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H_∞ Based on Adaptive Fuzzy Control design for Four Degree-of-Freedom of Drill-String System

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***Abstract*—The reduced-order models of a flexible drill-string systems allow for radial, bending, and torsion motions with stick-slip interactions between the drill-string and the outer shell is highly nonlinear and classified as under-actuated. Therefore, the drill-string system are very complex mechanical systems and have been the subject of research investigations for several decades. In this paper, a H_∞ based adaptive fuzzy control is proposed to control the four degree-of-freedom of the drill-string system. The advantage of employing an adaptive fuzzy system is using linear analytical results in place of estimating the dynamics of the drill-string system with an online update law. A robust control law combined with a variable structure (VS) and H_∞ control scheme is derived based on a Lyapunov criterion and the Riccai-inequality to overcome the system uncertainties, and external disturbances so that all signals of the closed-loop system are uniformly ultimately bounded (UUB). Simulations show that the proposed control scheme is effective in reducing the phenomenon of the drill-string system hitting with outer shell due to the drill-string system bending such that the rotation of a drill-string keep close to the radial center.**

Index Terms – Nonlinear systems; adaptive fuzzy control; H_∞ control; VS scheme; drill-string system.

I. Introduction

The drill-string system is a part of the rotary drill rig used for mining oil wells. A representative drill-rig system is illustrated in Fig. 1. The drill-string system combines with three subsystems, with one of them being the driving system, which includes the drill-string and bottom-hole-assembly used to carry out the drilling process. Structurally, the drill-string component, which is housed inside a drill pipe, includes the drill-string, drill collar, and drill bit. The upper and longer section of the drill-string is referred to as the drill-pipe. The lower part of the drill-string is called the Bottom Hole Assembly (BHA) [1]. The drill-string is mainly used for creating a bore hole, which runs miles below the ground. This system is surrounded by soil and rock, which can be considered as a core covered by outer shell pipes. When the drill rig is in operation, the driving torque is not always constant and there is contact between the drill-string and the outer shell as well as between the drill-string and the well bottom.

During drilling, the main vibrations and failures occur in the BHA where contact between the drill-string and the well [1], [2]. Most of drill-string systems of the drill-pipe are straight like the drill-string system of [3]. [4]-[5] developed the four degree-of-freedom reduced-order model to study the bending and torsion motions of the drill-string as well as the interactions with the outer shell. This kind of drill-string system is highly nonlinear and classified as under-actuated and a increase the difficulty of the controller design. Some researchers hold the structure of bit is a major cause of the stick-slip vibration, so they study to the mechanical structure [6]-[7]. Thus, various solutions have been proposed in the literature for controlling rotary system vibrations and to manipulate this problem of instability. Such as, adaptive control [3], sliding-mode control to conduct drill-string vibrations [3] and [8], classical controller as PID [9], and back-stepping control [10]. It is worth to mention, the dynamics of system are assumed known in the above literature, however, most of the dynamics of the paratical system are unknown. Thus, it is important work that how to design the controller for unknown dynamic system.

Adaptive fuzzy control techniques combine the advantages of fuzzy control and adaptive control methods resulting in an efficient algorithm to estimate the unknown nonlinear system [11]-[12]. It also designed to guarantee that the system outputs could track the anticipant signals. However, the matching error may diminish the tracking performance. Therefore, robust adaptive fuzzy scheme design for nonlinear systems with unknown or uncertain models have been attracted in last decade [13]-[14]. The VS scheme is a robust design methodology and used to deal with external disturbances, quickly varying parameters and unmodeled dynamics [13]. However, the external disturbance may be of finite-energy only, but not bounded. In recently, the H_∞ robust control theory has been developed and applied extensively in the efficient treatment of robust stabilisation and disturbance rejection problems, to solve the nonlinear control design problem under a state-space structure with bounded unknown or uncertain parameters and external disturbances such that the tracking error can be attenuated to a prescribed level and system performance can be improved [14].

In this paper, the proposed robust control law for drill-string system is based on VS adaptive fuzzy control. The adaptive fuzzy scheme uses a VS scheme to resolve the system uncertainties, and external disturbances such that H_∞ tracking performance is achieved. The control laws are derived based on a Lyapunov criteria and the Riccati-inequality such that all states of the system are UUB. Therefore, the effect can be reduced to a prescribed level to achieveH_∞tracking performance. Finally, simulations show that the proposed control scheme is effective in reducing the phenomenon of the drill-string system hitting with outer shell due to the drill-string

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system bending such that the rotation of a drill-string keep close to the radial center.

II. System Description and Problem Formulation

The drill-string systems is comprised of spatially continuous members and discrete elements, in an effort to understand the stick-slip interactions in this system, reduced-order models have been developed in this effort. A section of the spatially continuous and rotating drill-string is modeled as a system of two rotating sections with an unbalanced mass attached to one of them. The parameters used in the development of the four degree-of-freedom model and the diagrams of model are shown in Figs. 2 and 3. In this drill-string systems, Section 2 is referred to as the rotor, and Section 1 is referred to as the stator throughout this paper as shown in Fig. (3). The ρ is the radial or lateral displacement, θ is the rotation angle associated with the first section, ϕ is the rotation due to bending, and α is the rotation angle associated with the second section. In addition, m_b is the unbalanced mass located at a distance e from the axis of rotation of the second section, as shown in Fig. 3. A linear spring contact model is used to depict the interactions between the drill-string and the outer shell or borehole that is created. For the system shown in Fig. 2, by accounting for the continuous and the discrete elements, the Lagrangian can be formed as

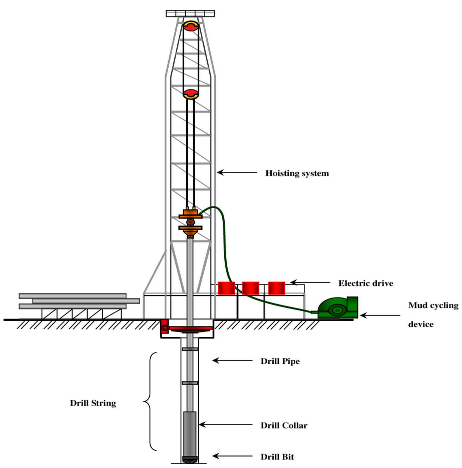

Fig. 1. Representative schematic of a rotary drill rig.

