An Application of Decentralized Control Theory to an Economic Policy Model

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Abstract: - We show an application of decentralized control theory to a macroeconomic model of unemployment and inflation. Fiscal and monetary policies are assumed to be designed by different institutions, namely the government and central bank respectively, and both policy-makers are assigned particular targets and may have different information. It is shown that the economic system can be stabilized by decentralized feedback. An interpretation in economic terms of the stabilizing controls and the conditions for stabilizability is given.

Key-words: - Decentralized control; large-scale systems; stability properties; stabilizability; stabilizing feedback; fixed modes; economic systems.

1 Introduction

Methods and models of system and control theory have been used in various ways to analyze economic policy problems and help economic policy makers achieve their goals; see, e.g., [4], [5], [1], [8], [13], [6]. This has led to important new insights into the possibilities and limitations of designing optimal economic policies, but also into the qualitative properties of the economic system to be influenced by the policy makers. Most of these studies start from the assumption that the economic system is to be controlled and stabilized by one single fully informed economic policy maker. In the real world, however, several institutions are involved in the policy-making process which have different instruments at hand and may have different targets and information. If we assume that these policy makers share the goal of stabilizing the economy but have different information about the economy or are responsible for different aspects of the stabilization policies, decentralized system and control theory could be used for analyzing such situations. This theory can be regarded as a dynamic generalization of team theory or as a generalization of control theory to the case of multiple controllers (cf. [15], [16], [17], [11], [7], [2], among many others).

In this paper, we show by example how decentralized control theory can be used to obtain insights into an economic policy problem. We consider a simple linear dynamic model of the trade-off between unemployment and inflation in which the government's fiscal policy and the central bank's monetary policy affect these policy targets. We assume that the government is primarily interested in reducing the rate of unemployment, and the central bank is mostly concerned about the rate of inflation. Both policy-making institutions aim at stabilizing the economy in the sense of driving it faster to its steady state. Decentralized stabilizability of the macroeconomic system under consideration is studied, using the notion of fixed modes introduced by [18]. Due to the simple character of the model, it can be solved analytically; more realistic larger models would allow for numerical solutions only.

2 The Economic Model

We consider a simple dynamic macroeconomic model of the trade-off between unemployment and inflation in continuous time. To simplify calculations, it is specified in linear functional form in growth rates. The rate of inflation $p(t)$ is determined by the expectations-augmented Phillips curve:

$$p(t) = g \cdot \log[Q(t) / Q_N(t)] + p^*(t), \quad g > 0, \qquad (1)$$

where $p^*(t)$ denotes the expected rate of inflation, $p(t) \equiv \dot{P}(t) / P(t) = d / dt \log[P(t)]$ the actual rate of inflation, $P(t)$ the general price level, $Q(t)$ real aggregate output and $Q_N(t)$ natural (potential, full-employment) real output. As a measure of aggregate excess demand in the goods market, we consider the Okun gap $(Q(t)-Q_N(t))/Q_N(t)$, which is approximated by $\log[Q(t)/Q_N(t)]$ in the neighborhood of $Q(t)=Q_N(t)$. The expected rate of inflation moves towards the actual rate of inflation by an adaptive process:

$$\dot{p}^*(t)=n[p(t)-p^*(t)],\quad n>0. \tag{2}$$

Financial markets are characterized by portfolio equilibrium given by a semi-logarithmic LM curve:

$$M(t)/P(t)=Q^b(t)\quad \exp[-cR(t)],\quad b,c>0, \tag{3}$$

$$\text{or } m(t)-p(t)=bq(t)-c\dot{R}(t), \tag{4}$$

where $M(t)$ is the nominal stock of money, $m(t)$ its growth rate, $R(t)$ the nominal rate of interest, and $q(t)\equiv\dot{Q}(t)/Q(t)$ the growth rate of real output or income; $\exp(\cdot)$ denotes the exponential function, $\log(\cdot)$ the natural logarithm.

