Demand forecasting in food retail: a comparison between the Holt-Winters and ARIMA models

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Abstract: - For business operations in retail companies which work with food products with short life cycle and perishables, the accuracy of forecast is of crucial importance because of the volatile demand pattern, influenced by an environment of rapid and dynamic response. In several studies in the literature, the choice of the most suitable forecasting model remains a central concern. In this context, this article aims to compare the performances between ARIMA and Holt-Winters (HW) models for the prediction of a time series formed by a group of perishable dairy products. As performance measures, metric analysis of the Mean Absolute Percentage Error (MAPE) and the Theil inequality index (U-Theil) were used. In this study, the HW model obtained better results regarding the performance metrics, having a better adjustment and capturing the linear behavior of the series.

Key-words:- demand forecasting, ARIMA, Holt-Winters, food products

1 Introduction

Currently, there are at least 70 different forecast models among linear and nonlinear methods for quantitative demand forecasting [14]. The models share the same basic concept, but follow paradigms from different areas. In the literature regarding prediction, it is well established that no quantitative model would be ideal for all situations under any circumstances [21]. Although several comparative studies have been described in the literature, the findings do not suggest what conditions make a method better than another [27]. Therefore, situations of complexity, seasonality and perishability, as occur in the retail of food products, require investigative studies about the most appropriate method for each condition [26].

Additionally, in order to make this corporate resource more robust, the present article aims the comparison between two lineardemand forecasting models: The Autoregressive Integrated Moving Average (ARIMA) model and the Holt-Winters (HW) model. These techniques have exceptional adaptive capacity to deal with the linearity in problem solving [17].

Several studies in the literature comparing linear and nonlinear forecasting methods [26]have been performed with aggregate data of retail products, both internationally [20, 7]and in Brazil [1, 26]. All these works show that any prediction model can be considered universally the best [17] and, so far, no research has conclusively correlated the characteristics which are decisive for the choice of a specific forecasting method [21]. As a result, the accuracy of each technique has been confirmed by the report of a large number of examples and the empirical comparisons remain the best tool for solving specific problems. Many theoretical and heuristic methods were developed and tested empirically in recent decades [2].In the next section will be addressed the empirical theoretical framework in order to support the models proposed in this study.

2 Theoretical framework

The review will be limited to the methodologies used in this work, therefore, will only be described two models of demand forecasting: ARIMA and Holt-Winters, as well as the performance metrics of accuracy of demand forecasting, which involve only the MAPE and U-Theil.

2.1 Holt-Winters method

The origin of the Holt-Winters (HW) method dates from the 1950s when renowned researchers met to satisfy a request from the Office of Naval Research, Planning and Control of Industrial Operations in the United States. The goal was to develop a high accuracy and low cost forecasting model that could integrate with the existing system [12]. In 1957s, the researcher Charles Holt showed that the forecasting method most often used at the time, the method of exponentially weighted moving average, could be used not only to smooththe level of a variable, but also to smooth he trend, seasonality and other components of a prediction[12].

The new model created by Holt could control multiplicative and additive seasonality, additive and multiplicative trend and standard errors. In addition, the new system was quick and easy programming, required minimal data storage, used simple initial conditions and robust parameters, and allowed automatic adaptation. This model was then studied and tested in several time series by the graduate student P.R. Winters, who found that the prediction formulas were surprisingly accurate [12]. Winters published his results in 1960s and the new model was called Holt-Winters [13] method. The formulas of Holt and Holt-Winters were quickly incorporated into commercial software systems for forecasting and more than 50 years later, still have been used in researches[11].

In general, it can be said that the Holt-Winters method, sometimes called the method of Winters or seasonal exponential smoothing, is a sophisticated extension of the exponential smoothing methodology, since it generalizes this methodology to deal with trend and seasonality. To do so, it considers α, γ, δ as the three smoothing parameters and p denotes the number of observations per seasonal cycle [13]. It is similar to the linear method of Holt, with an additional equation to deal with seasonality.

There are two different methods of Holt-Winters, differing by how the seasonality is modeled, classified as additive and multiplicative. This work will be applied only to the HW multiplicative model. In order to understand this classic forecasting model, this section will explain both methods.

