

Coupled Oscillators Energy Exchange Model Applied for the EEG Spectral Modeling and Forecasting

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Abstract: This research is driven by the need to find a valid approach for the most difficult problems remaining in the neuroscience – the explanation of the dynamic relationships between different brain regions and the explanation of the partial coherence of EEG signals. In this regard, our paper advocates for the field-theoretical approach, which is able to link experimentally observed human brain local EEG signal dynamics with the proposed coupled oscillator energy exchange model (COEEM). The reasoning behind the proposed COEEM model application is based on an energy exchange and synchronization simulation in a localized brain area using (i) the coupled oscillators approach, (ii) a novel coupled oscillators' phase-locking mechanism (PLM) and (iii) a unique and very narrow spectral band prognostication and superposition method. Based on the promising forecasting results obtained for the real EEG signals, we infer that the oscillatory model presented here is potentially able to explain the dynamic relationships between different brain regions and the explanation of the partial coherence of EEG signals.

Key-Words: coupled oscillators, energy exchange model, Kuramoto model, phase locking

1 Introduction

Recent years have witnessed an explosion of interest and activity in the area of human brain research. For instance, the Human Brain Project in the EU and the Brain Mapping Project (BRAIN initiative) in the US are just two examples of large scale research programs [1], [2], which are dedicated to brain simulation, neuroinformatics, high performance computing, medical informatics, brain imaging and mapping, neuromorphic computing, neurorobotics, etc. These advances in theoretical and experimental methods and techniques [3] not only help to reveal brain disease states but also broaden conceptual knowledge of processes taking place in the brain.

For many decades neuroscientists have been building various brain imaging methods (EEG/MEG/fMRI) [4] and computational neuroscience simulations models, which could simulate experimentally observed data [5, 6]. However, just few modeling results were obtained, which are attempting to simulate the oscillating phenomena of synchronized neural networks [7, 8]. But these theoretical simulations of coupled oscillators are poorly grounded on the experimental observations.

Despite such efforts, one of the most difficult problems remains – the explanation of the dynamic

relationships between different brain regions [9]. Another problem is the explanation of the partial coherence of EEG signals [10]. In this regard, some research focuses on analyses of the relationship among neural signals, using partial directed coherence [11]. One of the more promising areas of research close to our work is the study of the brain oscillation control of one single-neuron activity, which attempts to uncover the temporal relationship between brain oscillations and single-neuron activity [12]. Hence, in our research we made an attempt not only to refine the coupled oscillators' modeling approach but also to test its validity on the real EEG data, i.e. to check whether our model is capable to predict EEG dynamics for short and even for long (few seconds) periods.

Historically, linear regression models were mostly used for the forecasting of EEG time series. For instance, autoregressive (AR), moving average (MA), and autoregressive moving average (ARMA) models are still quite often employed [13, 14, 15]. In most cases, these models are applied for the forecasting of epilepsy or other mental diseases from the EEG time series analyses. Linear regression models, however, have certain limitations for the forecasting of highly nonlinear data [13].

After careful revision of the above-mentioned prognostication methods and models, we have elaborated a completely different approach, i.e. oscillation-based modeling and prognostication of short (several ms) and long time series (several seconds) of real EEG signals. We created and employed an iterative COEEM (coupled oscillator energy exchange model) scheme for the short time prognostication using fourth-order Runge-Kuta algorithms (RK4) with the non-filtered spectral range 1-512 Hz (including noise) EEG data. For the long time prognostication we used (i) filtered EEG data of 1-38 Hz (without power noise) and (ii) superposition of prognostication results for very narrow spectral bands of 0.01-0.1 Hz.

For the sake of clarity we would like to note, that the COEEM model was created in the context of a larger scheme of multi-agent systems (MAS) simulation research [16, 17], i.e. in the framework of the multidisciplinary OSIMAS paradigm, which aims to model social agents as oscillatory systems (see <http://osimas.ksu.lt>). Within the framework of this project we investigate opportunities to make use of a biologically inspired approach, where basic human mind states can be represented in the form of the EEG oscillations. Based on the EEG experimental findings, we are looking for ways to model (i) human (social agent) mind states in terms of distributions of characteristic oscillations and (ii) transitions between basic mind states in terms of redistributions of characteristic oscillations.

