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River Flow Forecasting Using An Inverse Wavelet Transform Neural Network Approach

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***Abstract*—We report a study of river flow modeling and forecast by using short-term and long-term training of a wavelet based neural network. We combine wavelet analysis and artificial neural networks to perform river flow forecasting of the Tittabawassee River. Our results are superior to some existing methods.**

***Index Terms*—Wavelet, neural network, multilayer perceptron**

I. Introduction

It was on May 20, 2020, that the Tittabawassee River, near Central Michigan University, crested at 35 feet. This peak gage height was more than 10 feet above flood level and less than the most dire projection of 38 feet. The previous record was 33.9 feet in 1986, according to the National Weather Service. Unprecedented flooding was ongoing that morning in Midland County, Michigan, after a pair of dams collapsed following record rainfall [1]. Thousands of residents were ordered to evacuate as flood waters gushed into the communities along the Tittabawassee River. The flash flood event inundated homes and businesses and prompted an emergency declaration from Michigan Governor Gretchen Whitmer. The flooding impacted a major Dow Chemical plant that lies along the river and where river water mixed with chemical containment ponds. Had the record discharge event been predicted in time, the dams could have been lowered greatly diminishing the risk of the catastrophic failure that occurred.

In fact, the river flow forecasting has great importance in water resources and hazard management. Floods affect countless lives, infrastructure, and property so it is crucial that we develop improved understanding of their prediction. There are many river flow forecasting models [10], [11]. The accuracy of models used for flood forecasting is critical. Wavelet analysis is a useful and powerful tool in performing time frequency analysis of a time series. Artificial Neural Networks (ANN) is using machine learning approach to perform river flow forecasting. In this paper, we combine wavelet analysis and ANN as a hybrid river flow forecasting model and call it WNN method. We use WNN method to perform river flow forecasting on the Tittabawassee River data.

We organize our paper as follows. We provide some theoretical background of wavelet analysis, neural networks, and WNN in Section 2. Our results are presented in Section 3, which is followed by discussions in Section 4 and conclusion in Section 5.

II. Theoretical Background

A. Wavelet Analysis

In this section, we give a brief account of wavelet analysis by recalling multiresolution analysis, scaling functions, wavelet functions, as well as continuous and discrete wavelet transforms.

A multiresolution analysis (MRA) [5] consists of a sequence of successive approximation spaces $\{V_j\}_{j\in Z}$ of $L^2(R)$ with the following properties:

(i) $V_j \subset V_{j+1}$,
(ii) $\lim_{j\to\infty} V_j = \bigcup_{j\in Z} V_j$ is dense in $L^2(R)$,
(iii) $\bigcap_{j\in Z} V_j = \{0\}$,
(iv) $f(x) \in V_j \iff f(2x) \in V_{j+1}$,
(v) $f(x) \in V_j \iff f(x + 2^{-j}k) \in V_j,\ \ \forall k \in Z$,
(vi) There exists a function $\phi \in V_0$ so that $\{\phi(x-j)\}_{j\in Z}$ is an orthonormal basis of V_0.

ϕ is called a *scaling function* that generates a MRA with the above properties. Through translation and dilation of ϕ, a Riesz basis $\{\phi_{j,k}(x)\}_{k\in Z}$ is obtained for the subspace $V_j \subset L^2(R)$ by the properties (iv)(v), where

$$\phi_{j,k}(x) = 2^{\frac{j}{2}}\phi(2^j x - k), \quad j,k \in Z. \tag{1}$$

This family can be generally expressed as $\phi_{m,n}(x) = \frac{1}{a^{\frac{m}{2}}}\phi(\frac{x-nb}{a^m})$.

