On Hybrid Autoregressive Integrated Moving Average Artificial Neural Network Time Series Analysis of the Nigeria External Reserves

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Abstract: The amount of the external reserves possessed by a country is an essential component in the measure of its economic status. In essence, a country with substantial amount of money in its external reserves would experience a buoyant and friendly economy among other nations. This study employs three different time series models to examine the status of the Nigeria External reserves for 22 years (1960-2022). These include the Auto Regressive Integrated Moving Average (ARIMA), Artificial Neural Networks (ANN) and the hybrid ARIMA-ANN models. ARIMA $(1,0,1)(0,0,2)_{12}$ emerged as the optimum model among other fitted linear models, having the smallest Akaike Information Criterion (AIC) value of 14365.84 and was used to fit the nonlinear Hybrid ARIMA-ANN model and for the prediction of the External Reserves series. The Mean Absolute Error (MAE), the Root Mean Square Error (RMSE) and the Mean Absolute Percentage Error (MAPE) values of (590.1479), (358.3421), and (24.10321) for the Hybrid ARIMA-ANN model were also the least in comparison with their corresponding values for the other models. These show that the Hybrid model has the least error value compared with those of the independent ARIMA and ANN models. Hence, the nonlinear Hybrid ARIMA-ANN model performed excellently in the estimation and the generation of the forecast values than the conventional linear ARIMA and the nonlinear ANN models.

Key-Words:- External Reserves, ARIMA model, Nonlinear, Neural Network, Hybrid model.

Received: October 19, 2022. Revised: October 19, 2023. Accepted: November 21, 2023. Published: December 19, 2023.

1. Introduction

External reserves, also referred to as foreign exchange reserves, are assets kept aside by a monetary institution in form of foreign currencies. They are usually held to support liabilities and influence monetary policies. They include deposits, treasury bills, bonds, foreign banknotes and other foreign government securities. They equally serve several other purposes but most importantly held to ensure that a government or any of its parastatals has backup funds in case of any loss in the value of its national currency. Foreign exchange reserves are also defined as international reserves. This implies that external reserves comprise of international assets of a financial institution but not foreign currency and securities held by the banks, public and private organizations, [3].

External reserves could also serve as antidotes to unforeseen possible financial crisis. According to [7], they are invested in liquid forms that are easily convertible to cash. It could be in form of short-term interest-yielding treasury bills and bonds, or in some other forms.

[8] stated that external reserves could be a result of situations in which foreign exchange payments

are less than the receipts. He further explained that foreign reserve is the net value of foreign exchange excesses of a country, accrued over a period of time. Therefore, many low-income and developing countries usually have most of their reserves from donations and foreign supports. In general, many countries reserve assets for different reasons depending on their economic status.

For Nigeria in particular, external reserves are primarily derived from the earnings of crude oil sales and export. From $3.40 billion in 1996, Nigeria's external reserves increased to about $28.28 billion in 2005 and then to about $47.00 billion in 2007, [3]. However, Nigeria's foreign reserves declined as a result of the decline in exports, exchange rate, and foreign currency inflows. Hence, the Nigerian Stock Exchange (NSE) market was adversely affected due to drastic fall in investors' confidence. This led to withdrawal of investors from the market which is obvious in the figures of the Nigeria's market capitalization, which dropped from 12,640 trillion Naira in March 2008 to 4,900 trillion Naira in March 2009. This is equivalent to a 62% loss from its previous value, [1].

[5] equally stated that the increase or decrease in a country's external reserves is a crucial aspect of her economy. However, a good exchange rate status forms a solid foundation for macroeconomic stability and sustained inflation, both of which are essential for a healthy economy, [14]. Investigation has equally shown that good fiscal and structural policies are of great importance for the sustenance of a robust economy, [13].

