes at the nodes (Dubois, Gottwald, Hajek, Kacprzyk, & Prade, 2005).

Now assume an extraneous injection of liquids into the banking system of D, with liquidity ratios of r_1 and r_2 in the NBFIs and the Banks respectively, as displayed in Table 2.

Stage		Deposit	"required" liquids	Advance	Re-deposit
1	Bank	D	Dr_2	$D(1-r_2)$	$D(1-r_2)s$
	NBFI				$D_1(1-r_2)(1-s)$
2	Bank	$D(1-r_2)s$	$D(1-r_2)s.r_2$	$D(1-r_2)^2s$	$D(1-r_2)^2s^2$
	NBFI	$D(1-r_2)(1-s)$	$D(1-r_2)(1-s)r_1$	$D(1-r_2)(1-s)(1-r_1)$	$D(1-r_2)(1-s)^2(1-r_1)$
	NBFI	$D(1-r_2)(1-s)^2(1-r_1) + D(1-r_2)^2s(1-s)$	…	…	…

Table 2: Introductory notation

In the 3rd stage these become $D(1-r_2)^2s^2 + D(1-r_2)(1-r_1)(1-s)s$ for the Banks and $D(1-r_2)(1-s)^2(1-r_1) + D(1-r_2)^2s(1-s)$ for the NBFIs. We prove later that the ultimate increase in deposits in the Banks (ΔB) and in the NBFIs (ΔN) are respectively

$$\Delta B = D + \frac{Ds(1-r_2)}{r_1 - s(r_1 - r_2)}, \tag{2.3}$$

and

$$\Delta N = \frac{D(1-s)(1-r_2)}{r_1 - s(r_1 - r_2)}, \tag{2.4}$$

For a total increase in deposits in both sectors of

$$\Delta(B+N) = D + \frac{Ds(1-r_2)}{r_1 - s(r_1 - r_2)} \tag{2.5}$$

which reduces to (2.2) when $r_1 = 1$ and $r_2 = r$.

3. Notational development

We see now that we need to employ suitable notation if we consider the increasing complexity of injections of liquids into the two systems as outlined in Table 3.

Stage		Deposit	"required" liquids	Advance	Re-deposit
1	Bank	$A_{1,1} = D_2$ ‡	D_2r_2	$D_2(1-r_2)$	$D_2(1-r_2)s$
	NBFI	$B_{1,1} = D_1$ ‡	D_1r_1	$D_1(1-r_1)$	$D_2(1-r_2)(1-s)$
2	Bank	$A_{2,1}$	$D_2(1-r_2)s.r_2$	$D_2(1-r_2)^2s$	$D_2(1-r_2)^2s^2$
	NBFI	$B_{2,1}$	$D_2(1-r_2)(1-s)r_1$	$D_2(1-r_2)(1-s)(1-r_1)$	$D_2(1-r_2)(1-s)^2(1-r_1)$

Table 3: Notational development (‡for notational convenience)

These continue in the 3rd stage as $A_{3,2} + A_{3,1}$ for the Banks and $B_{3,2} + B_{3,1}$ for the NBFIs, and in the 4th stage as $A_{4,4}+A_{4,3}+A_{4,2}+A_{4,1}$ for the Banks with $B_{4,4}+B_{4,3}+B_{4,2}+B_{4,1}$ for the NBFIs. At the 5th stage, there are 8 terms in each of the A (Bank) and B (NFBI) categories, so the notation needs to be further simplified in order to detect patterns. To do this we let

$$k_1 = (1-r_2)s$$
$$k_2 = (1-r_1)s$$
$$k_3 = (1-r_1)(1-s) = k_2u$$
$$k_4 = (1-r_2)(1-s) = k_1u$$

in which $u = (1-s)/s$, which we use later. Thus it is trivial but tedious to enumerate the first few cases in Table 4:

$A_{2,1} =$	$B_{3,2} =$	$B_{4,1} = B$	$A_{5,2} = B$	$A_{5,7} = A$	$B_{5,4} = B$
$B_{2,1} =$	$A_{4,1} = A$	$B_{4,2} = A$	$A_{5,3} = B$	$A_{5,8} = B$	$B_{5,5} = A$
$A_{3,1} =$	$A_{4,2} =$	$B_{4,3} = A$	$A_{5,4} = A$	$B_{5,1} = B$	$B_{5,6} = B$
$A_{3,2} =$	$A_{4,3} =$	$B_{4,4} = B$	$A_{5.,5} = B$	$B_{5,2} = A$	$B_{5,7} = B$
$B_{3,1} =$	$A_{4.4} =$	$A_{5,1} = A$	$A_{5,6} = A$	$B_{5,3} = A$	$B_{5,8} = A$

