

Frequentist and Bayesian methods of estimating parameters in a non-performing loan model

ABDULWAHAB ANIMOKU
 Middle East Technical University
 Institute of Applied Mathematics
 06800 Çankaya, Ankara
 TURKEY
 e168159@metu.edu.tr

Abstract: In the literature, several macroeconomic economic factors such as GDP, inflation rate, unemployment, and exchange rate have been identified to influence the level of non-performing loan ratio (NPL) in the banking sector. Other macroeconomic variables such as industry production index, stock exchange index, and oil price are also well documented to have strong explanatory power on NPLs. In this study, we examine the effects of some macroeconomic variables (exchange rates ($TL/\$$ and $TL/€$), industrial production index (IPI), stock exchange index (BIST100), and oil price) on NPL ratio. Further more, we focus on estimating the parameters related to the above variables in a non-performing loan ratio model via Frequentist approach and Bayesian analysis. In the Bayesian method, we provide uninformative and informative priors and a likelihood function that determines the posterior distributions of the parameters. Using Markov Chain Monte Carlo (MCMC) algorithm, we sample the estimates of the parameters from their posterior distributions. The results of the analysis show that the above mentioned macroeconomic variables examined in this study have significant effects on non-performing loan ratio.

Key-Words: Non-performing loan ratio, Bayesian analysis, Frequentist approach, FX, IPI, BIST100

1 Introduction

The healthiness of the banking sector is important for the growth and development of a country, therefore, there is need for efficiency in the financial system. For any financial institution, its performance is determined by the quality of its asset management. Hence, there is need for banks to manage their credit risk efficiently for profitability. Banks are the most important financial institution that mediates in the economic structure of a nation [8]. One of the primary duties of banks is to lend funds to the public. In doing so, they take into account the opportunity costs and credit risk involved. In case of a lending crunch, the stability and efficiency of the banks are threatened [11]. One of the key economic indicators of financial crisis is high level of non-performing loans (NPLs) and 2008 crisis was no exception as high default rate of mortgage loans largely contributed to this crisis. Many active research in this area have established the linkage between financial crisis and high level to NPLs. For example, the East Asia and sub-Saharan Africa financial crisis ([4, 27, 24]), the world financial bubble of 2008 ([21]), the financial crunch of the Turkish economy in 2000 ([28]) have all been linked to high NPLs. Further more, diverse studies on credit risk frictions and macroeconomic activities have shown that external

and interval factors adversely affect non-repayment of loans ([9, 22, 10]). The external factors refer to the macroeconomic variables such as GDP, employment rate, inflation rate, exchange rate, etc. The internal variables refer to the bank specific financial ratios such as bank size, capital adequacy ratio (CAR), operating expenses to operating income (ROA), etc. The objective of this paper is to determine the macroeconomic factors that affect the level of NPLs as well as estimate the parameters of these variables with robust numerical technique.

The paper contributes to the already abundant research in the literature on the macroeconomic variables having significant explanatory power on rising NPLs. To our knowledge, this paper is the first to estimate the parameters in NPL regressive model using Bayesian framework and Frequentist approach.

This paper addresses two empirical question. First, what macroeconomic variables explain the fluctuations in NPLs. Second, comparing robust numerical techniques that capture the significant relationship between NPL and the related macroeconomic variables in the right economic direction. The study involves sample from Turkish data for the period 2005-2015. To answer the first question, we use simple regression equation consisting of NPL ratio as the de-

pendent variable and exchange rates ($TL/\$$ and $TL/€$), industrial production index (IPI), stock exchange index (BIST100), and oil price as the independent variables. To answer the second question, we apply two well established robust numerical techniques (Bayesian and Frequentist approach) in estimating the parameters in the model.

The results show that the Frequentist approach which incorporates ordinary least squares in its methodology gives significant estimates for all the variables. These estimates are in line with most of the studies already available in literature except for the case of $TL/\$$ exchange rate which predicts a negative relationship with NPL ratio. In the Bayesian setting, we sample the parameters from their posterior distributions using MCMC algorithm because of the high dimensionality of the parameter space. In this respect, we incorporate two different sets of prior densities. For the first one, we assume uninformative prior allowing the parameters to take values from the real line. The results show that all the parameter estimates significantly explain NPL ratio except $TL/\$$ exchange rate which also predicts a negative relative relationship with NPL. For the second case, incorporating the knowledge about the direction of relationship each variable has with NPL, we found all the estimates to be significant in the expected direction.

