Computational Mathematics in Engineering Careers: Study of Electrical Circuits from the Perspective of Interdisciplinary Teaching

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Abstract: - The new challenges of the educational engineering framework pretend to encourage the interdepartmental integration and interaction between basic sciences and the module of applied technology subjects. This paper presents a learning approach of interdisciplinary learning in the teaching of trigonometric functions, with special emphasis on the sinusoidal function, showing how the transformation of this function is related to fundamental concepts in the training of electrical energy engineers. The product of trigonometric functions makes it possible to analyze the instantaneous power of passive linear electric circuits and highlight the importance of studying the phase angle between voltage and current associated with the inductive or capacitive nature of the circuit. The development of the contents in the class contemplates the interdisciplinary and integrative aspect that must be implemented from the basic cycle as a bridge towards the applications of mathematics, articulating the problems of Engineering and the use of technologies in teaching.

Key-Words: Interdisciplinary work, sinusoidal function, electrical circuits, laboratory, learning and simulation.

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1 Introduction

The university educational field is continuously conditioned by the demands of modern society that requires qualified creative professionals prepared to face the challenges that arise in a constantly changing world.

The new challenges of the educational engineering framework pretend to encourage the interdepartmental integration and interaction between basic sciences and the module of applied technology subjects.

In the search for an interdisciplinary training, the incorporation of technologies in higher education allows significant changes in the teaching and learning process that have an impact on engineering careers, especially on the Mathematics subjects.

The introduction of increasingly powerful computational tools and specific software allows making the necessary modifications to achieve this purpose [1].

This paper presents a learning approach to interdisciplinary learning in the teaching of trigonometric functions; special emphasis is placed on the sinusoidal function, showing how the transformation of this function is related to fundamental concepts in the training of electrical energy engineers.

The successive transformations from the sinusoidal function and the product of trigonometric functions allow us to analyze the instantaneous power of passive linear electric circuits and highlight the importance of studying the phase angle between voltage and current associated with the inductive or capacitive nature of the circuit.

The development of the contents contemplates the interdisciplinary and integrating aspect that must be given from the basic modules as a bridge towards the applications of mathematics, articulating the problems of Engineering and the use of technologies in teaching [2].

Particularly in the field of Mathematics, the professors who teach this subject are always willing to implement didactic strategies tending to transform teaching with the aim of facilitating the content learning.

The choice of real models in teaching influences the students' motivation as well as the acquisition of new knowledge.

Among other considerations, it is equally important to select engineering situations that encourage the student not only to learn but also to develop their creativity while using computer resources that optimize their conceptualization capacity.

2 Methodology

The seven principles for good practice in undergraduate education were developed by Chickering and Gamson [3]. The principles first appeared in the American Association for Higher Education (AAHE) Bulletin. As a result of a two year research in cooperation with AAHE and a number of higher education faculty and colleagues, it was concluded that good practice in undergraduate education:

- Encourages contact between students and faculty
- Encourages cooperation among students
- Encourages active learning
- Gives prompt feedback
- Emphasizes time on task
- Communicates high expectations
- Respects diverse talents and ways of learning.

These principles were initially conceived for face-to-face instruction but later, with the advent of newer information technologies, they were implemented for technology-based teaching [4].

Kontos described some of the best and most appropriate ways to use technologies such as video and computers to advance these seven principles. How does technology facilitate the above seven principles of good practice [5]?

New technologies are based on complex algorithms that manage to produce, through the use of appropriate technologies, tasks with a complexity unexpected 20 years ago. These algorithms have a strong mathematical basis and allow the conceptualization of alternative working methods capable of solving the engineering problems.

The study of this new mathematic required deepening and broadening the field of knowledge. To achieve the analysis and use of complex design systems and generate experimental non-linear models, the contributions of the new math will be critical, both during the engineering education stage and the professional development.

Engineers must use primarily reasoning during their professional activity in order to understand and solve problems with many logical components. Therefore during the education stage, it is necessary to focus on teaching and learning the development of processes that can give the future engineer the possibility to achieve intellectual autonomy [6].

The learning process based in engineering models is a methodology that allows the students acquire knowledge and key basic skills (competences) through the elaboration of projects that find the answers to an engineer speciality problem.