Several nations of the world do keep foreign reserves in order to maintain a satisfactory level of exchange rate, particularly with a view to having a stable and booming economy. Recently, there came up an argument on whether there is the need to increase the level of a nations' foreign reserves or not, and this dispute gets more interesting especially among developing countries like Nigeria. However, some personalities are of the view that putting scarce resources in reserve among several other local needs, such as education, health, agriculture, and infrastructure would not be a good decision towards a sustainable national development. Moreover, some others claim that the foreign reserve position is an integral part of a country's ranking in the global competitive market. Hence, a robust foreign reserves status would make the country look financially responsible and creditworthy to other countries, donors and creditors, [9].

2. Methodology

An Artificial Neural Network (ANN) model is a computational framework usually stimulated by the human brain's network structure. It comprises of different layers of interconnected nodes, usually referred to as neurons. These process and transform input data to produce desired output. It is also a nonlinear modeling technique that is relevant to different areas of studies with multiple ranges of applications.

ANN time series modeling has proven to be an appropriate substitute to the ARIMA models in forecasting time series observations and it has many distinguishing characteristics. According to [6], these characteristics include its universal approximation property. That is, it can evaluate any nonlinear continuous function up to different desired level of accuracy. [4] examined the ANN and Time Series Analysis (TSA) approaches to predict the level of electricity consumption by public transportation vehicles in Sofia, Bulgaria. Their predictive performances and error distributions were examined and they ascertained that the ANN model is much more accurate but would require more background information and calculation efforts, while the TSA model, shows a low demanding input entries and a lower estimation power against some errors. They further observed that the ANN model has a lower time period of prediction due to the fact that it requires many recent inputs to generate its output.

A single hidden feed forward layer ANN with one output node is mostly used in forecasting applications. The equation for a basic feed-forward ANN model can be described as follows. Let's consider a hidden layer feed-forward neural network with input features $X_1, X_2, ..., X_n$, h-hidden neurons in the hidden layer, and a single output neuron. The model can be represented as:

$$X_t = \phi_0 + \sum_{i=1}^{n} \phi_0 g(\theta_{0j} + \sum \theta_{ij} y_{t-i}) + \varepsilon_t \tag{1}$$

It comprises of the;

1. Input Layer: This layer simply passes the input features as they are to the hidden layer.
2. Hidden Layer: Here, every neuron in the hidden layer performs a weighted sum of its inputs, followed by an activation function. Let a_j be the activation of the j_{th} neuron in the hidden layer. The weighted sum for the j_{th} neuron can be represented as:

$$Z_j = \sum_{i=1}^{n} \omega_{ij} x_i + b_j \tag{2}$$

where:

ω_{ij} is the weight connecting the i^{th} input feature to the j^{th} hidden neuron.

b_j is the bias term for the j^{th} hidden neuron.

The activation function f is applied to the weighted sum Z_j to calculate the activation a_j;

$$a_j = f(z_j)$$

3. Output Layer: This comprises of neurons that process the activations from the hidden layer in a similar way. Let o be the output of the network, then the weighted sum for the output neuron is:

$$Z_0 = \sum_{j=1}^{h} \omega_{j0} a_j + b_0 \tag{3}$$

where:

ω_{j0} is the weight connecting the j_{th} neuron in the hidden layer to the output neuron.

b_0 is the bias term for the output neuron.

The final output o is obtained by applying an appropriate activation function g to the weighted sum Z_0.

$$o = g(z_0) \tag{4}$$

The activation functions f and g are typically nonlinear functions that incorporate nonlinearity into the model, and make it possible to capture complex relationships in the data.

Hybrid ARIMA–ANN model

[15] proposed a Hybrid ARIMA-ANN model to get more precise results as compared to using independent ARIMA and ANN. He stated that the Hybrid model combines two powerful techniques: the ARIMA model and the ANN model. This approach aims to leverage on the strengths of both methods to ensure improved forecast values. The ARIMA model usually captures the time series components, while the ANN captures potential nonlinear relationships and any inherent interactions. In most cases, a single model would not reveal all the characteristics of the observations. Thus, the rationale behind the combination of the models. [6].

Moreover, neither the ARIMA nor ANN is independently appropriate for all types of time series. This is because almost all real-life time observations combine both linear and nonlinear structures and properties. [15] also revealed this significant fact and proposed a hybrid method that applies ARIMA and ANN for modeling linear and nonlinear time series components respectively. According to him;

$$X_t = L_t + N_t \qquad (5)$$

where, X_t represents the series at time t. L_t and N_t are respectively the linear and nonlinear components at time t.