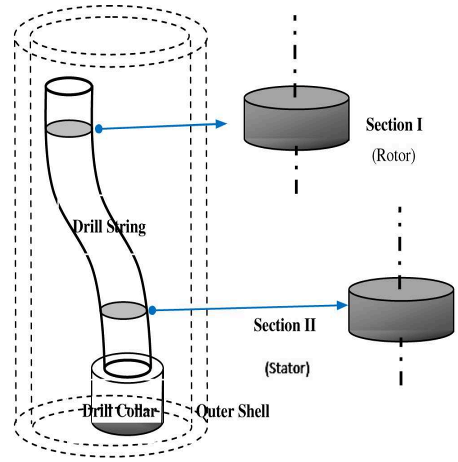

Fig. 2. Illustration of two section model.

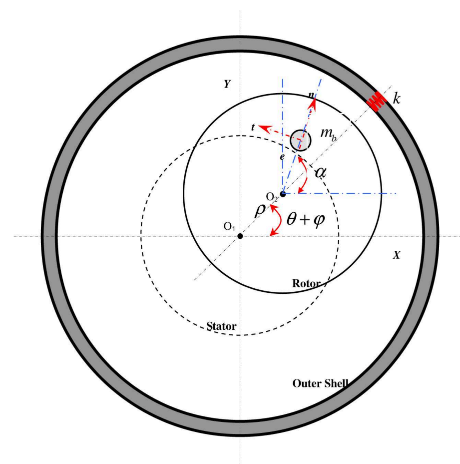

Fig. 3. Schematic of model with four degrees of freedom.

$$
\begin{aligned}
L_{\text{system}} &= L_{\text{continuous element}} + L_{\text{discrete element I+II}} \\
&= \int_0^l (T_{\text{continuous element}} - V_{\text{continuous element}})dz \\
&+ T_{\text{discrete element I}} + T_{\text{discrete element II}} \\
&- V_{\text{discrete element I}} - V_{\text{discrete element II}}
\end{aligned} \tag{1}
$$

where L represents the system Lagrangian, T and V are the kinetic and potential energy components, respectively, and z is a parameter along the axial direction of the drill-string of length l. Overlooking the inertia properties of the continuous element and taking into account only the stiffness properties of the continuous element and the unbalanced mass, the energy expressions for the system are formed as given next. The system kinetic energy is constructed as

$$
\begin{aligned}
T_{\text{total}} &\approx T_{\text{discrete element I}} + T_{\text{discrete element II}} \\
T_{\text{total}} &= \frac{1}{2}I_1\dot{\theta}^2 + \frac{1}{2}(m+m_b)(\dot{\rho}^2+\rho^2(\dot{\theta}+\dot{\varphi})^2) + \frac{1}{2}I_2\dot{\alpha}^2 \\
&+ \frac{1}{2}m_b e^2\dot{\alpha}^2 + m_b e\dot{\alpha}[\rho(\dot{\theta}+\dot{\varphi})\text{cos} - \dot{\rho}\text{sin} \\
&\times (\alpha-\theta-\varphi)]
\end{aligned} \tag{2}
$$

where the different inertia parameters are appropriately defined. The system potential energy is constructed as

$$
\begin{aligned}
V_{\text{total}} &= V_{\text{continuous element}} + V_{\text{discrete element I+II}} \\
&\approx \frac{1}{2}K_r(\rho-\rho_0)^2 + \frac{1}{2}K_t(\rho\varphi)^2 + \frac{1}{2}K_{tor}(\alpha-\theta)^2 \\
&+ \frac{1}{2}\lambda K_p[\rho - \frac{1}{2}(D-d)]^2
\end{aligned} \tag{3}
$$

where the different stiffness constants are appropriately defined and λ is a parameter used to capture the contact between the rotor and the outer shell. This parameter takes a value of 0 when there is no contact between the rotor and the outer shell and 1 when there is contact. The virtual work associated with the external forces and moments is given by

$$\delta W_{ext} = -\lambda F_t[R(\delta\alpha) + \rho(\delta\theta + \delta\varphi)] + M_{ext}\delta\alpha \tag{4}$$

A. Four Degree-of-freedom Model

By using the extended Hamilton's principle, the equations of motion are obtained as follows:

$$(m+m_b)\ddot{\rho} - (m+m_b)\rho(\dot{\theta}+\dot{\varphi})^2 + K_r(\rho-\rho_0) + \lambda K_p \times(\rho-\delta) + K_t\rho\varphi^2 - em_b[\ddot{\alpha}sin(\beta) + \dot{\alpha}^2cos(\beta)] = 0 \tag{5}$$

$$I_1\ddot{\theta} + (m+m_b)\rho^2(\ddot{\theta}+\ddot{\varphi}) + 2(m+m_b)\rho\dot{\rho}(\dot{\theta}+\dot{\varphi}) - K_{tor}(\alpha-\theta) - em_b\rho(\dot{\alpha}^2sin(\beta) - \ddot{\alpha}cos(\beta)) = -\lambda F_t\rho \tag{6}$$

$$(m+m_b)\rho(\ddot{\theta}+\ddot{\varphi}) + 2(m+m_b)\dot{\rho}(\dot{\theta}+\dot{\varphi}) + K_t\rho\varphi - em_b\left[dot\alpha^2sin(\beta) - \ddot{\alpha}cos(\beta)\right] = -\lambda F_t \tag{7}$$

$$em_b[-\ddot{\rho}sin(\beta)\rho(\ddot{\theta}+\ddot{\varphi})cos(\beta) + \rho(\dot{\theta}+\dot{\varphi})^2sin(\beta) + 2\dot{\rho}(\dot{\theta}+\dot{\varphi}) \times cos(\beta)] = M_{ext} - \lambda F_tR + (I_2+m_be^2)\ddot{\alpha} + K_{tor}(\alpha-\theta) \tag{8}$$

where

$$\beta = \alpha - (\theta+\varphi) \tag{9}$$

The motions of the drill-string system are described by (5)-(9) in terms of the coordinates ρ, θ, φ, and α. The force F_t is generated due to contact between the rotor and the outer shell.