2.1.1 Holt-Winters multiplicative method

The Holt-Winters multiplicative method is so termed because the seasonality is multiplied by the trend, as shown in the following equations [13]:

Indexl_t: $l_t = \alpha \frac{y_t}{s_{t-m}} + (1-\alpha)(l_{t-1} + b_{t-1})$ (1)

Trendb_t: $b_t = \beta^*(l_t - l_{t-1}) + (1-\beta^*)b_{t-1}$ (2)

Seasonality s_t : $s_t = \gamma y_t/(l_{t-1} + b_{t-1}) + (1-\gamma)s_{t-m}$ (3)

Forecast$\hat{y}_{t+h/t}$:$\hat{y}_{t+h/t} = (l_t + b_t h)s_{t-m+h_m^+}$ (4)

Whereis the extent of seasonality (for example, number of months or quarters in a year), l_t represents the series level, b_t denotesthe trend, s_t is the seasonal component, $\hat{y}_{t+h/t}$is the forecast for h periods ahead and $h_m^+ = [(h-1) \mathrm{mod}\, m] + 1$. The parameters ($\alpha$, β^*andγ) are usually restricted to the range 0 to 1.

2.1.2 Holt-Winters additive method

The Holt-Winters additive method is so termed because the seasonality is added to the series trend (which is represented by the sum of the level and the growth), as shown in the following equations[13]:

Indexl_t: $l_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1})$ (5)

Trendb_t: $b_t = \beta^*(l_t - l_{t-1}) + (1-\beta^*)b_{t-1}$ (6)

Seasonalitys_t: $s_t = \gamma(y_t - l_{t-1} + b_{t-1}) + (1-\gamma)s_{t-m}$ (7)

Forecast$\hat{y}_{t+h/t}$: $\hat{y}_{t+h/t} = l_t + b_t h + s_{t-m+h_m^+}$ (8)

Equations (2) and (6) are identical.The difference in the Holt-Winter additive method is that seasonality is added and subtracted rather than multiplied and divided, as in the multiplicative method. Critics to the Holt-Winter additive method claim that it does not generate good estimates for the level and to the seasonality of the time series. This limitation, however, can be corrected by a particular approach [15].

When the seasonal component is additive,the error correction calculation to update the local average level, l_t, the local trend, b_t, and the seasonal local index, s_t, become available as follows [5, 6]:

$$l_t = l_{t-1} + b_{t-1} + \alpha e_t \quad (9)$$

$$b_t = b_{t-1} + \alpha\gamma e_t \quad (10)$$

$$s = s_{t-p} + \delta(1-\alpha)e_t \quad (11)$$

Where $e_t = X_t - \hat{X}_{t-1}(1)$ representsthe forecast error one step ahead in the time *t*. Thus, the new forecast performed in the time*t* of$X_{t+k}$is given by:

$$\hat{X}_t(k) = l_t + kb_1 + s_{t-p+k} \quad (12)$$

for $k = 1,2, \dots, p$. There is a formula for the case of multiplicative seasonality.

As with all exponential smoothing methods, the components of Holt-Winters model (either additive or multiplicative) are obtained by setting the values of the constants α, β e γto estimate the initial values [13]. From these, it is possible to establish the specific values for the parameters to minimize the sum of squared errors of the forecast one step ahead [6, 23].

The calculation of the level l_t is a combination between the observed and the adjusted in the period *t*, removing the seasonal effect of the series. The trend b_t, as well as in the Holt method, is calculated by sequential difference of two consecutive levels, and the seasonality s_t is calculated by the combination between the observed and the adjusted seasonal index in period*t* [24].

Finally, it is important to note that different authors can use equations in its equivalent form and this is considered an additional source of confusion in the application of Holt-Winters model.

2.2Autoregressive integrated moving average (ARIMA) model

The ARIMA model has been extensively studied and applied in studies of forecast due to their attractive theoretical properties and because of the various empirical supporting evidences. In addition, ARIMA model has equivalence with most models of exponential smoothing, except for the multiplicative form of Holt-Winters [19].

The ARIMA model was popularized by George Box and GwilymJenkins in the 1970s, with application in time series analysis and forecasting. The underlying theories described by Box and Jenkins [3] and later by Box, Jenkins and Reinsell [4] are sophisticated, but easy to understand and apply.

Any forecasting method involves two steps [9]: (i) the analysis of time series and (ii) the selection of the forecasting model that best fits to the data set. Likewise, for ARIMA, is useda similar sequence of analysis and selection by decomposition methods and regression [3]. In this sense, this section is divided in two main parts: first, the basic concepts of autoregressive moving average models that support the ARIMA model are described, and then, the application of this model in time series forecasting.