For the goods market (the IS curve) we assume the following reduced form ad-hoc function, following various macroeconomic models (e.g. [14]):

$$\log Q(t)=a\log[D(t)/P(t)]+\log Z(t)-eR'(t),\ a,e>0, \tag{5}$$

where $D(t)$ is the nominal budget deficit and $Z(t)$ is an exogenous variable affecting the goods market. The real rate of interest $R'(t)$ is given by the Fisher equation:

$$R(t)=R'(t)+p^*(t). \tag{6}$$

Equation (5) can also be interpreted as a linearized relation between the growth rates:

$$q(t)=ad(t)-ap(t)-z(t)-e\dot{R}'(t), \tag{7}$$

with $d(t)\equiv\dot{D}(t)/D(t)$ and $z(t)\equiv\dot{Z}(t)/Z(t)$.

The rate of unemployment $u(t)$ depends on aggregate excess demand in the goods market through Okun's Law:

$$u(t)=u_N(t)-h\log[Q(t)/Q_N(t)],\quad h>0, \tag{8}$$

where $u_N(t)$ is the natural (full-employment) rate of unemployment. We assume the natural rate of unemployment and the growth rate of natural output to be constant:

$$\dot{u}_N(t)=\dot{q}_N(t)=0 \text{ with } q_N(t)\equiv[\dot{Q}_N(t)/Q_N(t)].$$

The above model has a long-run steady state where all variables are constant. Because of (2), in this equilibrium, inflationary expectations are fulfilled, $p^\infty=p^{*\infty}$, and due to (1) and (8) the hypothesis of the natural rate of unemployment holds: $Q^\infty=Q_N^\infty$ and $u^\infty=u_N$; moreover, $q^\infty=q_N$.

We assume that the growth of the nominal budget deficit $d(t)$ is determined by the government and is hence a fiscal policy instrument or control variable of the government; the central bank, on the other hand, determines the growth rate of money supply $m(t)$ as its monetary policy instrument. Both policy makers can influence the target variables rate of inflation $p(t)$ and rate of unemployment $u(t)$. For the long-run steady state we have from (4):

$$p^\infty=m^\infty-bq_N; \tag{9}$$

i.e. the long-run rate of inflation is determined by monetary policy according to the quantity theory of money. However, due to (7) we must have for $z^\infty=0$:

$$m^\infty=d^\infty+(b-(1/a))q_N. \tag{10}$$

Therefore monetary and fiscal policy must be set in a fixed relation to each other in order to achieve a long-run steady state or equilibrium in which they jointly determine the long-run rate of inflation. That is, in the short run the demand side of the economy (IS and LM curves) determines the rate of interest and real output, but in the long run these variables are independent of demand side influences, the latter influencing only the price level and the rate of inflation.

For a system-theoretic analysis of the model, we determine first its state space form (cf. e.g. [1]). To do so, we eliminate $R(t)$ and $R'(t)$ from (4), (6) and (7) to determine $q(t)$. Differentiation of (1) with respect to time gives:

$$\dot{p}(t)=g[q(t)-q_N]+\dot{p}^*(t) \tag{11}$$

and from (8) we get:

$$\dot{u}(t) = -h[q(t) - q_N]. \quad (12)$$

Substituting for $p^*(t)$ from (2) and for $q(t)$ from the reduced-form demand side relation calculated before, we arrive at the following state space representation of the model:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{W}(t) \quad (13)$$

with $\mathbf{x}(t) \equiv \begin{bmatrix} u(t) \\ p(t) \\ p^*(t) \end{bmatrix}$, $\mathbf{u}(t) \equiv \begin{bmatrix} d(t) \\ m(t) \end{bmatrix}$, $\mathbf{W}(t) \equiv \begin{bmatrix} w_1(t) \\ w_2(t) \\ 0 \end{bmatrix}$,

$$\mathbf{A} \equiv \begin{bmatrix} 0 & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}, \quad \mathbf{B} \equiv \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ 0 & 0 \end{bmatrix}.$$

Here we have $a_{12} = h[(ac + e - cen)/(be + c)]$, $a_{13} = cehn/(be + c)$, $b_{11} = -ach/(be + c)$, $b_{12} = -eh/(be + c)$, $w_1(t) = h[q_N - (c/(be + c))z(t)]$, $a_{22} = n[1 + ceg/(be + c)] - g((ac + e)/(be + c))$, $a_{23} = -n[1 + ceg/(be + c)]$, $b_{21} = acg/(be + c)$, $b_{22} = eg/(be + c)$, $w_2(t) = -g[q_N - (c/(be + c))z(t)]$, $a_{32} = n$ and $a_{33} = -n$. $\mathbf{x}(t)$ is the state vector, $\mathbf{u}(t)$ the control vector. $\mathbf{W}(t)$ contains the exogenous variables, which are irrelevant for the problem of stabilizing the system because they cannot be influenced by economic policies; hence they will be neglected in the following. We assume that the development of $z(t)$ does not induce a need for economic policy action during the adjustment process to the steady state.

