On Instability of a Dusty Stellar Atmosphere in Stern's Type Configuration

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Abstract: - The thermal-convective instability of a stellar atmosphere in the presence of a stable solute gradient in Stern's type configuration is studied in the presence of suspended particles. The criteria for monotonic instability are derived which are found to hold well in the presence of uniform rotation and uniform magnetic field, separately, on the thermosolutal-convective instability of a stellar atmosphere in the presence of suspended particles.

Key-Words: - Convection, suspended particles, solute gradient, uniform rotation, uniform magnetic field

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1 Introduction

Defouw, [1], has termed 'thermal-convective instability' as the instability in which motions are driven by buoyancy forces of a thermally unstable atmosphere. He has generalized the Schwarzschild criterion for convection to include departures from adiabatic motion and has shown that a thermally unstable atmosphere is also convectively unstable, irrespective of the atmospheric temperature gradient.
[1], has shown that an inviscid stellar atmosphere is unstable if

$$D = \frac{1}{C_P}\left(L_T - \rho\alpha L_\rho\right) + \kappa k^2 < 0, \qquad (1)$$

where L is the energy-lost function (the energy lost minus the energy gained per gram per second) and ρ, α, κ, k, c_P, L_T, L_ρ denote, respectively, the density, the coefficient of thermal expansion, the coefficient of thermometric conductivity, the wave number of the perturbation, the specific heat at constant pressure, the partial derivative of L with respect to temperature T and the partial derivative of L concerning density ρ; both evaluated in the equilibrium state. In general, the instability due to inequality (1) may be either oscillatory or monotonic. [1], has also shown that inequality (1) is a sufficient condition for monotonic instability in the presence of a magnetic field and rotation on thermal convective instability.

A detailed account of thermal convection, under varying assumptions of hydrodynamics and hydromagnetics, has been given by [2]. [3], has considered the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. In the stellar case, the physics is quite similar to [3], thermohaline configuration in that helium acts like salt in raising the density and in diffusing more slowly than heat. The thermohaline convection in a horizontal layer of viscous fluid heated from below and salted from above has been studied by [4]. In the thermohaline-convective instability problem, buoyancy forces can arise not only from density differences due to temperature variations but also from those due to variations in solute concentrations. The conditions under which convective motions are important in stellar atmospheres are usually far removed from the consideration of a single component fluid and rigid boundaries and, therefore, it is desirable to consider one gas component acted on by solute concentration gradient and free boundaries. Keeping such situations in mind, [5], have considered the thermosolutal-convective instability in a stellar atmosphere and have also studied the effects of uniform rotation and variable/uniform magnetic field on the instability. The criteria for monotonic instability are derived and are found to hold well also in the presence of the above effects. The onset of double-diffusive reaction-convection in a fluid layer with viscous fluid, heated and salted from below subject to chemical equilibrium on the boundaries has been investigated by [6]. [7], have studied the magnetohydrodynamic Veronis' thermohaline convection. The thermal-convective instability of a stellar atmosphere in the presence of a stable solute gradient in Stern's type configuration has been studied in the presence of radiative transfer effect by [8]. [9], has studied the thermal-convective instability of a composite rotating stellar atmosphere in the presence of a variable horizontal magnetic field to include, separately, the effects of medium permeability and solute gradient.

In geophysical situations, more often than not, the fluid is not pure but may instead be permeated with suspended (or dust) particles. The suspended particles are present in the stellar atmospheres and many astro-physical situations. Recent spacecraft observations have confirmed that dust particles play an important role in the dynamics of the Martian atmosphere as well as in the diurnal and surface variations in the temperature of the Martian weather, [10]. [11], have considered the effect of suspended particles on the onset of Benard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. The effect of suspended particles was found to destabilize the layer whereas the effect of a magnetic field was stabilizing. [12], have studied the stability of the shear flow of stratified fluids with fine dust and found the effect of fine dust to increase the region of instability. [13], have studied the onset of double-diffusive convection in a horizontal Brinkman cavity and analysis made on the linear stability of the quiescent state within a horizontal porous cavity subject to vertical gradients of temperature and solute. [14], has investigated the boundary roughness of a mounted obstacle that is inserted into an incompressible, external, and viscous flow field of a Newtonian fluid. [15], have investigated the instability of the plane interface between two viscoelastic superposed conducting fluids in the presence of suspended particles and variable horizontal magnetic fields through a porous medium. Coupled parallel flow of fluid with pressure-dependent viscosity through an inclined channel underlain by a porous layer of variable permeability and variable thickness has been studied, [16]. It is, therefore, of interest to study the presence of suspended particles in astrophysical situations.

