Feature extraction of the lesion in mammogram images using segmentation by minimizing the energy and orthogonal transformation adaptive

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Abstract: Segmentation and classification of breast masses in mammography play a crucial role in Computer Aided Diagnosis system (CAD) . In this paper we propose an approach consisting of two methods. The first is the main stage in image processing which is the mammograms segmentation. This method is based on the theory of all levels and minimization of the energy of the active contour which enables the selection of regions of interest of the mammograms images. While the second method is based on the theory of adaptive orthogonal transformation that will calculate the informative characteristics of regions of interest of mammography images. The characteristics obtained by this computing method allow the increase of the diagnostic certainty. To illustrate the effectiveness of the method we present the results of experiments carried out on the basis of images MIAS mammograms.

Key–Words:segmentation; mammography; active contours; levels set;classification

1 Introduction

Breast cancer is the second cause of death among women in the world [1], [2], [3]. Prevention of the disease is very difficult because the risk factors are either poorly understood or not influenced. Scientific studies have provided insight into the development of cancer, but it is not yet clear why such a person develops breast cancer. It should be noted that only 5-10% of breast cancers are hereditary origin related to the transmission of deleterious genes most commonly implicated are BRCA1 and BRCA2 (acronyms for breast cancer) associated with a predisposition to the disease[4]. Due to its delayed diagnosis, that often results into a heavy treatment, mutilating and costly which is accompanied by a high mortality rate. Various studies have confirmed that this is the early stage detection which can improve the prognosis and mammography in this case it is the best diagnostic technique [5]. However, it remains difficult for expert radiologists to provide a good interpretation of shots mammography due to low differences in densities of various tissues of the breast in the image. The interpretation of mammograms by radiologists is performed in the aim of finding the anomalies that indicate the changes associated with cancer. The indicators of probable presence of cancer are the microcalcifications and the opacities [6]. We analyse the mammography for the purpose of finding these indicators to characterize and classify Benign type and malignant, thus the importance of tools of aid to diagnostic (CAD) developed during these last years [7],[8]. Generally, CAD systems of aid to diagnostic based on a range of approaches including the steps of pre-processing, segmentation, extraction of classification parameters (size, shape, density and grouping in hearth) and finally the classification of anomalies suspect [7], [8], [9],[10],[11] and [12]. As general the segmentation of image mammography is a very important step in solving the problem of detecting breast cancer. Currently there are many different methods of segmentation, which are: the segmentation based on regions [13], [14], [15], the segmentation based on contours [16], [13], [18] and the segmentation with thresholding [19], [20], [21]. The analysis of these diversity methods allows concluding that the application of such a method or another can influence on rates of certainty of diagnostic. Given the low contrast and the highly textured nature of the images mammography, all segmentation methods proposed depend on the values of selected parameters (thresholds, mean values, variances, etc..) And the models exploited by these methods (probability of density, function of belonging ...). Therefore, a low error of estimation of these parameters or these models can lead to segmentation results of poor qualities in terms of error rate at the pixel level. This bad estimation may also jeopardize the detection of small parts contained in the mammography images. So, this opens in a path to propose another method that can lead to a correct interpretation. In this article we propose a method for segmentation the image mammography based on theory of Level Set and the principle of the minimization of the energy of active contours.

This paper is organized as follows: Section 2.1 describes the database used for evaluation, section 2.2 we give a reminder on the theory of level sets, section 2.3 and section 2.4 present the formulation of problem and method proposed for minimizing the energy for the segmentation of mammography image.Section 2.5 present the segmentation algorithm and we proposed in section 3 method for classification and detection of the lesion. The result is presented in section 4 and we end with a conclusion and perspective in section 5.

2 Materials and Method

2.1 Database

The mini-MIAS [22] database contains a total of data 322 MLO view mammography images. This database is divided into categories such margin: speculated, circumscribed or poorly defined. The images have a resolution of 1024 1024 pixels. From this data set, a total of 111 lesions was selected. These include 60 benign and 51 malignant masses. An example of a series of the image is given by Figure 1.

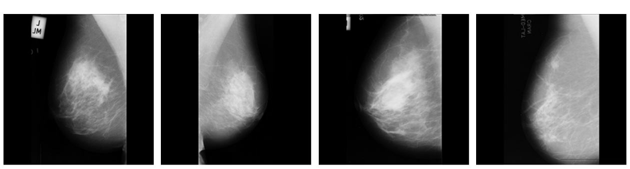

Figure 1: Example of mammography study.