The remaining part of the paper is structured as follows. Section 2 highlights the literature review on this topic. Section 3 explains the mathematical tools as well as the model specification used. In Section 4, we analyze the results obtained from the implementation of the methods and Section 5 concludes.

2 Literature Review

In this section, we discuss various authors who have studied on the effects of macroeconomic and firm specific variables that affect the level of NPL. The different monographs in literature on this topic cut across different regions; this shows the universality of this problem.

[26] studied the effects of internal and external factors on the loan losses of USA commercial banks using 1984-1987 data. By employing ordinary regression model, they conclude that both internal and external factors have significant effects on loan losses.

[25] studied the factors that significantly affect the level of NPL using 1985-1997 data for Spanish banks. They found that bank size, market power, real gross domestic product (GDP), rapid credit expansion, and capital ratio significantly explain the level of NPL by using dynamic model.

[14] used the data of Australian banks from the period of 1990-2001 to determine the effects of macroeconomic variables (real GDP, industrial growth, money growth, stock market indices, interest rate, business confidence index) on credit risk. They found that the level of NPL is strongly dependent on industrial production, interest rates, stock market indices, and business confidence index.

[10] studied the determinants of high NPL level and credit risk in sub-Saharan Africa. His empirical results show that economic growth, real exchange rate appreciation, the real interest rate, net interest margins and interbank loans have significant relationship with NPLs.

[22] analyzed the evolution of loan loss provisions and new bad debts over the business cycle for more than 200 Italian banks during the period 1985-2002. His research is to check whether the business cycle affects the level of NPL as well as which variables adversely determine NPLs in those cycles. By employing static and dynamic models, his results show that macroeconomic variables (stock exchange rate, interest rate, GDP) and bank specific variables (cost to income ratio, equity capital over total asset, return on assets, etc) significantly influence NPLs. Consequently, [19] determine that there is a significant link between default rates in Italian banks and business cycle using vector auto-regression. Their results show that default rate increase during recession and decrease during economic boom.

In order to investigate the macroeconomic variables affecting NPL in advanced economies over the period of 1998-2009, [21] used panel vector autoregressive (PVAR) model. She found that real GDP, unemployment, inflation, interest rates, change in housing and stock price indices, and private sector credit to GDP ratio explains the fluctuations in NPL.

[17] studied the dependency of NPL by loan types (business, consumer and mortgages) on macroeconomic variables using data from Greek banks during 2003-2009 period. They conclude that macroeconomic variables and management quality can significantly explain NPL with mortgages being the least affected by these factors.

Further more, [16] investigated the effects of macroeconomic variables on the NPLs for CESEE countries for the period 1998-2011 using a dynamic panel regression analysis. The result of this author shows that both macroeconomic and firm specific factors significantly affect NPL. However, the latter has low explanatory power compared to the former. In a related new study, [6] investigated the determinants of NPLs in Euro-area banking system for the period 1990-2015 using GMM estimations. The author found that bank-specific variables such as ROA, ROE

and the ratio of loans to deposits as proxies for management quality significantly explain NPLs. In addition, macroeconomic variables such as unemployment, income tax as % of GDP, government budget deficit/surplus and public debt as % of GDP, inflation, GDP growth and output gap have strong explanatory power for the level of NPLs.

Now, we discuss the studies done to understand the macroeconomic and firm specific variables that are linked with NPL levels in the case of Turkey. [12] investigated the effects of macroeconomic variables such as interest rate and public debt over GDP ratio on non-performing loan ratio of banks traded in BIST100. The author used econometric methods and found that the macroeconomic variables above strongly explain NPL ratio. Similar to the above study is the work of [15] who studied the relationship between NPL ratio and macroeconomic variables (exchange rates ($TL/\$$ and $TL/€$), industrial production index (IPI), stock exchange index (BIST100), and oil price). The authors conclude that there is a causal relationship between the macroeconomic variables on the NPL ratio. In another related work, [7] studied the impact of NPL levels on the balance sheets of banks in Turkey. Using vector auto-regressive (VAR) and Granger causality test, the author analyzed the relationship between NPL level and banks balance sheets. The author also gave some recommendations on how banks can manage their credit risk in order to avoid high level of NPLs. Finally, [28] investigated the macroeconomic factors that trigger financial crisis in Turkey during the period 1998-2012 using logit model to determine the parameters.

