Numerical-analytical solutions of predator-prey models

GILBERTO GONZÁLEZ-PARRA
Grupo de Matemática Multidisciplinar (GMM)
Dpto. Cálculo, Facultad Ingeniería
Universidad de los Andes
Hechicera, Mérida
VENEZUELA
gcarlos@ula.ve

ABRAHAM J. ARENAS
Departamento de Matemáticas y Estadística
Universidad de Córdoba
Grupo Tesseo, Universidad del Sinú
Montería, Córdoba
COLOMBIA
aarenas@correo.unicordoba.edu.co

MYLADIS R. COGOLLO
Departamento de Ciencia Básica
Universidad EAFIT
Medellín
COLOMBIA
mcogollo@eafit.edu.co

Abstract:- This paper deals with the construction of piecewise analytic approximate solutions for nonlinear initial value problems modeled by a system of nonlinear ordinary differential equations. In real world several biological and environmental parameters in the predator-prey model vary in time. Thus, non-autonomous systems are important to be studied. We show the effectiveness of the method for autonomous and non-autonomous predator-prey systems. The method we have used is called the differential transformation method which has some suitable properties such as accuracy, low computational cost, easiness of implementation and simulation as well as preserving properties of the exact theoretical solution of the problem. The accuracy of the method is checked by numerical comparison with fourth-order Runge-Kutta results applied to several predator-prey examples.

Key–Words: Differential transformation method, Population dynamics, Nonlinear differential system, Predator-prey system.

1 Introduction

The modeling biological systems is commonly based on systems of nonlinear ordinary differential equations. Mathematical models and their simulation are important to understand qualitatively and quantitatively these systems. The study of biological phenomena such as harvesting of populations and availability of biological resources is relevant for the ecological life and for several human activities such as forestry, fishery and others. Therefore, it is important to investigate models that include interactions between species. The predator-prey models are one of the most well known and were constructed independently by Lotka(1925) and Volterra(1926) [1]. There are many different kind of predator-prey models in the mathematical ecology literature including continuous and discrete models, and several works have been devoted to investigate these models regarding periodicity, global stability boundedness and others features [2]. It is important to remark that realistic models often require the effects of the changing environment giving rise to non-autonomous nonlinear ordinary differential equation systems. The aim of this paper is to investigate numerically the reliability and convenience of the differential transformation method (DTM) applied to predator-prey models governed by the following two-dimensional system of nonlinear ordinary differential equations

$$\begin{cases} \dot{x_1}(t) &= f(x_1(t), x_2(t), a_1(t), \cdots, a_n(t)), \\ \dot{x_2}(t) &= g(x_1(t), x_2(t), b_1(t), \cdots, b_n(t)), \end{cases} \tag{1}$$

where $x_1(t)$, $x_2(t)$ represent the population densities at time t of prey and predator respectively and the positive functions $a_i(t)$, $b_i(t)$ generally give relative measures of the effect of dimensional parameters [1]. Due to the structure of the functions f and g, the solution of system (1) is not trivial, therefore it is necessary to develop reliable numerical techniques to obtain their numerical solutions. The numerical solution of predator-prey models has been treated in several papers in order to investigate numerically the reliability and efficiency of different methods. For instance in [3, 4, 5, 6] the Adomian method has been tested numerically using a predator-prey model. Additionally, in [7] He's variational method was studied and applied to a predator-prey model. The nonstandard finite difference schemes has been applied also to the predator-prey model [8]. In [9] the DTM was applied to a predator-prey model with constant coefficients over a short time horizon. However in this paper, in order to illustrate the accuracy of the method, DTM is applied to autonomous and non-autonomous predator-prey models over long time horizons and the obtained results are compared with the fourth-order Runge-Kutta method, and when are available with the analytical exact solutions. It is shown that the DTM is easy to apply and its numerical solutions preserve the properties of the continuous models, such as periodic behaviors. In addition this method is applied directly to the nonlinear ordinary differential equation system without requiring linearization, discretization or perturbation. The DTM was first proposed by Zhou [10], and its main application therein is to solve initial value problems in electrical circuits and has been applied to solve a variety of problems that arise from differential equations [11, 12, 13, 14]. The Michaelis-Menten equation that describes the rate of depletion of the substrate of interest has been solved using the DTM in [15] and authors show that the DTM is accurate and easy to apply for this particular differential equation.