ARIMA is usually first fitted to the linear component and the corresponding predicted value, $\hat{L}_t$ at time t is estimated. The general equation for the Hybrid ARIMA-ANN model is given by equation 6:

$$\hat{X}_{(t+h|h)} = f(X_t, X_{t-1}, X_{t-2}, ..., X_1; \theta_{ARIMA,} \theta_{ANN}) \qquad (6)$$

where:

- $\hat{X}_{(t+h|h)}$ represents the forecasted value at time $t+h$ given information up to time t.
- $X_t, X_{t-1}, X_{t-2}, ..., X_1$ are past observations of the time series up to time t.
- $\theta_{ARIMA,}$ denotes the parameters of the ARIMA model of order (p, d, q) for autoregressive, differencing, and moving average components respectively.
- θ_{ANN} denotes the parameters of the ANN model, including weights, biases, and network architecture.

In practical application, [11] considered the modelling of electromagnetic propagation as a substitute to the Least Squares Method (LSM) and International Telecommunication Union (ITU) recommendation P.1546-5 for Amazon urbanized cities using the Hybrid ARIMA-ANN and estimated the electromagnetic wave propagation in densely forested urban areas.

In general, the process of building and training the Hybrid ARIMA-ANN model involves fitting the ARIMA component to the time series data and then utilizing its residuals (the difference between the actual observations and the ARIMA predictions) as inputs to an ANN. The ANN captures the patterns and relationships present in the residuals, thereby improving forecasting accuracy.

Using Monthly Gold prices dataset for 1989-2019, [2] compares the performances of the ARIMA model, ANN and hybrid models for time series forecasting. He equally used the dataset to train and test the forecasted values. Their performances were examined using the Mean Square Error (MSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). It was concluded that the hybrid model had improved accuracy for the forecast while neither ARIMA nor ANN model captures all the patterns in the series like the hybrid ARIMA-ANN model.

[10] also examined some time series forecasting models and the Hybrid ARIMA-ANN. They applied these approaches to examine the outbreak and trend of Covid-19 in India with a view to curtail its spread and transmission and to reduce the increasing number of cases in the country. They concluded that the Hybrid ARIMA-ANN model gave better accuracy, with the lowest RMSE than the other models.

[12] equally applied the ARIMA, ANN, and the Hybrid models to predict Tuberculosis (TB) cases among children below 15 years in some regions of Kenya. They concluded that although the three

models were able to predict the TB cases, the Hybrid ARIMA-ANN model exhibited the best predictive performance compared to independent ARIMA and ANN models.

3. Results and Findings

Table 1 shows the descriptive statistics of the monthly Adjusted External Reserves Position examined in this study. These are the mean, standard deviation, minimum, maximum and coefficient of variation of the data.

It shows that the External Reserves Position has mean of 23,127.15. The data also reveals that the variable's minimum and highest values for this research are 63.22 and 62,081.86, respectively. The median, Standard Deviation (SD), Variance, coefficient of variance, standard error of variance and range are 4,223.59, 16,670.43, 277,903,177.39, 1.27, 606.30 and 62,018.64 respectively.

Table 1: Descriptive Analysis for the Nigeria external reserves

Descriptive	Naira-Dollar
Mean	23127.15
CI. Mean 0.95	1190.23
Median	4223.59
Standard Deviation	16670.43
Var	277903177.39
Coefficient of variation	1.27
SE.mean	606.30
Range	62018.64
Minimum	63.22
Maximum	62081.86
Sum	9924121.99
Count	756

TIME PLOT

Time plot shows the movement over time of a particular variable. Fig. 1 shows the time plot of the exchange rate variables which span from 1960 to 2022. From the time plot, we observed seasonal movement from 1960 to 2022.