3 Methodology

In this section, we present the mathematical tools used in obtaining estimates of the parameters in the non-performing loan ratio model. Here, in line with various studies in literature, we choose non-performing loan ratio (non-performing loans divided by total loans) as the proxy to measure non-performing loans. Thence, non-performing loan ratio is taken to be the dependent variable in this study. Other macroeconomic variables such as: foreign exchange rates ($TL/\$$ and $TL/€$), industrial production index (IPI), stock exchange index (BIST100), and oil price, are taken to be independent variables. First, we present the mathematical model used in this study

$$y = \alpha_1 + \alpha_2 x_1 + \alpha_3 x_2 + \alpha_4 x_3 + \alpha_5 x_4 + \alpha_6 x_5 + \varepsilon, \quad (1)$$

where, y is the level of NPL ratio, α_1 is the regression intercept, x_1 represents the $TL/\$$ rate, x_2 represents the $TL/€$ rate, x_3 represents BIST100, x_4 is the

industrial production index, x_5 represents oil price, α_i , $i = 2, \dots, 6$ represents the predictive parameters for the independent variables respectively, and ε is the error term. Throughout this article, we have assumed that the model errors, $\varepsilon \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_0^2)$ are *iid* and normally distributed with mean 0 and unknown but fixed variance σ_0^2 . Thus, (1) can compactly be written as

$$y_i = f_i(\alpha) + \varepsilon_i, \text{ for } i = 1, \dots, n, \quad (2)$$

where $f(\cdot)$ is termed as the model response. Under the assumption that $\varepsilon \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_0^2)$, we can estimate the parameters α in ordinary least square (OLS) sense by

$$\hat{\alpha} = \underset{\alpha \in \mathcal{Q}}{\operatorname{argmin}} \sum_{i=1}^n [y_i - f_i(\alpha)]^2.$$

Next, we present the mathematical tools used in *Frequentist* approach and *Bayesian* framework.

3.0.1 Frequentist Approach

As with any other statistical modeling, the aim of Frequentist approach is to estimate the model parameters so that the model response fits the measuring data in an optimal sense [23]. In this approach, the parameters are viewed as *true*¹ but unknown deterministic parameters α . Given that the parameter estimator $\hat{\alpha}$ is a random vector, it has the mean, covariance and sampling distribution. Our aim is to construct the sampling distribution of the parameter α . Now, we write (1) in the matrix form as:

$$y = X\alpha_0 + \varepsilon, \quad (3)$$

where $y = [y_1, y_2, \dots, y_n]^T$ and $\varepsilon = [\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n]^T$, are random vectors, X is the $n \times 6$ deterministic and known matrix, and $\alpha_0 = [\alpha_1, \dots, \alpha_6]^T$ are the true but unknown parameters. In this setting, solving the normal equation under OLS gives the unknown true parameter estimate as

$$\alpha = (X^T X)^{-1} X^T y.$$

Next, we state the properties of the parameter estimator $\hat{\alpha}$ and error variance estimator $\hat{\sigma}$. Also, the *sampling* distribution of the estimator $\hat{\alpha}$ will be described.

Theorem 1 (Parameter estimator properties) *The parameter estimator $\hat{\alpha}$ has mean and covariance matrix given by*

$$\mathbb{E}(\hat{\alpha}) = \alpha_0, \text{ and } \mathbb{V}(\hat{\alpha}) = \sigma_0^2 (X^T X)^{-1}$$

¹In uncertainty quantification literature, “true” parameter represents a “fixed” but unknown parameter in a model [23].

Theorem 2 (Error variance estimator) *The unbiased error variance estimator $\hat{\sigma}$ is given by*

$$\hat{\sigma} = \frac{1}{n-6} \hat{R}^T \hat{R},$$

where $\hat{R} = y - X\hat{\alpha}$ is termed as the residual estimator

The proofs of Theorem 1 and Theorem 2 can be found in details in [23].