2.2 Level Set Theory

A level set is a vital category of deformable models. Level set theory, a formulation to implement active contours was proposed by Osher and Sethian [23]. They represent a contour implicitly via two dimensional Lipchitz continuous function $\varphi(x,y) : \Omega \to \mathbb{R}$, defined in the image plane. The function $\varphi(x,y)$ is called level set function, and a particular level, usually the zero level of $\varphi(x,y)$ is defined as the contour, such as

$$\mathcal{C} = \{(x,y) : \varphi(x,y) = 0\}, \forall (x,y) \in \Omega \quad (1)$$

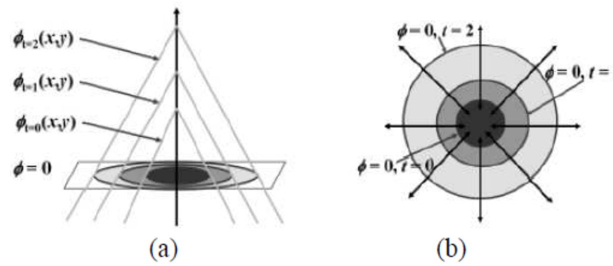

Figure 2: Level set evolution and the corresponding contour propagation: (a) Topological view of level sets $\varphi(x,y)$ evolution (b) the changes on the zero level sets

Fig.2 (a) shows the evolution of level set function $\varphi(x,y)$ and Fig.2 (b) shows the propagation of the corresponding contour $\mathcal{C}$. As the level set function $\varphi(x,y)$ increases from its initial stage, the corresponding set of contours $\mathcal{C}$, propagate towards outside. With this definition, the evolution of the contour is equivalent to the evolution of the level set function,i.e. $\frac{\partial \mathcal{C}}{\partial t} = \frac{\partial \varphi(x,y)}{\partial t}$ the advantage of using the zero level set is that a contour can be defined as the border between positive and negative areas, so the contours can be identified by just checking the sign of $\varphi(x,y)$ the initial level set function $\varphi_0(x,y) : \Omega \to \mathbb{R}$ may be provided by the signed distance from the initial contour such as

$$\varphi(x,y) = \{\varphi(x,y) : t = 0\} = \pm D((x,y), N_{xy}(\mathcal{C}_0)) \quad (2)$$

Where $\pm D(a,b)$ denotes a signed distance between a and b, and $N_{xy}(\mathcal{C}_0)$ denotes the nearest neighboring pixel on initial contours $\mathcal{C}_0 = \mathcal{C}\ (t=0)$ from (x,y) as a pixel (x,y) is located further inwards from the initial contours $\mathcal{C}_0$. The initial level set function is zero at the initial contour points given by $\varphi_0(x,y) = 0, \forall (x,y) \in 0$.

The deformation of the contour is generally represented in a numerical form as a partial differential equation. A formulation of contour evolution using the magnitude of the gradient of $\varphi(x, y)$ was initially proposed by Osher and Sethian [24] and is given by:

$$\frac{\partial\varphi(x,y)}{\partial t} = |\nabla\varphi(x,y)| = (\vartheta + \varepsilon k(\varphi(x,y))) \quad (3)$$

Where ϑ denotes a constant speed term to push or pull the contour,k denotes the mean curvature of the level set function $\varphi(x, y)$ and ε controls the balance between the regularity and robustness of the contour evolution.

2.3 Image Model and Problem Formulation

To cope with intensity inhomogeneity in image segmentation, we formulate our method based on an image model which describes the composition of images of the real-world, in which inhomogeneity of intensity is assigned to a component of an image. In this paper, we consider the following inhomogeneity intensity multiplicative model. From the physics of imaging in a variety of modalities (Mammography images), an observed image can be modeled as

$$I = bJ + n \quad (4)$$

Where J is the true image, b is the component that accounts for the intensity inhomogeneity, and n is additive noise. The component b is designated as a bias field (or shading image). The real image J measures an intrinsic physical property of the objects to be imaged, which is therefore assumed to be piecewise (approximately) constant. The bias field b is assumed to be slowly varying. The additive noise n can be considered to be zero-mean Gaussian noise.