The DTM develops from the differential equation system with initial conditions a recurrence equation system that finally leads to the solution of a system of algebraic equations as coefficients of a power series solution. It is important to remark that the DTM does not evaluate the derivatives symbolically; instead, it calculates the relative derivatives by an iteration procedure described by the transformed equations obtained from the original equations using differential transformation. In order to improve the rate of convergence and improve the accuracy of the calculations, it is convenient to divide the entire domain H into n sub-domains. The main advantage of domain split process is that only a few series terms are required to compose the solution in a small time interval H_i. Thus, the system of differential equations can then be solved in each sub-domain. Thus, after the recurrence equation system has been solved, each solution $x^j(t)$ can be obtained by a finite-term Taylor series. Unlike the conventional high order Taylor series method which requires a lot of symbolic computations, the DTM is performed iteratively [14]. However, the method have some drawbacks, which can be overcome by splitting the domain region into subintervals in order to obtain accurate solutions [12, 14]. In addition, some complex nonlinear models are difficult to be solved by the DTM. In [16] it has been proposed a new formula for these complex models. The DTM has been applied recently to integral equation systems [17]. Furthermore, the DTM was introduced recently in the area of random differential equations [18]. In particular has been used to solve the Riccati random differential equation in [19].

The number of sub-domains n has to be chosen in an appropriate form in order to obtain accurate solutions. Similar methods choose the value of n and the number of terms to obtain a given admissible global error given a priori. Other related works have proposed a strategy with few terms in each subinterval and a high number of sub-domains n [20]. Here, we follow a similar strategy only to show the effectiveness of the DTM for autonomous and non-autonomous predator-prey systems.

Since, several biological and environmental parameters in the predator-prey model vary in time, non-autonomous systems are important to be studied. In this paper three predator-prey models are considered in order to study numerically the reliability of the DTM applied to these type of models.

The organization of this paper is as follows. In Section 2, basic definitions of the DTM and some basic properties of the DTM are presented. Section 3 is devoted to present the numerical results of the application of the method to different predator-prey systems. Comparisons between the DTM and the fourth-order Runge-Kutta (RK4) solutions are shown. Finally in Section 4 discussion and conclusions are presented.

2 Basic definitions and properties of differential transformation method

For clarity of presentation of the DTM, we summarize the main issues of the method that may be found in [10].

Definition 1 *Let $x(t)$ be analytic in the time domain D, then it has derivatives of all orders with respect to time t. Put*

$$\varphi(t,k) = \frac{d^k x(t)}{dt^k}, \qquad \forall t \in D. \tag{2}$$

For $t = t_i$, then $\varphi(t,k) = \varphi(t_i,k)$, where k belongs to a set of non-negative integers, denoted as the K domain. Thus, (2) can be rewritten as

$$X(k) = \varphi(t_i,k) = \left[\frac{d^k x(t)}{dt^k}\right]_{t=t_i} \tag{3}$$

where $X(k)$ is called the spectrum of $x(t)$ at $t = t_i$.

Definition 2 *Suppose that $x(t)$ is analytic in the time domain D, then it can be represented as*

$$x(t) = \sum_{k=0}^{\infty} \frac{(t-t_i)^k}{k!} X(k). \tag{4}$$

Thus, the equation (4) represents the inverse transformation of $X(k)$.

