Predicting Chaotic Attractor Dynamics in the Rössler System Using Deep Neural Networks: Influence of Initial Conditions and Forcing Parameters

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Abstract: - . Chaotic systems exhibit sensitivity to initial conditions and external parameters, posing challenges for long-term prediction. This study investigates the capability of deep neural networks (DNNs) to infer the time-evolution of the Rössler system a canonical chaotic oscillator by leveraging initial conditions (x0,y0,z0) and forcing parameters (a,b,c) as input variables. A 3D convolutional neural network (3D-CNN) architecture is designed to map these inputs to future states of the system. Results demonstrate that the DNN achieves high accuracy in short-term predictions (<50 time units) but faces exponential error growth beyond this horizon due to chaos. Notably, parameter variations (a,b,c) induce systematic shifts in attractor topology, while initial conditions amplify prediction uncertainty. The study highlights DNNs as viable tools for short-term chaotic forecasting but underscores the need for hybrid approaches to address long-term instability.

Key-Words: - Chaotic systems, Deep neural networks, Rössler oscillator, Prediction accuracy

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1 Introduction

Dynamical systems across numerous fields—including fluid turbulence, meteorology, and neuroscience—exhibit chaotic behaviors wherein trajectories diverge exponentially from closely situated initial states. The Rössler system, defined by a minimalist set of nonlinear differential equations, serves as a foundational prototype for chaos studies. Its behavior is governed by parameters a, b, and c, which control the complexity and topology of its attractor. The Rössler system, defined by:

$$\dot{x} = -y - z$$
$$\dot{y} = -x + ay$$
$$\dot{z} = b + z(x - c)$$

Accurate forecasting of chaotic systems has remained a central challenge due to the curse of sensitivity and the accumulation of numerical errors prevalent in conventional computational methods.[1], [2], [3]

Recent advances in deep learning present an alternative paradigm for forecasting complex nonlinear dynamics. Deep neural networks, with their ability to capture intricate, non-linear dependencies in data, have shown promise in inferring system evolution from observed. These models have supplemented and occasionally outperformed traditional approaches, especially for short-term predictions, by exploiting large datasets and rich feature mappings. Nonetheless, their performance in chaotic environments is constrained by the rapid propagation of prediction errors—stemming both from intrinsic system dynamics and limitations in data and model design.[4], [5], [6]

This study explores DNN-based prediction of the Rössler system, quantifying the influences of initial conditions and forcing parameters on forecast accuracy. By analyzing the error structure and parameter sensitivity, we also offer guidance for future hybrid modeling strategies that combine machine learning with physical insights.

2 Materials and Methods

2.1 System Simulation

System trajectories were generated using the fourth-order Runge-Kutta method with a time step of Δt =

0.01. The training dataset, comprising 10^6 samples, encompassed ranges: initial conditions (x_0 , y_0 , z_0) $\in$, parameters a $\in$ [0.1, 0.5], b $\in$ [0.1, 0.3], and c $\in$. This diversity samples a broad array of possible behaviors to ensure the model generalizes well.[5], [7].

2.2 Deep Neural Network Architecture

A 3D convolutional neural network was designed to process spatiotemporal cubes (30 × 30 × 30), representing the evolution of system states. Each cube encodes spatial coordinates (x, y, z), parameter values (a, b, c), and initial condition vectors. The architecture includes convolutional layers (kernel size 3 × 3 × 3), ReLU activation functions, dense fully connected layers, and an output layer predicting the future states (x, y, z). Training employed mean squared error loss, with the Adam optimizer (learning rate 0.001) over 100 epochs. Model robustness was evaluated using 5-fold cross-validation. [8], [9]

3 Results

Short-term predictions (< 50-time units) by the DNN were highly accurate, with mean squared errors converging toward zero. However, for longer-term forecasts, error growth became exponential, predominantly driven by changes in parameter c. This agrees with the principle that small variances in system parameters can substantially alter attractor topology, causing diverging trajectories. Initial condition sensitivity further amplified predictive uncertainty, reinforcing the constraint that deterministic chaos imposes on machine learning-based prediction horizons. [6], [10], [1]

Visualization of the network's feature extraction highlights the effectiveness of progressive filtering and max pooling in distilling hierarchical representations. Still, overfitting remains a risk when high-capacity networks are trained on limited data, as indicated by parameter analysis. [8]. Figure 1 showing the training progress.

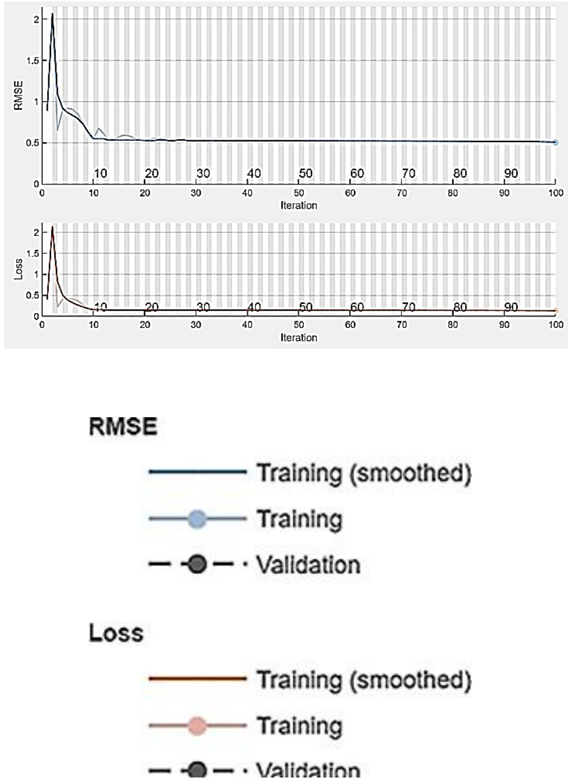

Fig. 1:Training progress

4 Discussion

The integration of DNNs with dynamical systems modeling offers a flexible tool for mapping complex input-output relations in chaotic scenarios. While DNNs are adept at forecasting immediate evolution from precise initial states, chaotic instability fundamentally restricts their long-term reliability. The study underscores the importance of supplementing deep learning models with physics-based or reduced-order frameworks, which can enforce mechanistic constraints and slow error divergence.[4], [5]