In this paper, we consider the image $I : \Omega \to \mathbb{R}$ as defined on a continuous domain function. The assumptions about the true image and the bias field can be stated more precisely as follows:

- b Bias field varies slowly, which means that b can be well approximated by a constant in a neighborhood of each point in the image domain. (a1)
- The real image J approximately takes about N distinct values constant $\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_N$ in disjoint regions $\Omega_1....\Omega_N$ respectively, where $\{\Omega_i\}_{i=1}^N$ forms a partition of the image domain, i.e $\Omega = \bigcup_{i=1}^N \Omega_i$ and $\Omega_i \bigcap \Omega_j = \emptyset$ for $i \neq j$.(a2)

Based on the model in (4) and the assumptions a1 and a2, we suggest a method for estimating regions $\{\Omega_i\}_{i=1}^N$, the constants $\{\mathcal{C}_i\}_{i=1}^N$, and the bias field $\hat{b}$. The resulting estimates of them are appointed by $\{\Omega_i\}_{i=1}^N$, the constants $\{\mathcal{C}_i\}_{i=1}^N$, and the bias field $\hat{b}$, respectively. The obtained bias field $\hat{b}$ should be slowly varying and the regions $\Omega_1...\Omega_N$ should satisfy certain regularity property to avoid spurious segmentation results caused by image noise. We will define a criterion for the search of this estimates based on the model and assumptions of the above image a1 and a2. This criterion will be defined in terms of the region, constants, and function, as energy in a variational framework, which is minimized to find the optimal regions $\{\Omega_i\}_{i=1}^N$, constants $\{\mathcal{C}_i\}_{i=1}^N$, and bias field $\hat{b}$. Consequently, the image segmentation and bias field estimation are simultaneously achieved.

2.4 Energy Formulation

Let Ω be the image domain, and $I : \Omega \to \mathbb{R}$ be a gray level image. In [25], a segmentation of the image I is carried out by finding a contour $\mathcal{C}$, which divides the area of the image domain into disjoint regions $\Omega_i...\Omega_N$, and a piecewise smooth function u that approximates the image I and is smooth within each region Ω_i. This can be formulated as a problem of minimizing the following Mumford-Shah functional

$$F^{MS}(u,\mathcal{C}) = \int_\Omega (I-u)^2 dx + u\int_{\Omega/\mathcal{C}} \nabla u^2 dx + \vartheta|\mathcal{C}| \quad (5)$$

Where $|\mathcal{C}|$ is the length of the contour $\mathcal{C}$.In the right part of (5), the first term is the term data, which requires u to be close to the image I, and the second term is the term of smoothness, which causes it to be smooth in each of the regions separated by the contour $\mathcal{C}$. The third term is introduced to regularize the contour $\mathcal{C}$.

To be written in a continuously the local intensity property combination indicates that the intensities in the neighborhood o_y can be classified into N clusters, with centers $m_i \cong b(y)\mathcal{C}_i, i = 1...N$, this allows to apply the K-means clustering standard to classify these local intensities.More specifically, the intensities $I(x)$ in the neighborhood o_y, the K-means algorithm is an iterative process to minimize the clustering criterion [26], which can form as

$$F_y = \sum_{i=1}^{N} \int_{o_y} |I(x) - m_i|^2 u_i(x) dx \quad (6)$$

We can rewrite as

$$F_y = \sum_{i=1}^{N} \int_{\Omega_i \bigcap \Omega_y} |I(x) - m_i|^2 dx \quad (7)$$

Therefore, we define the energy

$$\varepsilon \triangleq \int \varepsilon_y dy \quad (8)$$

i.e

$$\varepsilon = \int (\sum_{i=0}^{N} \int_{\Omega_i} k(y-x)|I(x) - b(y)\mathcal{C}_i|^2 dx) dy \quad (9)$$

In this paper, omit the domain Ω in the index integral symbol (as in the first integral above) if the integration is over the entire domain Ω the mammography Image segmentation by minimizing energy with respect to the region $\Omega_i ... \Omega_N$, constants $\mathcal{C}_i ... \mathcal{C}_N$ and bias field b. The kernel function K is chosen as a truncated Gaussian function defined by

$$K(u) = \{^{\frac{1}{a} e^{-\frac{|u|^2}{2\sigma}}}_{0, otherwise} \quad (10)$$

Where a is a normalization constant such that $\int K(u) = 1$, σ is the standard deviation (or the scale parameter) of the Gaussian function.