Table 4: Stages 2 to 5

Then at the nth stage, there are 2^{n-2} terms in each of the categories, $A_{n,i}, B_{n,i}, i = 1,2,4,...,2^{n-2}$, such that for

$$m = 1,2,3,...,2^{n-3}, \qquad \begin{cases} A_{n,m} = A_{n-1,m}(1-r_2)s, \\ B_{n,m} = B_{n-1,m}(1-r_1)(1-s \end{cases}$$

$$m = a+1, a+2,...,2^{n-2} \quad \begin{cases} A_{n,m} = B_{n-1,m-a}(1-r_1)s, \\ B_{n,m} = A_{n-1,m-a}(1-r_2)(1-s \end{cases}$$

in which $a = 2^{n-3}, n > 3.$

For example,

$$\begin{aligned} B_{5,3} &= A_{2,1}k_3k_4k_1 \\ &= A_{3,1}k_3k_4 \\ &= B_{4,3}k_3 \\ &= B_{4,3}(1-r_1)(1-s). \end{aligned}$$

4. Number theoretic interpretation

Let the total deposits after n stages be represented by

$$T_n = \sum_{i=1}^{2^{n-2}} \left(A_{n,i} + B_{n,i}\right),\ n \geq 2. \tag{4.1}$$

Then the sequence $\{T_n\}$ can be represented by the general term

$$T_n = \left(k_1 + uk_2\right)^{n-3} T_3 \tag{4.2}$$

in which

$$T_3 = (1+u)\left(k_1A_{2,1} + k_2B_{2,1}\right).$$

Proof of (4.2):

We use the principle of mathematical induction on n, and some results from Table 4.

$$\begin{aligned} T_4 &= \sum_{i=1}^{4}\left(A_{4,i} + B_{4,i}\right) \\ &= A_{2,1}\left((k_1k_1 + k_2k_4) + (k_3k_4 + k_4k_1)\right) + B_{2,1}\left((k_1k_2 + k_2k_3) + (k_3k_3 + k_4k_2)\right) \\ &= A_{2,1}\left((k_1k_1 + uk_2k_1) + (u^2k_1k_2 + uk_1k_1)\right) + B_{2,1}\left((k_1k_2 + uk_2k_2) + (u^2k_2k_2 + uk_1k_2)\right) \\ &= k_1A_{2,1}\left((k_1 + uk_2) + u(k_1 + uk_2)\right) + k_2B_{2,1}\left((k_1 + uk_2) + u(k_1 + uk_2)\right) \\ &= (1+u)(k_1 + uk_2)(k_1A_{2,1} + k_2B_{2,1}); \end{aligned}$$

assume the result is true for n = 5,6,…,m-1. In particular,

$$\begin{aligned} T_{m-1} &= \sum_{i=1}^{2^{m-3}}\left(A_{m-1,i} + B_{m-1,i}\right) \\ &= (1+u)\sum_{i=1}^{2^{m-4}}\left(k_1A_{m-2,i} + k_2B_{m-2,i}\right) \\ &= \left(k_1 + uk_2\right)^{m-4} T_3. \end{aligned}$$

Then

$$\begin{aligned} T_m &= \sum_{i=1}^{2^{m-2}}\left(A_{m,i} + B_{m,i}\right) \\ &= (1+u)\sum_{i=1}^{2^{m-3}}\left(k_1A_{m-1,i} + k_2B_{m-1,i}\right) \\ &= (1+u)\left(\sum_{i=1}^{2^{m-4}}\left((k_1^2 + uk_1k_2)A_{m-2,i} + (k_2^2 + uk_1k\right.\right. \\ &= (1+u)(k_1 + uk_2)\sum_{i=1}^{2^{m-4}}\left(k_1A_{m-2,i} + k_2B_{m-2,i}\right) \\ &= (k_1 + uk_2)T_{m-1} \\ &= (k_1 + uk_2)^{m-3} T_3, \end{aligned}$$

by the inductive hypothesis, and so the result is true in general.