Definition 3 *If $X(k)$ is defined as*

$$X(k) = M(k)\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_i} \tag{5}$$

where $k \in \mathbb{Z} \cup \{0\}$, then the function $x(t)$ can be described as

$$x(t) = \frac{1}{q(t)} \sum_{k=0}^{\infty} \frac{(t-t_i)^k}{k!} \frac{X(k)}{M(k)}, \tag{6}$$

where $M(k) \neq 0$ and $q(t) \neq 0$. $M(k)$ is the weighting factor and $q(t)$ is regarded as a kernel corresponding to $x(t)$.

Note, that if $M(k) = 1$ and $q(t) = 1$, then Eqs. (3) and (4) and (5) and (6) are equivalent. From the definitions above, we can see that the concept of differential transformation is based upon the Taylor series expansion. Note that, the original functions are denoted by lowercase and their transformed functions are indicated by uppercase letter. The DTM can solve a system of differential equations of the form $\dot{x}(t) = f\big(x(t),t\big)$ $t \in [a,b]$, with the initial condition $x(a) = x_a$, where $x(t) = (x^1(t), x^2(t), ..., x^j(t), ..., x^n(t))^T$ (T transposed) and that are well-posed. Thus, applying the DTM, a system of differential equations in the domain of interest can be transformed to a system of algebraic equations in the K domain and each $x^j(t)$ can be obtained by a finite-term Taylor series plus a remainder, i.e.,

$$\begin{aligned} x^j(t) &= \frac{1}{q(t)} \sum_{k=0}^{n} \frac{(t-t_i)^k}{k!} \frac{X^j(k)}{M(k)} + R_{n+1} \\ &= \sum_{k=0}^{n} \left(\frac{t}{H}\right)^k X^j(k) + R_{n+1}, \end{aligned} \tag{7}$$

where

$$R_{n+1} = \sum_{k=n+1}^{\infty} \left(\frac{t}{H}\right)^k X^j(k), \text{ and}$$
$$R_{n+1} \to 0, \text{ as } n \to \infty.$$

For practical problems of numerical simulation, the computation interval $[0,H]$ is not always small, and to accelerate the rate of convergence and improve the accuracy of the calculations, it is convenient to divide the entire domain H into n sub-domains. The main advantage of domain split process is that only a few Taylor series terms are required to compose the solution in a small time interval H_i, where $H = \sum_{i=1}^{n} H_i$. It is important to remark that, H_i can be chosen arbitrarily small if it is necessary. Thus, the system differential equation can then be solved in each subdomain. The approach described above is known as the D spectra method. Considering the function $x^j(t)$ in the first sub-domain ($0 \le t \le t_1$, $t_0 = 0$), the one-dimensional differential transformation is given by

$$x^j(t) = \sum_{k=0}^{n} \left(\frac{t}{H_0}\right)^k X_0^j(k), \tag{8}$$

where $X_0^j(0) = x_0^j(0)$. Therefore, the differential transformation and system dynamic equations can be solved for the first sub-domain and X_0^j can be solved entirely in the first sub-domain. The end point of function $x^j(t)$ in the first sub-domain is x_1^j, and the value of t is H_0. Thus, $x_1^j(t)$ is obtained by the DTM as

$$x_1^j(H_0) = x^j(H_0) = \sum_{k=0}^{n} X_0^j(k). \tag{9}$$

Since that $x_1^j(H_0)$ represents the initial condition in the second sub-domain, then $X_1^j(0) = x_1^j(H_0)$. In this way the function $x^j(t)$ can be expressed in the second sub-domain as

$$x_2^j(H_1) = x^j(H_1) = \sum_{k=0}^{n} X_1^j(k). \quad (10)$$

In general form, the function $x^j(t)$ can be expressed in the $i-1$ sub-domain as

$$x_i^j(H_i) = x_{i-1}^j(H_{i-1}) + \sum_{k=1}^{n} X_{i-1}^j(k) = X_{i-1}^j(0) + \sum_{k=1}^{n} X_{i-1}^j(k), \quad i = 1, 2, ..., n.$$

Using the D spectra method described above, the functions $x^j(t)$ can be obtained throughout the entire domain, for all j.