Remark 3 (Sampling Distribution for $\hat{\alpha}$) *For $\varepsilon \sim \mathcal{N}(0, \sigma_0^2)$, the sampling distribution for $\hat{\alpha}$ is given as,*

$$\hat{\alpha} \sim \mathcal{N}(\alpha_0, \sigma_0^2(X^T X)^{-1}).$$

Furthermore, if we let the δ_k , $k = 1, \dots, 6$ denote the k^{th} diagonal element of $(X^T X)^{-1}$ and α_{0_k} denote the k^{th} element of the true parameter vector α_0 , then

$$\hat{\alpha}_k \sim \mathcal{N}(\alpha_{0_k}, \sigma_0^2 \delta_k).$$

3.0.2 Bayesian framework

Here, we develop the mathematical tools necessary for the calibration of the model (2) in estimating the parameter α that is optimal. Unlike Frequentist approach, the Bayesian method involves the estimation of unknown stochastic parameter α . Herewith, we estimate α by obtaining its ‘‘posterior’’ distribution. The posterior distribution, denoted as $\pi(\alpha | y)$ is the probability density under which the experimental data y_i , $i = 1, \dots, n$ are most likely to be observed. To derive $\pi(\alpha | y)$, Bayes rule is used as follows:

$$\pi(\alpha | y) = \frac{\pi(y | \alpha)\pi_0(\alpha)}{\pi_y(y)}, \quad (4)$$

where $\pi_0(\alpha)$ represents the prior density, $\pi(y | \alpha)$ is the likelihood function, and $\pi_y(y)$ is the normalizing constant given by

$$\pi_y(y) = \int_{\mathbb{R}^p} \pi(y | \alpha)\pi_0(\alpha)d\alpha.$$

Considering the statistical model in (2), the likelihood function can be characterized by

$$\pi(y | \alpha) = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left\{-\frac{1}{2\sigma^2}SS(\alpha)\right\} \quad (5)$$

where

$$SS(\alpha) = \sum_{i=1}^n |y_i - f_i(\alpha)|^2$$

is the sums of squares error.

Furthermore, we consider two forms of initial distribution. The first one is an uninformative prior given by

$$\pi_0(\alpha) = \mathbb{1}_{(-\infty, \infty)}(\alpha) \quad (6)$$

This prior allows the values of the parameter to be sampled from the entire the real line. Second, we choose an informative prior based on the relationships that have been established in the literature on this research topic. Hence, we consider a robust and significant estimates when exchange rates ($TL/\$$ and $TL/€$) and oil price have positive relationship with NPL while industrial production index (IPI) and Borsa Istanbul stock index (BIST100) have negative relationship with NPL. Thence, we consider the prior distribution

$$\pi_0(\alpha) = \begin{cases} 1 & \text{if } \alpha_2, \alpha_3, \alpha_6 \geq 0 \text{ and } \alpha_4, \alpha_5 \leq 0 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

to enforce the parameter to satisfy the conditions described above. Next, we incorporate (5) and (6) as well as (5) and (7) into (4) to obtain the posterior densities for the parameter α as

$$\pi(\alpha | y) \propto \mathbb{1}_{(-\infty, \infty)} \left\{ -\frac{1}{2\sigma^2}SS(\alpha) \right\} \quad (8)$$

and

$$\pi(\alpha | y) \propto \mathbb{1}_{\{\alpha_2, \alpha_3, \alpha_6 \in [0, \infty) \text{ and } \alpha_4, \alpha_5 \in (-\infty, 0]\}} \left\{ -\frac{1}{2\sigma^2}SS(\alpha) \right\} \quad (9)$$

respectively.

4 Implementation and Results

In this section we analyze the results obtained by applying the methodology formulated and explained in Section 3.

First, we give the sources as well as transformation of the data used. The data used to test the algorithms consist of 123 observations of non-performing loans and related macroeconomic variables tested for the period between January 2005 to May 2015. We have chosen this time period because of availability of data for this period. The time series variables were taken from the Central Bank of Turkey (TCMB), Turkish Statistical Institution (TUIK) and Turkish Banking Supervisory Body (BDDK). In this study, we have used non-performing loan ratio (non-performing loan divided by the total number of loans) as a measure for the non-performing loan level. Following the recommendations of [15] and [12], the logarithmic change of the time series data was taken to

avoid stationary problem. In other to have a coherent measures for the dependent (non-performing loan ratio) and independent variables ((exchange rates ($TL/\$$ and $TL/€$), industrial production index (IPI), stock exchange index of 100 biggest companies in Turkey (BIST100), and oil price (OP)), we have taken the end of the month values for each variable.