3 Stability Properties of the Model

The dynamic system (13) is asymptotically stable iff it converges to the long-run steady state determined by $\mathbf{W}(t)$ for $t \to \infty$, where $\dot{\mathbf{x}}(t) = 0$. Without active policy measures, that is with $m(t) = 0$ and $d(t) = 0$ (or if $m(t)$ and $d(t)$ are exogenous variables depending on time only), this is the case iff $\mathbf{A}$ is a stable matrix, that is, iff all eigenvalues of $\mathbf{A}$ have negative real parts. Because of $\det \mathbf{A} = 0$ (det denotes the determinant) one eigenvalue is zero and the system is trivially not stable. The subsystem formed by the inflation variables $p(t)$ und $p^*(t)$ can be stable or unstable, depending on the parameters.

According to the Routh-Hurwitz conditions (see e.g. [10], p. 93), this subsystem is stable iff $a_{22} + a_{33} < 0$ and $a_{22}a_{33} - a_{23}a_{32} > 0$. The second condition is always fulfilled because

$$a_{22}a_{33} - a_{23}a_{32} = gn[(ac + e)/(be + c)]. \quad (14)$$

Both eigenvalues are hence either positive (the subsystem is totally unstable) or negative (it is totally stable). According to the first condition the subsystem is stable iff

$$S \equiv ac + e - cen > 0. \quad (15)$$

Because of $\partial S/\partial a = c > 0$, $\partial S/\partial n = -ce < 0$, $\partial S/\partial c = a - en > 0 \Leftrightarrow a/e > n$ and $\partial S/\partial e = 1 - cn > 0 \Leftrightarrow 1/c > n$, this is more likely to be fulfilled, ceteris paribus, the smaller the speed of adjustment of inflationary expectations n and the higher the reaction of aggregate demand to the budget deficit a. For large a and small n the subsystem is more likely to be stable the larger c (or e) is for small e (or c), and the smaller c (or e) is for large e (or c). The interest elasticities of the demand for money and for goods have therefore opposite effects on the stability of the price subsystem. Both a "fiscalist" (c large, e small) and a "monetarist" (c small, e large) scenario contribute to making the price subsystem stable. On the influence of the parameter n, see also [12]. As has to be expected, substituting rational for adaptive inflationary expectations (equation (2)) results in an unstable inflation equation; this can easily be seen by substituting $p(t) = p^*(t)$ and hence $q(t) = q_N$ into the demand system (4), (6) and (7).

The eigenvalues of $\mathbf{A}$ are obtained by solving the characteristic equation

$$\det(s^0\mathbf{I} - \mathbf{A}) = s^0[(s^0 - a_{22})(s^0 - a_{33}) - a_{23}a_{32}] = 0, \quad (16)$$

where $\mathbf{I}$ denotes the (three-dimensional) identity matrix and s^0 is an eigenvalue of $\mathbf{A}$. Obviously we have $s_1^0 = 0$; the eigenvalues $s_{2,3}^0$ are given by

$$s_{2,3}^0 = (g/(be + c))(cen - ac - e) \pm (1/2)\sqrt{g/(be + c)}\sqrt{[g/(be + c)](cen - ac - e)^2 - 4n(ac + e)} \quad (17)$$

or by

$$\begin{cases} s_2^0 + s_3^0 = [g/(be + c)](cen - ac - e), \\ s_2^0 \cdot s_3^0 = gn[(ac + e)/(be + c)]. \end{cases} \quad (18)$$

If s_2^0 and s_3^0 are positive ($S < 0$), economic policies will aim at stabilizing the system; if they are negative, the policies will aim at raising the speed of convergence.

