Stability Analysis of a Compressible Plasma

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Abstract:- In the presence of a uniform horizontal magnetic field, thermal instability of a compressible plasma is postulated to occur in the presence of effects of finite Larmor radius (FLR) and Hall currents. The dispersion relation is found by using the linear stability theory, Boussinesq approximation and the normal mode analysis approach, respectively. In the scenario of stationary convection, it was discovered that the compressibility has a stabilizing effect whereas FLR and Hall currents have stabilizing as well as destabilizing effects. For $\left(C_p\beta/g\right) < 1$, the system is stable. The magnetic field, FLR and Hall currents introduce oscillatory modes in the system for $\left(C_p\beta/g\right) > 1$. In addition to this, it has been discovered that the system is reliable for $\frac{1}{G-1}\frac{C_p\alpha\kappa}{\nu} \leq \frac{27\pi^4}{4}$andunder the condition $\frac{1}{G-1}\frac{C_p\alpha\kappa}{\nu} > \frac{27\pi^4}{4}$the system goes into an unstable state.

Keywords: - Finite Larmor radius, Hall currents, compressibility, thermal instability, plasma.

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1 Introduction

The thermal instability of a fluid layer heated from below plays an important role in geophysics, oceanography, atmospheric physics etc., and has been investigated by many authors, e.g. Be'nard [1], Rayleigh [2], Jeffreys [3]. A detailed account of the theoretical and experimental results of the onset of thermal instability (Be'nard convection) in a fluid layer under varying assumptions of hydrodynamics and hydromagnetics has been given in a treatise by Chandrasekhar [4]. The use of the Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in its association with the external force. The approximation is well justified in the case of incompressible fluids.

The properties of ionized space and laboratory magnetic fluids (plasmas) have been intensively investigated theoretically and experimentally in the past sixty years. One of the key aspects studied in this context is the stability of plasma structures. Usually, instabilities can be divided into two categories: macro- and micro-instabilities. Macro-instabilities occur with low frequencies compared to the plasma and cyclotron frequency and they are studied within the framework of magnetohydrodynamics (MHD). Physicists have understood the behaviour of macro-instabilities and they showed how to avoid the most destructive of them, but small-scale gradient driven micro-instabilities are still a serious obstacle for having a stable plasma for a large range of parameters. Micro-instabilities are described by models which include, e.g. finite Larmor radius (FLR) and collision less dissipative effects in plasmas. Time and length scales of micro-instabilities are comparable to the turbulent length scales and the length scales of transport coefficients. In general, the FLR effect is neglected. However, when the Larmor radius becomes comparable to the hydromagnetic length of the problem (e.g. wavelength) or the gyration frequency of ions in the magnetic field is of the same order as the wave frequency, finiteness of the Larmor radius must be taken into account. Strictly speaking, the space and time scale for the breakdown of hydromagnetics are on the respective scales of ion gyration about the field, and the ion Larmor frequency. Finite Larmor radius effect on plasma instabilities has been the subject of many investigations. In many astrophysical plasma situations such as in solar corona, interstellar and interplanetary plasmas the assumption of zero Larmor radius is not valid. The effects of finiteness of the ion Larmor radius, showing up in the form of a magnetic viscosity in the fluid equations, have been studied by Rosenbluth et al. [5], Roberts and Taylor [6], Vandakurov [7] and Jukes [8]. Melchior and Popowich [9] have considered the finite Larmor radius (FLR) effect on the Kelvin-Helmholtz instability of a fully ionized plasma, while the effect on the Rayleigh-Taylor instability has been studied by Singh and Hans [10]. Sharma [11] has studied the effect of a finite Larmor radius on the thermal instability of a plasma.Hernegger [12] investigated the stabilizing effect of FLR on thermal instability and showed that thermal criterion is changed by FLR for wave propagation perpendicular to the magnetic field. Sharma [13] investigated the stabilizing effect of FLR on thermal instability of rotating plasma. Ariel [14] discussed the stabilizing effect of FLR on thermal instability of conducting plasma layer of finite thickness surrounded by a non-conducting matter. Vaghela and Chhajlani [15] studied the stabilizing effect of FLR on magneto-thermal stability of resistive plasma through a porous medium with thermal conduction. Bhatia and Chhonkar [16] investigated the stabilizing effect of FLR on the instability of a rotating layer of self-gravitating plasma incorporating the effects of viscosity and Hall current. Vyas and Chhajlani [17] pointed out the stabilizing effect of FLR on the thermal instability of magnetized rotating plasma incorporating the effects of viscosity, finite electrical conductivity, porosity and thermal conductivity. Kaothekar and Chhajlani [18] investigated the problem of Jeans instability of self-gravitating rotating radiative plasma with finite Larmor radius corrections. The frictional effect of collisions of ionized with neutral atoms on Rayleigh-Taylor instability of a composite plasma in porous medium has been considered by Kumar and Mohan [19]. Kumar et al. [20] considered the Rayleigh-Taylor instability of an infinitely conducting plasma in porous medium taking account the finiteness of ion Larmor radius (FLR) in the presence of a horizontal magnetic field. Kumar and Singh [21] investigated the thermal convection of a plasma in porous medium to include simultaneously the effect of rotation and the finiteness of the ion Larmor radius (FLR) in the presence of a vertical magnetic field. The effect of finite Larmor radius of the ions on thermal convection of a plasma has been studied by Kumar and Gupta [22]. Kumar [23] investigated the thermal convection of a plasma in porous medium in the presence of finite Larmor radius (FLR) and Hall effects. Thus FLR effect is an important factor in the discussion of thermal convection and other hydromagnetic instabilities.