Next, we analyze the parameter α estimates using the Frequentist approach. Using Algorithm 1, the parameter estimates are given in Table 1. Substituting α into (1) gives

$$y = 0.0155 - 0.0764x_1 + 0.0253x_2 - 0.0159x_3 - 0.2351x_4 + 0.0348x_5 \quad (10)$$

(10) suggests that the level of NPL ratio increases as $TL/€$ rate and oil price increases; while it decreases as $TL/\$$ rate, BIST100 index, and industrial production index increases. Further more, the analysis in the frequentist approach shows that all of these coefficients are economically significant as seen from the t -values. Also, the sampling distribution obtained from this analysis can be written mathematically as

$$\begin{aligned} \alpha_1 &\sim \mathcal{N}(0.0155, 4.41 \times 10^{-6}) \\ \alpha_2 &\sim \mathcal{N}(-0.0764, 0.0087) \\ \alpha_3 &\sim \mathcal{N}(0.0253, 0.0057) \\ \alpha_4 &\sim \mathcal{N}(-0.0159, 0.0008) \\ \alpha_5 &\sim \mathcal{N}(-0.2351, 0.0119) \\ \alpha_3 &\sim \mathcal{N}(0.0348, 0.0035) \end{aligned} \quad (11)$$

using Remark 3. The plot of the sampling distribution of α_1 is given in Figure 1(a). Similar graphs can be obtained for the remaining parameters. In order to determine the covariance matrix of α , the variance of the errors is needed. The sample variance of the errors is estimated to be 4.4040×10^{-4} . Thence, the errors are *iid* and normally distributed with mean 0 and fixed variance 4.4040×10^{-4} given by

$$\varepsilon \stackrel{iid}{\sim} \mathcal{N}(0, 4.4040 \times 10^{-4}).$$

Consequently, the covariance matrix of the parameter is computed as

$$\text{Var} = \begin{pmatrix} 0.0000 & -0.0001 & -0.0000 & -0.0000 & -0.0001 & -0.0000 \\ -0.0001 & 0.0087 & -0.0027 & 0.0018 & 0.0016 & 0.0001 \\ -0.0000 & -0.0027 & 0.0057 & 0.0004 & -0.0010 & -0.0003 \\ -0.0000 & 0.0018 & 0.0004 & 0.0008 & 0.0001 & -0.0000 \\ -0.0001 & 0.0016 & -0.0010 & 0.0001 & 0.0119 & 0.0016 \\ -0.0000 & 0.0001 & -0.0003 & -0.0000 & 0.0016 & 0.0035 \end{pmatrix}$$

Next, we analyze the economic interpretation of the regression estimates of the parameters obtained via the Frequentist approach. First, we have obtained

a negative relation ($\alpha_2 = -0.0764$) between the NPL ratio and $TL/\$$ exchange rate which is significant (-3.8930×10^3). This does not support our hypothesis that depreciation of home currency will have negative effect on NPLs. However, the relationship observed might be because Turkey is a net exporter of goods to countries in the middle east in which she trades in American dollars with. Therefore, these companies can easily meet up with their loan obligations when the Turkish lira depreciates against US dollars due to strengthening in their financial capability. Our finding under the frequentist approach is supported by the work of [10, 2, 1] which explain that exchange rate is falsely identified to be positively linked with the level of NPLs. [29] also supports our findings in their analysis of effect of exchange rate on NPL in Slovakian banks. They find that companies that are net exporter of goods are favored by an increase in exchange rate than companies which are net importer of goods.

Second, the coefficient ($\alpha_3 = 0.0253$) of $TL/€$ exchange rate shows that $TL/€$ has positive relationship with NPL ratio which conforms with our hypothesis. The t-value (1.6314×10^3) also shows that this estimate is statistically significant. This is expected as Turkey is a net importer of goods from European countries which she trades in Euros with. For a company that borrows in euros to meet its financial trade obligations, when the $TL/€$ appreciates, it hurts the company's ability to repay the euro-loan. Hence, the probability of default on such loans is high which increases NPL levels for the banks. The works of [18, 5, 16] support our finding that there is a significant positive relationship between exchange rate and NPL.

Third, $\alpha_4 = -0.0159$ predicts a significant negative relationship between the level of NPL and market index. The stock index is characterize as the leading indicator of economic development by [13]. Therefore, when the Istanbul stock index (BIST100) price increases, it shows an increasing healthiness in economic structure of Turkey which is negatively related to the level of NPL. In this respect, our finding is in accordance with our hypothesis on the economic relationship between NPL level and market index. [27] also supports our finding using a data sample from January 2007 to March 2013 by concluding a significant negative relationship between NPL and BIST100 in Turkey.

Fourth, (10) shows that there is a negative and economically significant relationship between NPL ratio and industrial production index with coefficient ($\alpha_5 = -0.2351$). This suggests that as the companies produce and sell more goods there is an increased advantage of lower default rate on their loans. [27] who

uses *OLS* method in conjunction with other econometric techniques to study the effects of macro-economic variables (including IPI) on NPL level. The author also concludes that there is a significant negative relationship between NPL and industrial price index.