The operation properties of the differential transformation

Let us consider $q(t) = 1$, $M(k) = \frac{H^k}{k!}$ and $x^1(t)$, $x^2(t)$, $x^3(t)$ three uncorrelated functions with time t and $X^1(k)$, $X^2(k)$, $X^2(k)$ are the corresponding transformed functions. Let $c_1, c_2 \in \mathbf{R}$, in Table 1 we show a list of the transformation properties that are useful in this paper.

Table 1: Differential transformation conversion (i denotes the i-th split domain)

Original function $\Longleftrightarrow$	Transformed function
$c_1x^1(t) \pm c_2x^2(t)$	$c_1X^1(k) \pm c_2X^2(k)$
c_1	$c_1\delta(k)$
$x^1(t)x^2(t)$	$X^1(k) * X^2(k) = \sum_{l=0}^{k} X^1(l)X^2(k-l)$
$x^1(t)x^2(t)x^3(t)$	$\sum_{k_2=0}^{k} \sum_{k_1=0}^{k_2} X^3(k_1)X^2(k_2-k_1)X^3(k-k_2)$
$z(t) = x^1(t)/x^2(t)$	$Z(k) = \frac{X^1(k) - \sum_{l=0}^{k-1} Z(l)X^2(k-l)}{X^2(0)}$
$\frac{d^n x^1(t)}{dt^n}$	$\frac{(k+1)(k+2)\cdots(k+n)}{H_i^n} X^1(k+n)$
$x^1(t) = \cos(\omega t + \alpha)$	$\frac{(H_i\omega)^k}{k!} \cos\left(\frac{\pi k}{2} + \alpha + 2\pi i H_i\right)$

3 Numerical solutions on predator-prey systems

In this section, the differential transformation technique is applied to solve three different nonlinear differential equations systems representing predator-prey models. Thus, from the properties given in Section 2, the corresponding spectrum can be determined for the system (1) as

$$\begin{aligned} &\mathbf{X}_1(k+1) = \\ &\frac{H_i}{k+1}\mathbf{F}\Big(X_1(k), X_2(k), A_1(k), \cdots, A_n(k)\Big), \\ &\mathbf{X}_2(k+1) = \\ &\frac{H_i}{k+1}\mathbf{G}\Big(X_1(k), X_2(k), B_1(k), \cdots, B_n(k)\Big), \end{aligned} \quad (11)$$

where the initial conditions are given by $\mathbf{X}_1(0) = x_1(0)$ and $\mathbf{X}_2(0) = x_2(0)$.

3.1 Example 1

The first model presents the problem in which some rabbits and foxes are living together, where foxes eat the rabbits and rabbits eat clover, and there is an increase and decrease in the number of foxes and rabbits [3]. The model is represented analytically by the following ordinary differential equation system:

$$\begin{aligned} \dot{x_1}(t) &= a_1x_1(t) - a_2x_1(t)x_2(t), \\ \dot{x_2}(t) &= -b_1x_2(t) + b_2x_1(t)x_2(t). \end{aligned} \quad (12)$$

Thus, using the properties of the DTM the spectrum of system (12) is given by

$$\begin{aligned} \mathbf{X}_1(k+1) &= \frac{H_i}{k+1}\Big\{a_1\,\mathbf{X}_1(k) \\ &\quad - a_2 \sum_{k_1=0}^{k} \mathbf{X}_1(k_1)\,\mathbf{X}_2(k-k_1)\Big\}, \\ \mathbf{X}_2(k+1) &= \frac{H_i}{k+1}\Big\{-b_1\,\mathbf{X}_2(k) \\ &\quad + b_2 \sum_{k_1=0}^{k} \mathbf{X}_1(k_1)\,\mathbf{X}_2(k-k_1)\Big\}. \end{aligned} \quad (13)$$

This is a classic predator-prey system with periodic solution if $a_1b_1 > 0$. In Figure 1 it can be seen that the DTM reproduces the correct periodic behavior of the prey and predator populations. In Table 2 we present the absolute differences between the 3-term DTM solutions on time steps $h = 0.1, 0.001$ and the fourth-order Runge-Kutta solution on time step $h = 0.001$. These results show the numerical consistency of the DTM. Furthermore, as expected the accuracy of the solution is increased when the time step is decreased. For the time step size $h = 0.001$, DTM and Runge-Kutta present very well concordance.