It is, therefore, the motivation of the present study to re-examine the thermosolutal-convective instability in Stern's type configuration of a stellar atmosphere (the thermal-convective instability in the presence of stable solute gradient) in the presence of suspended particles and to seek the modification, if any, in the criteria for instability. The Coriolis forces and magnetic field play important roles in astrophysical situations. The effects of rotation and magnetic field (separately) are, therefore, also studied on the thermal-convective instability of a stellar atmosphere in the presence of a stable solute gradient in Stern's type configuration in the presence of suspended particles. These aspects form the subject matter of the present paper.

2 Descriptions of the Instability and Perturbation Equations

Consider an infinite horizontal fluid layer of thickness d heated from above and subjected to a stable solute concentration gradient so that the temperatures and solute concentrations at the bottom surface $z = 0$are T_0 and C_0 and at the upper surface $z = d$ are T_1 and C_1 respectively, the z -axis being taken as vertical. This layer is acted on by a gravity force $\vec{g}(0,\ 0,\ -g)$. Let $\rho, p, T, C, \vec{v}(u, v, w), \vec{u}(l, r, s), g, \alpha, \alpha', \nu, \kappa, and\ \kappa'$de note, respectively, the density, pressure, temperature, solute concentration, fluid velocity, particle velocity, gravitational acceleration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, kinematic viscosity, thermal diffusivity, and solute diffusivity. The suffix zero refers to values at the reference level $z = 0$. Let $N(\bar{x},\ t)$ stands for the number density of the particles $K_1 = 6\pi\rho\nu\varepsilon$, where ε is the particle radius, denotes the Stokes' drag, $\bar{x} =\ (x, y, z)$ and $\vec{\lambda} = (0,0,1)$. Then, following the Boussinesq approximation, which states that the inertial effects produced by density variation are negligible in comparison to its gravitational effects i.e. ρ can be taken as constant everywhere in the equations of motion except in the term with external force, the equations expressing the conservation of momentum, mass, solute mass concentration, and equation of state are

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\,.\,\nabla)\vec{v} = -\frac{1}{\rho_0}\nabla p + \nu\,\nabla^2\vec{v} + \frac{K_1 N_0}{\rho_0}(\vec{v} - \vec{u}) + \vec{g}\left(1 + \frac{\delta\rho}{\rho_0}\right), \quad (2)$$

$$\nabla . \vec{v} = 0, \quad (3)$$

$$\frac{\partial C}{\partial t} + (\vec{v}\,.\,\nabla)C = \kappa'\nabla^2 C\,, \quad (4)$$

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(C - C_0)]. \quad (5)$$

In the equations of motion for the gas, the presence of particles adds an extra force term, proportional to the velocity difference between particles and gas. Since the force exerted by the gas on the particles is equal and opposite to that exerted by the particles on the gas, there must be an extra force term equal in magnitude but opposite in sign in the equations of motion for the particles. Interparticle reactions are not considered for we assume that the distances between particles are quite large compared with their diameter. Because of the small size and large distances between particles, the effects of gravity, pressure, etc., are negligible. If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions, are

$$mN\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u}\right] = K_1 N(\vec{v} - \vec{u}), \quad (6)$$

$$\frac{\partial N}{\partial t} + \nabla.(N\vec{u}) = 0. \quad (7)$$

Let $C_v, C_{pt}, T,$ and K denote, respectively, the gas-specific heat at constant volume, particles-specific heat, the temperature, and the ‘effective’ thermal conductivity, which is the conductivity of the pure gas. Since the volume fraction of the particles is assumed extremely small, the effective properties of the suspension are taken to be those of the clean gas. If we assume that the particles and the gas are in thermal equilibrium, the first law of thermodynamics may be written in the form

$$C_v\left(\frac{\partial}{\partial t} + \vec{v}.\nabla\right)T + \frac{mN}{\rho}C_{pt}\left(\frac{\partial}{\partial t} + \vec{u}.\nabla\right)T = -L + \frac{K}{\rho}\nabla^2 T + \frac{p}{\rho^2}\frac{d\rho}{dt}\,. \quad (8)$$

The steady-state solution is

$$\vec{v} = 0, \vec{u} = 0, N = N_0(constant), T = T_0 + \beta z,$$
$$C = C_0 - \beta' z, \rho = \rho_0[1 - \alpha\beta z - \alpha'\beta' z]\,, \quad (9)$$

where

$$\beta = \frac{T_1 - T_0}{d}, and\ \beta' = \frac{C_0 - C_1}{d}$$

are the magnitudes of uniform temperature and concentration gradients.