Our suggested energy in (9) is expressed in terms of the regions $\Omega_1 ... \Omega_N$. It is difficult to draw a solution to the energy minimization problem from this expression. In this section, the energy ε is converted to a level set formulation by representing the disjoint regions $\Omega_1 ... \Omega_N$ with a number of level set functions, with a regularization term on these level set functions. In the level set formulation, the energy minimization can be solved by using well-established variational methods [27].

Note that for a given $\mathcal{C}$, there is more than one possible level set representation: if φ is a level set function for Ω. φ Can only represent two regions, inside and outside the contour $\mathcal{C}$, as $\Omega_1 = inside(\mathcal{C}) = \{\varphi > 0\}$ and $\Omega_2 = outside(\mathcal{C}) = \{\varphi < 0\}$, respectively. In this case, a level set function is used to represent the two regions Ω_1 and Ω_2. The regions Ω_1 and Ω_2 can be represented with their membership functions defined by

$$TwoPhase\{^{M_1(\varphi)=H(\varphi)}_{M_2(\varphi)=1-H(\varphi)} \quad (11)$$

Where $H(\bullet)$ is the Heaviside functional, φ represents the set of the level set functions such that $\varphi = \varphi$ for the Two-Phase model and $\varphi = \{\varphi_1, \varphi_2\}$ for the Four-Phase model.

$$FourPhase \begin{cases} M_1(\varphi) = H(\varphi_1)H(\varphi_2) \\ M_2(\varphi) = H(\varphi_1)(1 - H(\varphi_2)) \\ M_3(\varphi) = (1 - H(\varphi_1))H(\varphi_2) \\ M_4(\varphi) = (1 - H(\varphi_1))(1 - H(\varphi_2)) \end{cases} \quad (12)$$

Where

$$H(\varphi) = [\frac{1}{2}[1 + \frac{2}{\pi} arctan(\frac{\varphi}{\epsilon})]] \quad (13)$$

Thus, for the case of $N \geq 2$, the energy in (9) can be re-written as:

$$\varepsilon \triangleq \int (\sum_{i=0}^{N} k(y-x)|I(x) - b(y)\mathcal{C}_i|^2 M_i(\varphi(x)) dx) dy \quad (14)$$

2.5 Algorithm of segmentation

Enter :
I Initial Image
μ Regularization parameter
$\triangle t$ the Pas in time
γ Proportionality constant
$IterNum$ Number of iterations
υ speed constant
Initialisation :

$$\varphi_0 = \begin{cases} +1 \ where \ (x,y) \ is \ inside \ C, \\ -1 \ where \ (x,y) \ is \ outside \ C. \end{cases}$$

estimate :
For n ranging from 1 to IterNum do
. - Calculate the contour carte by Algorithm of Canny
. - Calculate the gradient carte $\varphi(x,y)$
. $\varphi(x,y) = \sqrt{\sum_{s \in \eta_s} |I_p - I_s|^2}$
. - Calculate the gradient g
. For i ranging from 1 to nl do
. For j ranging from 1 to nc do
. $g(\varphi(x,y)) = \frac{1}{1+\varphi(x,y)}$
. - Calculate the mean curvature term K
. For i ranging from 1 to nl do
. For j ranging from 1 to nc do
. $K(i,j) = div(\frac{\nabla\varphi(i,j)}{|\nabla\varphi(i,j)|})$

. - Calculate $\varphi^{n+1}(i,j)$

$$\varphi_{i,j}^{n+1} = \varphi_{i,j}^{n}$$
$$+ \quad \triangle \quad t \left[-\gamma\delta_\varepsilon(\varphi)(c_1 - c_2)\left(I(\varphi) - \frac{c_1+c_2}{2}\right)\right.$$

$$+ \mu \left(\triangle\varphi - div\left(\frac{\nabla\varphi}{|\nabla\varphi|}\right)\right) + \upsilon\delta_\varepsilon(\varphi) div\left(g.\frac{\nabla\varphi}{|\nabla\varphi|}\right)\Big]$$

. -end.

where
$c_1 = \frac{K_\sigma * [H_\varepsilon(\varphi)I]}{K_\varepsilon * H_\varepsilon(\varphi)}$

and
$c_2 = \frac{K_\sigma * I - K_\sigma * [H_\varepsilon(\varphi)I]}{K_\sigma * 1 - K_\sigma * H_\varepsilon(\varphi)}$

(1 is a function with a constant value of 1).

