Robust Optimization Model for Inventory Control Problems Production Raw Materials With Storage Costs and Orders Under Uncertainty

HENDRA CIPTA[1], RINA WIDYASARI[2]
[1,2]Department of Mathmatics, Universitas Islam Negeri Sumatera Utara,
Jln. Lapangan Golf, Desa Durian Jangak, Kecamatan Pancur Batu 20353
INDONESIA

Abstract: - Inventory is a major part of a company's operations which can have a negative impact if it was not managed properly. Inventory control must be planned and controlled effectively and efficiently. The main problem in inventory control usually occurs because there are parameters that contain data uncertainty, such as uncertain product demand. Because of this element of uncertainty, it definitely influenced decision makers, according to many studies have discussed how to control uncertainty or minimize its impact. Therefore, This research presented a robust optimization model that can overcome the element of uncertainty in controlling the inventory of production raw materials, namely the uncertainty in the amount of demand for production raw materials where ordering capacity can only be ordered at the maximum value to obtain an optimal solution. Completion of the model was done by changing the mixed integer programming (MIP) problem into a linear programming (LP) problem. Data simulation was carried out with the help of MATLAB to obtain optimal results from the model created. This research also provides indications for decision makers in a company to model and control the company's production raw materials if demand becomes uncertain. Completion of the model was done by changing the mixed integer programming (MIP) problem into a linear programming (LP) problem. Data simulation was carried out with the help of MATLAB to obtain optimal results from the model created. This research also provides indications for decision makers in a company to model and control the company's production raw materials if demand becomes uncertain. Completion of the model was done by changing the mixed integer programming (MIP) problem into a linear programming (LP) problem. Data simulation was carried out with the help of MATLAB to obtain optimal results from the model created. This research also provides indications for decision makers in a company to model and control the company's production raw materials if demand becomes uncertain.

Key-Words: Raw materials, Mixed Integer Programming (MIP), Linear Programming (LP), Inventory Control, Data Uncertainty, Robust optimization

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1 Introduction

Progress in the industrial sector is currently developing rapidly. Industry is increasingly advanced so that the problems that arise became increasingly complex [1]. Every developing industry needs experts to help and develop the industry in solving problems that arise [2]. There are many methods in the industrial sector that are used to improve and develop industry. In various ways, it is hoped that it was able to make the industry and business it runs grow and continue to run and generate profits. Every operation in the goods industry will ideally produce sufficient products but requires optimal profits with minimal costs, so that operations will continue to run [3]. In the goods industry, one of the things you need to pay attention to is the product, starting from the amount of production, quality and quantity. One way to optimize profits is to determine optimal production planning [4]-[5]. Companies can prepare production estimates that was carried out precisely and optimally with minimal costs. Production planning is one of the determining factors for a company's success [6]-[7].

Production planning is a process of determining the number of products to be produced and considerations obtained from previous data to match market demand [8]-[9]. Fulfilling market demand must also be followed by optimal use of available resources. Condition of resources such as raw

materials, labor, machines and other equipment needed for the production process. In preparing production planning, many things are considered in relation to optimizing production planning so that the lowest cost level is achieved in carrying out the production process. Optimizing the production planning process allows more than one goal to be achieved. However, the production planning process can sometimes experience obstacles, for example due to uncertainty in the goods planning process, running out of stock of production raw materials, uncertainty in production times and others.

Many unpredictable uncertainties occur in real life, and these risks can make decisions more difficult. Uncertainty arises as a result of a certain event or a decision. The outcome cannot be known if a decision has more than one effect [10]-[11]. For companies, uncertainty can affect the company's operational cycle or the impact can even cause bankruptcy. Therefore, uncertainty can cause decision makers to be unable to carry out long-term planning and carry out the investment process, resulting in lost opportunities for the company's development progress. This uncertainty also often results in a lack of optimal solutions to optimization problems. However, decision makers usually ignore the uncertainty of the data and base their decisions more on predetermined scenarios.

