# Optimization of Sharpley Value Method of Cost Allocation in a Bimodal Transport- Supply Chain Distribution Via Dynamic Programming

HARRISON OBIORA AMUJI*
Department of Statistics
Federal University of Technology, Owerri
PMB 1526, Owerri Imo State
NIGERIA

BRIDGET NWANYIBUIFE OKECHUKWU
Department of Statistics
Federal University of Technology, Owerri
PMB 1526, Owerri Imo State
NIGERIA

IHEANYI CHINEDU OBINWANNE
Department of Statistics
Federal University of Technology, Owerri
PMB 1526, Owerri Imo State
NIGERIA

VIVIAN NGOZI IKEOGU
Department of Logistics and Transport Technology
Federal University of Technology, Owerri
PMB 1526, Owerri Imo State
NIGERIA

SAMUEL INAKU EMRI
Department of Urban and Regional Planning
University of Cross River State
Calabar, Cross River State
NIGERIA

Abstract: - In this paper, we proposed a coalition between two modes of transportation, where one provides cargo train and the other provides trucks of capacity 453 tons respectively for the coalition. We have five grand coalitions and four coalitions. The coalitions were distributed along five paths across Nigeria and specialized in the distribution of agricultural produce from the north to the south. Each of the coalition was made up of four transport providers and composed of four legs. Since this is a cooperative game scenario, Sharpley's value method of cost allocation was used to obtain the gains accrued to the grand coalition. The coalition made a total savings of 1259.6 million naira within the period under study. The researchers further developed and applied a Dynamic programming model to the supply chain distribution and obtained an intelligent result. They discovered that if the five coalitions were distributed among the four legs, in this other (1, 1, 2, 1), the grand coalition would make an additional gain of 377.6 million naira. The distribution means that the allocation of the coalition to the third leg should be doubled while the other legs remain as they were. The introduction of the optimization method into the system brought additional revenue to the coalition and increased the total gain to 1637.3 million naira.

Key-Words: - Bimodal mode of transportation, Sharpley value method, Transport and logistics, Dynamic programming, Optimal decision policy, Gains to the grand coalition.

Received: June 13, 2023. Revised: February 16, 2024. Accepted: March 17, 2024. Published: May 20, 2024.

## 1 Introduction

Sharpley value method of cost allocation shares some similarities with dynamic programming [1]. It is a method based on some mathematical axiom, which distributes average cost based on the contribution of the participant to the grand coalition; the method is based on marginal cost. Also, dynamic programming is anchored on the principle of marginal cost, in which the current status is based on the immediate past event [2], meaning that the principle of optimality is dependent on marginal cost. Therefore, both methods share a common feature of marginal cost.

In the supply chain distribution, competition among the transport providers is obvious and in most cases counter-productive. This kind of competition among players brings us to competitive game theory. A competitive game theory is an activity between two or more rational players with strategies at the end each player receives a reward or suffers a loss [3], this is a competitive game with either pure or mixed strategies. When a player is using a specific course of action, it is said to be using a pure strategy and we have this kind in a static game, but when a player is using the strategy in a certain proportion such that the opponent keeps guessing on which the next move of the player would be, the player is said to be using a mixed strategy and we have such in a dynamic game. A competitive game is also called a zero-sum game because a loss to one player is a gain to the other player and each player would like to eliminate the other from existence, that is one of the criteria for dominance and remaining relevant in the market. In contrast, a cooperative game theory proposes cooperation among players, where each player will concentrate on an area of specialization for the overall interest of the players and the market and share collectively the proceeds or loss from the outcome of their cooperation. In this case, the Sharpley value method offers a unique approach to handling this kind of scenario in the cooperative game theory [4]. In a supply chain distribution powered by transportation, cooperation among different modes of transportation encourages specialization and cost reduction. This is the principle behind cooperative game theory.