2.2.1 ARIMA model for time series data

The regression model takes the form:

$$Y_t = b_0 + b_1X_1 + b_2X_2 + \cdots + b_pX_p + e_t (13)$$

Where Y is the predicted variable, X_1 to X_p are explanatory variables, b_0 to$b_p$are linear regression coefficients and $e_t$represents the error.If, however, these variables are defined as $X_1 = Y_{t-1}$, $X_2 = Y_{t-2}, X_3 = Y_{t-3}, \dots$, $X_p = Y_{t-p}$, the equation (13) become:

$$Y_t = b_0 + b_1Y_{t-1} + b_2Y_{t-2} + \cdots + b_pY_{t-p} + e_t \ (14)$$

Equation (14) still represents a regression equation, but differs from Equation (13) since it has different explanatory variables that are, in fact, previous values of the predictor variableY_t, called autoregressive (AR).

Just as it is possible to regress past values of a series again, there is a time series model that uses past errors as explanatory variables:

$$Y_t = b_0 + b_1e_{t-1} + b_2e_{t-2} + \cdots + b_pe_{t-q} + e_t (15)$$

In Equation (15), a dependency relationship is established between successive errors and the equation is called a moving average model (MA).

Many stationary random processes cannot be modeled purely as moving or as auto-regressive averages because they have qualities of both types of processes [4]. In this situation, the autoregressive (AR) can be effectively connected to the moving average model to form a common and general class of time series models called autoregressive moving average models (ARMA).

The ARMA model can only be used on stationary data. In practice, many of the time series are non-stationary, so that the characteristic of the underlying stochastic process changes over time. To extend the use of the ARMA model for non-stationary series is necessary to differentiate the data set. In this situation, the model is now called the autoregressive integrated moving average (ARIMA), name popularized by Box and Jenkins in 1970 [3].

It can be said that Y_t is stationary homogeneous of order*d* if$w_t = \Delta^d Y_t$is a stationary series. Considering thatΔ denotes the difference:

$$\Delta Y_t = Y_t - Y_{t-1} \quad (16a)$$

$$\Delta^2 Y_t = \Delta Y_t - \Delta Y_{t-1} \quad (16b)$$

And so on.

With a series w_t, is possible to come back to Y_t by the sum of w_t in a total of *d* times. It can be written as $Y_t = \sum^d w_t$, where $\sum$ the summation operator:

$$\sum w_t = \sum_{i=-\infty}^{t} w_i \quad (17a)$$

$$\sum^2 w_t = \sum_{j=-\infty}^{t} \sum_{i=-\infty}^{j} w_i \quad (17b)$$

And so on. It is worth noting that the summation operator $\sum$ is merely the inverse of the difference operator Δ. Since $\Delta Y_t = Y_t - Y_{t-1}$, it can be written that $\Delta = 1 - B$ and thus $\sum = \Delta^{-1} = (1-B)^{-1}$.

When calculating this sum for an effective series, begins the first observation of the original series without differentiating (Y_0) and then adds up successive values of the series in difference. Thus, if $w_t = \Delta Y_t$, Y_t can be calculated:

$$Y_t = \sum w_t = \sum_{i=-\infty}^{t} w_i = \sum_{i=-\infty}^{0} w_i + \sum_{i=1}^{t} w_i = Y_0 + w_1 + w_2 + \cdots + w_t \quad (18)$$

If Y_t was differentiated twice, such that $w_t = \Delta^2 Y$, would be possible to calculate Y_t from w_t by the sum of this termtwice.

After differentiating the series Y_t to obtain the stationary series w_t, is possible to model w_t as an ARMA process. If $w_t = \Delta^d Y_t$ is an ARMA(*p*,*q*) process, then it can be said that Y_t is an autoregressive integrated moving average processof order (*p*,*d*,*q*) or simply ARIMA(*p*,*d*,*q*). If $Y_t = \sum w_i$ increases linearly along the time, the series has a linear trend over time which is independent of the random disturbances, in other words, is deterministic [4].

Any homogeneous non-stationary time series can be modeled as an ARIMA process of order (*p*,*d*,*q*). The practical problem is to choose the most appropriate values for *p*, *d* e *q*, i.e. specify the ARIMA model [22, 4].This problem is solved in part by examining the autocorrelation function and partial autocorrelation function for the time series of interest. The first step is to determine the degree of homogeneity *d*, that is, the number of times that the series needs to be differentiated to produce a stationary series. Then it examines the correlation and partial autocorrelation function to determine possible specifications of *p* and *q* [22].

2.3 Evaluation Metrics

The mean absolute percentage error (MAPE) is the mean absolute error as a percentage of demand. This method presents problems when the series have values for closed (or equal to) zero. These problems can be avoided by including in the analysis only data with positive values, however, this artificial solution limits the application of the method in various situations [9] because of the heavy penalty on the positive errors compared to negative errors [11].Nevertheless, these authors also show that more scientific papers and discussions are still needed about the symmetric measures proposed so far. In practice, a MAPE value lower than 10% may suggest a forecast potentially very good, lower than 20%, potentially good and above 30%, potentially inaccurate [16]. The MAPE can be expressed as follows:

$$MAPE_n = \frac{\sum_{t=1}^{n} \left|\frac{E_t}{D_t}\right| 100}{n} \quad (19)$$

Where:
$|E_t|$ =absolute error value in the period *t*;
$|D_t|$ =absolute value of real demand in the period *t*;
n =all the periods.