For example,

$$\begin{aligned} T_5 &= \sum_{i=1}^{8}\left(A_{5,i} + B_{5,i}\right) \\ &= (1+u)\sum_{i=1}^{4}\left(k_1A_{4,i} + k_2B_{4,i}\right) \\ &= (1+u)(k_1 + uk_2)\sum_{i=1}^{2}(1+u)(k_1 + uk_2) \\ &= (1+u)(k_1 + uk_2)^2\left(k_1A_{2,i} + k_2B_{2,i}\right). \end{aligned}$$

From within the proof of (4.2) we have that

$$
\begin{aligned}
T_m &= \sum_{i=1}^{2^{m-2}} \left(A_{m,i} + B_{m,i}\right) \\
&= (1+u)\sum_{i=1}^{2^{m-3}} \left(k_1 A_{m-1,i} + k_2 B_{m-1,i}\right) \\
&= \sum_{i=1}^{2^{m-3}} \left(k_1 A_{m-1,i} + k_2 B_{m-1,i}\right) + u\sum_{i=1}^{2^{m-3}} \left(k_1 A_{m-1,i} + k
\end{aligned}
$$

from which we can obtain the coupled recurrence relations with a similar process of mathematical induction

$$\sum_{i=1}^{2^{n-2}} A_{n,i} = \sum_{i=1}^{2^{m-3}} \left(k_1 A_{m-1,i} + k_2 B_{m-1,i}\right) \tag{4.3}$$

and

$$\sum_{i=1}^{2^{n-2}} B_{n,i} = \sum_{i=1}^{2^{m-3}} \left(k_1 A_{m-1,i} + k_2 B_{m-1,i}\right) \tag{4.4}$$

This leads to the main mathematical result, namely,

$$\sum_{n=2}^{\infty} T_n = \frac{D(1-r_2)}{r_1 - s(r_1 - r_2)} \tag{4.5}$$

in which we assume for convergence that

$$
\begin{aligned}
|k_1 + uk_2| &= |s(r_1 - r_2) + (1-r_1)| \\
&\le s|r_1 - r_2| + |1 - r_1| \\
&< 1.
\end{aligned}
$$

Proof of (4.5): we take

$$A_{2,1} = D(1-r_1)s,$$
$$B_{2,1} = D(1-r_2)(1-s).$$

Now

$$
\begin{aligned}
\sum_{n=2}^{\infty} T_n &= T_2 + \sum_{n=3}^{\infty} T_n \\
&= \left(A_{2,1} + B_{2,1}\right) + \sum_{n=3}^{\infty} \left(k_1 + uk_2\right)^{n-3} T_3 \\
&= D(1-r_2) + \frac{T_3}{1 - k_1 - uk_2} \\
&= D(1-r_2) + \frac{(1+u)\left(k_1 A_{2,1} + k_2 B_{2,1}\right)}{1 - k_1 - uk_2} \\
&= D(1-r_2) + \frac{D\left(s(1-r_2)^2 + (1-s)(1-r_2)(1\right.}{r_1 - s(r_1 - r_2)} \\
&= D(1-r_2)\left(1 + \frac{s(1 - r_2 + (1-s)(1-r_1))}{r_1 - s(r_1 - r_2)}\right) \\
&= D(1-r_2)\left(1 - \frac{r_1 - s(r_1 - r_2) - 1}{r_1 - s(r_1 - r_2)}\right) \\
&= \frac{D(1-r_2)}{r_1 - s(r_1 - r_2)},
\end{aligned}
$$

as required.

It follows then that separately we have

$$\sum_{n=2}^{\infty}\sum_{i=1}^{2^{n-2}} B_{n,i} = \frac{D(1-s)(1-r_2)}{r_1 - s(r_1 - r_2)}, \tag{4.6}$$

and

$$\sum_{n=2}^{\infty}\sum_{i=1}^{2^{n-2}} A_{n,i} = \frac{Ds(1-r_2)}{r_1 - s(r_1 - r_2)}, \tag{4.7}$$

Proofs:
From the coupled recurrence relations (4.3) and (4.4),

$$
\begin{aligned}
\sum_{n=2}^{\infty}\sum_{i=1}^{2^{n-2}} A_{n,i} &= \frac{1}{u}\sum_{n=2}^{\infty}\sum_{i=1}^{2^{n-2}} B_{n,i} \\
&= \frac{1}{1+u}\sum_{n=2}^{\infty} T_n \\
&= \frac{Ds(1-r_2)}{r_1 - s(r_1 - r_2)}
\end{aligned}
$$

and similarly

$$\sum_{n=2}^{\infty}\sum_{i=1}^{2^{n-2}} B_{n,i} = \frac{u}{1+u}\sum_{n=2}^{\infty} T_n$$
$$= \frac{D(1-s)(1-r_2)}{r_1 - s(r_1 - r_2)},$$

as required.