In this paper, we propose a coalition in a bimodal transport system. The cooperation is between Rail transport via cargo trains and Road transport via trucks [5] that ship cargoes from origin to destinations. The train ships cargo from the origin to the terminal where the truck picks it up and supplies/distributes the cargo to various terminals where they are either stored or distributed immediately to the final consumers. Our interest is in the collaboration between these modes of transport and among the coalitions. We encourage this kind of cooperation not only for the economies of scale but for healthy competition and the overall good of the economy. We use the Sharpley value method to obtain the savings among different legs of the coalition and the sum of savings per coalition. But instead of stopping there to determine the percentage savings, our interest was to discover how more gains could be made on the savings using the optimization method. Using the optimization method, we seek to determine the optimal allocation of the coalitions to different legs to maximize savings using the Dynamic programming model. Our interest is to determine how best the coalition can be allocated to optimize savings for the grand coalition. Therefore, we developed a Dynamic programming model and the optimal distribution policy [6] that will allocate appropriate carriers to different legs (routes). This will help the players gain more insight about the coalition business thereby having control of it.

For a better understanding of the work, we organize the paper as follows: in section one, we treated the introduction of the work, the background of games theory and the types, coalition among modes of transportation, dynamic programming and its relationship with Sharpley value method of cost allocation in a cooperative game theory, the specific modes of transportation that collaborate in this study. In section two, we treated the literature review on the related area, even though we acknowledge that no direct work was done specifically on the topic of this paper. In section three, we treat materials and methods; here, we state the method that would be applied to achieve the aim of the paper. In section four, we presented data and analysed it using both the Sharpley value method in conjunction with the Dynamic programming model stated in section three, and finally, in section five, we treated the result and general discussions of the work.

## 2 Literature Review

The literature in this area is rare because no direct work was done on it, nevertheless, we reviewed some related works. Competition among players (transport providers) may be destructive among rivals who seek to provide the same service. However, the cooperation among different modes of transportation where each of them engages in different sections of the distribution is called horizontal cooperation. Each player performs a given assignment for the overall interest of the coalition. A lot of benefits are derived from horizontal cooperation [7] because instead of unhealthy rivalry, the players build an inclusive system that benefits the entire players and stabilizes the market. Collaboration among transport providers is cost-saving [8] with a reduction in delivery time. The gains of cooperative game theory, where players act in unison is an aspect of game theory that should be encouraged in real-life scenarios against a zero-sum game where each player sees the other as a rival. Cooperative game theory is a characteristic function of an n-person cooperative game with a set C of players assigned to each subset S of C the maximum value $v(S)$ that coalition S can guarantee itself by coordinating the strategies of its members [9]. The gains from the coalition are shared among the players.

Our focus in this paper is on the discrete serial dynamic programming structure [10] where one stage receives input from the previous stage and sends its output to precisely one preceding stage. This is a case where trains receive cargo from the shipper, and convey it to a terminal where trucks distribute them to various destinations [11] demonstrates the various applications of dynamic programming modelling in solving real-life problems in which supply chain distribution would not be an exception. Dynamic programming breaks the entire problem into different stages, with each stage having independent decisions and policy; the current stage is linked to the immediate past stage through the principle of optimality [12] and the recursive relationship is established through the backward pass. Dynamic programming is so flexible that it can be used to model so many complex problems that other optimization methods could not be used, it is a branch of decision theory. For example, [13] used Dynamic programming to model the ASUU strike and from it offer a solution on how to avert future ASUU strikes. We used dynamic programming to model the gains from the Sharpley value method from the bimodal cooperation among the transport providers.