When the fluids are compressible, the equations governing the system become quite complicated. To simplify the set of equations governing the flow of compressible fluids, Spiegel and Veronis [24] made the following assumptions:

(i) The depth of fluid layer is much smaller than the scale height as defined by them and
(ii) The fluctuations in temperature, pressure and density, introduced due to motion, do not exceed their static variations.

Under the above assumptions, the flow equations are the same as those for incompressible fluids except that the static temperature gradient is replaced by its excess over the adiabatic. The thermal instability in compressible fluids in the presence of rotation and magnetic field has been considered by Sharma [25]. Sharma [26] also studied the thermal instability of a compressible Hall plasma. Sharma and Sharma [27] considered the thermal instability of a partially ionized plasma in the presence of compressibility and collisional effects while the thermal instability of a compressible plasma with FLR has been studied by Sharma et al. [28]. Finite Larmor radius (FLR) effects are likely to be important in “weakly” unstable systems such as high beta stellarator, mirror machines, slowly rotating plasmas, large aspect tori etc. The Hall effects are likely to be important in many astrophysical situations as well as in flows of laboratory plasma. Sherman and Sutton [29] considered the effect of Hall currents on the efficiency of a magneto-fluid dynamic generator while Sato [30] and Tani [31] studied the incompressible viscous flow of an ionized gas with tensor conductivity in channels with consideration of Hall effect. Sonnerup [32] and Uberoi and Devanathan [33] investigated the effects of Hall current on the propagation of small amplitude waves taking compressibility into account.

Keeping in mind the importance in the physics of atmosphere and astrophysics especially in the case of ionosphere and outer layers of the sun’s atmosphere, the present paper is devoted to the study of thermal instability of a compressible plasma under the effects of finite Larmor radius (FLR) and Hall currents in the presence of a uniform horizontal magnetic field.

2 Formulation of the Problem and Perturbation Equations

Consider an infinite horizontal layer of compressible, viscous, heat-conducting and finite electrically conducting fluid of thickness d in which a uniform temperature gradient $\beta(=|dT/dz|)$ is maintained. Consider the cartesian coordinates (x,y,z) with origin on the lower boundary $z=0$ and the z-axis perpendicular to it along the vertical. The fluid is acted on by a horizontal magnetic fluid $\vec{H}(H,0,0)$ and gravity force $\vec{g}(0,0,-g)$.