B. Contact Parameter λ and Stick-Slip Interaction.

The stick-slip interactions between the drill-string and the outer shell are modeled according to previous studies [15]. The different cases considered here are as follows: (i) No contact between the outer edge of the string and the shell, i.e., λ=0, and the normal contact force $\boldsymbol{F}_{\text{normal}}$ is zero in this case, (ii) there is contact and only rotation, and no sliding, as shown in Fig. 4(b), and (iii) there is contact and pure sliding and no rotation, as shown in Fig. 4(c). In the present work, the possibility for combined rolling and slipping is not included. Eqs. (15)-(20) are used to determine and describe the contact between the drill-string and the outer shell, and the tangential force used in Eqs. (5)-(8) is determined from Eq. (13). The parameter δ in Eq. (10) is the radial separation between the outer shell and the drill-string, and this parameter is used to judge whether there is contact or not. The relative velocity $\boldsymbol{V}_{\text{relative}}$ between the two contacting surfaces is used to determine whether there is sliding or not. Eq. (15) is determined on the basis of a pure rolling mode. The maximum tangential force is denoted as $\boldsymbol{F}_{\text{t max}}$.

$$\delta = 0.5\cdot(D-d) \tag{10}$$

$$\lambda = \begin{cases} 0; & \rho \le \delta \\ 1; & \rho > \delta \end{cases} \tag{11}$$

$$\boldsymbol{F}_{\text{normal}} = \begin{cases} 0 & \rho \le \delta \\ \boldsymbol{K}_p\cdot(\rho-\delta) & \rho > \delta \end{cases} \tag{12}$$

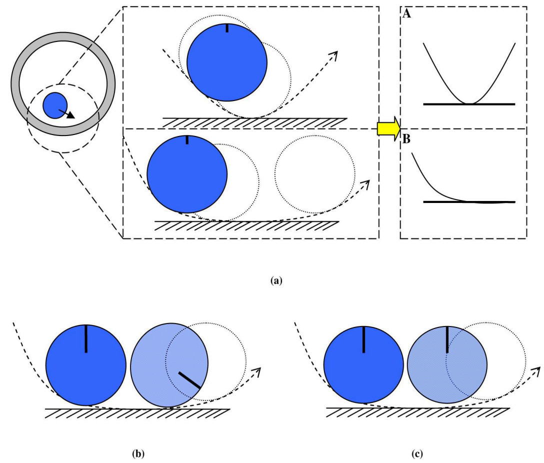

Fig. 4. Illustration of contact scenarios between drill-string and outer shell: (a) two contact scenarios, (b) rotation with no sliding, and (c) pure sliding with no rotation.

$$\boldsymbol{F}_t = \begin{cases} \boldsymbol{F}_{\text{tequ}}; & \boldsymbol{V}_{\text{relative}} = 0 \text{ and } |\boldsymbol{F}_{\text{t max}}| \le |\boldsymbol{F}_{\text{tequ}}| \\ \boldsymbol{F}_{\text{t max}}; & \text{else} \end{cases} \tag{13}$$

$$\boldsymbol{F}_{\text{t max}} = -\text{sgn}(\boldsymbol{V}_{\text{relative}})\cdot\mu\cdot\boldsymbol{F}_{\text{normal}} \tag{14}$$

$$\boldsymbol{F}_{\text{tequ}} = -\frac{\boldsymbol{M}_{\text{ext}}}{\frac{2\boldsymbol{I}_2}{m\cdot d} + 0.5\cdot d} \tag{15}$$

Either rotation of sliding is the phenomenon of drill-string hitting outer shell, it can be viewed as the external disturbance of system.

C. Dynamics of Four degree-of-freedom Drill-String System.

Consider the dynamical equation of the four degree-of-freedom drill-string system with disturbance is described as follows:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}(t) + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}(t) + \boldsymbol{W}(\boldsymbol{q}) = \boldsymbol{U}(t) + \boldsymbol{d}(t) \tag{16}$$

where $\boldsymbol{M}(\boldsymbol{q})$ is the 4×4 mass matrix, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the 4×4 damping matrix, $\boldsymbol{W}(\boldsymbol{q})$ is the 4×4 stiffness matrix, $\boldsymbol{U}$=$[0\ 0\ 0\ u]^\top$ is the input motor torque, and $\boldsymbol{d}$ = $[d_1\ d_2\ d_3\ d_4]^\top$ is the disturbance acting on the drill-string system. The matrix $\boldsymbol{M}(\boldsymbol{q})$ is positive-definite for every $\boldsymbol{q}$. $\boldsymbol{M}(\boldsymbol{q})$ and $\boldsymbol{M}(\boldsymbol{q})^{-1}$ are assumed uniformly bounded. Eq. (16) can be rewritten as

$$\ddot{\boldsymbol{q}}(t) = \boldsymbol{C}'(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}(t) + \boldsymbol{W}'(\boldsymbol{q}) + \boldsymbol{H}(\boldsymbol{q})\boldsymbol{U}(t) + \boldsymbol{d}'(t) \tag{17}$$

where $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$=$-\boldsymbol{M}(\boldsymbol{q})^{-1}\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$, $\boldsymbol{d}'(t)$=$\boldsymbol{M}(\boldsymbol{q})^{-1}\boldsymbol{d}(t)$, $\boldsymbol{W}'(\boldsymbol{q})$=$-\boldsymbol{M}(\boldsymbol{q})^{-1}\boldsymbol{W}(\boldsymbol{q})$, and $\boldsymbol{H}(\boldsymbol{q})$=$\boldsymbol{M}(\boldsymbol{q})^{-1}$.
In order to designs controller in later chapters, effortlessly, we

rewritten eq. (17) as follows:

$$\begin{cases}\dot{x}_{1_1} = x_{2_1}\\ \dot{x}_{1_2} = x_{2_2}\\ \dot{x}_{1_3} = x_{2_3}\\ \dot{x}_{1_4} = x_{2_4}\\ \dot{x}_{2_1} = f_1(x) + g_1(x)u + d'_1\\ \dot{x}_{2_2} = f_2(x) + g_2(x)u + d'_2\\ \dot{x}_{2_3} = f_3(x) + g_3(x)u + d'_3\\ \dot{x}_{2_4} = f_4(x) + g_4(x)u + d'_4\end{cases} \tag{18}$$