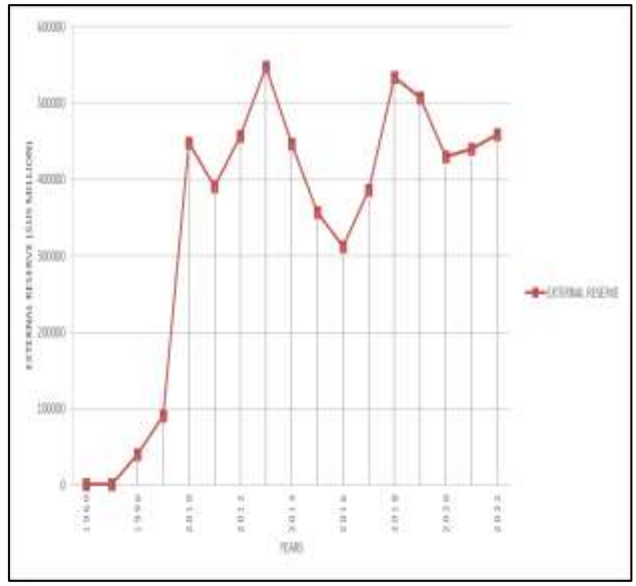

Fig. 1: Time plot for External Reserves Position

UNIT ROOT TEST

This test is used to determine the stationarity status of a set of time series observations. Dickey-Fuller and the Philip-Perron test statistics are obtained to establish the stationarity of the series for this study. The results indicate that the location of the external reserves is stationary and that the Augmented Dickey-Fuller test statistic and the Philip-Perron unit root test both have significant p-values of 0.01 and 0.01 at 5% level of significance, respectively. Hence, the original series is stationary.

DOI: 10.37394/232026.2023.5.21

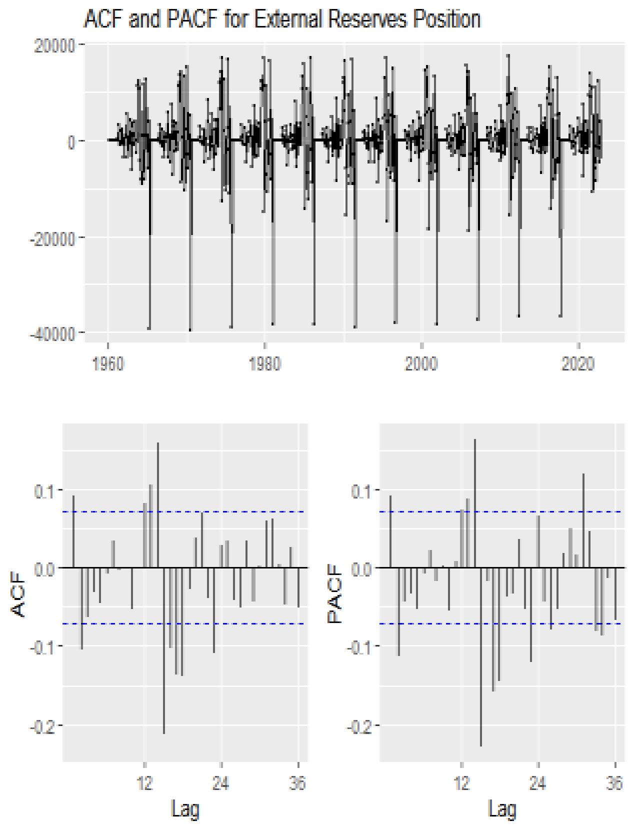

Fig 2: ACF and PACF for External Reserves Position

ARIMA results for External Reserves Position

The suggested ARIMA result for External Reserves Position is presented in Table 2 with their respective AIC values. The order with the lowest AIC shall be considered the best for the exchange rate variable.