Following Spiegel and Veronis [24] and Sharma et al. [28], the linearized hydromagnetic perturbation equations appropriate to the problem are

$$\frac{\partial \vec{q}}{\partial t} = -\left(\frac{1}{\rho_m}\right)\nabla\delta\overleftrightarrow{P} + \nu\nabla^2\vec{q} + \frac{1}{4\pi}\left(\nabla\times\vec{h}\right)\times\vec{H} - \vec{g}\alpha\theta\,, \quad (1)$$

$$\nabla.\vec{q} = 0\,, \quad (2)$$

$$\frac{\partial \vec{h}}{\partial t} = \nabla\times\left(\vec{q}\times\vec{H}\right) + \eta\nabla^2\vec{h} - \left(\frac{1}{4\pi Ne}\right)\nabla \times\left[\left(\nabla\times\vec{h}\right)\times\vec{H}\right], \quad (3)$$

$$\nabla.\vec{h} = 0\,, \quad (4)$$

$$\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{C_p}\right)w + \kappa\nabla^2\theta\,, \quad (5)$$

where $\vec{q}(u,v,w), \vec{h}(h_x, h_y, h_z), \delta p, \delta\rho$ and θ denote respectively the perturbations in velocity, magnetic field $\vec{H}$, pressure p, density ρ and temperature T. Here $\delta\overleftrightarrow{P}, \rho_m, \mu, \nu(=\mu/\rho_m), \kappa', \kappa(=\kappa'/\rho_m C_p), \eta, g/C_p, g, \alpha, N$ and e stand for stress tensor perturbation, constant space average of ρ, viscosity, kinematic viscosity, thermal conductivity, thermal diffusivity, resistivity, adiabatic gradient, acceleration due to gravity, coefficient of thermal expansion, electron number density and charge of an electron respectively.

For the horizontal magnetic field $\vec{H}(H,0,0)$, the stress tensor $\delta\overleftrightarrow{P}$, taking into account the finite ion gyration [Vandakurov [7]], has the components

$$\delta P_{xx} = \delta p\,, \delta P_{xy} = \delta P_{yx} = -2\rho\nu_0\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right),$$

$$\delta P_{xz} = \delta P_{zx} = 2\rho\nu_0\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \delta P_{yy} = \delta p - \rho\nu_0\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right),$$

$$\delta P_{yz} = \delta P_{zy} = \rho\nu_0\left(\frac{\partial v}{\partial y} - \frac{\partial w}{\partial z}\right), \delta P_{zz} = \delta p + \rho\nu_0\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right). \quad (6)$$

Here $\rho\nu_0 = NT/4\omega_H$ where N, T and ω_H denote respectively the number density, the ion temperature and the ion gyration frequency.

3 Dispersion Relation

We decompose the disturbances into normal modes and assume that the perturbed quantities are of the form

$$[w,\theta,h_z,\zeta,\xi] = [W(z),\Theta(z),K(z),Z(z),X(z)]exp\left(ik_x x + ik_y y + nt\right), \quad (7)$$

where k_x, k_y are the wave numbers along the x- and y- directions respectively, $k = \left(k_x^2 + k_y^2\right)^{1/2}$ is the resultant wave number and n is the frequency of oscillation. ζ and ξ stand for the z-components of vorticity and current density respectively.

Let $a = kd, \sigma = nd^2/\nu$, $p_1 = \nu/\kappa$, $p_2 = \nu/\eta$, $D = d/dz$ and x, y, z stand for the coordinates in the new unit of length d. Equations (1) – (6), using expression (7), give

$$(D^2 - a^2)(D^2 - a^2 - \sigma)W - \left(\frac{g\alpha d^2}{\nu}\right)a^2\Theta + \frac{ik_x H d^2}{4\pi\rho_m \nu}(D^2 - a^2)K + \left(\frac{ik_x \nu_0 d^2}{\nu}\right)\left(D^2 - a^2 + 3\frac{k_x^2}{k^2}a^2\right)Z = 0, \quad (8)$$

$$(D^2 - a^2 - \sigma)Z = \left(\frac{ik_x \nu_0}{\nu}\right)\left(D^2 - a^2 + 3\frac{k_x^2}{k^2}a^2\right)W - \frac{ik_x H d^2}{4\pi\rho_m \nu}X, \quad (9)$$

$$(D^2 - a^2 - p_2\sigma)X = -\left(\frac{ik_x H d^2}{\eta}\right)Z - \left(\frac{ik_x H}{4\pi N e\eta}\right)(D^2 - a^2)K, \quad (10)$$

$$(D^2 - a^2 - p_2\sigma)K = -\left(\frac{ik_x H d^2}{\eta}\right)W + \left(\frac{ik_x H d^2}{4\pi N e\eta}\right)X, \quad (11)$$

$$(D^2 - a^2 - p_1\sigma)\Theta = -\frac{d^2}{\kappa}\left(\beta - \frac{g}{C_p}\right)W. \quad (12)$$