where $f_j(\boldsymbol{x})$ and $g_j(\boldsymbol{x})$ are unknown but bounded non-linear continuous functions, d_j is the external disturbance, $j = 1, \cdots, 4$. All of the elements of $f_1, \cdots, f_4$, $g_1, \cdots, g_4$, and $d_1{}' \cdots, d_4{}'$ will be described in section IV. Let $\boldsymbol{x}$=$[\boldsymbol{x}_1^\top \boldsymbol{x}_2^\top]^\top$=$[\boldsymbol{q}^\top \dot{\boldsymbol{q}}^\top]^\top$=$[\rho\ \theta\ \varphi\ \alpha\ \dot{\rho}\ \dot{\theta}\ \dot{\varphi}\ \dot{\alpha}]^\top$=$[x_{1_1}\, x_{1_2}\, x_{1_3}\, x_{1_4}\, \dot{x}_{1_1}\, \dot{x}_{1_2}\, \dot{x}_{1_3}\, \dot{x}_{1_4}]^\top$=$[x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8]^\top$
Thus, Eq. (18) can be written as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_0\boldsymbol{x} + \boldsymbol{B}(\mathbf{F}(\boldsymbol{x}) + \mathbf{G}(\boldsymbol{x})u) + \boldsymbol{d}'(t) \tag{19}$$

where

$$\boldsymbol{A}_0 = \begin{bmatrix} 0_{(4)} & \mathbf{I}_{(4)} \\ 0_{(4)} & 0_{(4)} \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} 0_{(4)} \\ \mathbf{I}_{(4)} \end{bmatrix}$$

$$\mathbf{F}(\boldsymbol{x}) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \boldsymbol{C}'\boldsymbol{x}_1 + \boldsymbol{W}'\boldsymbol{x}_2,$$

$$\mathbf{G}(\boldsymbol{x})u = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{bmatrix} u = \boldsymbol{H}(\boldsymbol{x}) \begin{bmatrix} 0 \\ 0 \\ 0 \\ u \end{bmatrix} = \boldsymbol{H}(\boldsymbol{x})\boldsymbol{U}$$

and $\boldsymbol{d}'(t)$= $[d'_1\ d'_2\ d'_3\ d'_4]^\top$. The input matrix $\mathbf{G}(\boldsymbol{x})$ is assumed nonsingular and bounded for all $\boldsymbol{x} \in U_{\boldsymbol{x}}$, where $U_{\boldsymbol{x}} \subset \boldsymbol{R}^8$ is some compact set. Also, $\|\boldsymbol{d}'\| \leq \epsilon$, where ϵ is a positive constant. The proposed control law in this paper is based on VS adaptive fuzzy control. The purpose of the paper is to synthesize a fuzzy adaptive VS control scheme for drill-string velocity and rotation velocity (due to bending) of drill-string systems so that drill-string systems states can asymptotically track the given desired reference signals. Let $\boldsymbol{x}_r = [\boldsymbol{x}_{r_1}^\top \boldsymbol{x}_{r_2}^\top]^\top$=$[x_{r_{1_1}}\ x_{r_{1_2}}\ x_{r_{1_3}}\ x_{r_{1_4}}\ \dot{x}_{r_{1_1}}\ \dot{x}_{r_{1_2}}\ \dot{x}_{r_{1_3}}\ \dot{x}_{r_{1_4}}]^\top$=$[x_{r_1}\ x_{r_2}\ x_{r_3}\ x_{r_4}\ x_{r_5}\ x_{r_6}\ x_{r_7}\ x_{r_8}]^\top$ be the tracking reference signal and assume that, there is a compact set Ω_r such that $\boldsymbol{x}_r \in \Omega_r$, $\forall t \leq 0$. Define the output tracking error as

$$\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x}_r = [\boldsymbol{e}_1^\top\ \boldsymbol{e}_2^\top]^\top = [e_1\ e_2 \cdots\ e_8]^\top$$

Then, the error dynamic equation can be obtained as

$$\dot{\boldsymbol{e}} = \boldsymbol{A}_0\boldsymbol{e} + \boldsymbol{B}(\mathbf{F}(\boldsymbol{x}) + \mathbf{G}(\boldsymbol{x})u + \boldsymbol{d}' - \ddot{\boldsymbol{x}}_r) \tag{20}$$

where $\ddot{\boldsymbol{x}}_r = [\ddot{x}_{r_1}\ \ddot{x}_{r_2}\ \ddot{x}_{r_3}\ \ddot{x}_{r_4}]^\top$. Choose a feedback gain matrix $\boldsymbol{K}$ =$[\boldsymbol{k}_1\ \ \boldsymbol{k}_2]$, $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are 4×4 matrices such that the characteristic polynomial of $\boldsymbol{A} = \boldsymbol{A}_0 - \boldsymbol{B}\boldsymbol{K}$ to be Hurwitz.

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{e} + \boldsymbol{B}\big(\boldsymbol{K}\boldsymbol{e} + \mathbf{F}(\boldsymbol{x}) + \mathbf{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{d}' - \ddot{\boldsymbol{x}}_r\big) \tag{21}$$

III. INDIRECT ADAPTIVE FUZZY CONTROL

A fuzzy adaptive system is defined as an fuzzy logic system provided with a learning algorithm, where the fuzzy system is builded from a set of fuzzy IF-THEN rules using fuzzy logic laws in the following form:

$$\begin{aligned} R^k : \text{IF } x_1 \text{ is } C_1^k \text{ and } x_2 \text{ is } C_2^k \text{ and} \cdots \text{and } x_8 \text{ is } C_8^k,\\ \text{THEN } w \text{ is } E^k, k = 1, 2, \cdots, N \end{aligned} \tag{22}$$

where x_i, $i = 1, 2, \cdots, 8$, and w are the input and output of the fuzzy logic system, respectively, N is the number of fuzzy rules, and C_i^k, $i = 1, 2, \cdots, 8$, and E^k are fuzzy sets. Using the product inference engine, center-average defuzzifier, and strategy of singleton fuzzifier, the output of fuzzy system can be concluded as

$$w(x) = \frac{\sum_{k=1}^{N} \prod_{i=1}^{8} \mu_{C_i^k}(x_i)\bar{w}^k}{\sum_{k=1}^{N}(\prod_{i=1}^{8} \mu_{C_i^k}(x_i))} \tag{23}$$

where $\mu_{C_i^k}(x_i)$ is the membership function of the kth fuzzy set for the input and $\bar{w}^k$=$max_{w\in R}\ \mu_{E^k}(\boldsymbol{w})$, in which $\mu_{E^k}(\boldsymbol{w})$ is the membership function of the jth fuzzy set for the output. The fuzzy basis functions can be defined as

$$\xi^k = \frac{\prod_{i=1}^{8} \mu_{C_i^k}(x_i)}{\sum_{k=1}^{N}(\prod_{i=1}^{8} \mu_{C_i^k}(x_i))} \tag{24}$$