The Theil inequality index (U-Theil) is a relative measure, in percentage terms, of the discrepancies one step ahead committed with the forecast [24, 25]. This metric assumes a decisive role in the determination of use whether or not of a forecasting technique. This model evaluates the adjustment of the series referred to the original series. The closer to zero, the greater the range of adjustment provided in respect of the original series. In contrast, values near the unit indicate that the model was unable to make good predictions. Thus, the U-Theilcan be expressed as:

$$U - Theil = \frac{\sqrt{\frac{1}{N_y}\sum_{l=1}^{N_y}(y_l^* - y_l)^2}}{\sqrt{\frac{1}{N_y}\sum_{l=1}^{N_y}(y_l^*)^2} + \sqrt{\frac{1}{N_y}\sum_{l=1}^{N_y}(y_l)^2}} \quad (20)$$

Where y_l^* is the forecasted value for the period *t*, y_l is the observed value and *N* the number of observations.

3 Methodological procedures

This work can be classified as descriptive since there is a relationship of asymmetrical association between the variables forecast demand (independent) and error (dependent), in which one influences the other, but it is not the only condition for the phenomenon to occur [10]. To investigate the comparison between HW and ARIMA models, will be used historical demand data from a group of dairy products with short life cycle, in the period from 2005 to 2013, of a company that ranks among the three market leaders in their field of activity located in southern Brazil. This is a research ex-post factoas the "researcher has no control over the independent variable, which is the presumed factor of the phenomenon, because it has already occurred [10]".Regarding the temporal cutting, it is a sectional study. The research problem was approached quantitatively, justified by the nature of the object of study as well as the procedure used for data collection.

For a thorough comparative analysis of different methodologies, two quantitative forecasting models of linear demand will be analyzed: the model of Holt-Winters model (HW) and ARIMA. The MAPE and Theil inequality index (U-Theil) will be used as performance metrics, as presented in the previous section.The selected product group represents 70% of total sales in the southern region, composed of 50 SKUS (Stock KeepingUnits). The individual analysis of the various products shows no practical application in the management decision making process. Thus, many groups of similar products can be aggregated by predetermined criteria in a given time series and analyzed together [27, 28].

4Analyses and results

For a precise performance of a forecasting model, it is necessary to make adjustments to the parameters for each technique. Within this context, for the application of HW and ARIMA models, the data adjustments were generated by Minitab statistical software for 12 periods (seasonal). In defining the components of level, trend and seasonality, was used the statistical program Solver, a tool contained in the Excel software. As demonstrated in Table 1, for the HW model, the best fittingα, β and γwere 0,20/ 0,30/ 0,40. For the ARIMA model, due to the lower value of MAPE and U-Theil,the following adjustment parameters (*p*, *d*, *q*) were considered, shown in Table 1.

Table 1 - ARIMA and HW parameters

Adjustment parameters						
Models	**α**	**β**	**γ**	***p***	***d***	***q***
ARIMA				1	1	2
Holt-Winters	0,20	0,30	0,40			

Source: Research data

The forecast adjustments were made from 2005 to 2012 in order to project future forecast for the year 2013, in the period 97 to 108, as shown in Figure 1. Figure 1 graphically shows the results obtained using the ARIMA and HW models compared to the actual demand of retail food products under study.

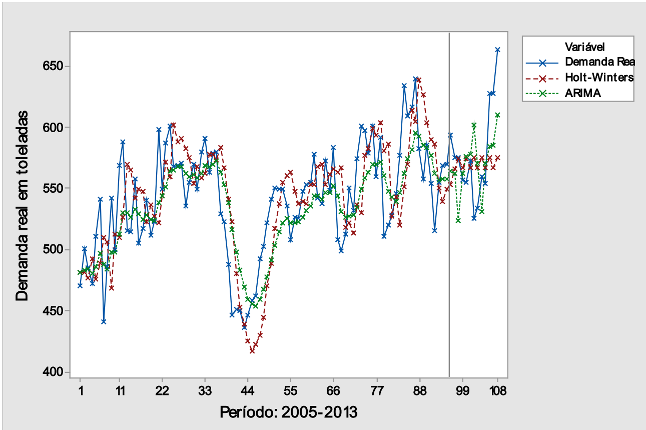

Fig. 1 – Actual demand and application of ARIMA and HW models
Source: Research data and Minitab software

Figure 2 demonstrates the residue analysis based on the performance of the HW model and Figure 3 shows the errors correlation based on the ARIMA model performance.Based on the residue analyses of Figures 1 and 2, it was observed that the model ran within the required parameters based on time series.