In order to establish relationship between experimentally measured EEG signal oscillations and conceptually described oscillating agents in the OSIMAS paradigm, we have created the coupled oscillations based COEEM model. Investigation and further refinements of this biologically inspired experimental model helped to define features of our artificially constructed oscillating agent model (OAM) in the OSIMAS paradigm. Hence, oscillation based modeling of human brain EEG signals oscillations, using a refined Kuramoto model [10], not only helps to specify the oscillating agent model [16, 17] but also significantly contributes to EEG prognostication research, which is the major topic of this particular paper.

This article is organized as follows: Section 2 describes the COEEM setup and phase-locking mechanism. Section 3 describes the COEEM model and the very narrow spectral band prognostication technique. Section 4 describes the experimental setup and our findings. Finally, Section 5 provides conclusions.

2 The COEEM Setup and Phase-Locking Mechanism

The coupled oscillator energy exchange model (COEEM) estimates energy fluctuations in the localized brain area. In this regard, the COEEM model is a coupled oscillator energy exchange model. Equations (1) using the COEEM model describe the evolution of complex amplitudes of oscillators. The energy of each oscillator is proportional to the modulus squared of the complex amplitude. In general, energy exchange takes place between all oscillators, despite internal or external division. In the COEEM model, though, we model energy exchange between the inner and outer system of oscillators. Hence, the EEG signal is modeled as the total energy change of an internal oscillator system over time.

In the COEEM model, neuron is modeled as an oscillator, which has a phase and a complex amplitude. Thus, modeling the energy exchange between groups of neurons is replaced by the model of the energy exchange between groups of oscillators. These oscillators are coupled with each other in a relationship function that can be freely chosen. This model is similar to the Kuramoto model, where the same coupling equations of oscillators are used. As in the Kuramoto model, we use the sinus coupling function of oscillators. In short, the COEEM model investigates systems of coupled oscillators, where each oscillator is characterized by an angular frequency w_i , see (1). This frequency is also emitted in a form of the field, which enables the energy exchange between the oscillators. Therefore, coupling constants were chosen equal to w_j , see (1). The COEEM model equations are as follows:

$$\frac{\partial \theta_i}{\partial t} = w_i + \sum_{j=1, j \neq i}^N w_j \sin(\theta_i - \theta_j), \quad (1)$$

$$\frac{\partial A_i}{\partial t} = A_i w_i + \sum_{j=1, j \neq i}^N A_j w_j \sin(\theta_i - \theta_j),$$

where A_i is the complex amplitude of the i -th oscillator (the amplitude A_i is itself a complex number), θ_i – the phase of the i -th oscillator, w_i – the angular frequency of the i -th oscillator, and t – the temporal coordinate.

Therefore, here we have $2N$ linear equations for the phases and complex amplitudes of the oscillators. The system Eq. (1) of $2N$ linear equations describes the temporal evolutions of the

phases and the complex amplitudes of the oscillators. The system Eq. (1) consists of two phases: the oscillator phase θ_i and the radiated field complex amplitude phase $\arg(A_i)$. In this case, the energy of an oscillator is proportional to the modulus squared of the complex amplitude A_i (A_i describes the EEG signal). We are modeling this signal as a radiated field (A_i is proportional to radiated field) using the system of internal oscillators. In this case, oscillators are coupled via the phase differences between them. The phase differences between phases of the radiated fields ($\sin(\arg(A_i) - \arg(A_j))$) are not used for the coupling. In this way, we can get the temporal evolution of the energy of each oscillator. Thus, we can evaluate the energy redistribution between oscillator groups after a certain time.

The phase difference between two oscillators describes the energy exchange mechanism between them: the energy exchange process takes place between oscillators where the phase difference is not equal to zero (if the phase difference is zero, then energy exchange is absent).

Hence, we study the temporal EEG signal dynamics recorded from a single channel. Since the surface area of one EEG channel electrode is relatively small when compared to the total surface area of the head, we can say that the analyzed EEG signal is generated by only a very small part of all neurons. Therefore, we explore two different size groups of oscillators, which form a single closed oscillator group, i.e. a closed system U consisting of two open systems W and w :

$$\begin{aligned} U &= W + w, \\ N_U &= N_W + N_w, \end{aligned} \quad (2)$$

where U is a closed global system, W – an external open system, w – an internal open system, N_U – the number of global oscillators, N_W – the number of external oscillators, N_w – the number of internal oscillators.