Denoting $\boldsymbol{\xi}(x)$=$[\xi^1(x)\xi^2(x) \cdots \xi^N(x)]^\top$, the learning algorithm adjusts the parameters of the fuzzy system based on the training information $\boldsymbol{\theta}$=$[\bar{w}^1\bar{w}^2 \cdots \bar{w}^N]^\top$. Then equation (23) can be written as

$$w(x) = \boldsymbol{\theta}^\top\boldsymbol{\xi}(x) \tag{25}$$

Since f_j and g_j, $j = 1, \cdots, 4$, in (18) are unknown, they can be approximated by the fuzzy system (25) and expressed as follows:

$$\begin{aligned}\hat{f}_j(\boldsymbol{x}) = \boldsymbol{\theta}_{f_j}^\top\boldsymbol{\xi}_{f_j}(\boldsymbol{x})\\ \hat{g}_j(\boldsymbol{x}) = \boldsymbol{\theta}_{g_j}^\top\boldsymbol{\xi}_{g_j}(\boldsymbol{x})\end{aligned} \tag{26}$$

where $\boldsymbol{\theta}_{f_j}$ and $\boldsymbol{\theta}_{g_j}$ are the adjustable parameter vectors, $\boldsymbol{\xi}_{f_j}(\boldsymbol{x})$ and $\boldsymbol{\xi}_{g_j}(\boldsymbol{x})$ are the vector of fuzzy basis functions. Define

$$\hat{\mathbf{F}}(\boldsymbol{x}) = [\hat{f}_1(\boldsymbol{x}) \cdots \hat{f}_4(\boldsymbol{x})]^\top = \boldsymbol{\Xi}_f(\boldsymbol{x})\boldsymbol{\Theta}_f \tag{27}$$

$$\hat{\mathbf{G}}(\boldsymbol{x}) = [\hat{g_1}(\boldsymbol{x}) \cdots \hat{g_4}(\boldsymbol{x})]^\top = \boldsymbol{\Xi}_g(\boldsymbol{x})\boldsymbol{\Theta}_g \tag{28}$$

where $\boldsymbol{\Xi}_f(\mathbf{x}) = diag\{\boldsymbol{\xi}_{f_1}^\top, \boldsymbol{\xi}_{f_2}^\top, \boldsymbol{\xi}_{f_3}^\top, \boldsymbol{\xi}_{f_4}^\top$, $\boldsymbol{\Xi}_g(\mathbf{x}) = diag\{\boldsymbol{\xi}_{g_1}^\top, \boldsymbol{\xi}_{g_2}^\top, \boldsymbol{\xi}_{g_3}^\top, \boldsymbol{\xi}_{g_4}^\top$, $\boldsymbol{\Theta}_f = [\boldsymbol{\theta}_{f_1}^\top\boldsymbol{\theta}_{f_2}^\top\boldsymbol{\theta}_{f_3}^\top\boldsymbol{\theta}_{f_4}^\top]^\top$, and $\boldsymbol{\Theta}_g = [\boldsymbol{\theta}_{g_1}^\top\boldsymbol{\theta}_{g_2}^\top\boldsymbol{\theta}_{g_3}^\top\boldsymbol{\theta}_{g_4}^\top]^\top$. Then, according to the universal approximation theorem, there exists optimal approximation parameters $\boldsymbol{\Theta}_f^*$ and $\boldsymbol{\Theta}_g^*$ which lead to minimum approximation errors for $\mathbf{F}(\boldsymbol{x})$ and $\mathbf{G}(\boldsymbol{x})$, respectively, and expressed as follows:

$$\Delta\mathbf{F}(\boldsymbol{x}) = \mathbf{F}(\boldsymbol{x}) - \hat{\mathbf{F}}(\boldsymbol{x})|\boldsymbol{\Theta}_f^*) \tag{29}$$

$$\Delta\mathbf{G}(\boldsymbol{x}) = \mathbf{F}(\boldsymbol{x}) - \hat{\mathbf{G}}(\boldsymbol{x})|\boldsymbol{\Theta}_g^*) \tag{30}$$

where $\Delta\mathbf{F}(\boldsymbol{x})$ and $\Delta\mathbf{G}(\boldsymbol{x})$ are the minimum approximation errors assumed to be bounded, and

$$\boldsymbol{\Theta}_f^* = arg \min_{\boldsymbol{\Theta}_f \in \Omega_x} \left\{ \sup_{\mathbf{x}\in U_x} |\hat{\mathbf{F}}(\mathbf{x})|\boldsymbol{\Theta}_f) - \mathbf{F}(\mathbf{x}) \right\}$$

$$\boldsymbol{\Theta}_g^* = arg \min_{\boldsymbol{\Theta}_f \in \Omega_x} \left\{ \sup_{\mathbf{x}\in U_x} |\hat{\mathbf{G}}(\mathbf{x})|\boldsymbol{\Theta}_g) - \mathbf{G}(\mathbf{x}) \right\}$$

The approximation errors are defined as

$$\tilde{\boldsymbol{\Theta}}_f = \boldsymbol{\Theta}_f^* - \boldsymbol{\Theta}_f \tag{31}$$

$$\tilde{\boldsymbol{\Theta}}_g = \boldsymbol{\Theta}_g^* - \boldsymbol{\Theta}_g \tag{32}$$