We now consider a small perturbation on the steady state solution and let $\delta\rho, \delta p, \theta, \gamma, \vec{v}, \vec{u}$ and N denote, respectively, the perturbations in densityρ, pressure$p$, temperature$T$, solute concentration C, the velocity of the gas, the velocity of particles, and the number density of the particles N_0.

Then equations (2) - (4) and (6)-(7) on linearization give

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p + \nu\nabla^2\vec{v} - \vec{g}(\alpha\theta - \alpha'\gamma) + \frac{K_1 N_0}{\rho_0}(\vec{u} - \vec{v}), \quad (10)$$

$$\nabla\,.\,\vec{v} = 0, \quad (11)$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\vec{u} = \vec{v}\,, \quad (12)$$

$$\frac{\partial M}{\partial t} + \nabla.\vec{u} = 0, \quad (13)$$

$$\frac{\partial \gamma}{\partial t} - \kappa'\nabla^2\gamma = \beta' w, \quad (14)$$

where $\tau = m/K_1$ and $M = N/N_0$. In writing equation (10), use has been made of the equation of state (5) wherefrom the change in density $\delta\rho$, caused by the perturbations θ and γ in temperature and concentration is given by

$$\delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma). \quad (15)$$

Following, [1], the linearized perturbation form of equation (8) is

$$\left(1+\tau\frac{\partial}{\partial t}\right)\left[B\frac{\partial}{\partial t}+\frac{1}{C_p}\left(L_T-\rho\alpha L_\rho\right)-\kappa\nabla^2\right]\theta = -\left[\beta\left(B+\tau\frac{\partial}{\partial t}\right) + \frac{g}{C_p}\left(1+\tau\frac{\partial}{\partial t}\right)\right]w, \qquad (16)$$

where $B=1+b, b=mN_0C_{pt}/\rho_0C_v$, and C_p is the gas-specific heat at constant pressure.

We now consider the case in which both boundaries are free as well as perfect conductors of both heat and solute concentration. The density changes arise principally from thermal effects. The case of two free boundaries is the most appropriate for stellar atmospheres as pointed out by [17]. The boundary conditions appropriate for the problem are

$$w={}^{\partial^2 w}\!/_{\partial z^2}=\theta=\gamma=0. \qquad (17)$$

3 The Dispersion Relation

We shall now analyze an arbitrary perturbation into a complete set of normal modes and by seeking solutions whose dependence on space and time coordinates is of the form

$$exp\left(ik_x x+ik_y y+nt\right)\sin k_z z\,, \qquad (18)$$

where n is the growth rate, $k_z={}^{s'\pi}\!/_{d}$, (s' being any integer and d is the thickness of the layer) and $k=\left(k_x^2+k_y^2+k_z^2\right)^{1/2}$ is the wave number of the perturbation.

Eliminating $\vec{u}$ and δp from equations (10) – (12), we obtain

$$\left[\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nu\nabla^2\right)+\frac{K_1N_0\tau}{\rho_0}\frac{\partial}{\partial t}\right]\nabla^2 w = g\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\alpha\theta - \alpha'\gamma), \qquad (19)$$

Now, eliminating θ,γ from equations (19), (14), and (16), and using (18), we obtain the dispersion relation

$$\tau Bn^4+(\tau D+BC'+\tau\kappa' k^2B)n^3 + \left[\nu k^2B+DC'+\tau\Gamma\left(\beta+\frac{g}{C_p}\right) + \kappa' k^2(D\tau+BC')\right]n^2 + \left[\nu k^2(D+B)+DC' + \Gamma\left(\beta B+\frac{g}{C_p}\right)+\kappa' k^2\tau\Gamma\left(\beta+\frac{g}{C_p}\right) + \Gamma'\beta' B\right]n + \left[(\nu\kappa' k^4+\Gamma'\beta')D + \kappa' k^2\Gamma\left(\beta B+\frac{g}{C_p}\right)\right]=0, \qquad (20)$$

where

$$\Gamma=g\alpha\frac{k_x^2+k_y^2}{k^2},\Gamma'=\frac{g\alpha'\left(k_x^2+k_y^2\right)}{k^2}\ and\ C' = 1+\nu k^2\tau+\frac{K_1N_0\tau}{\rho_0}. \qquad (21)$$