As a result, systems based on deterministic data may not be able to effectively address uncertain scenarios. Uncertainty is a problem that cannot be avoided because uncertainty is a complex problem. Many studies have proposed methods to control the impact of uncertainty. One area of optimization that is able to solve problems related to uncertainty in controlling production raw material inventories is robust optimization. One field of optimization to address issues with uncertainty is robust optimization. This addresses the uncertainty inherent in the parameters in deterministic optimization problems. With robust optimization, With robust optimization, the robust solutions obtained can help decision making in avoiding losses from data uncertainty [12].

In order to solve the issue of data uncertainty, robust optimization ensures that the best possible solution is both feasible and optimal in the worst-case scenario for a parameter. In inventory control problems, there is a random variable in the demand level formulation where the demand changes every time period. Robust optimization also assumes that uncertain data has a convex set and a limit set called the uncertainty set and aims to find a solution that remains feasible for all possible data parameters until it reaches the worst case. Specifically, robust optimization plays an important role in things that are easy to do tractable and provide an overview of the optimal solution [13]-[14].

To solve this problem, Soyster [15] first proposed the concept of robust optimization based on scenario analysis and planning objectives in optimizing systems, variables and penalty costs. Most robust optimization studies only discuss single uncertainty optimization. However, in real conditions, more than one uncertainty can influence real-life decision making, and these factors mutually influence each other, giving rise to very complex consequences for decision makers. For example, the coronavirus has caused major disruption to the decline in global raw material production and transportation systems, requiring consideration of various uncertainties.

Various studies related to production planning and distribution of production results have been carried out. Octavia [16] uses the Mixed Integer Linear Programming (MILP) model which integrates production, distribution and marketing involving materials and sales results. Che, Z. H [17], considers production and transportation reliability and develops a multi-objective robust optimization model with two uncertainties. Additionally, a nondominated sorting genetic algorithm is proposed to solve the proposed multi-objective robust optimization model. The relationship between robustness parameter values and budget is explored by considering the proposed robust optimization model with production and transportation uncertainties. Ehsan, et. al. [18], in his research, proposed a new multi-product and multi-period robust model in planning and pricing. Demand is considered uncertain and dependent on price. So for each price, there is a different possible range of demand. Unfulfilled requests are considered lost, therefore, backlogging is not allowed. The goal is to maximize profits over the planning period, which consists of a limited number of periods. Hector [19], In his work, retailers are exposed to the unpredictability of stationary consumer demand at each time period in a single product, multi-period centralized system. Every unmet demand scenario is examined from two perspectives: lost sales and backlog. Three methods are utilized to predict and optimize the need for daily inventory replenishment from direct suppliers throughout the network: (1) deterministic linear programming, (2) multi-stage stochastic linear programming, and (3) reinforcement learning.

Heibatolah [20], In his research, he created an inventory model that takes into account distinct demand, coordinated prices, and various delivery

policies in the production inventory system of a single supplier for a single buyer. There can be no shortages, and there is no limit to the planned horizon. This study framework's primary objective is to provide decision-makers with information on orders, prices, and delivery quantities in order to maximize system profits overall. Lijun [7], the study focuses on a class of robust optimization problems where the uncertainty parameters of the constraints and objectives are the same. Existing methods tackle this issue for each objective and constraint independently in the worst case, and then reformulate the model according to each multiple form constraint in the worst case. Since it is impossible to achieve the worst case concurrently, this strategy may lead to unclear parameter values in the optimal solution, which may not equal the worst case of each constraint.

From the various literature described above, this research will model robust optimization in controlling the supply of production raw materials in the presence of sources of uncertainty. The main problem in this research is controlling production raw materials in the form of a mixed integer programming (MIP) problem as the initial problem. Then, due to the uncertainty in the amount of demand for production raw materials where the ordering capacity can only be ordered with the maximum value, a robust optimization model was created that can handle the given problem.