The students work in an independent way and are makers of their own learning, hence they develops autonomy and accountability, as they are in charge of planning, task organization and working out the results to answer the target questions.

Teachers' work is to set up and guide the learning conditions facilitating the incorporation of curricular contents and offering support along the learning experience.

This methodological proposal intends to show the mathematical content based on the searching of models that simulates the situation which is required to formulate, or the technical situation in mathematical terms whereby a simplified situation is presented, that situation is then translated in mathematical terminology, and is worked with that model [7].

3 Objectives

The objectives taken into account for carry out the cited experience in this paper are related with the new curricular design specified for the Mathematical teaching in the engineering careers. These objectives are endorsed by current regulations and the national document pretends:

- To elaborate experiences where the student practices a self-administrated and collaborative work with the aim of internalizing the concepts of these subjects by designing an engineer-related situation [1].
- To develop the abstraction, generalization and particularization skills. Those competences reinforce the deductive and inductive thinking by the use and application of models based on electrical process.
- To transform the traditional classroom into a classroom workshop where the student can test learning generated by techniques of interaction between the teacher as passive subject and the student as active subject of knowledge.

This approach facilitates a class design which takes into account a learning transformation which can make the student leave the historical place that used to have in classroom so as to occupy another space with a different work dynamic, i.e. a space to interact with their partners and with the work approach [8].

The development of non-simple linear models analyzed by theoretical arguments using techniques, analytical process and graphic representation permits working out problems of application, interpreting the obtained results in the context of situation and identifying their elements by means of different semiotic representations and communicating by the appropriate mathematical language.

The use of a symbolical calculus tool and computational resources links Mathematics with real models, and develop students' competences by contributing to the development of the student basic skills [7].

4 Theoretical Learning Space

The interdisciplinary teaching approach aims to relate the contents and concepts taught in class with real engineering projects so that students acquire the necessary tools to spot the relevant variables in a problem.

In this way, students are able to interpret and propose solutions to different alternatives, increasing their capacity for analysis, which leads to decision-making based on the solutions found.

The proposed didactic activities consist of a first stage that is carrying out a theoretical investigation by the students with teacher tutoring, second making an analogy of the theoretical concepts with the physical parameters of the subject under study and finally interpreting the graphs of the functions analyzed with technological resources [2], [5].

Along these stages, the students look for bibliography to scan the theoretical concepts that they will need to develop the topic.

In the classroom, all the students with the teachers' agreement write a common document with the proposed concepts: Instantaneous Power, Active Power and Reactive Power.

4.1 Instantaneous Power

The tension (v) applied to a lineal circuit of passive elements, and the resulting electric current (i) are functions that depend on time. Also, the electric current value depends on the elements that integrates the circuit. Fig. 1 shows a simplified scheme of the circuit.

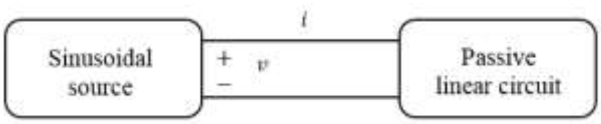

Fig. 1 A Simplified circuit scheme

The Instantaneous Power is the product in each instant of the current tension, and it is expressed by Equation (1):

$$p(t) = v(t).i(t) \quad (1)$$

p can be both positive or negative according to the considered instant. If $p(t) > 0$ the energy transference produced from Power Source to the circuit, while if $p(t) < 0$ the energy transference is in an opposite way, i.e. from the circuit to the Power Source. This energy return to the Power Source is produced due to the energy accumulation on magnetic and electric circuit fields [9].

The v tension and the i current are sinusoidal functions to the entrance of any passive circuit, and are generally out of phase to each other.

The expressions that determinates the tension and current are shown in the equations (2) and (3).

$$v(t) = V_{max}\, sen(wt) \quad (2)$$

Observing (2) and (3), the students determine that the tension and current are out of phase by φ radians.

$$i(t) = I_{max}\, sen(wt + \varphi) \quad (3)$$

Where φ names the angle's power factor and it will be positive or negative, according to their capacitive or inductive circuit nature. The φ angle's cosine has the name of Power factor and influences the electrical circuit behaviour.