The following VS scheme adaptive fuzzy-based control law can be obtained

$$u = \hat{\mathbf{G}}^*(\boldsymbol{x}|\boldsymbol{\Theta}_g)\big(-\hat{\mathbf{F}}(\boldsymbol{x}|\boldsymbol{\Theta}_f) + \ddot{\boldsymbol{x}}_r - \boldsymbol{K}^\top \boldsymbol{e} + \boldsymbol{u}_h + \boldsymbol{u}_s\big) \tag{33}$$

where $\hat{\mathbf{G}}^*$ denotes the pseudo-inverse of $\hat{\mathbf{G}}$, i.e., $\hat{\mathbf{G}}\hat{\mathbf{G}}^*\hat{\mathbf{G}} = \hat{\mathbf{G}}$, $\boldsymbol{u}_h$, and $\boldsymbol{u}_s$ are compensators described later for disturbance, and uncertainties, respectively. Then, substituting (33) into eq. (20), the error dynamic equation can be obtained as

$$\begin{aligned}\dot{\boldsymbol{e}} &= \boldsymbol{A}\boldsymbol{e} + \boldsymbol{B}\big(-\boldsymbol{\Xi}_f^\top \tilde{\boldsymbol{\Theta}}_f - \boldsymbol{\Xi}_g^\top \tilde{\boldsymbol{\Theta}}_g u + \Delta\mathbf{F} + \Delta\mathbf{G}u + \boldsymbol{u}_s + \boldsymbol{u}_h + \boldsymbol{d}'\big) \\ &= \boldsymbol{A}\boldsymbol{e} + \boldsymbol{B}\big(-\boldsymbol{\Xi}_f^\top \tilde{\boldsymbol{\Theta}}_f - \boldsymbol{\Xi}_g^\top \tilde{\boldsymbol{\Theta}}_g u + \Delta\mathbf{F} + \Delta\mathbf{G}\hat{\mathbf{G}}^*(\boldsymbol{x}|\boldsymbol{\Theta}_g) \\ &\quad \times (-\hat{\mathbf{F}}(\boldsymbol{x}|\boldsymbol{\Theta}_f) + \ddot{\boldsymbol{x}}_r - \boldsymbol{K}^\top \boldsymbol{e} + \boldsymbol{u}_h + \boldsymbol{u}_s) + \boldsymbol{u}_h + \boldsymbol{u}_s + \boldsymbol{d}'\big)\end{aligned} \tag{34}$$

Throughout this research, we need the following assumptions:

Assumption 1 : There are exist positive constants $\kappa_f > 0$, $\kappa_g > 0$, and a positive function $0 \le \alpha_g(\mathbf{x}) < 1$ such that $|\Delta\mathbf{F}_j| \le \kappa_f$, $|\lambda_j(\Delta\mathbf{G}(\mathbf{x})\hat{\mathbf{G}}^*(\mathbf{x}|\boldsymbol{\Theta}_g))| \le \kappa_g$, and $|\Delta\mathbf{G}(\mathbf{x})\hat{\mathbf{G}}^*(\mathbf{x}|\boldsymbol{\Theta}_g)_j| \le \alpha_g(\mathbf{x})$, where $\Delta\mathbf{F}_j$ is the jth element of $\Delta\mathbf{F}$, $\lambda_j(\Delta\mathbf{G}(\mathbf{x})\hat{\mathbf{G}}^*(\mathbf{x}|\boldsymbol{\Theta}_g))$ is the jth eigenvalue of $\Delta\mathbf{G}(\mathbf{x})\hat{\mathbf{G}}^*(\mathbf{x}|\boldsymbol{\Theta}_g)$, and $\Delta\mathbf{G}(\mathbf{x})\hat{\mathbf{G}}^*(\mathbf{x}|\boldsymbol{\Theta}_g)_j$ is the jth element of $\Delta\mathbf{G}(\mathbf{x})\hat{\mathbf{G}}^*(\mathbf{x}|\boldsymbol{\Theta}_g)$, $j = 1, \ldots, 4$.

Theorem 1 : H_∞ Performance: An H_∞ performance is considered as follows [16], [17]:

$$\begin{aligned}\int_0^t \boldsymbol{e}^\top \boldsymbol{Q}\boldsymbol{e}\; dt &\le \boldsymbol{e}^\top(0)\boldsymbol{P}\boldsymbol{e}(0) + \frac{1}{\gamma_f}\boldsymbol{\Theta}_f^\top(0)\boldsymbol{\Theta}_f(0) \\ &\quad + \frac{1}{\gamma_g}\boldsymbol{\Theta}_g^\top(0)\boldsymbol{\Theta}_g(0) + \rho^2 \int_0^t (\boldsymbol{d}'^\top \boldsymbol{d}')\; dt.\end{aligned} \tag{35}$$

where $\boldsymbol{Q} > 0$, $\boldsymbol{P} = \boldsymbol{P}^\top > 0$, and ρ^2 is the prescribed attenuation value which denote the worst case effect of the external disturbances $\boldsymbol{d}'$ on the tracking error $\boldsymbol{e}$. The physical meaning of performance in (35) is that the effects of $\boldsymbol{d}'$ on $\boldsymbol{e}$. It must be attenuated below a desired level ρ from the energy viewpoint, whether what $\boldsymbol{d}'$ is, i.e., the L_2-gain from $\boldsymbol{d}'$ to $\boldsymbol{e}$ must be equal to or less than a prescribed value ρ^2. In general, ρ is chosen as a positive small value less than one for attenuation of $\boldsymbol{d}'$. ■