Since $V_0 \subset V_1$, there is a set of coefficients $\{a_k\}_{k\in Z}$, so that ϕ satisfies the two–scale equation or refinement equation

$$\phi(x) = \sum_k a_k \phi(2x-k). \tag{2}$$

For every $j \in Z$, we define W_j to be the orthonormal complement of V_j in V_{j+1}, we then have

$$V_{j+1} = V_j \bigoplus W_j \tag{3}$$

and

$$W_j \perp W_{j'} \quad if \quad j \neq j'. \tag{4}$$

It follows that, for $j > J$

$$V_j = V_J \bigoplus \left(\bigoplus_{k=0}^{J-j+1} W_{J-k} \right). \tag{5}$$

By virtue of (ii) and (iii) above, this implies

$$L^2(R) = \bigoplus_{j \in Z} W_j \tag{6}$$

which is a decomposition of $L^2(R)$ into mutually orthogonal subspaces. It turns out that a basis for W_0 can be obtained by dilating and translating a single function $\psi(x)$ called basic (mother) wavelet which is defined by (wavelet equation)

$$\psi(x) = \sum_k b_k \phi(2x - k) \tag{7}$$

where $b_k = (-1)^k a_{-k+1}$. In fact, $\{\psi_{j,k}(x) = 2^{\frac{j}{2}} \psi(2^j x - k)\}_{k \in Z}$ forms an orthonormal basis for W_j.

Let $P_j,\ Q_j$ denote the orthogonal projection $L^2 \to V_j,\ L^2 \to W_j$, respectively. Then

$$P_j f(x) = \sum_k \alpha_{j,k} \phi_{j,k}(x), \tag{8}$$

$$Q_j f(x) = \sum_k \beta_{j,k} \psi_{j,k}(x), \tag{9}$$

where the coefficients $\alpha_{j,k},\ \beta_{j,k}$ are given by the inner product:

$$\alpha_{j,k} = < f,\ \phi_{j,k} > = \int_{-\infty}^{\infty} f(x) \phi_{j,k}(x) dx, \tag{10}$$

$$\beta_{j,k} = < f,\ \psi_{j,k} > = \int_{-\infty}^{\infty} f(x) \psi_{j,k}(x) dx. \tag{11}$$

$P_j f$ converges to f in the L^2 norm which is the best approximation of f in V_j.

More precisely, the above coefficients can be obtained by applying wavelet transforms which are defined as follows.

The continuous wavelet transform is defined as:

$$[w_\psi x(t)](a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi^* \Big(\frac{t - b}{a} \Big) dt \qquad a > 0, b \in R, \tag{12}$$

where the symbol * represents the complex conjugate, $x(t)$ is the given signal (river discharge) and ψ is a wavelet.
The discrete wavelet transform is defined as:

$$[Dw_\psi x(n)](a, b) = \sum_{n \in Z} x(n) g_{j,k}(n),$$

$$a = 2^j, b = k2^j, j \in N, k \in Z, \tag{13}$$

where g's are the coefficients of the wavelet equation associated with ψ.

B. Neural Network

Artificial neural networks (ANN) are a machine learning technique with flexible mathematical structure which is capable of identifying complex non-linear relationships between input and output data. The objective of ANN is to transform the inputs into meaningful outputs. It is a computational model inspired by the functionality of the human brain. ANN has become a popular hydrological forecasting tool and is based on the multilayer perceptron (MLP) structure. The model basically consists of neurons and nodes. The neurons are organized in layers, with each neuron only connected with neurons in the adjacent layers. Each node receives a weighted input which is the output from every node in the previous layer. By having many layers and many functions, a nonlinear process is created. The complexity and power of ANN is achieved by the interaction of several neurons through the nonlinear process.

C. Wavelet Neural Network

We combined wavelet analysis and ANN as an optimal way to do river flow forecasting and denote it as WNN method [3], [4], [8]–[10]. The idea is to use approximation and details components of original time series as input to ANN in order to forecast daily discharge. Each wavelet transform with associated wavelet can be applied to the given signal and produces different level of information. The outcome may differ by the choices of wavelets and approximation levels. The forecasting is obtained by first training a set of non-linear auto-regressive networks, where we use MLP for the hidden layer and linear transfer for the output layer. The training is accomplished using the scaled conjugate gradient method and a genetic algorithm for ideal MLP weight selection. The set of trained networks are then used to generate a historical simulation of testing data to demonstrate the performance of the training. After simulation, the networks are closed off from external data, and recursively fed next-step predictions to produce a multi-step forecast. The combined results of the trained networks are averaged and reported as the simulation and forecast.