Table 2: Different fitted seasonal ARIMA models for the External Reserves

ARIMA	AIC
ARIMA(2,0,2)(1,0,1)[12] with non-zero mean	14381.63
ARIMA(0,0,0) with non-zero mean	15597.09
ARIMA(1,0,0)(1,0,0)[12] with non-zero mean	14386.77
ARIMA(0,0,1)(0,0,1)[12] with non-zero mean	14944.79
ARIMA(0,0,0) with zero mean	15928.1
ARIMA(2,0,2)(0,0,1)[12] with non-zero mean	14368.03
ARIMA(2,0,2) with non-zero mean	14370.42
ARIMA(2,0,2)(0,0,2)[12] with non-zero mean	14369.39
ARIMA(2,0,2)(1,0,0)[12] with non-zero mean	14379.65
ARIMA(2,0,2)(1,0,2)[12] with non-zero mean	14383.26
ARIMA(1,0,2)(0,0,1)[12] with non-zero mean	14365.01
ARIMA(1,0,2) with non-zero mean	14367.46
ARIMA(1,0,2)(1,0,1)[12] with non-zero mean	14378.58
ARIMA(1,0,2)(0,0,2)[12] with non-zero mean	14366.37
ARIMA(1,0,2)(1,0,0)[12] with non-zero mean	14376.57
ARIMA(1,0,2)(1,0,2)[12] with non-zero mean	14380.21
ARIMA(0,0,2)(0,0,1)[12] with non-zero mean	14680.27
ARIMA(1,0,1)(0,0,1)[12] with non-zero mean	**14364.48**
ARIMA(1,0,1) with non-zero mean	14366.12
ARIMA(1,0,1)(1,0,1)[12] with non-zero mean	14377.99
ARIMA(1,0,1)(0,0,2)[12] with non-zero mean	14365.73
ARIMA(1,0,1)(1,0,0)[12] with non-zero mean	14376.02
ARIMA(1,0,1)(1,0,2)[12] with non-zero mean	14379.52
ARIMA(1,0,0)(0,0,1)[12] with non-zero mean	14375.11
ARIMA(2,0,1)(0,0,1)[12] with non-zero mean	14366.18
ARIMA(0,0,0)(0,0,1)[12] with non-zero mean	15567.17
ARIMA(2,0,0)(0,0,1)[12] with non-zero mean	14367.00

ARIMA(1,0,1)(0,0,1)[12] with zero mean	14377.18

From Table 2, it could be deduced that the External Reserves Position is best modelled with ARIMA(1,0,1)(0,0,2)$_{12}$ with the smallest AIC value of 14365.84 and was used to fit the nonlinear Hybrid model for the prediction of External Reserves Position.

Table 3: ARIMA estimate for the External Reserves Position

	Coefficients	s.e.
ar1	0.8795	0.0198
ma1	0.1573	0.0432
sma1	0.0716	0.0373
Mean	12661.305	2638.497

sigma^2 = 47366095, log likelihood= -7177.88

AIC=14364.48, AICc=14364.84, BIC=14388.51

Table 3 shows the estimate value of the coefficients of the fitted seasonal ARIMA model and their corresponding standard deviation 0f 0.0198, 0.0432, 0.0373. The residual plot of the model together with the ACF and PACF plots are displayed in Figure 3.