Eliminating Θ, K, X and Z between equations (8) – (12), we obtain

$$\begin{aligned}&[(D^2 - a^2 - p_2\sigma)^2(D^2 - a^2 - \sigma) + Qk_x^2d^2(D^2 - a^2 - p_2\sigma) - Mk_x^2d^2(D^2 - a^2)(D^2 - a^2 - \sigma)]\left[Ra^2\left(\frac{G-1}{G}\right) + (D^2 - a^2)(D^2 - a^2 - \sigma)(D^2 - a^2 - p_1\sigma)\right]W \\ &+ Qk_x^2d^2(D^2 - a^2)(D^2 - a^2 - p_1\sigma)\left[\{(D^2 - a^2 - \sigma)(D^2 - a^2 - p_2\sigma) + Qk_x^2d^2\} - M^{1/2}N^{1/2}k_x^2d^2(D^2 - a^2 - p_2\sigma) - M^{1/2}N^{1/2}k_x^2d^2\left(D^2 - a^2 + 3\frac{k_x^2}{k^2}a^2\right)\right]W \\ &- Nk_x^2d^2(D^2 - a^2 - p_1\sigma)\left(D^2 - a^2 + 3\frac{k_x^2}{k^2}a^2\right)^2[(D^2 - a^2 - p_2\sigma)^2 - Mk_x^2d^2(D^2 - a^2)]W = 0, \quad (13)\end{aligned}$$

where $Q = H^2d^2/4\pi\rho_m\nu\eta$ is the Chandrsekhar number, $R = g\alpha\beta d^4/\nu\kappa$ is the Rayleigh number, $M = (H/4\pi Ne\eta)^2$ is the non-dimensional number accounting for Hall currents, $N = (\nu_0/\nu)^2$ is a non-dimensional number accounting for FLR effect and $= C_p\beta/g$. Consider the case in which both the boundaries are free and the medium adjoining the fluid is non-conducting. The appropriate boundary conditions for this case are [Chandrasekhar [4]]

$$\left.\begin{array}{c} W = D^2W = 0, \Theta = 0, DZ = 0, X = 0 \\ at\ z = 0\ and\ 1 \\ and\ \vec{h}\ is\ continuous \end{array}\right\}. \quad (14)$$

The case of two free boundaries, though little artificial, is the most appropriate for stellar atmospheres [Spiegel [34]]. Using the boundary conditions (14), one can show that all the even derivatives of W must vanish for $z = 0$ and 1 and hence the proper solution of (13) characterizing the lowest mode is

$$W = W_0 \sin \pi z\,, \quad (15)$$

where W_0 is a constant. Substituting (15) in (13) and letting $a^2 = \pi^2 x, R_1 = R/\pi^4, Q_1 = Q/\pi^2, k_x = k\cos\theta$ and $i\sigma_1 = \sigma/\pi^2$, we obtain the dispersion relation

$$\begin{aligned} R_1 &= \left(\frac{G}{G-1}\right)\Bigg[\left(\frac{1+x}{x}\right)(1+x+i\sigma_1)(1+x \\ &+ ip_1\sigma_1) \\ &+ \big[Q_1 cos^2\theta\,(1+x)(1+x \\ &+ ip_1\sigma_1)\{(1+x+i\sigma_1)(1+x+ip_2\sigma_1) \\ &+ Q_1 x cos^2\theta\} \\ &+ N cos^2\theta(1+x \\ &+ ip_1\sigma_1)(1+x \\ &- 3x cos^2\theta)^2\{(1+x+ip_2\sigma_1)^2 \\ &+ Mx(1+x)cos^2\theta\} \\ &+ M^{1/2}N^{1/2}Q_1 x(1+x)(1+x \\ &+ ip_1\sigma_1)cos^4\theta\{(1+x+ip_2\sigma_1) \\ &+ (1+x-3x cos^2\theta)\}\big]\big[(1+x-ip_2\sigma_1)^2(1 \\ &+ x+i\sigma_1) + Q_1 x cos^2\theta(1+x+ip_2\sigma_1) \\ &+ Mx cos^2\theta(1+x)(1+x \\ &+ i\sigma_1)\big]^{-1}\Bigg]. \end{aligned} \quad (16)$$

Equation (16) is the required dispersion relation studying the effects of FLR and Hall currents on thermal instability of a compressible plasma. In the absence of Hall currents ($M \to 0$), equation (16) reduces to the dispersion relation (Sharma et al. [28]).