Theorem 2 : Consider the four degree-of-freedom of drill-string systems (18) and the modeling system (20). Let the variable structure adaptive fuzzy-based control law be given as in (33) with

$$\boldsymbol{u}_h = \frac{1}{2(1-\kappa_g)}\boldsymbol{R}^{-1}\boldsymbol{B}^\top \boldsymbol{P}\boldsymbol{e} \tag{36}$$

$$\boldsymbol{u}_s = -\frac{M_e(\boldsymbol{x})}{1-\kappa_g} sgn(\boldsymbol{B}^\top \boldsymbol{P}\boldsymbol{e}) \tag{37}$$

where $\boldsymbol{R}$ denotes the robust H_∞ control gain, $M_e \triangleq |(\Delta\mathbf{F} + \Delta\mathbf{G}\hat{\mathbf{G}}^*(\boldsymbol{x}|\boldsymbol{\Theta}_g)(-\hat{\mathbf{F}} + \ddot{\boldsymbol{x}}_r - \boldsymbol{K}^\top \boldsymbol{e}))_j|$ is the absolute value of the qth element of $\Delta\mathbf{F} + \Delta\mathbf{G}\hat{\mathbf{G}}^*(\boldsymbol{x}|\boldsymbol{\Theta}_g)(-\hat{\mathbf{F}} + \ddot{\boldsymbol{x}}_r - \boldsymbol{K}^\top \boldsymbol{e})$, $j = 1, \ldots, 4,$, and $\boldsymbol{P} = \boldsymbol{P}^\top > 0$ is a symmetric positive definite matrix and satisfying the Riccati-like equation as follow

$$\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^\top \boldsymbol{P} + \boldsymbol{Q} + \boldsymbol{P}\boldsymbol{B}(\frac{1}{\rho^2}\boldsymbol{I} - \boldsymbol{R}^{-1})\boldsymbol{B}^\top \boldsymbol{P} \le 0 \tag{38}$$

$\boldsymbol{Q} = \boldsymbol{Q}^\top > 0$ is a weighting matrix, and $0 < \rho < 1$ denotes a prescribed attenuation level, and the adaptive parameter adjustment laws are chosen as follow:

$$\dot{\tilde{\boldsymbol{\Theta}}}_f^\top = \gamma_f \boldsymbol{\Xi}_f^\top \boldsymbol{B}^\top \boldsymbol{P}\boldsymbol{e} \tag{39}$$

$$\dot{\tilde{\boldsymbol{\Theta}}}_g^\top = \gamma_g \boldsymbol{\Xi}_g^\top \boldsymbol{B}^\top \boldsymbol{P}\boldsymbol{e}u \tag{40}$$

Proof: Consider the Lyapunov Function Candidate

$$V = \frac{1}{2}\boldsymbol{e}^\top \boldsymbol{P}\boldsymbol{e} + \frac{1}{2\gamma_f}\tilde{\boldsymbol{\Theta}}_f^\top \tilde{\boldsymbol{\Theta}}_f + \frac{1}{2\gamma_g}\tilde{\boldsymbol{\Theta}}_g^\top \tilde{\boldsymbol{\Theta}}_g \tag{41}$$

The time derivative of V is

$$\dot{V} = \frac{1}{2}\dot{\boldsymbol{e}}^\top \boldsymbol{P}\boldsymbol{e} + \frac{1}{2}\boldsymbol{e}^\top \boldsymbol{P}\dot{\boldsymbol{e}} + \frac{1}{\gamma_f}\dot{\tilde{\boldsymbol{\Theta}}}_f^\top \tilde{\boldsymbol{\Theta}}_f + \frac{1}{\gamma_g}\dot{\tilde{\boldsymbol{\Theta}}}_g^\top \tilde{\boldsymbol{\Theta}}_g \tag{42}$$

Substituting (34) into (42) leads to:

$$\begin{aligned}\dot{V} &= \frac{1}{2}(\boldsymbol{A}\boldsymbol{e} + \boldsymbol{B}(-\boldsymbol{\Xi}_f^\top \tilde{\boldsymbol{\Theta}}_f - \boldsymbol{\Xi}_g^\top \tilde{\boldsymbol{\Theta}}_g u + \Delta\mathbf{F} + \Delta\mathbf{G}u + \boldsymbol{u}_h + \boldsymbol{u}_s \\ &\quad + \boldsymbol{d}'))^\top \mathbf{P}\mathbf{e} + \frac{1}{2}\boldsymbol{e}^\top \boldsymbol{P}(\boldsymbol{A}\boldsymbol{e} + \boldsymbol{B}(-\boldsymbol{\Xi}_f^\top \tilde{\boldsymbol{\Theta}}_f - \boldsymbol{\Xi}_g^\top \tilde{\boldsymbol{\Theta}}_g u + \Delta\mathbf{F} \\ &\quad + \Delta\mathbf{G}u + \boldsymbol{u}_h + \boldsymbol{u}_s + \boldsymbol{d}')) + \frac{1}{\gamma_f}\dot{\tilde{\boldsymbol{\Theta}}}_f^\top \tilde{\boldsymbol{\Theta}}_f + \frac{1}{\gamma_g}\dot{\tilde{\boldsymbol{\Theta}}}_g^\top \tilde{\boldsymbol{\Theta}}_g \\ &= \frac{1}{2}\boldsymbol{e}^\top(\boldsymbol{A}^\top \boldsymbol{P} + \boldsymbol{P}\boldsymbol{A})\boldsymbol{e} - \boldsymbol{e}^\top \boldsymbol{P}\boldsymbol{B}\boldsymbol{\Xi}_f^\top \tilde{\boldsymbol{\Theta}}_f - \boldsymbol{e}^\top \boldsymbol{P}\boldsymbol{B}\boldsymbol{\Xi}_g^\top \tilde{\boldsymbol{\Theta}}_g u \\ &\quad + \boldsymbol{e}^\top \boldsymbol{P}\boldsymbol{B}(\Delta\mathbf{F} + \Delta\mathbf{G}\hat{\mathbf{G}}^*(x|\boldsymbol{\Theta}_g)(-\mathbf{F} + \ddot{\boldsymbol{x}}_r - \mathbf{K}^\top \boldsymbol{e})) \\ &\quad + \boldsymbol{e}^\top \boldsymbol{P}\boldsymbol{B}(\mathbf{I} + \Delta\mathbf{G}\mathbf{G}^*(x|\boldsymbol{\Theta}_g))(\boldsymbol{u}_s + \boldsymbol{u}_h) + \frac{1}{\gamma_f}\dot{\tilde{\boldsymbol{\Theta}}}_f^\top \tilde{\boldsymbol{\Theta}}_f \\ &\quad + \boldsymbol{e}^\top \boldsymbol{P}\boldsymbol{B}\boldsymbol{d}' + \frac{1}{\gamma_g}\dot{\tilde{\boldsymbol{\Theta}}}_g^\top \tilde{\boldsymbol{\Theta}}_g\end{aligned} \tag{43}$$