Note that for a closed system the energy conservation law is valid and that for each of the two open systems the energy conservation law is not valid:

$$E_U = E_W + E_w, \quad (3)$$

where E_U – the energy of the global system, E_W – the energy of the external system, E_w – the energy of the internal system.

One of the two open oscillator groups we call the internal system, and the other – the external system. Note, that the number of external oscillators is much

higher. In this case, we calculate the energy exchange between the internal and the external oscillators, where each oscillator exchanges energy with any other remaining oscillator.

Principal scheme of the COEEM performance investigation includes calibration, optimization, validation and forecasting stages, see Fig. 1. Calibration was performed for a different number of iterations (see iterative scheme below). For our further estimates, we used a setup with the number of iterations 16 because under such conditions the COEEM curve is sufficiently calibrated to the experimental EEG curve. Then, we applied optimization stage to find an optimal number of calibrated external oscillators. For this particular EEG data set it equaled 5. Validation and forecasting stages are described in the next sections.

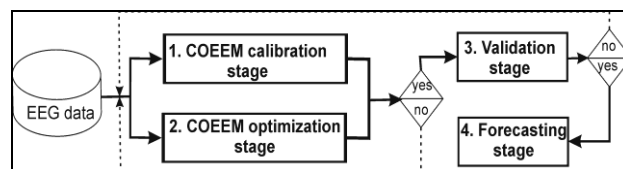


Fig. 1. Principal scheme of the COEEM performance investigation

COEEM has not been performed to model the connectivity distant brain areas. In fact, COEEM model calculations are localized at a chosen EEG channel area around 1cm in diameter. Therefore, there is no sense in calculating signal delays. In principle, the COEEM model is not designed for the accurate topological imaging of brain activity across large areas of the skull. Instead, it provides an approximate model, which allows us to prognosticate the temporal evolution of a real EEG signal at the chosen localized area in the brain.

The time-evolution of the simulated EEG data is modeled as the total energy fluctuations of an internal oscillator (or their system). The dependence of energy on time is derived as follows: the modulus squared of the EEG data is integrated over the intervals of a certain short time. For the internal oscillators, the dependence of energy on time is obtained by summing up the energies of all the internal oscillators at certain moments. The COEEM model uses experimentally measured EEG data and finds the time dependences of the amplitudes and phases of the external and internal oscillators.

The iterative scheme is as follows:

Step 1. The phases and amplitudes of all the oscillators are assigned random values. The distribution functions of random values are similar to the uniform distribution. That is, all random

numbers are generated with approximately equal probability.

Step 2. The amplitudes of all the oscillators are normalized to the total energy of all the internal oscillators. Hence, we then get the amplitudes and phases of all the oscillators at $t=t_1$.

Step 3. We solve the COEEM model equations by using the RK4 method (see the next section). The solution gives the amplitudes and phases of all the oscillators after time-step Δt at $t=t_2$.

Step 4. The amplitudes of all the internal oscillators are replaced in such a way that the total energy of the internal oscillators is equal to the value of the EEG power at $t=t_2$:

$$A'_k = \sqrt{\frac{E_{\text{EEG}}(t_2)}{N_w}} \exp[i \arg(A_k(t_2))] \quad (4)$$

Step 5. We solve the COEEM model equations by using the RK4 method (back propagation: $\Delta t := -\Delta t$). After this step, we have the modified amplitudes and phases of all the oscillators at $t=t_1$.

Step 6. We reset these modified amplitudes of the internal oscillators to the initial values at $t=t_1$ ¹.

In the third section, we discuss how to improve COEEM model for prognostication of long periods. Whereas, our simulation results are presented in the fourth section.

3 COEEM Used with Narrow Spectral Bands Superposition Approach

COEEM model originally was designed for prognostication of EEG signals of broad spectral ranges but short periods (several milliseconds). In this section, though, we use the COEEM model for prognostication of EEG signals with close to constant frequencies and maximums of amplitudes for long periods (several seconds). Actually, EEG signals' amplitudes and frequencies have small fluctuations only for very narrow spectral widths (bands). Therefore, our idea is to apply COEEM prognostication model for broad spectral ranges, using superposition of narrow spectral bands prognostication results, see Fig. 2.