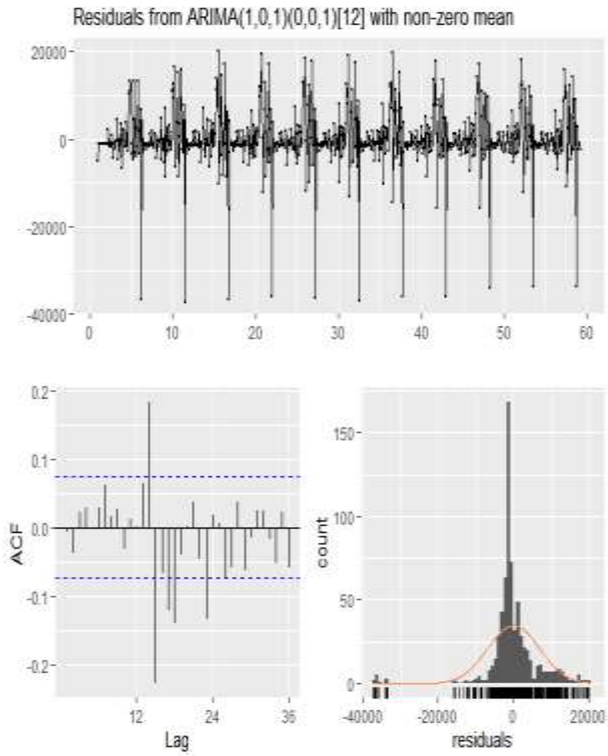

Fig. 3: Residual plot for ARIMA(1,0,1)(0,0,1) of External Reserves Position

NEURAL NETWORK RESULT

This section discusses the result obtained using Artificial Neural Network (ANN) model for the External Reserves Position. Table 4 shows the ANN output for the time series data. The average linear output units for 20 networks, each of which is a 28-14-1 network with 421 weight possibilities was calculated to be 94.32.

Table 4: ANN model Error output for External Reserves Position

ME	RMSE	MAE	MPE	MAPE	MASE
28.6	681.0	363.0	-2.50	89.3	0.0241

Table 4 shows that ANN model External Reserves Position as Neural Network Autoregressive model, NNAR(28,14,1)[12] with average of 20 networks each of which is 25 inputs, 1 hidden neuron and 13 outputs. The table also shows that the model produced RMSE, MAE and MAPE of 681.0, 363.0 and 89.3respectively.

HYBRID ARIMA-ANN results for External Reserves Position

The coefficients of the fitted Hybrid ARIMA-ANN model and their corresponding p-values are as shown in table 5.

Table 5: Coefficients of the fitted HYBRID ARIMA-ANN

ARIMA coefficients	Value	P-value
ar1	8.794616e-01	0.000
ma1	1.573133e-01	2.727041e-04
sma1	7.164627e-02	5.475226e-02
Intercept	1.266130e+04	1.597146e-06

Estimated error values for the Hybrid ARIMA-ANN model

Table 6: Estimated error values for Hybrid ARIMA-ANN

RMSE	MAE	MAPE
590.1479	358.3421	24.10321

Table 6 shows Estimated error values for Hybrid ARIMA-ANN model, ARIMA(1,0,1)(0,0,1)NNAR(28,14,1) with average of 20 networks each of which is 28 inputs, 1 hidden neuron and 14 outputs. The table also shows that the model produced RMSE, MAE and MAPE of 590.1479, 358.3421 and 24.10321 respectively.

Model comparison using the External Reserves

In this section, we examined the performance of the three models used to model Nigeria External Reserves Position data.

Table 7: Error status using the External Reserves Position data

Model	RMSE	MAE	MAPE
ANN	681.0	363.0	89.3
ARIMA	6862.61	4004.3	678.36
ARIMA-ANN	590.1479	358.3421	24.10321

Table 7 shows the error status for the purpose of comparison in search for the optimum model among the three. Hybrid ARIMA-ANN outperformed ARIMA and ANN as it has the lowest Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) values.

Forecast Values

Based on the optimum Hybrid model, Table 8 shows the forecast table for the External Reserves Position.

Table 8: Forecast results for External Reserves Position

Actual	Predicted	Error	Error %
150.23	6486.04	6335.81	4217.41
146.59	5775.946	5629.356	3840.2
148.29	6396.03	6247.74	4213.19
124.51	7802.64	7678.13	6166.68
149.65	258.21	108.56	72.54
159.52	1256.35	1096.83	687.58
137.61	2610.95	2473.34	1797.35
69.45	4032.09	3962.64	5705.75
66.02	6153.73	6087.71	9221.01
85.11	5415.61	5330.5	6263.07
108.24	3142.53	3034.29	2803.3
161.94	8563.89	8401.95	5188.31

It shows the actual, predicted, error values as well as the error percent figures for the forecast plot of the External Reserves Position.

4. Conclusion

The Nigeria external reserves have experienced a dramatic growth between the year 1980 to 2010 after which it began to undergo unstable and dwindling up and down movements. Moreover, there has not been a remarkable increase since its decline in 2018. Hence, the government of Nigeria would be encouraged to ensure more commitment to ensure significant growth in its external reserves for improved economic status of its citizens. Moreover, the Hybrid ARIMA-ANN has performed excellently in the investigation of the status of the Nigeria External Reserves than the independent ARIMA and ANN time series models.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.