Finally, we examine the relationship between NPL and oil price. The results suggest a positive significant relationship between oil prices (in Turkish liras) and NPL ratio. The estimate ($\alpha_6 = 0.0348$) predicts that one unit change (increase/decrease) in oil price leads to 0.0348 change (increase/decrease) in the same direction. Since Turkey imports all of its oil and gas products, as oil price increases, there is an increased cost for companies who import oil products and this in turn decrease their capability to meet loan obligations. Our result is supported by [29] who report that an increase in oil price generally puts an upward pressure on most firm's ability to repay loans.

Now, we consider the results obtained via the Bayesian method of estimating the regression coefficients. We run an MCMC chain of 10000 estimates to sample α from its posterior distributions in (8) and (9). The MCMC chains for α_1 can be seen in Figure 1(b) and Figure 1(c). Similar procedure can be done for the other parameters. The 10000th chain of the parameter is taken as this constitutes the stationary distribution of the parameter. We consider the 10000th chain of the sample estimates drawn from the posterior distributions (8) and (9). Using Algorithm 2, the parameter estimates are given in Table 2. The parameter estimates when plugged into (1) give

$$y = 0.0150 - 0.0779x_1 + 0.0533x_2 - 0.0093x_3 - 0.2579x_4 + 0.0481x_5 \quad (12)$$

and

$$y = 0.0111 + 0.0313x_1 + 0.0296x_2 - 0.0291x_3 - 0.2193x_4 + 0.1438x_5 \quad (13)$$

for (8) and (9) respectively. The estimates in (12) is similar in magnitude as well as direction to that of the Frequentist approach. However, in the case of Bayesian approach all of the coefficients are significant except $\alpha_2 = -0.0779$ with t-value and p-value of -1.3289 and 0.1839 respectively. The work of [3, 20] which conclude that a negative relationship between NPL and exchange rate is insignificant supports our findings here. However, the estimates in (13) are all economically significant with $\alpha_2 = 0.03313$ as seen in Table 2. The Bayesian approach allows for incorporation of prior knowledge that gives significant estimates. It can also be understood through the Bayesian method that uninformative prior provides evidence

that negative relationship between NPL and exchange rate is insignificant which Frequentist approach fails to deduce. The posterior distributions of parameter α in (8) and (9) can be seen in Figure 1(d) and Figure 1(e) respectively. It can be observed that the distributions of each parameter using (9) appear better in comparison to (8). We attribute this differences to the insignificance of α_2 in the estimates obtained from (8).

Now, we compare the fitness of the model response and the data as well as the distribution of the residuals for each method implemented in this paper. The graphs are given in Figure 2. It can be observed that the model responses from both Frequentist and Bayesian approaches fit well to the data with random residuals. This suggests a good fit of the model (1) to the problem investigated in the paper. Also, the sums of squared errors was calculated for each method. The sums of squared errors using the estimates in the Frequentist approach is 0.50 while that of the Bayesian method with posterior distribution (8) gives 0.501 and with (9) produces 0.0512. The low *sums of squared errors* also supports the good fitness of the models as well as methodologies implemented in estimating the parameters.

5 Conclusion

In this paper, our results have corroborated previous research in the determinants of NPLs. We have shown consistent and robust methods of estimating parameters in a non-performing loan model via Frequentist and Bayesian frameworks. Using Turkish data for the period January 2005 to May 2015, we constructed the sampling distribution and model parameter densities under Frequentist and Bayesian frameworks respectively. Our results demonstrate that macroeconomic variables such as euro exchange rates and oil price have significant positive relationships while industrial production index and borsa Istanbul stock index (BIST100) have negative relationships with NPLs. Although, we found that dollar exchange rate has strong negative influence on NPL in the Frequentist approach it is found to be insignificant in the Bayesian framework with non-informative prior. When informative prior is incorporated into the posterior distribution of the parameter, we found a significant positive relationship between dollar exchange rate and NPL ratio. This is the first empirical study that uses Bayesian analysis to investigate the effect of macroeconomic variables on NPL and the results show consistent and robust methodology that are applicable by the bank analysts.