4 Important Theorems and Discussion

Theorem 1: The system is stable for $G < 1$.

Proof: Multiplying equation (8) by W^*, the complex conjugate of W, integrating over the range of z, and making use of equations (9) – (12) together with the boundary conditions (14), we obtain

$$I_1 + \sigma I_2 + \frac{C_p \alpha \kappa a^2}{\nu(1-G)}(I_3 + p_2 \sigma^* I_4) + \frac{\eta}{4\pi\rho_m \nu}(I_5 + p_2 \sigma^* I_6) + \frac{\eta d^2}{4\pi\rho_m \nu}(I_7 + p_2 \sigma I_8) + d^2(I_9 + \sigma^* I_{10}) = 0\,, \quad (17)$$

where

$$I_1 = \int_0^1 (|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2)dz,$$

$$I_2 = \int_0^1 (|DW|^2 + a^2|W|^2)dz\,,$$

$$I_3 = \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2)dz\,,$$

$$I_4 = \int_0^1 (|\Theta|^2)dz\,,$$

$$I_5 = \int_0^1 (|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2)dz\,,$$

$$I_6 = \int_0^1 (|DK|^2 + a^2|K|^2)dz\,,$$

$$I_7 = \int_0^1 (|DX|^2 + a^2|X|^2)dz,$$

$$I_8 = \int_0^1 (|X|^2)dz\,,$$

$$I_9 = \int_0^1 (|DZ|^2 + a^2|Z|^2)dz\,,$$

$$I_{10} = \int_0^1 (|Z|^2)dz\,. \quad (18)$$

The integrals $I_1 - I_{10}$ are all positive definite.

Putting $\sigma = \sigma_r + i\sigma_i$ and equating the real and imaginary parts of equation (17), we obtain

$$\sigma_r \left[I_2 + \frac{C_p \alpha \kappa a^2}{\nu(1-G)} p_1 I_4 + \frac{\eta}{4\pi\rho_m \nu} p_2 (I_6 + d^2 I_8) + d^2 I_{10} \right] = -\left[I_1 + \frac{C_p \alpha \kappa a^2}{\nu(1-G)} I_3 + \frac{\eta}{4\pi\rho_m \nu}(I_5 + d^2 I_7) + d^2 I_9 \right], \quad (19)$$

and

$$\sigma_i \left[I_2 - \frac{C_p \alpha \kappa a^2}{\nu(1-G)} p_1 I_4 - \frac{\eta}{4\pi\rho_m \nu} p_2 (I_6 - d^2 I_8) - d^2 I_{10} \right] = 0\,. \quad (20)$$

It is evident from equation (19) that σ_r is negative if $G < 1$. The system is therefore stable for $G < 1$.

Theorem 2: The modes may be oscillatory or non-oscillatory in contrast to the case of no magnetic field and in the absence of Hall currents and finite Larmor radius where modes are non-oscillatory, for $G > 1$.

Proof: It is clear from equation (20) that, for $G > 1$, σ_i may be zero or non-zero, meaning that the modes may be oscillatory or non-oscillatory. The oscillatory modes are introduced due to the presence of magnetic field (and hence the presence of Hall currents and FLR effects).

In the absence of a magnetic field and hence absence of Hall currents and FLR effects, equation (20) gives

$$\sigma_i \left[I_2 + \frac{C_p \alpha \kappa a^2}{\nu(G-1)} p_1 I_4 \right] = 0\,, \quad (21)$$

and the terms in brackets are positive when $G > 1$. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied, but in the presence of Hall currents, magnetic field and finite Larmor radius effects, the oscillatory modes come into play.

Theorem 3: The system is stable for $\frac{1}{G-1}\frac{C_p\alpha\kappa}{\nu} \leq \frac{27\pi^4}{4}$ and under the condition $\frac{1}{G-1}\frac{C_p\alpha\kappa}{\nu} > \frac{27\pi^4}{4}$, thesystem becomes unstable.