2 Literature Review

2.1 Linear Programming

Linear Programming is used to allocate limited resources to achieve optimal results. The general form of linear programming is [21]-[22]:

$$Opt\ Z = \sum_{j=1}^{n} C_j x_j \tag{1}$$

subject to

$$\sum_{i=1}^{m} a_{ij} x_j \{\leq, =, \geq\} b_i \tag{2}$$

$$x_j \geq 0 \tag{3}$$

where

Z : objective function value

C_j : value per unit of activity

x_j : the number of activities j with $j = 1,2,3,\ldots,n$

a_{ij} : the amount of resource i consumed by the activity

b_i : number of resources i

2.2 Robust Optimization

The robust optimization approach is a method developed by Ben-Tal [12], [23] for the formulation of uncertain linear optimization:

$$\min_x \left\{ c^T x : Ax \leq d \right\}_{(c,A,d)\in U} \tag{4}$$

where $c \in \mathbb{R}^n, A \in \mathbb{R}^{m\times n}$ and $d \in \mathbb{R}^m$ an uncertain coefficient, U indicates the set of uncertainties. The basic paradigm of robust optimization is based on the following three assumptions [13],[24]-[25]:

1) All decision variables $x \in \mathbb{R}^n$ represents a here and now decision, where the decision variable must obtain a certain numerical value as a result of solving the problem before the actual data.
2) If the actual data falls inside a predefined uncertainty set U, the decision maker bears complete responsibility for the decision's outcomes.
3) The decision maker cannot accept constraints violations when the data falls inside a predefined uncertainty set U because the constraints of the uncertainty problem are hard.

2.3 Mixed Integer Programming (MIP) on Inventory Problems

According to Bertsimas and Thiele [26] and in the [27]-[28] The inventory problem is a MIP problem which is formulated:

$$\min \sum_{k=0}^{T-1} cu_k + Kv_k + y_k \tag{5}$$

subject to

$$y_k \geq h\left(x_0 + \sum_{k=0}^{t-1} \left(u_k - d_k \right) \right), k = 0,1,\ldots,t-1 \tag{6}$$

$$y_k \geq -p\left(x_0 + \sum_{k=0}^{t-1} \left(u_k - d_k \right) \right), k = 0,1,\ldots,t-1 \tag{7}$$

$$0 \leq u_k \leq Mv_k, v_k \in \{0,1\}, k = 0,1,\ldots,t-1 \tag{8}$$

where

c : the cost of purchasing goods

u_k : orders during k-period

K : costs of procuring or ordering goods

a_{ij} : the amount of resource i consumed by activities

v_k : binary variable 0 or 1

y_k : total inventory costs during period-k

h : storage costs

x_0 : stock that existed during the initial period

u_i : stock ordered during period-i
d_i : product demand during period-i
p : cost of shortage
m : a very large positive number

Robust problems are linear programming problems if there are no fixed costs $(K=0)$ and the mixed-integer programming (MIP) problem if fixed costs are given by $(K>0)$. The expansion of the robust model for ordering capacity can only be ordered with the maximum value, so additional constraints are given, namely:

$$u_k \le d, \forall k \tag{9}$$

The extension of the robust model assumes that inventory can only be stored with a maximum value of *G*, so additional constraints are given:

$$x_0 + \sum_{i=0}^{k}\left(u_i - d_i\right) \tag{10}$$

$$d_i = \bar{d}_i + d_k.z_i \tag{11}$$

3 Methodology

The data used in this research is secondary data from the production inventory of building materials at CV. Sumatra Jaya. This secondary data will be used in the simulation model created. Secondary data simulations will be discussed in the results section.

The procedural steps in this research will be described as follows:

1) Calculate the mean value $(\bar{x})$ of the number of production requests d_k during the *k*-th period
2) Calculate the deviation scale s_k
3) Determine whether the deviation scale s_k meets the constraints, if it does not, an optimal deviation scale s_k^* will be sought.
4) Calculate the average value $\bar{d}_k$ of the number of production requests during the *k*-th period.
5) Calculating variables p_k, q_k, v_k, w_k, r_k from the constraint function

4 Result and Discussion

4.1 Initial Inventory Demand Model

According to Bartsekas [19],[29], Since inventory ordered at the start of the kth period is shipped in the *k*-th period as well, before the period $k+1$ begins, all orders have a wait time of zero [30]. Demand in period x_{k+1} written with,

$$x_{k+1} = x_k + u_k - d_k, \quad k = 0,1,...,t-1 \tag{12}$$

where

x_k : inventory on hand when the time period started *k*-th
u_k : order during period-*k*
d_k : demand during the *k*-th period

