Table 2. Errors between RK4 and SDM approximate solutions for model Eq. (1).

Time(Months)	S_c(RK4 – SDM)	I_c(RK4 – SDM)	R_c(RK4 – SDM)
1	0.0004	0.0011	0.0005
2	0.0012	0.0030	0.0010
3	0.0011	0.0009	0.0016
4	0.0006	0.0018	0.0019
5	0.0009	0.0011	0.0019
6	0.0011	0.0012	0.0010
7	0.0011	0.0018	0.0039
8	0.0033	0.0006	0.0045
9	0.0030	0.0004	0.0081
10	0.0011	0.0011	0.0011
11	0.0031	0.0007	0.0012
12	0.0018	0.0075	0.0007

The graphical representations in Figure 1 and Figure 2 depict the model fit for cumulative and active COVID-19 cases in Nigeria throughout the year 2020. Notably, a consistent upward trend is evident over time, attributed to a significant lack of adherence to COVID-19 protocols. This underscores the imperative for stringent enforcement of non-pharmaceutical interventions to curb the rapid spread of the disease. Additionally, the examination of Table 1 and Table 2 reveals a harmonious agreement between the two numerical methods, displaying minimal errors. Furthermore, it is noteworthy that the Sumudu Decomposition Method (SDM) exhibits better performance in both efficiency and convergence when compared to the Runge–Kutta fourth-order (RK4) method, while Figure 3 and Figure 4 describes the effect of recovery rate ϕ on R_{cr} in 12 months the host community. It is observed that the curve converges to the disease – free and endemic equilibrium when $R_{cr} < 1$ *and* $R_{cr} \approx 10.84$. When the recovery rate through treatment is increased, that is, $\phi = 0.8429$, R_{cr}reduces but not below unity. This highlights the challenge of completely eradicating the virus through treatment alone. The relevance of umerical modeling techniques and recovery rate through treatment for the containment of the spread of COVID-19 has been shown in this work. Though effective treatment is needed to ameliorate the impact of the virus, preventive measures is essential to reduce the spread due to the endemic nature of the disease.

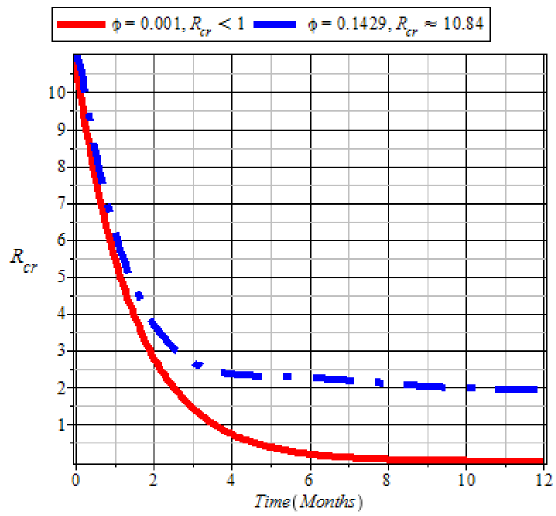

Fig. 3: Behavior of recovery rate $\phi = 0.1429$ on R_{cr} when R_{cr}<1 and $R_{cr} > 1$

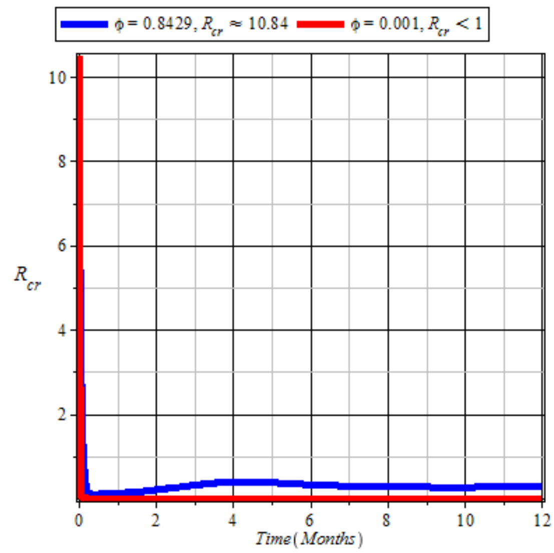

Fig. 4: Behavior of recovery rate $\phi = 0.8429$ on R_{cr} when R_{cr}<1 and $R_{cr} > 1$

5 Conclusion

The compartmental model has been used in this study to analyze COVID-19 cumulative and active cases in Nigeria throughout the year 2020. The simulations, based on fitted and estimated parameters from existing literature, showed a basic reproduction number R_{cr} of approximately 10.84. This finding reveals the endemic nature of COVID-19 in Nigeria, with an average infection rate of at least 10 individuals. Furthermore, the investigation into the impact of the recovery rate on R_{cr} showed that an increase in the recovery rate through treatment can reduce R_{cr}, although it remains above unity. This suggests that treatment alone may not be sufficient to effectively combat the disease. The numerical implementation of the model equations using the Sumudu Decomposition Method (SDM) and the Runge–Kutta fourth-order (RK4) method demonstrated their efficiency, with SDM exhibiting better convergence. Consequently, health policy-makers in Nigeria are advised to intensify the implementation of Non-Pharmaceutical Interventions (NPIs) recommended by the World Health Organization (WHO). This strategic scaling up of NPIs is crucial to mitigate the spread of COVID-19 and reduce R_{cr} below unity, ultimately aiming to eliminate the disease. Mathematically, this study suggests potential extensions into spatial, fractional order, stochastic, and optimal control problems. These avenues of research could further enhance our understanding of the dynamics of COVID-19 and contribute to the development of more effective strategies for disease control and prevention.

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