III. Results

A. Model Performance

In this paper, we use Mean squared error (MSE), Correlation coefficient (R), and Root mean squared relative error (RMSRE) for performance, we recall their definitions as follows:

MSE	$\frac{1}{n}\sum_{t=1}^{n} e_t^2$
R	$\frac{N\sum xy - (\sum x \sum y)}{\sqrt{[N\sum x^2 - (\sum x)^2][N\sum y^2 - (\sum y)^2]}}$
RMSRE	$\sqrt{\frac{1}{n}\sum_{t=1}^{n}(\frac{e_t}{y_t})^2}$

where e_t is error of each component.

B. Data

USGS river discharge Dataset [2] used for training was daily mean river discharge measured in cubic feet per second obtained from USGS station 04156000 Tittabawassee River at Midland, Michigan. Time period used for this study was April 1st, 1936 to June 7th, 2017. Additional data beyond June 7th was used to provide a comparison for the predicted forecast.

C. Outcomes

1) Training: The structure of the process consists of three layers which are the input layer, output layer, and two hidden layers. The integers for each method, (m, n, h, k) denote k input layers having m inputs, k output layers with one neuron and 2 hidden layers having n and h neurons, respectively. For the IDWT (Inverse Discrete Wavelet Transform) method $m = 1$ and $k = 2$ is indicating that the entire process is run twice, once for details and once for approximation.

In this study the Daubechies orthogonal wavelet family $db4$ is used. This is motivated by the successful application of this family in other work [4], [11].

ANN(1,8,8,1) is multistep prediction using raw discharge data with two hidden layers of size 8. The WNN(11,8,8,1) refers to a neuro-wavelet, where the inputs are the approximation and details of a level 10 discrete wavelet transform. IDWT(1,8,8,2) is a new method being developed for this work and is refered to as inverse discrete wavelet transform of level 1. Each method was trained using twenty years of either raw daily discharge signal (ANN) or wavelet filtered signal (WNN/IDWT), using block data division. The data was divided as 52% for training , 15% for validation, and the final 33% reserved as testing data not used for training. In Fig. 1, the days count back from with $t = 1$ as June 8, 2017 and $t = -2409$ is from 33% of the twenty years of training.

We calculate the performance of ANN, WNN, and IDWT for the simulation until June 8, 2017 versus 33% testing data as a measure of the performance of the network training (see Tab. I and Fig. 1).

TABLE I
SIMULATION* UNTIL JUNE 8, 2017 VERSUS 33% TESTING DATA.

1 year training			
Method	***MSE***	***R***	***RMSRE***
ANN(1,8,8,1)	3.10E+06	0.85775	1.90E-03
WNN(11,8,8,1)	1.29E+07	0.15186	5.28E-03
IDWT(1,8,8,2)	4.93E-18	1.00000	1.40E-15
20 year training			
Method	***MSE***	***R***	***RMSRE***
ANN(1,8,8,1)	7.13E+05	0.94764	3.99E-05
WNN(11,8,8,1)	7.93E+05	0.95988	7.72E-05
IDWT(1,8,8,2)	3.09E-18	1.00000	9.39E-17

*Model is average of 100 trials.

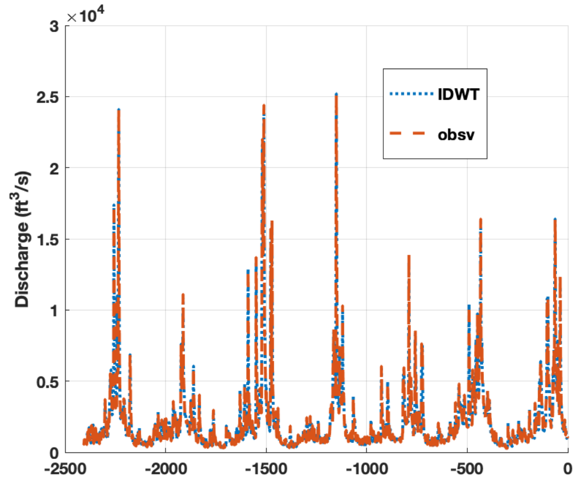

Fig. 1. Simulation until June 8, 2017 versus 33% testing data.