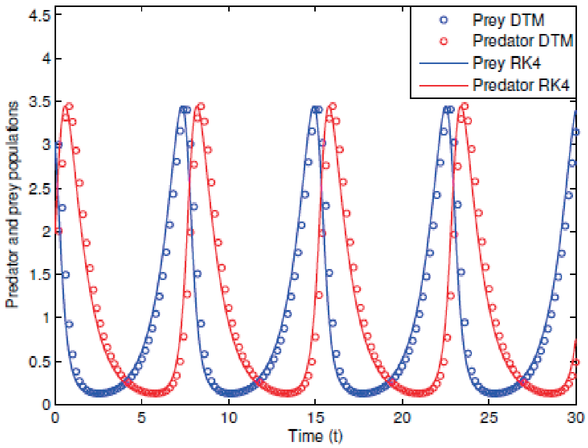

Fig. 1: Dynamics of the model (12), when $a_1 = 1$, $a_2 = 1, b_1 = 1, b_2 = 1, x_1(0) = 3$ and $x_2(0) = 2$. DTM solution is obtained using 3 terms and $H_i = 0.2$.

3.2 Example 2

The second model considers the problem that the predator in the model is not of commercial importance. The prey is subjected to constant effort harvesting and the harvesting activity does not affect the predator population directly. Predator population is indirectly reduced by the availability of the prey to the predator. Furthermore a simple logistic growth for prey population is assumed [6]. This model is represented by the following system,

$$\dot{x_1}(t) = x_1(t)(1 - x_1(t)) - bz(t) - rx_1(t), \quad (14)$$

$$\dot{x_2}(t) = cz(t) - ex_2(t), \quad (15)$$

where

$$z(t) = \frac{x_1(t)x_2(t)}{x_1(t) + x_2(t)}.$$

From (14) and using the properties of the DTM

Table 2: Comparison of the solutions obtained with 3-term DTM ($H_i = 0.1, 0.001$) and $RK4$ method ($h = 0.001$) for system (12).

$\Delta = |DTM_{0.1} - RK4_{0.001}|$

$Time$	Δx_1	Δx_2
0.00	$.0000E + 00$	$.0000E + 00$
1.00	$.4076E - 02$	$.1069E - 02$
2.00	$.4545E - 02$	$.1041E - 02$
3.00	$.4226E - 02$	$.7633E - 03$
4.00	$.3726E - 02$	$.4890E - 03$
5.00	$.3240E - 02$	$.2857E - 03$
6.00	$.2833E - 02$	$.1562E - 03$
7.00	$.2510E - 02$	$.8176E - 04$
8.00	$.2251E - 02$	$.4168E - 04$
9.00	$.2035E - 02$	$.2094E - 04$
10.00	$.1849E - 02$	$.1044E - 04$

$\Delta = |DTM_{0.001} - RK4_{0.001}|$

$Time$	Δx_1	Δx_2
0.00	$.0000E + 00$	$.0000E + 00$
1.00	$.4421E - 04$	$.1155E - 04$
2.00	$.4887E - 04$	$.1111E - 04$
3.00	$.4524E - 04$	$.8067E - 05$
4.00	$.3982E - 04$	$.5121E - 05$
5.00	$.3465E - 04$	$.2970E - 05$
6.00	$.3037E - 04$	$.1615E - 05$
7.00	$.2699E - 04$	$.8417E - 06$
8.00	$.2427E - 04$	$.4280E - 06$
9.00	$.2200E - 04$	$.2147E - 06$
10.00	$.2003E - 04$	$.1069E - 06$