From de equations (1), (2) and (3) combinations, a new expression of the instantaneous power given in the equation is obtained (4).

$$p(t) = V_{max}\, I_{max}\, sen(wt)\, sen(wt + \varphi) \quad (4)$$

If it is applied to (4), the established relations by the trigonometric identities results a new expression of p given in the equation (5).

If $A = 0{,}5\, V_{max}\, I_{max}$, then

$$p(t) = A\, (cos\varphi - cos(2wt + \varphi)) \quad (5)$$

In (5) the student can interpret that p results in a periodic function of angular frequency equal to the double $(2wt)$ of current frequency and tension (wt) and it is characterized for possessing two components:

- A constant component dependent of Power factor $0{,}5\, V_{max}\, I_{max}\, cos(\varphi)$ and another pulsating amplitude $0{,}5\, V_{max}\, I_{max}$
- and angular frequency $2wt$ [10].

4.2. Active Power

Active Power P is defined as the average value of Instantaneous Power p at $T = 2\frac{\pi}{w}$ period:

$$P = \frac{1}{T}\int_0^T p(t)dt \quad (6)$$

According to expression of p given in (5), P results in as expressed in (7)

$$P = 0,5\, V_{max}\, I_{max}\, cos\varphi \quad (7)$$

Equation (7) corresponds with the first term of general expression in (5). The students observe that (7) is independent of time.

In the study of electrical engineering and circuit theory it is common to represent the tension and current with expressions that has values denominated effectives.

So, the effective tension is defined [10]:

$$V_{ef} = \frac{V_{max}}{\sqrt{2}} \quad (8)$$

And effective current as:

$$I_{ef} = \frac{I_{max}}{\sqrt{2}} \quad (9)$$

From the combination of equation (8) and (9), emerge a new expression of the Active Power as a function of the effective values of tension and current as expressed in (10) [9].

$$P = V_{ef}\, I_{ef}\, cos\varphi \quad (10)$$

4.2. Reactive Power

If φ angle represents the phase shift between the angle (argument) of tension θ_v and the angle (argument) of current θ_i, results:

$$\varphi = \theta_v - \theta_i \quad (11)$$

If the tension argument is replaced as a function of current argument, it is obtained that:

$$v(t) = V_{max}\, sen(wt + \theta_v) \quad (12)$$
$$v(t) = V_{max}\, sen(wt + \theta_i + \varphi) \quad (13)$$

Resulting

$$v(t) = V_{max}\operatorname{sen}\varphi \,.\cos(wt + \theta_i) + \\ + V_{max}\cos\varphi\,.\operatorname{sen}(wt + \theta_i) \quad (14)$$

The first term of equation (14) is found in quadrature with i, while the second term is in phase with the same.

The Equation (1) can be written in term of equation (14), resulting in equation (15)

$$p(t) = V_{max}\, I_{max}\, sen\varphi\, cos(wt + \theta_i) \\ sen(wt + \theta_i) + \\ + V_{max}\, I_{max}\, cos\,\varphi\,.sen^2(wt + \theta_i) \quad (15)$$

If the equation (8) is applied again in (15) and besides trigonometric relations between involved angle arguments are applied, the equation which results is (16)

$$p(t) = V_{ef}\, I_{ef}\cos\varphi\,.[1 - cos2(wt + \theta_i) \\ -V_{ef}\, I_{ef}\, sen\,\varphi\, cos\left(2(wt + \theta_i) + \frac{\pi}{2}\right) \quad (16)$$

The coefficient of the first term of equation (16) is the Active Power P, while the coefficient of second term $V_{ef}\, I_{ef}\, sen\varphi$ is denominated Reactive Power and is designated by letter Q [9]-[10].

Q indicates that the second term comes from tension v component in quadrature with the current i. In this way, the instant power value is expressed as:

$$p(t) = P.[1 - \cos 2(wt + \theta_i)] - \\ -Q.\cos\left(2(wt + \theta_i) + \frac{\pi}{2}\right) \quad (17)$$

If power is the variation of energy in the unity of time, so the first term represents the energy that is in transit from Power Source to the network (unidirectional Power), while the second term represents the energy that interchanges between the Power Source and the Network (Interchange Power) [9], [10].