2) Prediction: After training a set of 100 networks by genetic algorithm and performing testing simulation, each network is used to produce a 30 step ahead forecast beginning on June 8, 2017. We calculate the performance of ANN, WNN and IDWT (Inverse discrete wavelet transform of level 1) for these forecasts until and present the results here (see Tab II and Fig. 2). In Fig. 2, we present the 14-day river flow forecast for June 8, 2017 with compared with future data. Results displayed are based on the 20 year training outcomes.

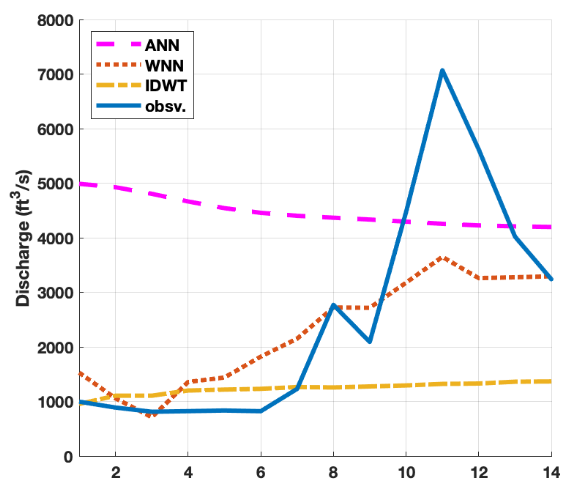

Fig. 2. 14-day river flow forecast for June 8, 2017.

IV. DISCUSSION

A. Simulation

Our modeling results here are for the simulation until June 8, 2017 with each method. All method exhibit similar or better performance to the comparable results in [11].

Note that wavelet-aware IDWT is able to use inverse wavelet transform to obtain "perfect reconstruction" of training signal, giving R=1.00000. While this does improve the simulation

TABLE II
30 DAY PREDICTION* FOR JUNE 8, 2017.

Method	*MSE*	*R*	*RMSRE*
1 year training			
ANN(1,8,8,1)	1.03E+08	0.46657	0.03089
WNN(11,8,8,1)	9.93E+07	0.01621	0.03013
IDWT(1,8,8,2)	1.05E+08	0.13229	0.03118
20 year training			
ANN(1,8,8,1)	1.04E+08	0.12906	0.03096
WNN(11,8,8,1)	8.01E+07	0.65714	0.02666
IDWT(1,8,8,2)	1.03E+08	0.17872	0.03078

*Forecast is average of 100 trials.

ability, it makes it difficult to use the simulation results to assess the quality of the potential forecast performance. These performance statistics are calculated for the 33% testing block. Thus for the other networks R appears to be a good indicator when the network has been sufficient training.

A large R value in Tab I indicates that the networks are ready to be tested predictively. It does not however appear to be an indication of the quality of the forecast.

The WNN forecast appears to gain quality with additional training and results suggest a critical turn-on number for sufficient training for predictive ability.

B. Forecast

Our results indicate that, using historical discharge data alone, ANN has little to no predictive ability, however the wavelet methods IDWT/WNN do offer some capability to forecast future discharges. The WNN performs the best likely to due the higher level wavelet decomposition incorporating more information of the signal. The fact that the IDWT at only level 1 shows a physically correct result in the first 7 days of the 14-day forecast is a good suggestion that this method will improve with more wavelet filtration.

V. CONCLUSION

Overall, WNN model for using longer training produces better results than other methods. Wavelet analysis improves the ability of a forecasting model by capturing useful information on various resolution levels of time series. ANN has great ability in modeling and forecasting time series. Combining both methods, we obtain better results than other methods. It is interesting to know what improvements can be made by changing various settings [6], [7] in the model such as level of resolutions, number of neurons, increasing the number of layers and different time periods.

Additionally, from this study it appears that R is a poor indicator for predictive power and that a better indicator of hydrological efficiency will be used in future work. This is evident by the R values of the ANN results and that methods' poor ability to produce a physically accurate hydrological prediction.

In future work we will expand the IDWT method to include higher level wavelet information. We also look to extend all methods to include climate information in the form of temperature and precipitation data.

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