The applications of game theory - cooperative and zero-sum game – are so wide, provided the subject to be modelled has an element of competition or decision making. Game theory was used in the modelling of the cryptocurrency [14] market and its players. The study was specifically for Bitcoin and blockchains. The author was motivated to model the Bitcoin problem with a zero-sum game bearing in mind that a loss to one player is a gain to its opponent. Game theory is mostly concerned with decision-making, in other words, game theory is within the domain of decision theory and applied in solving various economic problems [15]. Game theory, be it zero-sum or cooperative, has one common objective, that is, to apply the rational decision to win over the opponent, depending on what the opponent is. Apart from the empirical application of game theory, other researchers [16] did extensive work on the theoretical application of game theory and viewed it as a complex mathematical theory developed specifically for decision-making. They observed that game theory is associated with the conflict situation that requires rationality to win or minimize losses. They noted some areas of application of games such as social life, political and economic life; and that the players include political parties, government authorities, firms or businesses, prison inmates and professional sports franchises, etc. To further demonstrate the application of Game theory, [17] applied it between doctors and patients to determine an equilibrium point which is the value of the game.

Some researchers [18] did review work, where more insights about game theory were given. They compared the popularity of game theory and entrepreneurship using different journal indexing databases such as EBSCO, Google Scholar, etc., and the review showed that game theory was more popular than entrepreneurship based on the search made. Also, similar work [19] was done on bibliometric analysis of game theory to compare the popularity of game theory with energy and natural resources. The research was based on published works in Web of Science (WOS) indexing. The result showed that game theory was more popular. Hence, game theory is gaining more popularity since virtually all human endeavours involve rationality and competition. In this study, we seek to optimize the gains in cooperative game theory offered by the Sharpley value method via dynamic programming.

## 3 Materials and Methods

In this paper, we proposed a bimodal transportation mode with a grand coalition of five. The transport providers are generally categorized into two groups with one providing cargo trains and the other providing trucks of capacities 453 tons respectively. The two transportation modes are concerned with the distribution and supply of agricultural produce from northern to southern Nigeria. We have observed that this collaboration between the transportation modes and among players falls within the purview of cooperative game theory. Shapley value is the most fair and efficient cost allocation method based on cooperative game theory [20]. According to [5], cooperative game theory is a pair (N, v) with a finite set of players v: 2N ↦ R, and the function that assigns a real-valued payoff v(S) to each coalition S ⊆ N with v (∅) = 0. If we let |S| be the number of members in the coalition S and N \ {k}, be the grand coalition and xi denotes the share of the grand coalition's payoff that a player i ∈ N receives; then, the Shapley value of player k is defined as:

$$\varphi_k(N,v) - \frac{1}{N!}\sum_{S\subseteq N\setminus\{i\}}(|S|-1)!\,(|N|-|S|)!\,[v(S\cup\{k\}-v(S)] \qquad (1)$$

Which can be written as;

$$Z_k(N,v) = C_i + y_i \qquad (2)$$

where $Z_k(N,v)$ is the total cost of transportation, $C_i$ $is$ the new cost obtained from Sharpley value method and $y_i$ is the savings for the ith partner of the coalition. However, our focus is to optimize the allocation of the coalitions to various legs of the coalition to further maximize the savings through the optimal allocation policy developed in this paper.

To optimize the gains in equation (2), we carefully observed that equation (2) is a special case of the Dynamic programming model where the total cost, $Z_k(N,v)$, is composed of new cost plus marginal cost (savings); hence, we can write equation (2) as;

$$f_{Z_k}(N,v) = \max_{x_i}\left[R_i(v) + f_{C_i}*(N-v)\right] \qquad (3)$$

Equation (3) is a Dynamic programming model, See [13], where $f_{Z_k}(N,v)$ is the optimization function of two variables (stage and state variables); $R_i(v)$ is the function that assigns values to gains due to the collaboration (marginal cost); $f_{C_i}*$ is the new cost from Sharpley value method.

The optimal allocation policy is determined as follows; let $N_i$ be the stage variables, i = 1, . . ., n; $v_i$ be the state variables and $d_i$* be the optimal decision variable at each stage, then the optimal allocation policy is:

$$\left.\begin{aligned} &N_1 = n_1;\ v_1^* = d_1\\ &N_2 = (n_1 - d_1) = d_{k2};\ v_2^* = d_2,\\ &\quad\cdots\cdots\cdots\\ &N_{n-1} = (n_{-1} - d_{kn-1});\ v_{n-1}^* = d_{n-1}\end{aligned}\right] \qquad (4)$$

$$N_n = (n - d_{n-1});\ v_n^* = d_n$$

Therefore, the allocation of the coalition to different legs in the following order ($d_1$, $d_2$, …, $d_n$), will yield additional gain to the grand coalition. Hence, the total gain for the grand coalition will be (K + m) = ($P_k$) million naira; where K = gains from the Sharpley value method, m = gains from the optimization method, and $P_k$ = the total gains in millions of naira.