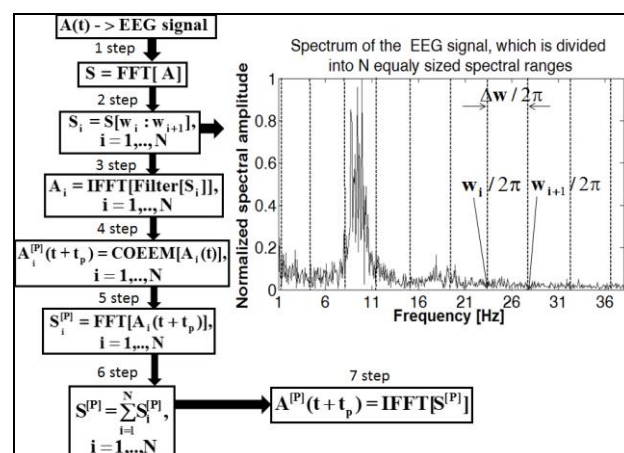


Fig. 2. Principal scheme of narrow spectral bands superposition approach, which is used for long time prognostication.

This methodology consists of the following steps:

Step 1. Original EEG signal is transformed to the spectral representation using fast Fourier transformation (FFT).

Step 2. The obtained spectral representation is divided into N equally sized narrow spectral bands.

Step 3. After filtration and inverse fast Fourier transformation (IFFT) for each narrow spectral band, we obtain N temporal EEG signals.

Step 4. Each temporal EEG signal (N) is prognosticated employing COEEM.

Step 5. Fast Fourier transformation is applied for each prognosticated temporal EEG signal in order to get N spectral representations.

Step 6. Superposition is applied for the obtained N spectral representations.

Step 7. After inverse fast Fourier transformation of the superposed spectral representation, we obtain prognosticated temporal EEG signal for the wide spectral range.

In the section below, we present COEEM model applications with and without the above-described principal scheme.

4 The Results of the COEEM Simulation for Short and Long Periods

Our EEG experimental results, discussed below, were obtained using the BioSemi ActiveTwo Mk2 system with 64 channels. The time-density of the recorded signals from the head surface of participants was 1024 points per second for all 64 channels.

¹ We cannot replace the amplitudes of internal oscillators because the initial conditions of internal oscillators must remain the same in the first layer.

For the illustration of obtained results below, we picked few experimental EEG signal sets: (A), (B), (C) and (D). EEG signal (A) corresponds to the state of thinking; EEG signal (B) corresponds to the state of deep relaxation; EEG signal (C) corresponds to the state of deep relaxation with oddball audio signals; EEG signal (D) corresponds to the state of thinking with oddball audio signals.

Below, we offer a step-by-step description of the optimization procedure and the results of the short time prognostication (up to 92 ms) using COEEM model for the non-filtered EEG signal (A).

In the calibration stage, the COEEM simulated curve became sufficiently close to the experimental EEG curve. In this way, we found the time dependences of the amplitudes and phases of the external and internal oscillators.

After proper calibration, we can state that the COEEM simulated curve, which is generated by one internal oscillator, well matches with the experimental EEG curve, see Fig. 3. Our findings revealed that each external uncalibrated oscillator could radically change the COEEM generated signal.

Hence, we calibrated all external oscillators using the chosen EEG data range [0, 1388] ms. After proper calibration, the COEEM simulated curve was fitted to the experimental curve in the chosen data range. At the end of the calibration step we depicted the data range [268, 1000] ms for validation of the prognostication results. Using the test and trial method, we observed better prognostication results applying this particular data range, though it can be freely chosen.

It should also be noted that not all calibrated external oscillators were used for the prognostication. For this reason, we applied another optimization step to find an optimal number of calibrated external oscillators. We used time interval [1388, 1398] ms to find an optimal number of calibrated oscillators. For this particular EEG data set it equaled 5.

The best prognostication estimates in terms of Pearson correlation coefficient and MSE were obtained when the number of the calibrated external oscillators equals to 5. The best obtained correlation coefficient for the prognostication period 35 ms equals 0.80 and for period 92 ms equals 0.76. In other cases the correlation coefficients are less than 0.19.

Next, we present results of the prognostication for the experimental EEG data using the optimized (calibrated) COEEM model. Prognostication results were obtained starting from the 1398 ms time mark, see Fig. 3.