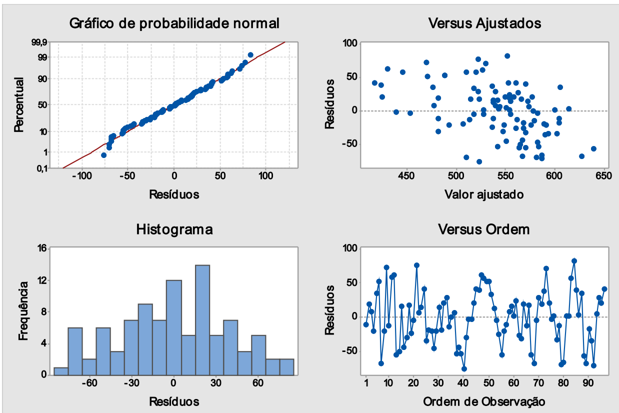

Fig. 2 - Residue analyses for the HW model
Source: Research data and Minitab software

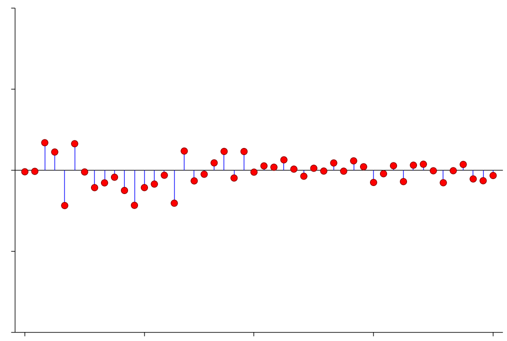

Fig. 3 - Correlation and autocorrelation residue analyses in the ARIMA model
Source: Research data and Minitab software

The Figure 3 demonstrates that the applied model is within the parameters for implementing the ARIMA technique.

Table 2 –Performance metrics results

Model	MAPE	U-Theil
ARIMA	5,66	0,031
Holt-Winters	4,97	0,019

Source: Research data and Minitab software

Based on Table 2 and on model performances, the HW model obtained better results comparing to ARIMA model, according to MAPE and U-Theil. The actual demand in 2013(Period 97 to 108), according to data shown in Figure 1, was 6.928,19 tons of products. The HW model was able to predict with an accuracy of 98,9%, with a MAPE of 4,97% andU-Theil0,019. The ARIMA achieved an accuracy of 97,4%, with 5,66% MAPE and U-Theil0,031.

Within this context, the two models reached a compatible performance for the time series in analysis. For Lewis [16], a MAPE below 10% is considered a good forecast. For the U-Theilmetric, the closer to unity the worse is the forecast, and the closer to zero, the better is the forecast. Comparing both models, even with a minimal difference, the HW got a better performance for the specific situation under study.

5Conclusions

This study aimed to compare the ARIMA and Holt-Winters models based on MAPE and U-Theil, applied to demand forecasting of a time series formed by a group of perishable dairy products with a short life cycle, in a retail company southern Brazil. The intention of comparing two classical forecasting techniques in food retailing assumed that the ARIMA model has equivalence with most models of exponential smoothing, except with the multiplicative form of the Holt-Winters model and for this reason the comparison was motivated.

The ARIMA forecasting model does not always have the desired accuracy for a given range. In the forecast ranges, the uncertainty in the estimation of the parameters is not shown. Consequently, the intervals are narrower than they should be, preventing the addition of further uncertainty. In addition, ARIMA model has many assumptions that cannot be met. One of the most important assumptions is that the historical pattern of the data will not change during the forecast period. If the model assumptions are true, we obtain an excellent forecast, otherwise the results are not satisfactory. Despite these constraints, scientific evidence shows that ARIMA methodology is competitive in terms of accuracy [17, 18, 19]and therefore remains intensively studied in order to provide support for assumptions of each case.

The Holt-Winters model is simple and can provide accurate forecasting results as those obtained with more complex techniques. This method is popular, easy to use and generally works well in practical applications. However, it is recommended that the horizon of the predictions made with the Holt-Winters method does not exceed the seasonal cycle of the series, because the predictions with a larger horizon tend to have reduced accuracy. Given the limitations of this study, it is suggested as future research, the comparison between natural computing based models, as well as the particle swarms techniques for optimization [8].

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