# 4 Data Presentation and Analysis

## 4.1 Data Presentation

We present the data from the origin to destination, bimodal transportation route, transport providers, unit cost, and total unit cost in Appendix 4. Again, we presented the computation of savings on each leg of the coalitions in Appendix 5. We present Tables 1 and 2 as Appendix 1 and 2 and their computation were based on Appendix 4 and 5. In Appendix 6, we present the computational tables for the problem from where we determine the optimal decision policy.

### 4.2.1 Data Analysis 1

In this subsection, we present two types of analysis, namely, the analysis based on the

Sharpley value method presented in Appendix 1 and the analysis based on the dynamic programming model presented in Appendix 2.

In the analysis in Appendix 1, we saw that there were five grand coalition (|N|) and four operators per leg and each of the legs has uniform weight (wi). We observed that the first coalition (S1) made 21.86% savings from the supply chain distribution through collaboration thereby bringing down the actual cost from 1727 million to 1349.4 million naira. The second coalition (S2) made a savings of 15% thereby bringing down the cost from 1726 million to 1467.1 million, etc. We have noted that each coalition from (S2) to (S5) made a gain of 15% thereby reducing the cost by 15% respectively. In general, due to the collaboration of these transport providers, the cost of shipping for one month was reduced from 7607 million to 63347.4 thereby saving 1259.6 million naira for the grand coalition. This kind of collaboration is encouraged and it is capable of stabilizing the prices of staple food especially now Nigeria is facing a food crisis and galloping inflation on her staple food items.

### 4.2.2. Analysis 2:

In the second analysis, we apply a dynamic programming model to further analyse the supply chain distribution to determine how best to allocate coalitions for optimal gains. The data in Appendix 2 was used for the analysis. In Appendix 3, we present the data in a Dynamic programming format.

From Appendix 3, Xi represents the legs (routes) and Si represents the coalitions. Our interest is to find the best way to allocate the coalitions to the routes to maximize savings for the grand coalition. We use the Dynamic programming model in equation (3) and Optimal decision policy in equation (4) to arrive at the optimal decision, see [13].

#### 4.2.2.1. Optimal Decision Policy

For optimal solution, we apply equation (4) to arrive at:

$S_1 = 5, \quad x^*{}_1 = 1$

$S_2 = 5 - 1 = 4, \quad x^*{}_2 = 1$

$S_3 = 4 - 1 = 3, \quad x^*{}_3 = 2$

$S_4 = 3 - 2 = 1, \quad x^*{}_4 = 1$

For a better understanding of the above optimal decision policy, refer to Appendix 6. The optimal decision variables, $x^*$, were selected from the Tables starting from stage 1 to stage 4. Our interest in this section is to allocate the coalition to different legs for optimal gains to the grand coalition; we have four legs and five coalitions. From our analysis, we observed that if we allocate the five coalitions to the four legs in this order (1, 1, 2, 1), we will maximize the savings by 377.6 million naira. This is an intelligent decision as we can see that the appropriate allocation of these coalitions using the optimization method will bring in more savings in addition to the earlier savings through the Sharpley value method. Therefore, the total gain for the grand coalition (1259.6 +377.7) is 1637.3 million naira.