As for the notion of stabilization, we use the following terminology and assumptions: The economic policy makers cannot or do not want to observe all variables of the system. For instance, inflationary expectations in general are not directly observable. We assume that only certain linear combinations of the variables contained in $\mathbf{x}(t)$ are observed. Then the policy maker gets as the output of the system (13):

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t), \tag{19}$$

where $\mathbf{C}$ is a known matrix. Economic policy makers react to the observation or output $\mathbf{y}(t)$ by determining the values of the instruments (control). Hence the control variables are fed back into the observation vector by a linear control law:

$$\mathbf{u}(t) = \mathbf{K}\mathbf{y}(t) = \mathbf{K}\mathbf{C}\mathbf{x}(t), \tag{20}$$

resulting in the overall closed-loop dynamics of the system with endogenous $\mathbf{u}(t)$ according to (20):

$$\dot{\mathbf{x}}(t) = (\mathbf{A} + \mathbf{BKC})\mathbf{x}(t) + \mathbf{W}(t). \tag{21}$$

The economic policy problem now consists in the choice of $\mathbf{K}$ such that the entire system (21) is stable.

For centralized dynamic systems we know from the results of [19] and [3] that for controllable and observable systems of the form (13) and (19), there exist feedbacks generating a stable closed-loop system, that is, a system with all eigenvalues having negative real parts. It is even possible to design the feedback such as to place all eigenvalues of the overall system at pre-specified values, although for $\mathbf{C} \neq \mathbf{I}$ in general dynamic instead of static compensators (as in (20)) are required. For the economic policy problem of stabilization this means that in the case of centralized decision making, it is possible to achieve some desired stability behavior of the politico-economic system resulting from including the feedback of the instrument variables in the economic system (13) by an appropriate choice of the eigenvalues, provided the system is controllable and observable. In particular, all eigenvalues of the closed-loop system can be placed in the left complex half plane to make the politico-economic system stable. Stabilizability hence has a close connection to the property of controllability, which also has a neat economic policy interpretation in the context of the existence, uniqueness and design problems in the theory of economic policy ([13]). On the other hand, it must be noted that we use stabilization in a particular (qualitative) sense here, related to the adjustment dynamics towards a steady state; often in the economics literature, stabilization policies refer to a quantifiable goal to be achieved by solving an optimization problem.

4 Decentralized Stabilizability

Now let us take into account that government and central bank are not one homogeneous decision-making body but make decisions separately and not necessarily in a coordinated way. This is in particular the case in the European Economic and Monetary Union where there is one central bank and several governments, but it also holds in countries like the US where the Fed has a considerable degree of independence from the government. Therefore fiscal and monetary policy must be distinguished analytically. One possibility of doing so consists in assuming that both policy makers have access to different observations, each of them receiving its own output $\mathbf{y}_i(t), i = 1,2$, from the system, where

$$\mathbf{y}_i(t) = \mathbf{C}_i\mathbf{x}(t). \tag{22}$$

This assumption can be justified by presuming that each of the two institutions has its own economic advisor observing (and interpreting) the economic system, possibly according to different economic theories. Another interpretation of the decentralized information pattern (22) could start from the observation that government and central bank are interested in particular target variables only and neglect other ones. For example, in many countries governments are primarily interested in reducing unemployment only, while central banks often devote their entire attention to the price level and inflation. Then it is quite natural to assume that the government feeds its decisions back to the rate of unemployment only, while the central bank does so for the rate of inflation exclusively. For example, in the European Economic and Monetary Union, the European Central Bank interprets its mandate as aiming exclusively at price stability in the Euro zone, and similar provisions are contained in the statutes of central banks in many other countries. To some extent, this can be an efficient assignment of targets to instruments and to policy-making bodies in the sense of Mundell [9].

We now start from the following system for the case of two decision makers in our model:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_1\mathbf{u}_1(t) + \mathbf{B}_2\mathbf{u}_2(t) \tag{23}$$

with the information pattern (22). Here we denote the government as decision maker 1 and the central bank as decision maker 2: $\mathbf{u}_1(t) \equiv d(t)$, $\mathbf{u}_2(t) \equiv m(t)$ and

$$\mathbf{B}_1 \equiv \begin{bmatrix} b_{11} \\ b_{21} \\ 0 \end{bmatrix}, \ \mathbf{B}_2 \equiv \begin{bmatrix} b_{21} \\ b_{22} \\ 0 \end{bmatrix}.$$