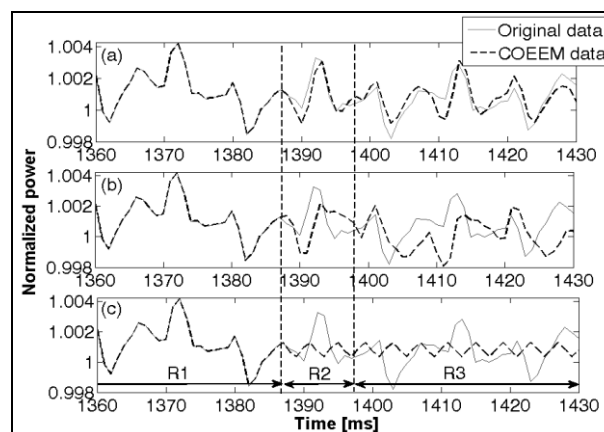


Fig. 3. Calibration (R1), optimization (R2) and prognostication (R3) of the COEEM curve vs. the original non-filtered EEG data. Number of the calibrated external oscillators equals to 5 in case (a), to 15 in case (b), and to 0 in case (c).

As we already mentioned, in order to effectively prognosticate EEG signals for long periods of time, we (i) filtered out noise (leaving 1-38 Hz spectral range), (ii) applied COEEM for the narrow spectral bandwidths (0.01-0.1Hz) prognostication, (iii) performed superposition of the obtained (prognosticated) narrow bandwidth spectra, (iv) used an inverse Fourier transformation to obtain prognostication of the temporal EEG signal in the spectral range (1-38 Hz), see Fig. 2.

The prognostication correlation coefficient for this narrow bandwidth equals 0.9944. In this way, we got good prognostication results for just one narrow spectral bandwidth. For instance, in the case of the 1-38 Hz spectral range, we would have 370 such narrow spectral bands. All these narrow spectral bands have to be prognosticated using COEEM. Then the prognostication results are superpositioned (see Fig. 2) and with the help of the Fourier transform inverted back to the time scale.

We did this procedure for longer, i.e. 3s duration filtered (1-38Hz) EEG signals. We used R1=12s for calibration, R2=1s for optimization and R3=3s for prognostication. The whole spectral range (1-38 Hz) was divided into the 0.07 Hz almost monochromatic spectral bandwidths. Then we made prognostications for each narrow band and superpositioned these results to get a wide (1-38Hz) spectral range. Below the presented prognostication results illustrate different brain waves and mind states for the whole spectral range, see Fig. 4.

Correlation coefficients between the prognosticated curve and the original EEG signal at frequency range 1-4 Hz for cases (A), (B) and (C) are respectively 0.9590, 0.9840 and 0.9961, see

Table I. Very similar values of correlation coefficients are obtained for the same EEG signals at the other frequency ranges: from 8 to 13 Hz and from 1 to 38 Hz, see Table I. As we can see from Table I, the largest value of the prognostication correlation coefficient equals 0.9961 at the frequency range 1-4 Hz and the smallest value equals 0.9297 at the frequency range 8-13 Hz.

In short, high values of the correlation coefficients show that prognoses of a 3 second time period at different ranges of frequencies for all three EEG signals (A), (B) and (C) is quite accurate, i.e. we obtain good matching between the COEEM prognosticated data and the original EEG data, Fig. 4 and Table I.

Table 1. Correlation coefficients for various states and frequency ranges (prognostication period 3 s)

States	Correlation coefficients		
	1-4 Hz	8-13 Hz	1-38 Hz
(A)	0.959	0.9589	0.9821
(B)	0.984	0.9297	0.9562
(C)	0.9961	0.9952	0.9893

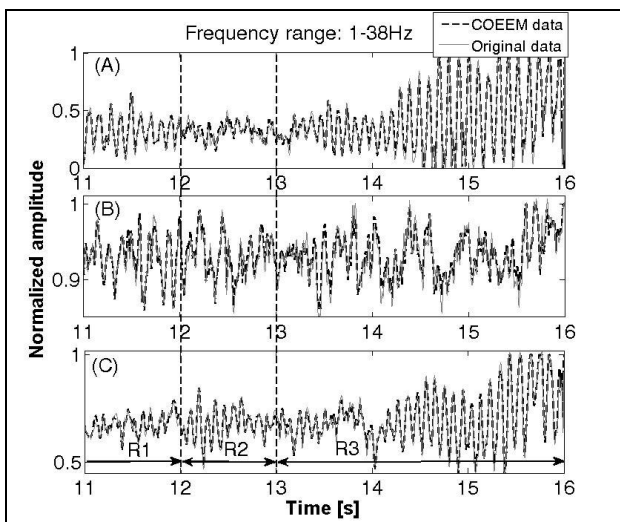


Fig. 4. Filtered EEG signals (A), (B) and (C) in the frequency range 1-38 Hz: calibration (R1), optimization (R2) and prognostication (R3) range. Prognostication period 3 s.