## 5. Discussion and Recommendation

### 5.1. Discussion

In this paper, we proposed a coalition between two modes of transportation (Bimodal mode of transportation). The coalition was between two groups of transportation, where one provided cargo train and the other provided trucks of capacity 453 tons respectively. There were five grand coalitions |N| and four coalitions |S|. The coalitions were distributed along five paths across Nigeria. The first coalition, S1 covers Katsina-Enugu, the second coalition, S2 covers Katsina-Jos, the third coalition, S3 covers Kaduna-Calabar, the fourth coalition, S4 covers Katsina-Benin and the fifth coalition, S5 covers Agbor-Owerri. Each of the coalition is made up of four transport providers (players). Each of the coalition is composed of four legs. Since this is a kind of cooperative game scenario where each player cooperates to enhance the efficiency of the distribution of the agricultural produce, the Sharpley value method of allocation of cost was appropriate for the study and was applied to obtain the gains accrued to the grand coalition. With the collaboration, the coalition was able to make some gains from each leg and we observed that the total savings for the grand coalition was 1259.6 million naira within the proposed period. But we did not stop there, we developed and applied the Dynamic programming model to the supply chain distribution problem and obtained an intelligent result. we discovered that if the five coalitions were distributed among the four legs, in this other (1, 1, 2, 1), the grand coalition would make an additional gain of 377.6 million naira. The distribution means that the allocation of the coalition to the third leg should be doubled while the other legs remain as they were. This arrangement will bring additional revenue and the total gain for the grand coalition was 1637.3 million naira. More optimization work should be done in conjunction with the Sharpley value method of cost or savings allocation using different optimization methods. This will enhance efficiency and encourage the application of optimization methods in this rare area of research.

## 5.2. Recommendations

From the findings in this paper, we recommend as follows:

1. Efforts should be made to encourage coalition among transport and logistics providers because such cooperation eliminates destructive competition inherent in the real-world system. By the application of the Sharpley value method, the players are encouraged to work as a team.

2. Cooperation of this kind can stabilize the prices of goods, especially agricultural produce. Nigeria is facing challenges of the high cost of transportation which resulted in the removal of fuel subsidy and for this reason, there is galloping inflation. This kind of cooperation can cushion the effect of the high cost of transportation thereby stabilizing the prices of food items.

3. At each time, optimization methods should be encouraged and brought into supply chain distribution because we can see from the paper that resources can be better allocated and managed using optimization methods, especially Dynamic programming.

4. Transport providers are encouraged to form coalitions in different areas of supply chain distribution because it is profitable and brings gains to the grand coalition. The gains can be shared using the Sharpley value method.

*References:*

[1] Aziz, H., Cahan, C., Gretton, C., Kilby, P., Mattei, N., Walsh, T., A Study of Proxies for Shapley Allocations of Transport Costs, (2014), pp.1-51.

[2] Amuji, H. O., Ugwuanyim, G. U., Ogbonna, C. J., Iwu, H. C., Okechukwu, B. N., The usefulness of dynamic programming in course allocation in the Nigerian Universities. *Open Journal of Optimization*, Vol. 6(4), (2017), pp.176-186.

[3] Amuji1, H. O., Ugwuanyim, G. U., Anyiam, K. E., Application of game theory in maintaining the academic standard in the Nigerian Universities, *World Scientific News: An International Scientific Journal*, Vol.125, (2019), pp.72-82.

[4] Malawski, M., Wieczorek, A., Sosnowska, H., Konkurencja i kooperacja - teoria gier w ekonomii i naukach społecznych (Competition and Cooperation–Theory of Games in Economics and Social Science), *Warszawa: Wydawnictwo Naukowe PWN*, (2006), pp.127-150.

[5] Kayikci, Y., Analysis of Cost Allocation Methods in International Sea-Rail Multimodal Freight Transportation, *Journal*

*of Yasar University*, Vol.15(57), (2020), pp.129-142.

[6] Amuji, H. O., Onwuegbuchunam, D. E., Aponjolosun, M. O., Okeke, K. O., Mbachu, J. C. Ojutalayo, J. F., The dynamic programming model for optimal allocation of laden containers to Nigerian seaports, *Journal of Sustainable Development of Transport and Logistics*, Vol. 7(2), (2022), pp.69-79.