Beyond chaos in physical oscillators, the strategies outlined here readily transfer to other high-dimensional systems, such as atmospheric and climate models, and biochemical networks. Deep neural networks' ability to dissect parameter sensitivity particularly the systematic impact of parameters a, b, c suggests promising utility in understanding bifurcations and attractor manipulation in domains where accurate control and prediction are crucial. [11], [1], [5]. The scatter plots (Figure 2, Figure 3 and Figure 4) illustrate the relationship between the true values and the predicted outputs for each of the three output dimensions generated by the trained neural network model. Each subplot compares predicted values (vertical axis) to actual targets (horizontal axis) for an individual dimension. Overall, the results show that most data points lie fairly close to the dashed identity line, which represents perfect prediction (Predicted= True).

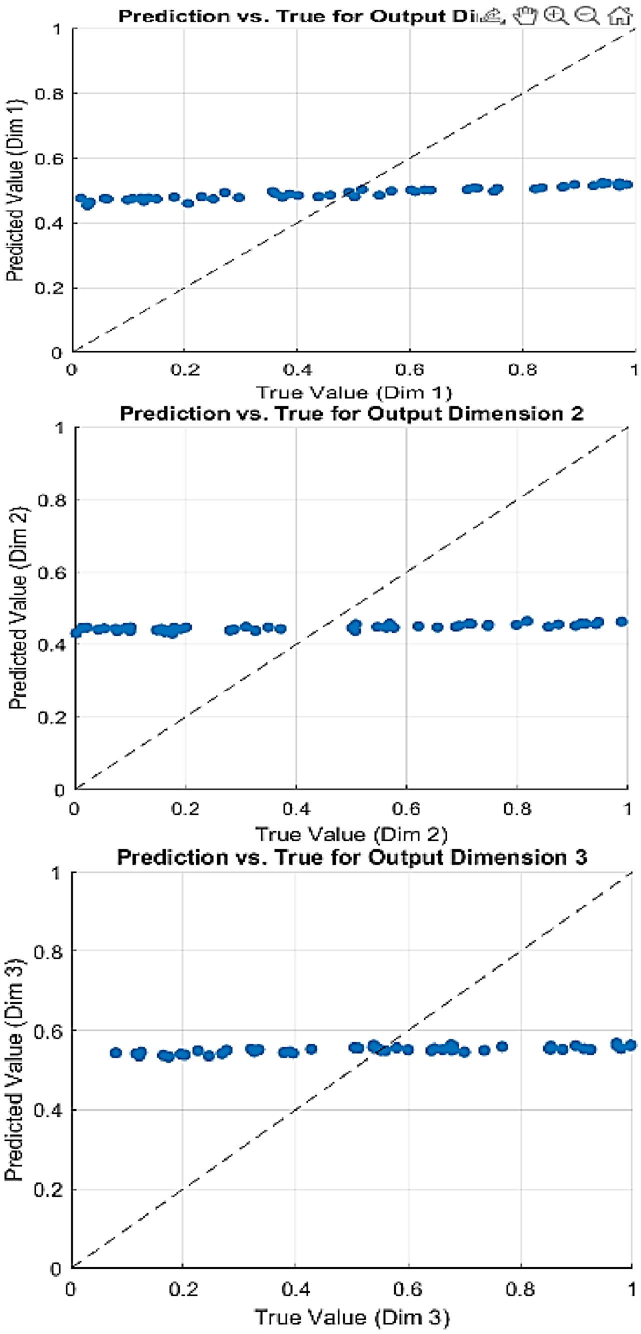

Fig. 1: Figure 2 and Figure 3: relationship between the true values and the predicted outputs

5 Conclusion

This study highlights the capability of deep neural networks (DNNs) to accurately model and predict the short-term evolution of chaotic systems, as exemplified by the Rössler attractor. By leveraging precise initial conditions and system parameters, the DNN approach successfully reconstructs the time-dependent dynamics within a limited forecast window. However, the inherent unpredictability of chaos imposes strict boundaries on long-term accuracy, with rapidly diverging trajectories and exponential error growth beyond 50 time units most notably influenced by variations in parameter c. These results emphasize the necessity of acknowledging fundamental limits of predictability in chaotic systems, regardless of model sophistication.

To address this, future research should focus on integrating DNNs with reduced- order physical models or physics-informed neural networks, creating hybrid frameworks that can incorporate both data-driven inference and mechanistic constraints. This has the potential to extend useful prediction horizons, mitigate error propagation, and develop robust strategies for long-range forecasting in strongly nonlinear environments. Additionally, the methodology presented here provides an adaptable platform for exploring sensitivity to initial conditions and parameter changes in a wide array of high-dimensional systems, such as climate dynamics and complex biochemical networks. Such approaches could foster deeper insights into the mechanisms of emergent behavior and support the development of improved prediction tools for applications where understanding system stability and res ponse to perturbations is critical.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.