5 Theoretical-practical-technological learning space

The Computing and Multidisciplinary Laboratory of Basic Sciences is the area where the experience takes place. This activity will be carried out with teachers who belong to the laboratory in collaboration with teachers from the Department of Electrical Engineering.

The programming resources are used with the Geogebra digital mathematical tool. This platform has a series of applications which, based on a mathematical understanding, allows the demonstration of the concept of instantaneous power and its dependence on the power factor graphically. At the same time, the intervals and conditions where the energy transfer is from the source to the circuit or in the reverse direction are shown graphically.

These applications work dynamically and allow teachers and students to analyze the response of the circuits in different conditions from cursors that allow them to modify their parameters in real time with the option of doing them automatically in animation mode [5].

6 Results

6.1 Tension and Current analysis

With the supplied data to the students, they determine the temporal expression of tension (v), current (i), phase shift (angle φ) and the Power factor (f_p). The phase shift angle between v and i results $\varphi = 0{,}96 rad.$, in consequence of Power factor $f_p = 0{,}5736$, and with those values, the particular expression v and i which are expressed onto equations (18) and (19) can be modelled

$$v(t) = 1{,}4\, sen\,(wt) \quad (18)$$
$$i(t) = 1{,}3 \cos(wt + 0{,}96) \quad (19)$$

The students elaborate an interactive graphic from Fig. 2, corresponding to waves v and i by programming in the Geogebra platform. This platform has a function called a slider that allows you to modify the amplitudes of the waves, the phase shift (angle φ) and from the cursor denominated wt it , to generate the rotation from the phasors, at the same time it runs through the temporal evolution indicating the value which corresponds to each instant.

The temporal expression of v and I determines the value of each state in real time, the same way the out of phase angle is shown as well as the calculus of the power factor.

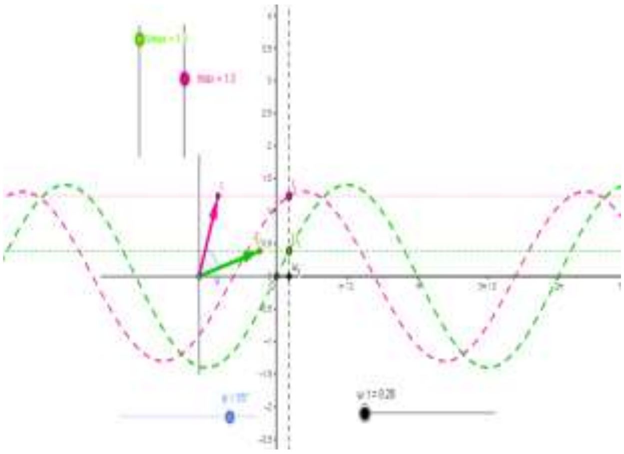

Fig. 2 Current and Tension on circuit.

The temporal expression of v and i determines the corresponding value to each state in real time, in the same way that the phase angle and the calculation of the power factor are shown.

It is intended to strengthen the study of sinusoidal functions, highlighting the impact that the modification of its parameters has on the waveform and its electrical interpretation as a representation of current and voltage in passive linear electrical circuits.

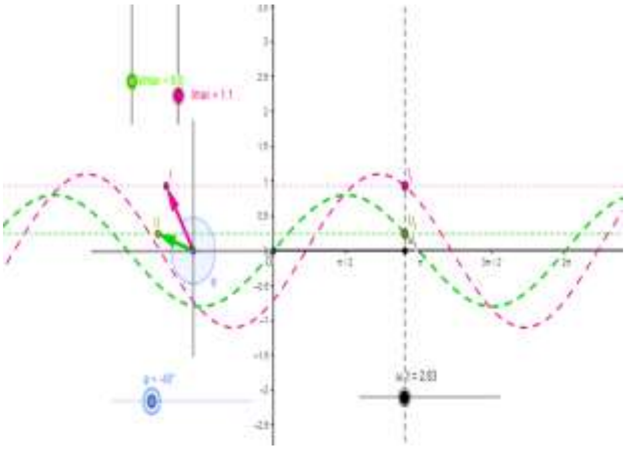

Fig. 3 Current and Tension on circuit.