Proof: From equation (20) it is clear that σ_i is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero.

If $\sigma_i \neq 0$, equation (19) upon utilizing (20) and the Rayleigh-Ritz inequality gives

$$\left[\frac{27\pi^4}{4} - \frac{1}{G-1}\frac{C_p\alpha\kappa}{\nu}\right]\int_0^1 |W|^2 dz + \frac{\pi^2 + a^2}{a^2}\left\{d^2 I_9 + \frac{\eta}{2\pi\rho_m\nu} p_2 d^2 \sigma_r I_8 + \frac{\eta}{4\pi\rho_m\nu}(d^2 I_7 + I_5) + 2\sigma_r I_2\right\} \leq 0, \tag{22}$$

since the minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ with respect to a^2 is $\frac{27\pi^4}{4}$.

Now, let $\sigma_r \geq 0$, we necessarily have from inequality (22) that

$$\frac{1}{G-1}\frac{C_p\alpha\kappa}{\nu} > \frac{27\pi^4}{4}. \tag{23}$$

Hence, if

$$\frac{1}{G-1}\frac{C_p\alpha\kappa}{\nu} \leq \frac{27\pi^4}{4}, \tag{24}$$

then $\sigma_r < 0$. Therefore, the system is stable.

Thus, under the condition (24), the system is stable and under condition (23) the system becomes unstable.

Theorem 4: For stationary convection case:

- **(I)** In the absence of Hall currents, finite Larmor radius has a stabilizing effect and in the presence of FLR and Hall currents, the finite Larmor radius effect may be both stabilizing as well as destabilizing effects on the system.
- **(II)** In the absence of FLR, Hall currents always has a destabilizing effect but in the presence of FLR and Hall effects, the Hall currents may have both destabilizing as well as stabilizing effects on the system.

Proof: When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (16) reduces to

$$R_1 = \left(\frac{G}{G-1}\right)\left[\frac{(1+x)^3}{x} + \left[Q_1cos^2\theta(1+x)\{(1+x)^2+Q_1xcos^2\theta\} + M^{1/2}N^{1/2}Q_1xcos^4\theta(1+x)\{(1+x) + (1+x-3xcos^2\theta)\} + Ncos^2\theta(1+x)(1+x-3xcos^2\theta)^2(1+x + Mxcos^2\theta)\right][(1+x)^2+Q_1xcos^2\theta + Mx(1 + x)cos^2\theta]^{-1}\right], \quad (25)$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters Q_1 , M , N and G.

Let $\overline{R_c}$ and R_c denote respectively the critical Rayleigh numbers in the presence and in the absence of compressibility. For fixed values of Q_1 , M and N, let the non-dimensional number G accounting for the compressibility effects be also kept fixed, then we find that

$$\overline{R_c} = \left(\frac{G}{G-1}\right)R_c . \quad (26)$$

The effect of compressibility is thus to postpone the onset of thermal instability. Hence compressibility has a stabilizing effect. $G > 1$ is relevant here. The cases $G < 1$ and $G = 0$ correspond to negative and infinite values of critical Rayleigh numbers in the presence of compressibility which are not relevant in the present study.

To investigate the effects of finite Larmor radius and Hall currents, we examine the natures of

$$\frac{dR_1}{dN} \; and \; \frac{dR_1}{dM}$$

analytically.

(I) Equation (25) yields

$$\frac{dR_1}{dN} = \left(\frac{G}{G-1}\right)\left[(1+x)(1+x-3xcos^2\theta)^2(1 + x + Mxcos^2\theta)cos^2\theta + \frac{1}{2}\left(\frac{M}{N}\right)^{1/2} Q_1x(1+x)cos^4\theta\{(1+x) + (1+x-3xcos^2\theta)\}\right][(1+x)^2 + Q_1xcos^2\theta + Mxcos^2\theta(1+x)]^{-1} , \quad (27)$$

which is positive if $2(1+x) > 3xcos^2\theta$ i.e. the wave number range satisfying

$$\cos\theta < \left\{\frac{2(1+x)}{3x}\right\}^{1/2} .$$

This shows that FLR has a stabilizing effect for the above wave-number range. In the absence of Hall currents $(M \to 0)$, FLR always has a stabilizing effect. But in the presence of FLR and Hall effects on thermal instability, the FLR effect may be both stabilizing as well as destabilizing but completely stabilizes the above wave-number range.