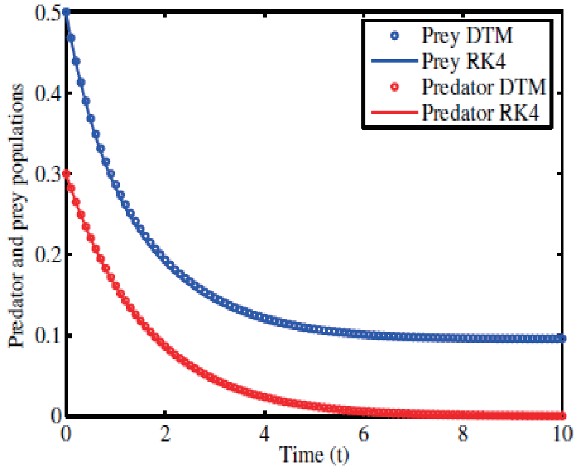

Fig. 2: Dynamics of the model (14), when $b = 0.8$, $c = 0.2$, $e = 0.5$, $r = 0.9$, $x_1(0) = 0.5$ and $x_2(0) = 0.3$. DTM solution is computed using 3 terms and $H_i = 0.1$.

one gets the recurrence system,

$$\begin{aligned}\mathbf{X}_1(k+1) &= \frac{H_i}{k+1}\Big\{(1-r)\mathbf{X}_1(k) - b\,\mathbf{Z}(k) \\ &\quad - \sum_{k_1=0}^{k} \mathbf{X}_1(k_1)\,\mathbf{X}_1(k-k_1)\Big\}, \\ \mathbf{X}_2(k+1) &= \frac{H_i}{k+1}\Big\{-c\,\mathbf{Z}(k) - e\mathbf{X}_2(k)\Big\}\end{aligned}$$

where

$$\begin{aligned}\mathbf{Z}(k) &= \frac{\mathbf{X}_1(k)\mathbf{X}_2(0)}{\mathbf{X}_1(0)+\mathbf{X}_2(0)} \\ &+ \sum_{k_1=0}^{k-1}\Bigg(\frac{\big(\mathbf{X}_1(k_1)+\mathbf{Z}(k_1)\big)\mathbf{X}_2(k-k_1)}{\mathbf{X}_1(0)+\mathbf{X}_2(0)} \\ &+ \frac{\mathbf{Z}(k_1)\mathbf{X}_1(k-k_1)}{\mathbf{X}_1(0)+\mathbf{X}_2(0)}\Bigg),\end{aligned}$$

for $k \geq 1$ and,

$$\mathbf{Z}(0) = \frac{\mathbf{X}_1(0)\mathbf{X}_2(0)}{\mathbf{X}_1(0)+\mathbf{X}_2(0)}.$$

In Figure 2 a noteworthy observation is that prey and predator species can become extinct simultaneously for some values of the parameters, regardless of the initial values. The obtained solution with DTM reproduces the correct dynamics of model (14). In Table 3 we present the absolute differences between the 3-term DTM solutions on time steps $h = 0.1, 0.001$ and the fourth-order Runge-Kutta solution on time step $h = 0.001$. These results show well concordance between both methods. As in the previous example the accuracy of the solution is increased when the time step is decreased. For the time step size $h = 0.001$, DTM and fourth-order Runge-Kutta presents very well concordance.