Algorithm 1 Algorithm for obtaining the sampling distribution of α

1. The least squares estimates for α are obtained using MATLAB function *lsqlin.m*
2. The error variance estimates are obtained using $\sigma = \frac{1}{n-p} R^T R$, where $R = y - X\alpha$.
3. The covariance matrix estimates are obtained by $\mathbb{V}(\alpha) = \sigma^2 (X^T X)^{-1}$
4. Using $\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$ obtain the sampling distribution estimates for α given by $\alpha_k \sim \mathcal{N}(\alpha_{0k}, \sigma^2 \delta_k)$, $k = 1, \dots, 6$, where $\delta_k = (X^T X)^{-1}_{kk}$.
5. Compute the *t-test* values for the sampling distribution estimates for α_k , $k = 1, \dots, 6$ using MATLAB command *ttest.m*
6. Plot the sampling distribution for α_k , $k = 1, \dots, 6$.

Algorithm 2 MCMC algorithm in Bayesian framework

1. Choose initial parameter $\vec{\alpha}^0$ such that $\pi(\vec{\alpha}^0 | V) > 0$; $\dim(\vec{\alpha}^0) = p$
2. For $k = 1, \dots, M$
 - (a) for $z \sim \mathcal{N}(0, \mathcal{I}_p)$, construct the candidate

$$\vec{\alpha}^* = \vec{\alpha}^{k-1} + Wz,$$

where $WW^T = V$ (Cholesky decomposition) for the covariance matrix V of the parameter $\vec{\alpha}$. Sometimes V is replaced with the diagonal matrix D whose elements contains the variances of each parameter in $\vec{\alpha}$.

- (b) $\vec{\alpha}^* \sim \mathcal{N}(\vec{\alpha}^{k-1}, V)$
- (c) Compute the probability $r(\vec{\alpha}^* | \vec{\alpha}^{k-1})$ such that

$$r(\vec{\alpha}^* | \vec{\alpha}^{k-1}) = \frac{\pi(\vec{\alpha}^* | V)}{\pi(\vec{\alpha}^{k-1} | V)} = \frac{\pi(V | \vec{\alpha}^*) \pi_0(\vec{\alpha}^*)}{\pi(V | \vec{\alpha}^{k-1}) \pi_0(\vec{\alpha}^{k-1})};$$

- i. set $\vec{\alpha}^k = \vec{\alpha}^*$ with probability $\alpha = \min(1, r)$
- ii. Otherwise $\vec{\alpha}^k = \vec{\alpha}^{k-1}$

Table 1: Sampling distribution estimates for α

coefficient	Mean	Std	t-value	p-value
α_1	0.0155	0.0021	3.5353×10^4	0
α_2	-0.0764	0.0932	-3.8930×10^3	0
α_3	0.0253	0.0754	1.6314×10^3	0
α_4	-0.0159	0.0287	-2.6296×10^3	0
α_5	-0.2351	0.1091	-1.0355×10^4	0
α_6	0.0348	0.0594	2.7825×10^3	0

Table 2: Bayesian estimates for α

coefficient	Estimates from (8)	t-value	p-value	Estimates from (9)	t-value	p-value
α_1	0.0150	975.2690	0	0.0111	708.8928	0
α_2	-0.0779	-1.3289	0.1839	0.0313	124.6310	0
α_3	0.0533	1104.1	0	0.0296	130.1250	0
α_4	-0.0093	-1702.8	0	-0.0291	124.0262	0
α_5	-0.2579	-530.2763	0	-0.2193	-255.4759	0
α_6	0.0481	572.4270	0	0.1438	106.9926	0