The problem of stabilization policy now consists in finding independent (in general, dynamic) feedbacks for each of the two decision makers to achieve a stable closed-loop politico-economic system. A general form of such a control law would be:

$$\begin{aligned} &\mathbf{u}_i(t) = \mathbf{Q}_i\mathbf{z}_i(t) + \mathbf{K}_i\mathbf{y}_i(t) + \mathbf{L}_i\mathbf{v}_i(t), \\ &\dot{\mathbf{z}}_i(t) = \mathbf{S}_i\mathbf{z}_i(t) + \mathbf{R}_i\mathbf{y}_i(t) + \mathbf{G}_i\mathbf{v}_i(t), \quad i = 1,2. \end{aligned} \tag{24}$$

Here $\mathbf{z}_i(t)$ is the state of the i-th compensator (controller), $\mathbf{v}_i(t)$ is an external control input of the i-th decision maker, and $\mathbf{Q}_i$, $\mathbf{K}_i$, $\mathbf{L}_i$, $\mathbf{S}_i$, $\mathbf{R}_i$ and $\mathbf{G}_i$ are matrices of appropriate dimensions. The problem of choosing these matrices to achieve a closed-loop system consisting of (24) with (23) and (22) with only stable eigenvalues was first formulated and solved to a certain extent by Wang and Davison [18]. They provided the following necessary and sufficient condition for the existence of a decentralized control law stabilizing the system in the sense considered here: the system is stabilizable by decentralized feedback iff all fixed modes are located in the left complex half-plane (have negative real parts). Fixed modes are the common eigenvalues of the combined system (22)–(24), that is, of $\mathbf{A} + \mathbf{BKC}$, where all feasible (block-diagonal) matrices $\mathbf{K}$ are taken into account, and

$$\mathbf{B} = [\mathbf{B}_1, \mathbf{B}_2], \ \mathbf{C} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \end{bmatrix} \text{ and } \mathbf{K} = \begin{bmatrix} \mathbf{K}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_2 \end{bmatrix}.$$

The set of fixed modes is thus given by the intersection $\{ s^f \mid \det(s^f\mathbf{I} - \mathbf{A} - \mathbf{BKC}) = 0$ for all feasible $\mathbf{K} \}$. If we want to know whether a given system is stabilizable by decentralized feedback, we therefore have to determine its fixed modes. If there is an element with a non-negative real part among them, the system cannot be stabilized by decentralized feedback.

This procedure can be applied to our simple model. In order to find out whether the economic system represented by this model has fixed modes, we have to determine s^d from $\det(s^d\mathbf{I} - \mathbf{A} - \mathbf{BKC}) = 0$ for general block-diagonal $\mathbf{K}$ and a given information pattern $\mathbf{C}_1, \mathbf{C}_2$. The fixed modes, if they exist, must be eigenvalues of $\mathbf{A}$ (because $\mathbf{K} = \mathbf{0}$ is feasible according to the criterion). Let us consider first a very general information pattern:

$$\mathbf{C}_1 = \begin{bmatrix} c_{11}^1 & c_{12}^1 & c_{13}^1 \\ c_{21}^1 & c_{22}^1 & c_{23}^1 \\ c_{31}^1 & c_{32}^1 & c_{33}^1 \end{bmatrix}, \ \mathbf{C}_2 = \begin{bmatrix} c_{11}^2 & c_{12}^2 & c_{13}^2 \\ c_{21}^2 & c_{22}^2 & c_{23}^2 \\ c_{31}^2 & c_{32}^2 & c_{33}^2 \end{bmatrix}.$$

The matrix $\mathbf{K}$ has the structure:

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{21} & k_{22} & k_{23} \end{bmatrix}.$$

Then $\mathbf{BKC}$ is given by

$$\mathbf{BKC} = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

with $l_{kr} = \sum_{j=1}^{2} b_{kj} \sum_{i=1}^{3} c_{ir}^j k_{jr}; \quad k = 1,2; \quad r = 1,2,3$.