3.3 Example 3

The last considered model is a Lotka-Volterra model represented by a nonautonomous ordinary differential equation system. In this model time varying values for the growth rate of the prey, the efficiency of the predator is ability to capture prey, the death rate of the predator and the growth rate of the predator are considered. It is important to remark that since in this problem coefficients are time varying careful attention must be paid in order to obtain the correct recurrence equation system of the model. This model has also been used to test power series, Adomian and hybrid methods in other works [4, 7]. The aforementioned model is described by the following ordinary differential equation system,

$$\begin{aligned}\dot{x_1}(t) &= a_1(t)x_1(t) - a_2(t)x_1(t)x_2(t), \\ \dot{x_2}(t) &= -b_1(t)x_2(t) + b_2(t)x_1(t)x_2(t).\end{aligned} \tag{16}$$

Thus, the spectrum of (16) is given by

$$\begin{aligned}\mathbf{X}_1(k+1) &= \frac{H_i}{k+1}\Big\{\sum_{k_1=0}^{k} \mathbf{A}_1(k_1)\,\mathbf{X}_1(k-k_1) \\ &\quad - \sum_{k_1,k_2=0}^{k,k_1} \mathbf{A}_2(k_1)\mathbf{X}_1(k_1-k_2)\,\mathbf{X}_2(k-k_1)\Big\}, \\ \mathbf{X}_2(k+1) &= \frac{H_i}{k+1}\Big\{-\sum_{k_1=0}^{k} \mathbf{B}_1(k_1)\,\mathbf{X}_2(k-k_1) \\ &\quad + \sum_{k_1,k_2=0}^{k,k_1} \mathbf{B}_2(k_1)\mathbf{X}_1(k_1-k_2)\,\mathbf{X}_2(k-k_1)\Big\}.\end{aligned}$$

For the numerical simulations of the model (16), we take $a_1(t) = 4 + \tan(t)$, $a_2(t) = \exp(2t)$, $b_1(t) = -2$, $b_2(t) = \cos(t)$, $x_1(0) = -4$ and $x_2(0) = 4$. The exact solution for these coefficients is $x_1(t) = -\frac{4}{\cos(t)}$, $x_2(t) = 4\exp(-2t)$. In Figure 3 it can be observed that the DTM reproduce the correct dynamic behavior of predator-prey system (16) and the obtained solution has well accuracy. In Table 4 it is presented the absolute differences between the analytical and the DTM(5 and 10 terms) solution.

Table 3: Comparison of the solutions obtained with 3-term DTM ($H_i = 0.1, 0.001$) and $RK4$ method ($h = 0.001$) for system (14).

$\Delta = \vert DTM_{0.1} - RK4_{0.001}\vert$		
$Time$	Δx_1	Δx_2
0.00	$.0000E+00$	$.0000E+00$
1.00	$.4076E-02$	$.1069E-02$
2.00	$.4545E-02$	$.1041E-02$
3.00	$.4226E-02$	$.7633E-03$
4.00	$.3726E-02$	$.4890E-03$
5.00	$.3240E-02$	$.2857E-03$
6.00	$.2833E-02$	$.1562E-03$
7.00	$.2510E-02$	$.8176E-04$
8.00	$.2251E-02$	$.4168E-04$
9.00	$.2035E-02$	$.2094E-04$
10.00	$.1849E-02$	$.1044E-04$

$\Delta = \vert DTM_{0.001} - RK4_{0.001}\vert$		
$Time$	Δx_1	Δx_2
0.00	$.0000E+00$	$.0000E+00$
1.00	$.4421E-04$	$.1155E-04$
2.00	$.4887E-04$	$.1111E-04$
3.00	$.4524E-04$	$.8067E-05$
4.00	$.3982E-04$	$.5121E-05$
5.00	$.3465E-04$	$.2970E-05$
6.00	$.3037E-04$	$.1615E-05$
7.00	$.2699E-04$	$.8417E-06$
8.00	$.2427E-04$	$.4280E-06$
9.00	$.2200E-04$	$.2147E-06$
10.00	$.2003E-04$	$.1069E-06$

As it can be observed, the accuracy of the DTM increase when number of terms are increased, as it was expected. In this example, we increase the number of terms since the system is nonautonomous and is more complex. However, the computational work necessary to solve numerically this example with the DTM is less than the multistage Adomian method and comparable to the fourth-order Runge-Kutta method.