Equation (11) can be rewritten as,

$$x_{k+1} = x_0 + \sum_{k=0}^{T-1}\left(u_k - d_k\right),\ k = 0,1,...,t-1 \tag{13}$$

The problem in this research is the uncertainty in the amount of demand for production raw materials d_k by defining a deviation scale from d_k namely,

$$s_k = \frac{d_k - \bar{d}_k}{d_k} \tag{14}$$

where

s_k : deviation scale
$\bar{d}_k$: average value of d_k
d_k : maximum deviation

which has value $[-1,1]$. In this case there is uncertainty in demand in each period, so it is necessary to add a constraint,

$$\sum_{k=0}^{t-1}\left|s_k\right| \le \zeta, k = 0,1,...,t-1 \tag{15}$$

with ζ value 1. Robust can be understood as the worst case scenario since the significant degree of uncertainty outweighs the significant scale of variance in cumulative demand.

Next, will be create a model of the cost function, where the cost function that occurs in period k consists of two parts, namely (1) purchasing costs (c) and (2) storage costs and shortages resulting at the end of the period. $R\left(x_k + u_k - d_k\right)$ which is calculated at the end of the period after the stock order is ordered u_k sent and request d_k realized.
Costs for purchases are considered non-existent if no requests or orders occur. So, purchasing costs can be formed:

$$c = \begin{Bmatrix} K + cu & \text{if } u > 0 \\ 0 & \text{if } u = 0 \end{Bmatrix} \tag{16}$$

with $c > 0$ for variable costs and $K \geq 0$ for procurement costs. If $K > 0$, then the cost of procuring goods is at the time the order occurs. Both storage costs and shortages represent $R(x_k + u_k - d_k)$.

4.2 Mixed-Integer Programming (MIP) Models for Carrying and Ordering Cost Functions

With the storage cost and shortage functions, we can model ordering costs into a mixed-integer programming (MIP) problem as follows:

$$\min \sum_{k=0}^{T-1} cu_k + Kv_k + y_k \quad (17)$$

subject to

$$y_k \geq h\left(x_0 + \sum_{k=0}^{T-1} \left(u_k - d_k\right) \right), k = 0,1,...,t-1 \quad (18)$$

$$y_k \geq -p\left(x_0 + \sum_{k=0}^{T-1} \left(u_k - d_k\right) \right), k = 0,1,...,t-1 \quad (19)$$

$$0 \leq u_k \leq Mv_k, v_k \in \{0,1\}, k = 0,1,...,t-1 \quad (20)$$

where

$$d_k = \bar{d}_k + d_k . z_k \quad (21)$$

Uncertainty data only affects Equations (17)-(21) in mixed-integer programming (MIP) problems. So that the stock equation available in the period $k+1$ is:

$$x_{k+1} = \hat{x}_{k+1} - \sum_{k=0}^{T-1} \hat{d}_k z_k \quad (22)$$

With $\hat{x}_{k+1}$ is stock that is still stored. By substituting equation (2) into equation (22), obtained:

$$x_{k+1} = x_0 + \sum_{k=0}^{T-1} \left(u_k - \bar{d}_k\right) \quad (23)$$

for all k. In the worst case, there are costs with data uncertainty, so we must maximize the deviation scale for several constraints for each k storage costs or shortage costs which are expressed in the following equation function:

$$\max \sum_{k=0}^{T-1} \hat{d}_k z_k \quad (24)$$

subject to

$$\sum_{k=0}^{T-1} z_k \leq \zeta \quad (25)$$

$$0 \leq z_k \leq 1, \ \forall k \quad (26)$$

4.3 Robust Optimization Model for Inventory Control in the Existence of Uncertainty

By substituting equations (17)-(21), a robust formulation for inventory control problems is produced:

$$\min \sum_{k=0}^{T-1} cu_k + Kv_k + y_k \quad (27)$$

subject to

$$y_k \geq h\left(x_0 + \sum_{k=0}^{T-1} \left(u_k - \bar{d}_k\right) + q_k \zeta_k + \sum_{k=0}^{T-1} r_k \right), \quad (28)$$
$$k = 0,1,...,t-1$$

$$y_k \geq p\left(-x_0 - \sum_{k=0}^{T-1} \left(u_k - \bar{d}_k\right) + q_k \zeta_k + \sum_{k=0}^{T-1} r_k \right), \quad (29)$$
$$k = 0,1,...,t-1$$

$$q_k + r_k \geq \hat{d}_k, \forall k \quad (30)$$

$$q_k \geq 0, +r_k \geq 0, \forall k \quad (31)$$

$$0 \leq u_k \leq Mv_k, v_k \in \{0,1\}, k = 0,1,...,t-1 \quad (32)$$

where M is a very large positive number. Variable q_k and r_k used to measure cost sensitivity to very small changes in key parameters of a robust approach. In each period $q_k \zeta_k + \sum_{k=0}^{T-1} r_k$ represents the deviation of a worst case cumulative demand from that value and a limit to the magnitude of the uncertainty. In this case, the number of products that can be produced each month is limited and the warehouse capacity is also limited, so it is necessary to provide additional constraints for equations (18) and (19), namely:

$$u_k \leq d, \forall k \quad (33)$$

$$x_0 + \sum_{k=0}^{t-1} \left(u_k - d_k\right) \leq G \quad (34)$$

where d is the number of products that can be produced each month and G is the warehouse capacity to store product inventory. By taking into account the effect of data uncertainty in Equations (18)-(21) then equation (34) with a given worst case deviation becomes:

$$x_0 + \sum_{k=0}^{t-1} \left(u_k - \bar{d}_k\right) + u_k \zeta_k + \sum_{k-0}^{t-1} r_k \leq G \quad (35)$$

So the inventory problem model uses a robust optimization approach as follows:

$$\min \sum_{k=0}^{t-1} \left(cu_k + Kv_k + y_k \right) \tag{36}$$

subject to

$$y_k \geq h\left(x_0 + \sum_{k=0}^{t-1} \left(u_k - \bar{d}_k \right) + q_k \zeta_k + \sum_{k=0}^{t-1} r_k \right), \tag{37}$$

$$k = 0,1,...,t-1$$

$$y_k \geq p\left(-x_0 - \sum_{k=0}^{t-1} \left(u_k - \bar{d}_k \right) + q_k \zeta_k + \sum_{k=0}^{t-1} r_k \right), \tag{38}$$

$$k = 0,1,...,t-1$$

$$q_k + r_k \geq \hat{d}_k, \forall k \tag{39}$$

$$u_k \leq d, \forall k \tag{40}$$

$$x_0 + \sum_{k=0}^{t-1} \left(u_k - \bar{d}_k \right) + q_k \zeta_k + \sum_{k=0}^{t-1} r_k \leq G \tag{41}$$

$$q_k \geq 0, r_k \geq 0, \forall k \tag{42}$$

$$0 \leq u_k \leq M v_k, v_k \in \{0,1\}, k = 0,1,...,t-1 \tag{43}$$

5 Numerical Simulation

5.1 Initial Data

The initial data used is data on demand for building material production at CV. Sumatra Jaya as seen in the table 1.

Table 1. Data on demand for building materials production

Month	k	d_k
January	0	30524
February	1	23780
March	2	29400
April	3	26760
May	4	27882
June	5	23244
July	6	25500
August	7	29000
September	8	23582
October	9	31500
November	10	30800
December	11	28750

where k is the ordering period (in years) and d_k is the demand for building material products in the k-th period. Details of costs for production raw material supplies are given which can be seen in table 2.

Table 2. Data on demand for building materials production

Inventory Costs	Price/pair/year
Purchase costs (c)	20900
Procurement costs (K)	350500
Storage costs (h)	850,650
Shortfall cost (p)	629,625

5.2 Model Simulation

Based on table 1, data on demand for building material production is uncertain in terms of value $z_k \in [-1,1]$. So that the value was searched $\bar{d}$ using the equation:

$$\bar{d} = \frac{\sum_{k=0}^{11} d_k}{k+1} \tag{44}$$

So that value is obtained $\bar{d} = 27560.167$. Then the value was searched $\hat{d}_k$ which is the difference in value d_k in each period $\bar{d}$. Value results $\hat{d}_k$ seen in table 3.