The characteristic equation of this system is given by

$$\begin{aligned} &\det(s^d\mathbf{I} - \mathbf{A} - \mathbf{BKC}) = s^{d3} - (a_{22} + a_{23} + l_{11} + l_{22})s^{d2} \\ &+ [a_{22}a_{33} - a_{23}a_{32} + l_{11}l_{22} + l_{12}l_{21} \\ &+ (a_{22} + a_{33})l_{11} + a_{33}l_{22} - a_{12}l_{21} - a_{32}l_{23}]s^d \\ &+ a_{12}a_{33}l_{21} - a_{13}a_{32}l_{21} + a_{22}a_{33}l_{11} + a_{23}a_{32}l_{11} \\ &- a_{32}l_{21}l_{13} + a_{32}l_{11}l_{23} + a_{33}l_{12}l_{21} - a_{33}l_{11}l_{22} = 0. \end{aligned} \tag{25}$$

In this general form, it is not easily possible to determine the fixed modes, although it is possible to show that from the relations between the coefficients a_{ij} and b_{kl} $s^d = 0$ follows. Therefore we consider a more specific information pattern:

$$\mathbf{C}_1 = [1 \quad 0 \quad 0], \quad \mathbf{C}_2 = [0 \quad 1 \quad 0]. \tag{26}$$

This means that the government observes the rate of unemployment only and the central bank observes the rate of inflation only. This can be justified by an assignment of the respective targets to the two institutions, as explained above, meaning that each institution observes (is interested in) only the particular target variable assigned to it. Neither policy maker observes the expected rate of inflation. The government receives as output from the system $\mathbf{y}_1(t) = u(t)$, the central bank receives $\mathbf{y}_2(t) = p(t)$. The matrix $\mathbf{K}$ now has the structure

$$\mathbf{K} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}. \tag{27}$$

We obtain:

$$\mathbf{BKC} = \begin{bmatrix} b_{11}k_1 & b_{12}k_2 & 0 \\ b_{21}k_1 & b_{22}k_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{28}$$

$$\mathbf{A} + \mathbf{BKC} = \begin{bmatrix} b_{11}k_1 & a_{12} + b_{12}k_2 & a_{13} \\ b_{21}k_1 & a_{22} + b_{22}k_2 & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}, \tag{29}$$

and

$$\begin{aligned} &\det(s^d\mathbf{I} - \mathbf{A} - \mathbf{BKC}) = s^{d3} - (a_{22} + a_{33} + b_{11}k_1 + b_{22}k_2)s^{d2} \\ &+ [a_{22}a_{33} - a_{23}a_{32} + a_{22}b_{11}k_1 + a_{33}b_{22}k_2 \\ &+ b_{11}b_{22}k_1k_2 + a_{33}b_{11}k_1 - a_{12}b_{21}k_1 - b_{12}b_{21}k_1k_2]s^d \\ &+ a_{12}a_{33}b_{21}k_1 - a_{13}a_{32}b_{21}k_1 - a_{22}a_{33}b_{11}k_1 \\ &+ a_{23}a_{32}b_{11}k_1 - a_{33}b_{11}b_{22}k_1k_2 + a_{33}b_{12}b_{21}k_1k_2 = 0. \end{aligned} \tag{30}$$

From this follows:

$$s_1^d + s_2^d + s_3^d = (a_{22} + a_{33}) + (b_{11}k_1 + b_{22}k_2), \tag{31}$$

$$\begin{aligned} &s_1^d \cdot s_2^d + s_1^d \cdot s_3^d + s_2^d \cdot s_3^d = (a_{22}a_{33} - a_{23}a_{32}) \\ &+ (a_{22} + a_{33})b_{11}k_1 - a_{12}b_{21}k_1 + a_{33}b_{22}k_2 + (b_{11}b_{22} - b_{12}b_{21})k_1k_2, \end{aligned} \tag{32}$$

$$\begin{aligned} &s_1^d \cdot s_2^d \cdot s_3^d = (a_{22}a_{33} - a_{23}a_{32})b_{11}k_1 - (a_{12}a_{33} - a_{13}a_{32})b_{21}k_1 \\ &+ a_{33}(b_{11}b_{22} - b_{12}b_{21})k_1k_2. \end{aligned} \tag{33}$$