4 Discussion and Further Extensions

Theorem 1: A criterion that the thermosolutal-convective instability of a stellar atmosphere in the presence of suspended (or dust) particles is unstable if

$$D<0\ and\ |(\nu\kappa' k^4+\Gamma'\beta')D| > \kappa' k^2\Gamma\left(\beta\overline{1+b}+\frac{g}{C_p}\right).$$

Proof: Taking the dispersion relation (20), when

$$D<0\ and\ |(\nu\kappa' k^4+\Gamma'\beta')D| > \kappa' k^2\Gamma\left(\beta\overline{1+b}+\frac{g}{C_p}\right), \qquad (22)$$

the constant term in relation (20) is negative. Equation (20), therefore, involves one change of sign and hence contains one positive real root. The occurrence of a positive root implies monotonic instability.

We thus obtain a criterion that the thermosolutal-convective instability of a stellar atmosphere in the presence of suspended (or dust) particles is unstable if

$$D < 0 \; and \; |(\nu\kappa' k^4 + \Gamma'\beta')D| > \kappa' k^2 \Gamma\left(\beta\overline{1+b} + \frac{g}{C_p}\right), \quad (23)$$

Hence the result.

Further Extension-1: Here we consider the same problem as described above except that the system is in a state of uniform rotation. Since the volume fraction of the particles is assumed extremely small and the particles are assumed to be far apart from one another, the Coriolis force on the particles is also negligible. On the right-hand side of the equation of motion (10) of the gas, the Coriolis force term $2\rho_0(\vec{v}\times\vec{\Omega})$ is added and equations (11) – (16) remain unaltered.

Theorem 2: The criterion for monotonic instability (23) also holds well in the presence of rotation and suspended particles on thermosolutal-convective instability in Stern's type configuration in a stellar atmosphere.

Proof: Here we consider an infinite horizontal gas-particle layer of thickness $d$heated from above, solute concentrated from below and acted on by a uniform rotation $\vec{\Omega}(0,0,\Omega)$ and gravity force $\vec{g}(0,0,-g)$. The linearized perturbation equations, for the problem under consideration, then become

$$\left[\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nu\nabla^2\right)+\frac{K_1N_0\tau}{\rho_0}\frac{\partial}{\partial t}\right]u = -\frac{1}{\rho_0}\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial}{\partial x}\delta p + 2\Omega\left(1+\tau\frac{\partial}{\partial t}\right)v, \quad (24)$$

$$\left[\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nu\nabla^2\right)+\frac{K_1N_0\tau}{\rho_0}\frac{\partial}{\partial t}\right]v = -\frac{1}{\rho_0}\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial}{\partial y}\delta p - 2\Omega\left(1+\tau\frac{\partial}{\partial t}\right)u, \quad (25)$$

$$\left[\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nu\nabla^2\right)+\frac{K_1N_0\tau}{\rho_0}\frac{\partial}{\partial t}\right]w = -\frac{1}{\rho_0}\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial}{\partial z}\delta p + g\left(1+\tau\frac{\partial}{\partial t}\right)(\alpha\theta-\alpha'\gamma), \quad (26)$$

together with equations (11), (14), and (16). Eliminating δp from equations (24) – (26) and using (11), we obtain

$$\left[\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nu\nabla^2\right)+\frac{K_1N_0\tau}{\rho_0}\frac{\partial}{\partial t}\right]\nabla^2 w = -2\Omega\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial\zeta}{\partial z} + g\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\alpha\theta - \alpha'\gamma), \quad (27)$$

where

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

denotes the z-component of vorticity.

Equations (24) and (25) yield

$$\left[\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nu\nabla^2\right)+\frac{K_1N_0\tau}{\rho_0}\frac{\partial}{\partial t}\right]\zeta = 2\Omega\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial z}. \quad (28)$$