As an illustration, consider the simplified situation of the balance sheet in Table 1, with the assumptions that the banking system will gain 0.4 (= s) of any potential deposit while the NBFIs gain $1 - s = 0.6$ of any potential deposit. Again assume that the Banking System receives an extraneous injection of 100 of liquids. Then the ultimate equilibrium position with the addition to deposits can be summarized as

Banking System:
$$100 + \frac{100 \times 0.4 \times (1-0.3)}{0.1 - 0.4 \times (0.1 - 0.3)} = 255.6$$

NBFIs: $\dfrac{100(1-0.4)(1-0.3)}{0.1 - 0.4 \times (0.1 - 0.3)} = 233.3$

in which $s|r_1 - r_2| + |1 - r_1| = 0.98.$

Banking System (r_2 = 0.3)				NBFIs (r_1 = 0.1)			
Liabilities		Assets		Liabilities		Assets	
Deposits	1,255.6	Liquids	376.7	Deposits	733.3	Liquids	73.3
		Advances	878.9			Advances	660.0
	1,255.6		1,255.6		733.3		733.3

Table 5: Extension of Table 1

The total liquids in the system is the total before the extraneous injection (350) plus the extraneous injection (100). The changed 'shares' of system liquids is a result of the assumptions about liquidity ratios and the re-deposit ratio.

5. Findings and Conclusion

Finally, the combinatorial context for these sorting processes is briefly outlined. A relevant feature of the context is a formula which permits the selection of the coefficients of the powers of the liquidity ratios at any stage of the process. Knowledge of these is useful for self-managed superannuation funds. The number theoretic processes are essentially extensions of infinite sums of geometric series (the 'long run'), such as the well known

$$\frac{1}{1-v} = \sum_{j=0}^{\infty} v^j . \qquad (5.1)$$

Replace v by $v/(1-w)$ and

$$\frac{1}{1-v-w} = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\binom{j+k}{k} v^j w^k . \qquad (5.2)$$

We next replace w by $w/(1-x)$ to obtain:

$$\frac{1}{(1-v)(1-x)-w} = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\binom{j+k}{k}\binom{k+r}{r} v \qquad (5.3)$$

Similarly, we need the counter-intuitive result

$$\sum_{r=0}^{\infty}\sum_{t=0}^{r}\binom{r}{t} x^t = \sum_{r=0}^{\infty}\sum_{t=0}^{\infty}\binom{r+t}{t} x^t \qquad (5.4)$$

which we prove by reversing the order of summation:

$$\sum_{r=0}^{\infty}\sum_{t=0}^{r}\binom{r}{t} x^t = 1 + (1+x) + (1+2x+x^2) + (1+3x+6$$
$$+ \ldots + \left(1 + nx + \binom{n}{t}x^2 + \binom{n}{3}x^3 + \ldots +\right.$$
$$= (1 + x + x^2 + x^3 + \ldots + x^n) + (1 + 2x + 3$$
$$+ \left(1 + 3x + 6x^2 + 10x^3 + \ldots + \binom{n+2}{n} x\right.$$
$$= \sum_{r=0}^{\infty}\sum_{t=0}^{\infty}\binom{r+t}{t} x^t .$$

We now apply (5.3) to the expression and get:

$$\frac{D(1-r_2)}{r_1-s(r_1-r_2)}=\frac{D(1-r_2)}{(1-s(1-r_2))-(1-s)(1-r_2)} \quad (5.5)$$

$$=-D(1-r_2)\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\binom{j+k}{k}\binom{k+}{r}$$

$$=-D(1-r_2)\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\sum_{t=0}^{r}\sum_{u=0}^{t}(-1)^{u+}$$

from which we can pick out the powers of r_1 and r_2.

This new model (5.5) has provided a result which yields a realistic approach (2.5) to the credit creation multiplier than the more customary (2.1). This leads in the final section to consideration of the estimation of liquidity ratios, which are one (of many) actuarial factors in superannuation calculations. Used correctly in the financial sector it can provide the regulators and the stakeholders a better view to the credit creation scenario with an increased observation on the micro factors of the credit creation.

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