For the effective implementation of the proposed approach, we also investigated how prognostication results depend on the chosen narrow spectral bandwidth, which is used to get almost

monochromatic waves for the narrow band prognostication, see Fig. 5.

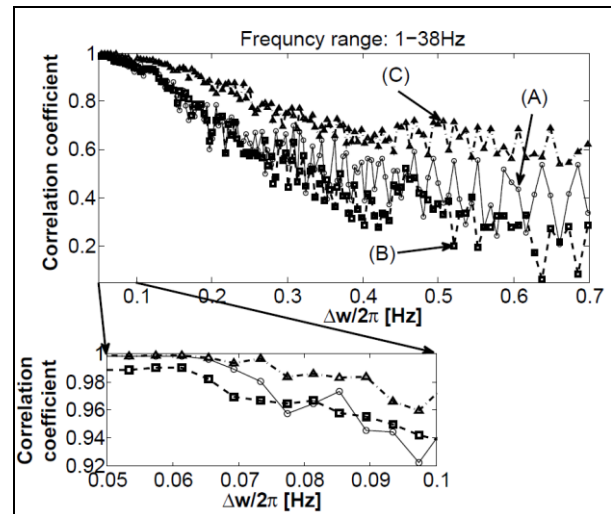


Fig. 5. Dependence of prognostication correlation coefficient from the size of spectral bandwidth for states (A), (B) and (C) at frequency range 1-38 Hz and prognostication time period 3s.

Fig. 5 indicates that the correlation coefficient acquires values between 0.92 and 0.99, when the spectral bandwidth is less than 0.1 Hz (see bottom graph). It therefore seems obvious, that for better prognostication results we have to choose a narrow bandwidth, which in this illustrated example ranges from 0.05 to 0.07 Hz, see Fig. 5. Contrary, in the case of a 0.7 Hz bandwidth, prognostication correlation may be lower than 0.2. As we can see from the same figure, very similar tendencies hold for all cases (A), (B) and (C), see Fig. 5.

Next, we present a few prognostication results, applying the same procedure of superposition to filtered (1-38Hz) very long, i.e. 28s time periods. Fig. 6 illustrates prognostication correlation 0.9829 between the COEEM and real EEG data curves. The bottom graph in Fig. 6 illustrates almost perfect matching between the prognosticated and real EEG signal (C) at the end of the prognostication period.

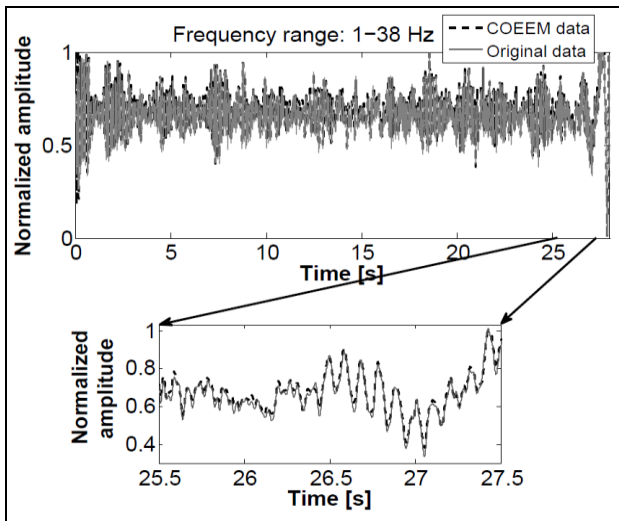


Fig. 6. Prognostication of the EEG signal (C) at frequency range 1-38 Hz for $R_3=28$ s time period ($R_1=36s, R_2=1s$).

The above example illustrates that the prognostication correlation coefficient is very high 0.9829 even for quite long prognostication periods.

Another example at Fig. 7 illustrates prognostication correlation 0.9640 between the COEEM and real EEG data curves for the signal (D).