Eliminating $\theta, \zeta \; and \; \gamma$ from equations (27), (28), (14), (16) and using expression (18), we obtain the dispersion relation

$$
\begin{aligned}
&B\tau^2 n^6 + (D\tau^2 + 2BC'\tau + \kappa' k^2\tau^2 B)n^5 \\
&+ \left[B\left(C'^2 + 2\nu k^2\tau + \frac{4\Omega^2 k_z^2\tau^2}{k^2}\right) + 2C'D\tau\right. \\
&+ \Gamma\left(\beta + \frac{g}{C_p}\right)\tau^2 + \Gamma'\beta'\tau^2 \\
&\left. + \kappa' k^2(D\tau^2 + 2BC'\tau)\right] n^4 \\
&+ \left[B\left(2\nu k^2 C' + \frac{8\Omega^2 k_z^2}{k^2}\right)\right. \\
&+ D\left(C'^2 + 2\nu k^2\tau + \frac{4\Omega^2 k_z^2\tau^2}{k^2}\right) + \Gamma\left(\beta + \frac{g}{C_p}\right)C'\tau \\
&+ \Gamma\left(\beta B + \frac{g}{C_p}\right)\tau + \Gamma'\beta'(B + BC' + D\tau)\tau \\
&+ \kappa' k^2\left\{B\left(C'^2 + 2\nu k^2\tau + \frac{4\Omega^2 k_z^2\tau^2}{k^2}\right) + 2DC'\tau\right. \\
&\left.\left. + \Gamma\left(\beta + \frac{g}{C_p}\right)\tau^2\right\}\right] n^3 \\
&+ \left[B\left(\nu^2 k^4 + \frac{4\Omega^2 k_z^2}{k^2}\right) + D\left(2\nu k^2 C' + \frac{8\Omega^2 k_z^2\tau}{k^2}\right)\right. \\
&+ \Gamma\left(\nu k^2\tau\overline{\beta + \frac{g}{C_p}} + C'\overline{\beta B + \frac{g}{C_p}}\right) \\
&+ \kappa' k^2\left\{B\left(2\nu k^2 C' + \frac{8\Omega^2 k_z^2\tau}{k^2}\right)\right. \\
&+ D\left(C'^2 + 2\nu k^2 + \frac{4\Omega^2 k_z^2\tau^2}{k^2}\right) \\
&\left. + \Gamma\left(\overline{\beta B + \frac{g}{C_p}} + C'\overline{\beta + \frac{g}{C_p}}\right)\tau\right\} \\
&\left. + \Gamma'\beta'(D\tau + DC'\tau + BC' + \nu k^2\tau B)\right] n^2 \\
&+ \left[D\left(\nu^2 k^4 + \frac{4\Omega^2 k_z^2}{k^2}\right) + \nu k^2\Gamma\left(\beta B + \frac{g}{C_p}\right)\right. \\
&+ \kappa' k^2\left\{B\left(\nu^2 k^4 + \frac{4\Omega^2 k_z^2}{k^2}\right)\right. \\
&+ 2D\left(\nu k^2 C' + \frac{4\Omega^2 k_z^2\tau}{k^2}\right) \\
&\left. + \Gamma\left(C'\overline{\beta B + \frac{g}{C_p}} + \nu k^2\tau\right)\right\} \\
&\left. + \Gamma'\beta'(DC' + \nu k^2\tau D + \nu k^2 B)\right] n \\
&+ \nu k^2\left[(\nu\kappa' k^4 + \Gamma'\beta')D + \kappa' k^2\Gamma\left(\beta B + \frac{g}{C_p}\right)\right. \\
&\left. + \frac{4\Omega^2 k_z^2}{k^2}\kappa' k^2 D\right] \\
&= 0. \qquad (29)
\end{aligned}
$$

When (23) is satisfied, the constant term in equation (29) is negative. The product of the roots must be negative. Therefore at least one root of equation (29) is positive and one root is negative. The occurrence of a positive root implies monotonic instability. The criterion for monotonic instability (23) thus holds well in the presence of rotation and suspended particles on thermosolutal-convective instability in a stellar atmosphere.

Further Extension-2: Further we consider an infinite horizontal viscous and finitely conducting gas-particle layer subjected to a stable solute concentration gradient and acted on by a uniform vertical magnetic field $\vec{H}(0,0,H)$ and gravity force $\vec{g}(0,0,-g)$. This layer is heated from above such that a steady temperature gradient $\beta\left(= {dT}/{dz}\right)$ is maintained.

Now we prove the following Theorem:

Theorem 3: The criterion for monotonic instability (23) derived for thermosolutal-convective instability of a stellar atmosphere in the presence of suspended particles also holds good in the presence of uniform magnetic field and suspended particles on thermosolutal-convective instability in Stern's type configuration in a stellar atmosphere.