$H_\varepsilon(\varphi)$ is the Heaviside function.
σ_ε is the Dirac function smoothed given by

$$\sigma_\varepsilon(\varphi) = H'(\varphi) = \left[\frac{1}{2}[1 + \frac{2}{\pi} arctan(\frac{\varphi}{\varepsilon})]\right]' = \frac{1}{\pi}.\frac{\varepsilon}{\varepsilon^2 + \varphi^2}$$

3 The Proposed Method for Classification

The first part of the approach proposed in this article has allowed the segmentation and selection of regions of interest of mammography images of various types of disease. The classification of mammograms depending on the type of disease images requires the extraction of the most relevant features in these regions of interest [28],[11].

the calculation of characteristics vectors informative regions is ensured by the use of an adaptable orthogonal transformation [29]for each class regions (type of disease). The spectrum of informative characteristics of a given area (class) obtained decreases rapidly. In other words, the analysis of the spectra obtained allows a considerable difference between the classes of regions. This property will subsequently develop a simple decision rule which ensured a high certainty of classification.

The principle of the proposed method is to synthesize an adaptable operator orthogonal transformation to generate functions of bases parameterizable. Using these transformations [29],[30] is favored by the ability to adapt to the shape of their basic functions depending on the nature of the standard vector formed by mammograms each class images. In other words, for each type of disease a system of basis functions are associated parameterizable for showing his images. In addition, these functions of base parameterizable meet the criteria for completeness of the system ensuring the transformation of vectors without loss of information content. The system of basis functions formed is expressed as a factorisable orthonormal matrix operator, which therefore allows a transformation with a fast calculation algorithm:

$$Y = \frac{1}{N}HX \quad (15)$$

where:
- $X = [x_1, x_2, ..., x_N]^T$ is the vector representing the region of interest in the segmented image mammography given initial vector transform (of size $N = 2^n$).
- $Y = [y_1, y_2, ..., y_N]^T$ is the vector of informational characteristics calculated by the spectral operator orthogonal H of dimension N x N.
Factorization of Good [31] showed a possibility of representing the matrix operator H as product G_i (16) Sparse matrix with a higher proportion of zero which has allowed the construction the quick transformation algorithms of Fourier , Haar and Walsh. The matrices $G_i(i = 1, ..., n)$ are constructed by blocks of matrices $V_{i,j}$ of minimum dimension that is called spectral nuclei [29]:

$$G_i = \begin{bmatrix} \alpha_{i1} & 0 & \cdots & 0 & \gamma_{i1} & 0 & \cdots & 0 \\ \beta_{i1} & 0 & & 0 & \delta_{i1} & 0 & & 0 \\ 0 & \alpha_{i2} & 0 & \cdots & 0 & \gamma_{i2} & \cdots & 0 \\ 0 & \beta_{i2} & 0 & & 0 & \delta_{i2} & & 0 \\ & & \ddots & & & & \ddots & \\ 0 & \cdots & 0 & \alpha_{i\frac{N}{2}} & 0 & \cdots & 0 & \gamma_{i\frac{N}{2}} \\ 0 & \cdots & 0 & \beta_{i\frac{N}{2}} & 0 & \cdots & 0 & \delta_{i\frac{N}{2}} \end{bmatrix} \quad (16)$$

With

$$V_{i,j} = \begin{bmatrix} \alpha_{ij} \cdots & \gamma_{ij} \\ \beta_{ij} \cdots & \delta_{ij} \end{bmatrix} = \begin{bmatrix} \cos(\varphi_{i,j}) & \cdots & w_{i,j}\sin(\varphi_{i,j}) \\ \sin(\varphi_{i,j}) & \cdots & -w_{i,j}\cos(\varphi_{i,j}) \end{bmatrix},$$
$$w_{i,j} = \exp(j\theta_{i,j}), \quad \varphi \in [0, 2\pi], \quad \theta \in [0, 2\pi].$$

Hence equation (15) can be written as follows:

$$Y = \frac{1}{N}HX = \frac{1}{N}G_1G_2....G_NX = \frac{1}{N}\Pi_{i=1}^{N}G_i \ (17)$$