If the different parameters involved in equations (18) and (19) are changed, a graph like the one in Fig. 3 can be obtained.

6.2 Instantaneous Power Analysis

In Fig. 4 the Instantaneous Power waveform is presented, highlighting the intervals in which the energy transfer occurs from the source to the circuit or vice versa.

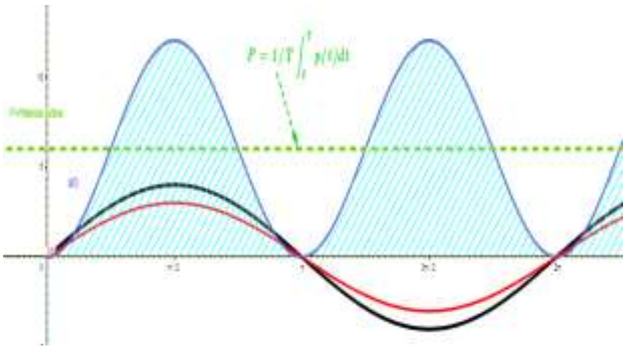

Fig. 4 Instantaneous Power

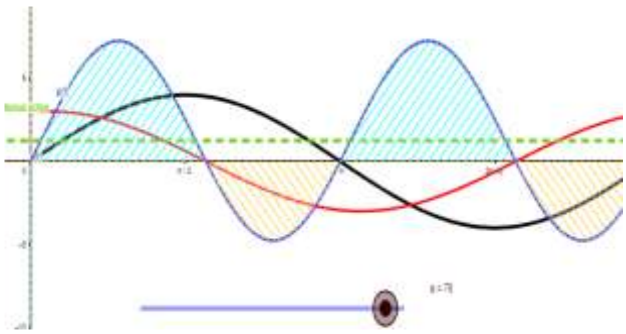

Fig. 5 Instantaneous Power

At the same time, the Active Power is highlighted as the average value of the Instantaneous Power.

The graphic of Fig. 5 is shown below, indicating how the instantaneous power varies for another value of the phase angle

6.3 Reactive Power Analysis

Once the Instantaneous, Active and Reactive Power have been explained, the students begin their work so that they interpret the physical manifestation of power in its different aspects mathematically.

The students previously develop the topic shifting of functions in the theory class by means of successive transformations of the same.

For this purpose, students are asked to analyze equation (17) through a debate, moderated by the teachers in charge of the class, which allows posing and interpreting students' different opinions.

The observations made by the students regarding equation (17) are:

- The first term from the equation corresponds to a cosine function where its argument is multiplied for the real number 2, producing in that way a horizontal compression that is translated on a frequency twice as the original.
- The angle θ_i adding to the argument generates a horizontal translation to the left when $\theta_i > 0$, while if $\theta_i > 0$ the translation occurs to the right.
- The real number 1 in the first term of equation, makes that the term would never be negative, reason why the amplitude in the power is not modified.
- The second term of the equation corresponds to a cosine function with a lateral translation of $\frac{\pi}{2}$ units. Due the adding of $\frac{\pi}{2}$ units in the argument, the expression $cos\left(2(wt + \theta_i) + \frac{\pi}{2}\right)$ transforms in $sen\big(2(wt + \theta_i)\big)$
- As in the first term, the real number 2 that multiplies the argument generates a horizontal compression that is translated into the doubling of the frequency.
- Unlike the first term, in the second term the average value is zero since it corresponds to a sinusoidal function without vertical translations.

4 Conclusion

In this experience it is corroborated that the changes introduced in the teaching context make the laboratories in the area of Mathematics possible, a fact that not only provides a meaningful framework to the learning context by encouraging the treatment of systems numerically and symbolically but also helps giving the field of knowledge required in the subject the possibility of moving forward.

Thus, interdisciplinary teaching in engineering careers should be developed since the approach foundation is that students learn Mathematics considering the advantages offered by the appearance of new technologies and simulation software that analyze various aspects of an engineering system.

In this way, meaningful learning is guaranteed from the first stages of the engineering studies, encouraging academic training not only to train suitable professionals but also ensuring that future engineers will be able to form highly efficient work teams.

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