Proof: Let $\vec{h}(h_x, h_y, h_z)$ denote the perturbation in a magnetic field $\vec{H}$. Then the linearized perturbation equations appropriate to the problem are

$$
\begin{aligned}
\frac{\partial\vec{v}}{\partial t} = &-\frac{1}{\rho_0}\nabla\delta p + \vec{g}(\alpha\theta - \alpha'\gamma) + \nu\nabla^2\vec{v} \\
&+ \frac{K_1 N_0}{\rho_0}(\vec{u} - \vec{v}) + \frac{1}{4\pi\rho_0}(\nabla\times\vec{h}) \\
&\times\vec{H}, \qquad (30)
\end{aligned}
$$

$$\frac{\partial\vec{h}}{\partial t} = \nabla\times(\vec{v}\times\vec{H}) + \eta\nabla^2\vec{h}, \qquad (31)$$

$$\nabla.\vec{h} = 0, \qquad (32)$$

where $\eta = 1/4\pi\sigma$ (σ being electrical conductivity) is the electrical resistivity. Equations (11), (12), (14), and (16) remain unchanged.

From equations (11), (12), and (30)-(32), we obtain

$$\left[\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nu\nabla^2\right)+\frac{K_1N_0\tau}{\rho_0}\frac{\partial}{\partial t}\right](\nabla^2 w) = g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\alpha\theta-\alpha'\gamma) + \frac{H}{4\pi\rho_0}\left(1 + \tau\frac{\partial}{\partial t}\right)\frac{\partial}{\partial z}\nabla^2 h_z, \quad (33)$$

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z = H\frac{\partial w}{\partial z}. \quad (34)$$

Eliminating θ, γ and h_z from equations (14), (16), (33), (34) and using expression (18), we obtain the dispersion relation

$$A_5n^5+A_4n^4+A_3n^3+A_2n^2+A_1n+A_0 = 0, \quad (35)$$

where

$$A_5 = B\tau$$

$$A_4 = BC'+\tau(D+\eta k^2B)+\kappa'k^2\tau B$$

$$A_3 = \nu k^2B+\eta k^2\tau D+C'(D+\eta k^2B)+k_z^2V_A^2\tau B + \Gamma\left(\beta+\frac{g}{C_p}\right)\tau+\tau D + \kappa'k^2B(\eta k^2\tau+C')+\Gamma'\beta'B\tau$$

$$A_2 = \nu k^2(D+\eta k^2B)+\eta k^2C'D+k_z^2V_A^2(B+C'D) + \Gamma\left\{\beta B+\frac{g}{C_p}+\eta k^2\tau\left(\beta+\frac{g}{C_p}\right)\right\} + \kappa'k^2\left\{\eta k^2\tau D+\nu k^2B+C'D + \Gamma\left(\beta+\frac{g}{C_p}\right)+\eta k^2BC'\right\} + \Gamma'\beta'\{B(1+\eta k^2\tau)+D\tau\}$$

$$A_1 = \nu\eta k^4D+k_z^2V_A^2D+\eta k^2\Gamma\left(\beta B+\frac{g}{C_p}\right) + \Gamma'\beta'(\eta k^2\tau D+D+\eta k^2B) + \kappa'k^2\left\{\eta k^2DC'+\nu k^2(D+\eta k^2B) + k_z^2V_A^2(B+D\tau) + \Gamma\left(\beta B+\frac{g}{C_p}+\eta k^2\tau\overline{\beta+\frac{g}{C_p}}\right)\right\}$$

$$A_0 = k_z^2V_A^2\kappa'k^2D + \eta k^2\left\{(\nu\kappa'k^4+\Gamma'\beta')D + \kappa'k^2\Gamma\left(\beta B+\frac{g}{C_p}\right)\right\}, \quad (36)$$

where

$V_A^2 = H^2/4\pi\rho_0$ denotes the square of the Alfv'en velocity.

If the criteria (23) are satisfied i.e. when

$$D<0 \; and \; |(\nu\kappa'k^4+\Gamma'\beta')D| > \kappa'k^2\Gamma\left(\beta\overline{1+b}+\frac{g}{C_p}\right), \quad (37)$$

the constant term in equation (36) is negative. Equation (36), therefore, involves one change of sign and hence contains one positive real root. The occurrence of a positive root implies monotonic instability.

The criteria for monotonic instability derived for thermosolutal-convective instability of a stellar atmosphere in the presence of suspended particles are, thus, also found to hold good in the presence, separately, of uniform rotation and uniform magnetic field on the problem under consideration.

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