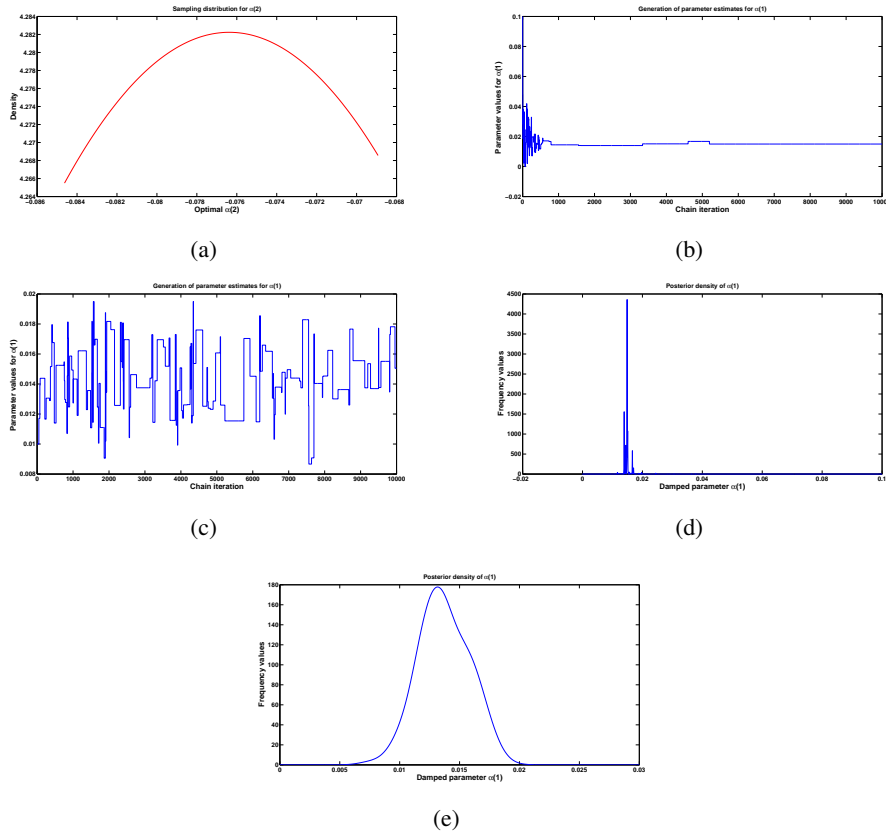


Figure 1: Analysis for α_1 ; similar analysis can be done for $\alpha_2, \dots, \alpha_6$: (a) sampling distribution for α_1 , (b) MCMC chain for α_1 from (8), (c) MCMC chain for α_1 from (9), (d) Posterior density for α_1 in (8), (e) Posterior density for α_1 in (9),

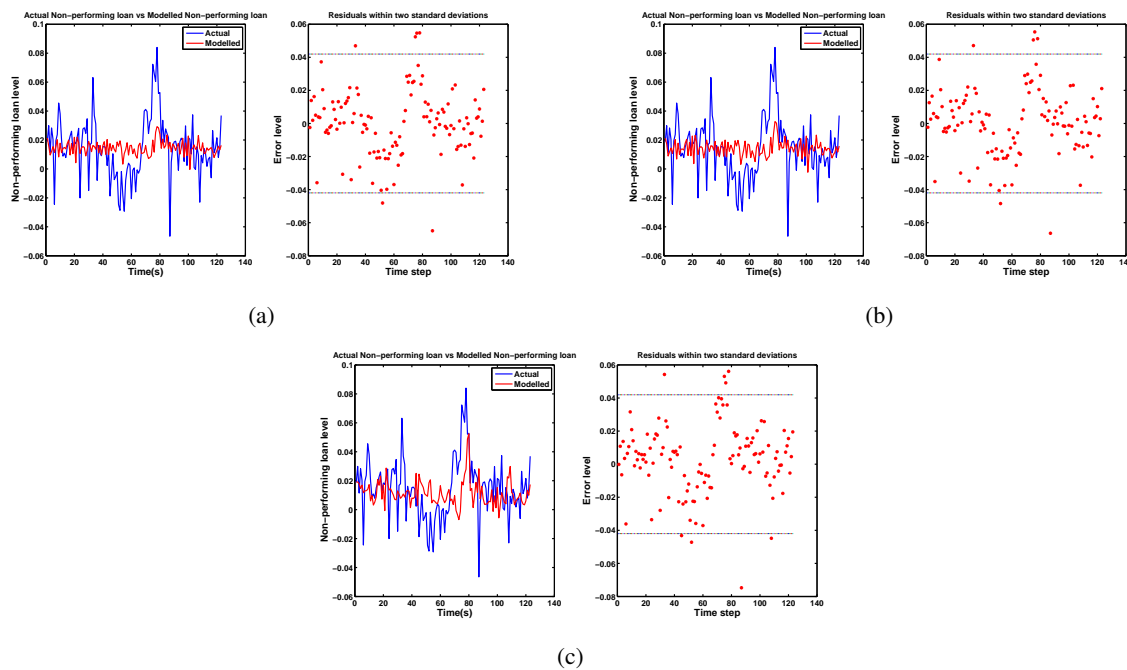


Figure 2: Non-performing loan ratio vs Model response with residuals within two standard deviations; (a) Frequentist approach, (b) Bayesian approach with posterior distribution in (8), (c) Bayesian approach with posterior distribution in (9)

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