[7] Cruijissen, F., Cools, M., Dullaert, W., Horizontal cooperation in logistics: opportunities and impediments, *Transport Research Part E: Logistics and Transport Review*, Vol.43(2), (2007), pp.129-142.

[8] Wang, Y., A collaborative approach based on Shapley value for carriers in the supply chain distribution, *Heliyon*, Vol.9, (2023), e17967.

[9] Thomas, L., *Games, Theory & Applications*. Chichester: Ellis Horwood (1986) p.86.

[10] Augustine, O. E., Barry, R. M., Non-Serial Dynamic Programming: A Survey, *Palgrave Macmillan Journals*, Vol. 25, (1974), pp.253-265.

[11] Adelson, R. M., Norman, J. M., Laporte, G., A dynamic programming formulation with diverse applications, *Operational Research Quarterly*, (1976), pp.119-121.

[12] Amuji, H. O., Onwuegbuchunam, D. E., Okoroji, L. I., Nwachi, C. C., Mbachu, J. C., Development/Application of Dynamic Programming Model for Students' Academic Planning and Performance in the Universities, *Greener Journal of Science, Engineering and Technological Research*, Vol.12(1), (2023), pp.20-25.

[13] Amuji, H. O., Olewuezi, N. P., Ohaeri, E. O., Ikeogu, V. N., Okoh, J. O., On Solution to ASUU Strike and Consolidated University Academic Salary Structure II (CONUASS II) in the Nigerian Universities Using Optimization Method, *International Journal of Applied Mathematics, Computational Science and Systems Engineering*, Vol.6, (2024), pp. 1-13.

[14] Moustapha, B. A., With a transaction fee market and without a block size limit in Bitcoin network; there exists a Nash equilibrium points of the mining game, *IJGTT.*, Vol. 6(1), (2020), pp.1-23.

[15] Allayarov, S., Kilicheva, F., Rakhimova, K., Mamasadikov, A., Khamrayeva,S., Durmanov, A., Game Theory and Its Optimum Application for Solving Economic Problems, *Turkish Journal of Computer and Mathematics Education* Vol.12(11), (2021), 3432- 3441.

[16] RUFAI, A. A., MARTINS, B. A., ADEBANWA, A. A., Review of Strategies in Games Theory and Its Application in Decision Making, *IOSR-JEF.*, Vol. 12(1), (2021), pp.06-22.

[17] Song, L., Yu, Z., He, Q., Evolutionary game theory and simulations based on doctor and patient medical malpractice, *PLoS ONE*, Vol.18(3), (2023), e0282434.

[18] Abdallah, M. H. A., Game theory in entrepreneurship: a review of the literature, *Journal of Business and Socio-economic Development*, Vol. 4(1), (2024), pp. 81-94.

[19] Dong, Y., Dong, Z., Bibliometric Analysis of Game Theory on Energy and Natural Resource, *Sustainability*, Vol. 15, (2023), 1-19.

[20] Shapley, L., A Value for n-person games, Albert William Tucker and Harold W. Kuhn (eds.), Contributions to the Theory of Games, *Princeton University Press*, (1953), pp.31–40.

Appendix 1: Coalitions, total unit cost, grand coalition, Sharpley weight (wi), savings, percentage savings, and the new cost.

| Si | Unit Cost | Number of operators \|S\| for each route from terminal to terminal | Total number of operators \|N\| in grand coalition | $\frac{(\lvert S\rvert-1)!(\lvert N\rvert-\lvert S\rvert)!}{N!}$ | $\sum_{\forall i} y_i$ | Savings (%) | New Cost |
|---|---|---|---|---|---|---|---|
| S1 | 1727 | 4 | 5 | 0.05 | 377.6 | 21.86 | 1349.4 |
| S2 | 1726 | 4 | 5 | 0.05 | 258.9 | 15.00 | 1467.1 |
| S3 | 1219 | 4 | 5 | 0.05 | 182.85 | 15.00 | 1036.15 |
| S4 | 1770 | 4 | 5 | 0.05 | 265.5 | 15.00 | 1504.5 |
| S5 | 1165 | 4 | 5 | 0.05 | 174.75 | 15.00 | 990.25 |
| **Total** | 7607 | | | | 1259.6 | 81.86 | 6347.4 |