Table 3. Value $\hat{d}_k$ in the k-period

k	d_k	$\hat{d}_k$	z_k
0	30524	2963.833	1
1	23780	-3780.167	0.326
2	29400	1839.833	0.129
3	26760	-800.167	0.0335
4	27882	321.833	0.386
5	23244	-4316.167	0.2665
6	25500	-2060.167	0.008956
7	29000	1439.833	0.423
8	23582	-3978.167	0.1509
9	31500	3939.833	0.1454
10	30800	3239.833	0.1865
11	28750	1189.833	0.0318

Based on table 3 above, the values obtained $\sum_{i=0}^{k} |z_k| = 3.0875$. Based on equation (15), value z_k It is not yet said to be optimal so the value was sought again z_k^* optimally using equation (24) with constraints in equations (25)-(26). After obtaining the value z_k^* optimal, then the value was searched for $\bar{d}_k$ in each kth period with $\bar{d}_k^* = d_k - \left(\hat{d}_k \times z_k \right)$.

Results z_k^* and $\bar{d}_k^*$. The optimal one was given in table 4.

Table 4. Value z_k^* And $\bar{d}_k^*$ optimal

k	z_k^*	$\bar{d}_k^*$
0	0.082999	2717.8381
1	0.082999	30837.7500
2	0.082999	29247.2956
3	0.082999	26826.4130
4	0.082999	27855.2881
5	0.082999	23602.2375
6	0.082999	25670.9917
7	0.082999	28880.4952
8	0.082999	23912.1838
9	0.082999	31172.9977
10	0.082999	30531.0970
11	0.082999	28651.2452

The Mixed-Integer Programming (MIP) model problem, namely equations (17)-((26), was converted into an inventory problem model using a robust optimization approach from the model equations (36)-(43). To get the results from the model, we will use MATLAB's help. From the results of calculations using the help of MATLAB, the total inventory cost is IDR 3,440,750,083 with each constraint for each period, namely the variable p_k, q_k, v_k, w_k, r_k as seen in the table 5.

Table 5. Calculation results of the robust optimization approach

k	p_k	q_k	v_k	w_k	r_k
0	26500	14868.667	0.0001868	1	176548609,01
1	26500	14868.667	0.0002064	1	368507796,91
2	26500	14868.667	0.0002244	1	195608766,04
3	26500	14868.667	0.0002535	1	189112657,31
4	26500	14868.667	0.00028775	1	163247323,78
5	26500	14868.667	0.0003386	1	198804577,32
6	26500	14868.667	0.0003865	1	213552948,23
7	26500	14868.667	0.0001754	1	173355205,08
8	26500	14868.667	0.0008456	1	443443914,50
9	26500	14868.667	0.0001233	1	295569984,81
10	26500	14868.667	0.0002388	1	278504480,24
11	26500	6868.36	0.002450	1	15340609,45

The results of this calculation are lower than the total inventory cost calculation carried out by CV. Sumatra Jaya, namely IDR. 3,680,685,380. The following phase is the validation stage, which comes after determining the overall cost of production raw material inventory. Placing the optimization decision result variables within the specified restrictions completes the validation stage. If the variable meets the given constraints, then the calculation results are said to be valid. Validation results using MATLAB.

5 Conclusion

From the results of the research carried out, the total cost of raw material inventory for building material production was obtained using robust optimization on CV. Sumatra Jaya is IDR. 3,440,750,083 for one year. This result is less than the calculation carried out manually by CV. Sumatra Jaya, namely IDR. Rp. 3,680,685,380 for one year. So it can be said that the robust optimization formulation here still assumes lead time = 0, meaning that not all products can be assumed to have lead time = 0. Future follow-up research is expected to analyze the lead time assumptions used and add other variables besides those mentioned in this research.

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