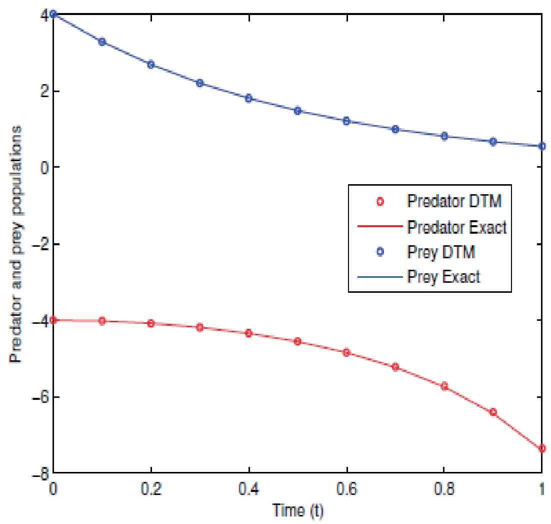

Fig. 3: Dynamics of the model (16) using 5-term DTM with $H_i = 0.1$.

4 Discussion and conclusions

In this paper, the DTM has been applied to predator-prey nonlinear ordinary differential equations models. In order to obtain very accurate solutions, the domain region has been splitted in subintervals and the approximating solutions are obtained in a sequence of time intervals. The DTM develops from the differential equation system with initial conditions a recurrence equation system that finally leads to the solution of a system of algebraic equations as coefficients of a power series solution. Moreover, the DTM does not evaluate the derivatives symbolically and this give advantages over other methods such as Taylor, power series or Adomian method.

In order to illustrate the efficiency and reliability of the DTM three different predator-prey models were considered. The obtained results of the present method are in excellent agreement with those obtained by the fourth-order Runge-Kutta method and with the analytical solutions when these were available. The calculated results show the reliability and efficiency of the method. The method has the advantage of giving a

Table 4: Absolute errors of the DTM (5-term and 10-term) solutions to system (16).

$Time$	$\|DTMx_1 - Exact\|$	$\|DTMx_2 - Exact\|$
	5-term	5-term
0.00	$.0000E+00$	$.0000E+00$
0.10	$.1761E-07$	$.1761E-07$
0.20	$.4259E-07$	$.4259E-07$
0.30	$.8947E-07$	$.8947E-07$
0.40	$.1925E-06$	$.1925E-06$
0.50	$.4483E-06$	$.4483E-06$
0.60	$.1174E-05$	$.1174E-05$
0.70	$.3627E-05$	$.3627E-05$
0.80	$.1422E-04$	$.1422E-04$
0.90	$.8145E-04$	$.8145E-04$
1.00	$.9237E-03$	$.9237E-03$
$Time$	$\|DTMx_1 - Exact\|$	$\|DTMx_2 - Exact\|$
	10-term	10-term
0.00	$.0000E+00$	$.0000E+00$
0.10	$.4440E-15$	$.4440E-15$
0.20	$.3996E-14$	$.3996E-14$
0.30	$.1998E-13$	$.1998E-13$
0.40	$.8792E-13$	$.8792E-13$
0.50	$.4054E-12$	$.4054E-12$
0.60	$.2246E-11$	$.2246E-11$
0.70	$.1668E-10$	$.1668E-10$
0.80	$.1909E-09$	$.1909E-09$
0.90	$.4308E-08$	$.4308E-08$
1.00	$.3257E-06$	$.3257E-06$

functional form of the solution within each time interval. Furthermore, the analytical form allows to study in a easier way the effect that biological parameters have on the dynamics of predators and preys. This is not possible in purely numerical techniques like the Runge-Kutta method, which provides solution only at discrete times.

Based on the numerical results it can be concluded that the DTM is a mathematical tool which enables to find accurate analytical solutions for predator-prey models represented by nonlinear ordinary differential equation systems. Furthermore, high accuracy can be obtained without using large computer power.

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