Appendix 2: Coalitions, unit cost savings, and total cost savings per leg

| | Leg1 | Leg2 | Leg3 | Leg4 | Total |
|---|---|---|---|---|---|
| S1 | 44.75 | 72.25 | 71.8 | 188.8 | 377.6 |
| S2 | 44.6 | 72.15 | 71.8 | 70.35 | 258.9 |
| S3 | 46.25 | 45.9 | 45.65 | 45.05 | 182.85 |
| S4 | 48 | 72.8 | 72.5 | 72.2 | 265.5 |
| S5 | 44 | 43.7 | 43.6 | 43.45 | 174.75 |

Appendix 3: Presentation of data in Table 2 in Dynamic programming format.

| Si/Xi | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 44.75 | 72.25 | 71.8 | 188.8 |
| 2 | 44.6 | 72.15 | 71.8 | 70.35 |
| 3 | 46.25 | 45.9 | 45.65 | 45.05 |
| 4 | 48 | 72.8 | 72.5 | 72.2 |
| 5 | 44 | 43.7 | 43.6 | 43.45 |

Appendix 4: Route and combination of Trains and trucks from different transport providers, cost, and frequency of supply per month.

| Si | Origin-Destination (O-D) | Bimodal freight transport route from terminal to terminal | Transport providers | No. of operators | Unit Cost(m) | Frequency | Unit Cost(m) | Frequency | Unit Cost(m) | Frequency | Unit Cost(m) | Frequency | Total Unit Cost (b) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| S1 | Katsina - Enugu | Katsina(+Tn) - Kaduna(+Tk) - Nsukka(+Tk) – Enugu(+Tk) | Nigeria Train; Admiral Trucker; Country Service Solution; Vennis Truck | 4 | 832 | 2 | 282 | 4 | 291 | 5 | 322 | 3 | 1727 |
| S2 | Katsina - Jos | Katsina(+Tn)- Kano(+Tk)- Kogi(+Tk)- Jos(+Tk) | Nigeria Train; Eccnosy Inte. Solution; Admiral Trucker; Maverick Int. Soltn. | 4 | 834 | 3 | 283 | 3 | 290 | 4 | 319 | 4 | 1726 |
| S3 | Kaduna - Calabar | Kaduna(+Tk)- Agbor(+Tk)- Onitsha(+Tk) - Calabar(+Tk) - | Admiral Trucker; Country Service Solution; Eccnosy Int. Soln; Vennis Trucker | 4 | 294 | 3 | 301 | 4 | 306 | 3 | 318 | 4 | 1219 |
| S4 | Katsina - Benin | Katsina(+Tn)- Kano(+Tk) – Lagos(+Tk)- Benin(+Tk) | Nigeria Train; Admiral Trucker; Country Service Solution; Maverick Int. Soln. | 4 | 810 | 2 | 314 | 3 | 320 | 4 | 326 | 3 | 1770 |
| S5 | Lagos - Sokoto | Lagos(+Tr) - Kaduna(+Tr) - Kastina(+Tr) - Skoto(+Tk) | Admiral Trucker; Country Service Solution; Eccnosy Int. Soln; Vennis Trucker | 3 | 285 | 5 | 291 | 5 | 293 | 3 | 296 | 3 | 1165 |