To find out whether there are fixed modes in this system, we want to know whether the eigenvalues of the uncontrolled system $(s_i^0, i = 1,2,3)$ can be eigenvalues of the decentralized closed-loop system $(s_i^d, i = 1,2,3)$. For $s_1^0 = 0$ this would mean $s_1^d = 0$, which is possible only if the right-hand side of (33) is identically equal to zero. From substituting for a_{ij} and b_{kl} into (33) it can be seen that this is indeed the case for arbitrary values of k_1 and k_2. Hence $s_1^d = 0$ is a fixed mode. This means that stabilization policies cannot affect the rate of unemployment in a stabilizing way if each of the two policy makers only takes care of its target proper, that is, under information pattern (26). With $s_1^d = 0$ we obtain from (31) and (32):

$$\begin{cases} s_2^d + s_3^d = \left(\dfrac{g}{be+c}\right)(cen - ac - e) - \left(\dfrac{ach}{be+c}\right)k_1 + \left(\dfrac{eg}{be+c}\right)k_2, \\ s_2^d \cdot s_3^d = \left(\dfrac{gn}{be+c}\right)(ac + e - k_2 e). \end{cases} \tag{34}$$

Comparing (34) with (18) shows that

$$\begin{cases} s_2^d + s_3^d = s_2^0 + s_3^0 - Ek_1 + Fk_2, \quad E, F > 0 \\ s_2^d \cdot s_3^d = s_2^0 \cdot s_3^0 - Gk_2. \end{cases} \tag{35}$$

s_2^d and s_3^d therefore cannot be fixed modes because they can be shifted by feedback. It is always possible to obtain eigenvalues such that $s_2^d \cdot s_3^d > s_2^0 \cdot s_3^0 > 0$ and $s_2^d + s_3^d < s_2^0 + s_3^0$ and in particular $s_2^d + s_3^d < 0$ by appropriate choice of $k_1 > 0$ and $k_2 < 0$. $k_1 > 0$ means that fiscal policy by the government feeds back positively into the target variable of the government: the budget deficit is raised when unemployment rises. $k_2 < 0$ means a negative feedback of monetary policy into the target variable of the central bank, the rate of inflation: money supply growth is reduced when inflation rises. This perfectly accords with the idea of a countercyclical stabilization policy as practiced in many countries. However, one should take into account that, due to the fixed mode s_1^d, the rate of unemployment is affected only indirectly by this policy: in the steady state, we have $u(t) = u_N$, and when the steady state is not yet reached, it is possible to exert influence on the rate of unemployment only by influencing $p(t)$ and $p^*(t)$. Thus the government, by setting $k_1 > 0$, contributes to stabilizing the price subsystem and not the rate of unemployment.

In the present case, stabilizing the price subsystem is even possible by using static compensators. If the uncontrolled system is unstable, i.e. if $S < 0$ in (15), stabilization just requires

$$\left(\frac{g}{be+c}\right)(cen - ac - e) - \left(\frac{ach}{be+c}\right)k_1 - \left(\frac{eg}{be+c}\right)|k_2| < 0. \tag{36}$$

If $\varepsilon \equiv -gS > 0$, $k_1 > 0$ and $k_2 < 0$ have to be chosen such that

$$achk_1 + eg \cdot |k_2| > \varepsilon . \tag{37}$$

This means that the government's efforts to stabilize are made easier ceteris paribus (smaller k_1 is sufficient) by larger values of a (the reaction of aggregate demand to the budget deficit), c (the interest elasticity of the demand for money), and h (the strength of the Okun relation), hence by a fiscalist scenario. The central bank's efforts to stabilize are made easier ceteris paribus by larger values of e (the interest elasticity of aggregate demand) and g (the strength of the price level's reactions to goods market disequilibria), hence by a monetarist scenario. In any case the politico-economic system can be stabilized by such policies. As stabilization policies fulfilling (37) are of course not unique, the remaining degrees of freedom in determining k_1 and k_2 can be used to fix the desired rate of inflation for the steady state p^∞, taking into account the consistency of budgetary and monetary policies (for $z^\infty = 0$ expressed by (10)).

5 Concluding Remark

In this paper, we have shown that decentralized system and control theory can contribute to the qualitative analysis of economic models and for problems of economic policy. By interpreting the conditions for decentralized stabilizability in economic terms light can be shed on the desirability of target assignments and on the possibilities and frontiers of stabilization policies conducted by independent decision makers under partial information. Next, extensions to more complicated (and more realistic) models should be attempted, including those with nonlinearities and stochastic elements. In general, it will not be possible to execute these tasks analytically. However, there exist implementations of computer algorithms which enable the determination of the fixed modes and a systematic analysis of stabilizing feedback control laws ([7]). We hope to have shown that these developments, which originated in the electrical and electronic engineering literature, can provide insights into economic problems, too.

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