By defining the parameters $\varphi_{i,j}$ and $\theta_{i,j}$ can train operators orthogonal of transformations with basic functions complex , and $\theta_{i,j} = 0$ operators with real functions. Adapting Operator H in (15) is provided by the condition:

$$\frac{1}{N}H_aZ_{sd} = Y_c = [Y_{c,1}, 0, 0, ..., 0], Y_{c,1} \neq 0 \ (18)$$

where:
- Y_c is the target that builds the basis for adjusting the vector operator H_a.
- Z_{sd} represents the calculated by means of the estimates of the statistical characteristics of images of a given class standard vector.
- H_a is adaptable to synthesize operator.
Synthesis adaptable operator H_a based standard Z_{sd} (for a given class) is to calculate the angular parameters $\varphi_{i,j}$ matrices G_i according to condition (18). The procedure for calculation of the parameters is based on an iterative algorithm to calculate the target vector Y_c by step according to the equation:

$$Y_i = G_i Y_{(i-1)} \quad (19)$$

The final calculation of the vector Y_c allows obtaining adaptable operator H_a.

4 Results and Discussion

This section presents the results of experiments performed on 111 selected mammography database mini-MIAS images. To evaluate the effectiveness of the proposed method, we used the information provided by the Mammographic Image Analysis Society database (MIAS) including the class of anomalous images and coordinates of their centers of regions of interest. To test our method, the algorithm of segmentation is applied to each image containing a mass lesion, then verified if the algorithm detects a region of interest containing the abnormally and segment the mass, finally calculates the percentage of abnormally detection on all cases. The example for result of segmantation is presented a figure 3.

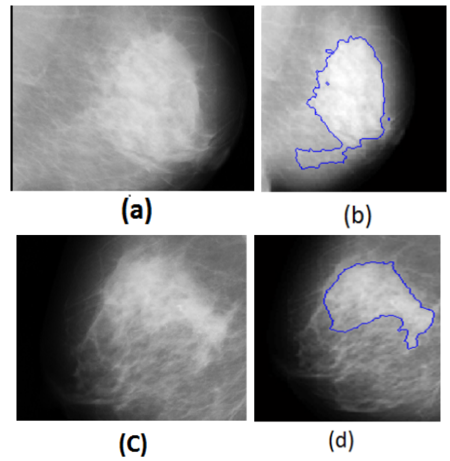

Figure 3: (a) The original image with Calcification benign. (b) Detection the region of interest (contains the abnormally) (c) The original image with Calcification malignant. (d) Detection the region of interest (contains the abnormally)

The result of experiment carried on the 111 mammogram image to detect the region of interest that contains the disease and segment the mass gives the percentage of precision equal to 92.27%.

To illustrate the effectiveness of the proposed method of calculate the informative characteristics of regions of interest of mammography images, Figure 4 shows the result of an experiment at diagnosis of two types of mammograms Image 'malignant' and 'benign'.

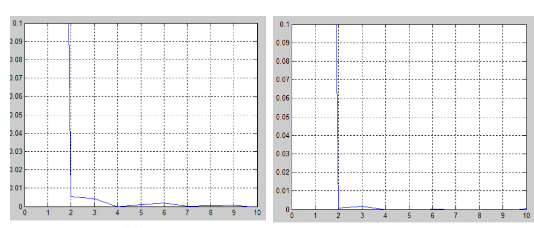

Figure 4: (a) Spectre of projection the regions of interest with Calcification benign in new base. (b) Spectre of projection the regions of interest with Calcification malignant in new base.

Figure 4 reflects a considerable distinction of vectors informative characteristics of regions in different classes mammography images (benign, malignant) . The projection vectors of the regions of interest in their system basic function provides adaptable vectors informative features that converge quickly to the standard vector class. While the projection regions of other classes results in obtaining wider spectra characteristics sufficiently distinguished and compared to other classes (other types of diseases). The results obtained by the developed method, illustrated in Figure 4 indicate its effectiveness and show that it ensures a high distinction that will help to make the classification of mammograms image using a decision making or calculates similarity.

5 Conclusion and Perspective

In this work we presented an approach consisting of two methods.The first is segmentation of masses in digital mammograms, this approach consists of three main stages which are the (ROI) detection, edge extraction and mass segmentation. The experimentation gives a percentage of 92.27% for all cases studied. The second method consisting to calculate the informative characteristics of regions of interest of mammography images. The results of the algorithm can contribute to solving the main problem in mammography image processing such as diagnostic and classification. The efficiency of the proposed method confirms the possibility of its use in improving the computer-aided diagnosis.

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