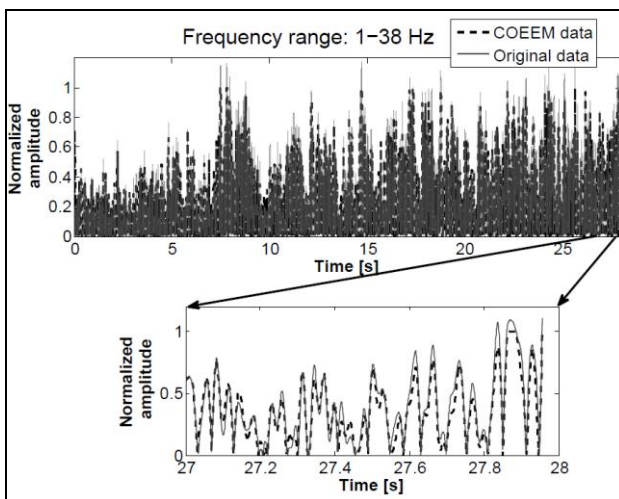


Fig. 7. Prognostication of the EEG signal (D) at frequency range 1-38 Hz for $R_3=28$ s time period ($R_1=36s, R_2=1s$).

Regarding the relation between the prognostication period and the correlation coefficient, we conducted an additional investigation, which is presented in the Fig. 8.

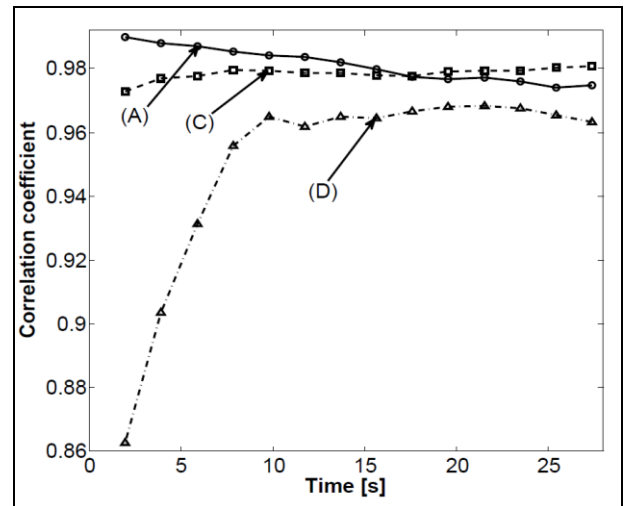


Fig. 8. Relation between prognostication period and correlation coefficient for EEG signals (A), (C) and (D) at frequency range 1-38 Hz.

In sum, based on the results presented in this section, the authors argue that the proposed COEEM model is outperforming the other EEG prognostication results provided at [13, 14]. Our online virtual lab for the interactive testing and modeling of the proposed COEEM approach is available at <http://vlab.vva.lt/> (login as Guest, password: guest555).

Hence, coupled oscillators energy exchange model (COEEM) reveals huge potential not only for applications in the computational neuroscience simulations but also in practical cases related with the recognition and prediction of various kind of cognitive diseases like epilepsy, sleep disorders, encephalopathies, stroke and other focal brain disorders [4,18,19,20].

5 Concluding Remarks

This article introduces the novel coupled oscillator energy exchange model (COEEM) which simulates experimentally observed human brain EEG signal dynamics. The reasoning behind the COEEM model construction is based on an energy exchange and synchronization simulation in a localized brain area.

Our research provides not only refinement details of the coupled oscillators' modeling approach but also tests its validity on the real EEG data. In this way, we perform the robust test whether our model is capable to predict EEG dynamics for short and even long (few seconds) periods.

In the first experimental research stage, short time EEG signal prognostications (up to 92 ms) of non-filtered EEG signals provided correlations to the order of 0.76-0.8.

Hence, in the second research stage, we applied this improved COEEM scheme for prognostication of filtered (1-38Hz) 3s time periods. In short, quite high values of the correlation coefficients (0.92-0.99) at different ranges of frequencies were obtained for all three investigated EEG signals (A), (B) and (C).

We also investigated how prognostication results depend on the chosen narrow spectral bandwidth. We found that for better prognostication results we have to choose as narrow a bandwidth as possible, i.e. in our case in the range of 0.05 to 0.07 Hz.

We also applied the improved COEEM scheme for the prognostication of filtered (1-38Hz) very long, i.e. 28s time periods. In sum, the prognostication correlation coefficient remained very high (0.96-0.98) even for quite long prognostication periods.

Like all pioneering studies, COEEM needs thorough further investigation with other sets of EEG data. However, initial experimental validation results provide potentially very intriguing insights about the oscillatory nature of the mind states and very promising coupled oscillator based research direction in the field of computational neuroscience simulations.

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