Appendix 5: Computation of savings on each leg of the coalition

| | Total Unit Cost (TL) | Unit Cost (U) | TL - U | Wi | Savings (yi) |
|---|---|---|---|---|---|
| | 1727 | 832 | 895 | 0.05 | 44.75 |
| S1 | 1727 | 282 | 1445 | 0.05 | 72.25 |
| | 1727 | 291 | 1436 | 0.05 | 71.8 |
| | 1727 | 322 | 1405 | 0.05 | 188.8 |
| | | | | | 377.6 |
| | 1726 | 834 | 892 | 0.05 | 44.6 |
| | 1726 | 283 | 1443 | 0.05 | 72.15 |
| S2 | 1726 | 290 | 1436 | 0.05 | 71.8 |
| | 1726 | 319 | 1407 | 0.05 | 70.35 |
| | | | | | 258.9 |
| | 1219 | 294 | 925 | 0.05 | 46.25 |
| | 1219 | 301 | 918 | 0.05 | 45.9 |
| S3 | 1219 | 306 | 913 | 0.05 | 45.65 |
| | 1219 | 318 | 901 | 0.05 | 45.05 |
| | | | | | 182.85 |
| | 1770 | 810 | 960 | 0.05 | 48 |
| | 1770 | 314 | 1456 | 0.05 | 72.8 |
| S4 | 1770 | 320 | 1450 | 0.05 | 72.5 |
| | 1770 | 326 | 1444 | 0.05 | 72.2 |
| | | | | | 265.5 |

| | Total Unit Cost (TL) | Unit Cost (U) | TL - U | Wi | Savings (yi) |
|---|---|---|---|---|---|
| | 1165 | 285 | 880 | 0.05 | 44 |
| S5 | 1165 | 291 | 874 | 0.05 | 43.7 |
| | 1165 | 293 | 872 | 0.05 | 43.6 |
| | 1165 | 296 | 869 | 0.05 | 43.45 |
| | | | | | 174.75 |

Appendix 6.

For n = 4, we have the Table below:

for n = 4

| S4 | f*4 | X*4 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 188.8 | 1 |
| 2 | 70.35 | 2 |
| 3 | 45.05 | 3 |
| 4 | 72.2 | 4 |
| 5 | 43.45 | 5 |

for n = 3

| S3/X3 | 0 | 1 | 2 | 3 | 4 | 5 | f*3 | X*3 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | | | | | | 0 | 0 |
| 1 | 188.8 | 71.8 | | | | | 188.8 | 0 |
| 2 | 70.35 | 260.6 | 71.8 | | | | 260.6 | 1 |
| 3 | 45.05 | 142.15 | 260.6 | 45.65 | | | 260.6 | 2 |
| 4 | 72.2 | 116.85 | 142.15 | 234.45 | 72.5 | | 234.45 | 3 |
| 5 | 43.45 | 144 | 116.85 | 116 | 261.3 | 43.6 | 261.3 | 4 |

for n = 2

| S2/X2 | 0 | 1 | 2 | 3 | 4 | 5 | f*2 | X*2 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | | | | | | 0 | 0 |
| 1 | 188.8 | 72.25 | | | | | 188.8 | 0 |
| 2 | 260.6 | 261.05 | 72.15 | | | | 261.05 | 1 |
| 3 | 260.6 | 332.85 | 260.95 | 45.9 | | | 332.85 | 1 |
| 4 | 234.45 | 332.85 | 332.75 | 234.7 | 72.8 | | 332.85 | 1 |
| 5 | 261.3 | 306.7 | 332.75 | 306.5 | 261.6 | 43.7 | 332.75 | 2 |

for n =1

| S1/X1 | 0 | 1 | 2 | 3 | 4 | 5 | f*1 | X*1 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | | | | | | 0 | 0 |
| 1 | 188.8 | 44.75 | | | | | 188.8 | 0 |
| 2 | 261.05 | 233.55 | 44.6 | | | | 261.05 | 0 |
| 3 | 332.85 | 305.8 | 233.4 | 46.25 | | | 332.85 | 0 |
| 4 | 332.85 | 377.6 | 305.65 | 235.05 | 48 | | 377.6 | 1 |
| 5 | 332.75 | 377.6 | 377.45 | 307.3 | 236.8 | 44 | 377.6 | 1 |

**Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)**
The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

**Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself**
No funding was received for conducting this study.

**Conflict of Interest**
The authors have no conflicts of interest to declare that are relevant to the content of this article.