(II) It is evident from equation (25) that

$$\frac{dR_1}{dM} = \Bigg[Q_1x(1+x)cos^4\,\theta\{(1+x)^2 + Q_1xcos^2\theta\}\left\{\frac{1}{2}\left(\frac{N}{M}\right)^{1/2}(1+x-3xcos^2\theta) + (1+x)\frac{(N^{1/2}-2M^{1/2})}{2M}\right\} + N^{1/2}Q_1x^2(1+x)cos^6\theta\left\{N^{1/2}(1+x-3xcos^2\theta)^2 - \frac{M^{1/2}}{2}(1+x)(2\overline{1+x}-3xcos^2\theta)\right\}[Q_1x(1+x)cos^4\,\theta]\{(1+x)^2 + Q_1xcos^2\theta\}\left\{\frac{1}{2}\left(\frac{N}{M}\right)^{1/2}(1+x-3xcos^2\theta) + (1+x)\frac{(N^{1/2}-2M^{1/2})}{2M}\right\} + N^{1/2}Q_1x^2(1+x)cos^6\theta\left\{N^{1/2}(1+x-3xcos^2\theta)^2 - \frac{M^{1/2}}{2}(1+x)(2\overline{1+x}-3xcos^2\theta)\right\}\Bigg][(1+x)^2 + Q_1xcos^2\theta + Mxcos^2\theta(1+x)^2]^{-2}, \quad (28)$$

which is positive if $N > 4M$ and $\cos\theta > [2(1+x)/3x]^{1/2}$ i.e. if

$$\frac{\nu_0}{\nu} > \frac{2cH}{4\pi Ne\eta}\ and \cos\theta > \left\{\frac{2(1+x)}{3x}\right\}^{1/2}.$$

In the absence of FLR, equation (28) yields that dR_1/dM is always negative thus indicating the destabilizing effect of Hall currents. In the presence of FLR and Hall effects, the Hall currents may have both destabilizing as well as stabilizing effects and there is competition between the destabilizing role of Hall currents and stabilizing role of FLR but completely stabilizes the above wave-number range if $\nu_0/\nu > 2[cH/(4\pi Ne\eta)]$.

5 Conclusions

An attempt has been made to investigate the effects of compressibility, FLR and Hall currents on the thermal instability of a plasma in the presence of a uniform horizontal magnetic field under the linear stability theory. It has been shown by Sato [30] and Tani [31] that inclusion of Hall currents gives rise to a cross-flow i.e. a flow at right angles to the primary flow in a channel in the presence of a transverse magnetic field. Tani [31] found that Hall effect produces a cross-flow of double-swirl pattern in incompressible flow through a straight channel with arbitrary cross-section. This breakdown of the primary flow and the formation of a secondary flow may be attributed to the inherent instability of the primary flow in the presence of Hall current. Our stability analysis lends support to this finding. The investigation of thermal instability is motivated by its direct relevance to soil sciences, groundwater hydrology, geophysical, astrophysical and biometrics. The main conclusions from the analysis of this paper are as follows:

- The system is found to be stable for $(C_p\beta/g) < 1$.

- The magnetic field, finite Larmor radius and Hall currents introduce oscillatory modes in the system for $(C_p\beta/g) > 1$.
- It is observed that the system is stable for $\frac{1}{G-1}\frac{C_p\alpha\kappa}{\nu} \leq \frac{27\pi^4}{4}$ and under the condition $\frac{1}{G-1}\frac{C_p\alpha\kappa}{\nu} > \frac{27\pi^4}{4}$, the system becomes unstable.
- FLR may have a stabilizing or destabilizing effect, but a completely stabilizing one for a certain wave-number range $\cos\theta < \left\{\frac{2(1+x)}{3x}\right\}^{1/2}$.
- In the absence of Hall currents, FLR always has a stabilizing effect.
- In the absence of FLR, the Hall currents has a destabilizing effect.
- In the presence of FLR and Hall effects, the Hall currents may have both destabilizing as well as stabilizing effects and there is competition between the destabilizing role of Hall currents and stabilizing role of FLR but completely stabilizes the above wave-number range if $\nu_0/\nu > 2[cH/(4\pi N e\eta)]$.

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