Substituting adaptive laws (39) and (40) into (43) leads to:

$$\begin{aligned}\dot{V} &\le \frac{1}{2}\boldsymbol{e}^\top(\boldsymbol{A}^\top \boldsymbol{P} + \boldsymbol{P}\boldsymbol{A})\boldsymbol{e} + \boldsymbol{e}^\top \boldsymbol{P}\boldsymbol{B}(\Delta\mathbf{F} + \Delta\mathbf{G}\hat{\mathbf{G}}^*(x|\boldsymbol{\Theta}_g) \\ &\quad \times (-\mathbf{F} + \ddot{\boldsymbol{x}}_r - \mathbf{K}^\top \boldsymbol{e})) + \boldsymbol{e}^\top \boldsymbol{P}\boldsymbol{B}(\mathbf{I} + \Delta\mathbf{G}\mathbf{G}^*(x|\boldsymbol{\Theta}_g)) \\ &\quad \times (\boldsymbol{u}_h + \boldsymbol{u}_s) + \boldsymbol{e}^\top \boldsymbol{P}\boldsymbol{B}\boldsymbol{d}'\end{aligned} \tag{44}$$

By completing the squares and using the control input $\mathbf{u}_h$ in (44) and $|\lambda_j(\Delta\mathbf{G}(\mathbf{x})\hat{\mathbf{G}}^*(\mathbf{x}|\boldsymbol{\Theta}_g))| \le \kappa_g$, we have

TABLE I
SYSTEM PARAMETER VALUES

Quantities	Variable	Value	Units
Mass of rotor	m	7.08×10^{-1}	kg
Unbalanced mass on rotor	m_b	7×10^{-3}	kg
Stator moment of inertia	I_1	5.9×10^{-3}	kgm^2
Rotor moment of inertia	I_2	1.9×10^{-3}	kgm^2
Bending stiffness	K_r	27.2	Nm^{-1}
Bending stiffness	K_t	27.2	Nm^{-1}
Torsion stiffness	K_{tor}	4.69	Nm^{-1}rad^{-1}
Outer shell stiffness	K_p	2.7×10^5	Nm^{-1}
Outer shell inner diameter	D	1.91×10^{-1}	kg
Rotor diameter	d	1.52×10^{-1}	kg
Initial position of rotor	ρ_0	1×10^{-3}	kg
Motor torque	τ	1.02×10^{-2}	Nm

$$\dot{V} \leq \frac{1}{2}\boldsymbol{e}^\top(\boldsymbol{A}^\top\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A})\boldsymbol{e}+\boldsymbol{e}^\top\boldsymbol{P}\boldsymbol{B}(\Delta\mathbf{F}+\Delta\mathbf{G}\hat{\mathbf{G}}^*(x|\boldsymbol{\Theta}_g) \times(-\mathbf{F}+\ddot{\boldsymbol{x}}_r-\mathbf{K}^\top\boldsymbol{e}))+\boldsymbol{e}^\top\boldsymbol{P}\boldsymbol{B}(\mathbf{I}+\Delta\mathbf{G}\mathbf{G}^*(x|\boldsymbol{\Theta}_g))\boldsymbol{u}_s + \frac{1}{2}\boldsymbol{e}^\top\boldsymbol{P}\boldsymbol{B}(\frac{1}{\rho^2}\mathbf{I}-\boldsymbol{R}^{-1})\boldsymbol{B}^\top\boldsymbol{P}\boldsymbol{e}+\rho^2\frac{\boldsymbol{d}'^\top\boldsymbol{d}'}{2} \tag{45}$$

Substituting (37) into (45) leads to

$$\dot{V} \leq \frac{1}{2}\boldsymbol{e}^\top(\boldsymbol{A}^\top\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A})\boldsymbol{e}+\frac{1}{2}\boldsymbol{e}^\top\boldsymbol{P}\boldsymbol{B}(\frac{1}{\rho^2}\mathbf{I}-\boldsymbol{R}^{-1}) \times\boldsymbol{B}^\top\boldsymbol{P}\boldsymbol{e}+\rho^2\frac{\boldsymbol{d}'^\top\boldsymbol{d}'}{2} \tag{46}$$

Hence, we have (47)

$$\dot{V} \leq -\frac{1}{2}\boldsymbol{e}^\top\boldsymbol{Q}\boldsymbol{e}+\frac{1}{2}\rho^2\boldsymbol{d}'^\top\boldsymbol{d}' \tag{47}$$

Therefore, whenever

$$\|\boldsymbol{e}\| \geq \frac{\rho\sqrt{\boldsymbol{d}'^\top\boldsymbol{d}'}}{\sqrt{\lambda_{min}(\boldsymbol{Q})}} \tag{48}$$

we have $\dot{V} \leq 0$, where $\lambda_{min}(\boldsymbol{Q})$ denotes the minimum eigenvalue of $\boldsymbol{Q}$. Therefore, the overall system satisfies the following relationship:

$$\int_0^t \boldsymbol{e}^\top\boldsymbol{Q}\boldsymbol{e}\,dt \leq \boldsymbol{e}^\top(0)\boldsymbol{P}\boldsymbol{e}(0)+\frac{1}{\gamma_f}\boldsymbol{\Theta}_f^\top(0)\boldsymbol{\Theta}_f(0) + \frac{1}{\gamma_g}\boldsymbol{\Theta}_g^\top(0)\boldsymbol{\Theta}_g(0)+\rho^2\int_0^t \boldsymbol{d}'^\top\boldsymbol{d}'\,dt. \tag{49}$$

In light of the Lyapunov stability theory functional differential equation and since ρ is the design constant serving as an attenuation level, it can be concluded that for any $t \geq t_0$, $\boldsymbol{e}$, $\tilde{\boldsymbol{\Theta}}_f$ and $\tilde{\boldsymbol{\Theta}}_g$ are uniformly ultimately bounded (UUB) and the H_∞ tracking performance is within the prescribed attenuation level ρ. This completes the proof. ∎

The flow chart of the overall VS adaptive fuzzy control system is shown in Fig. 5.

IV. SIMULATION RESULTS

In this section, a four degree-of-freedom of drill-string systems is used to as simulation example. The system parameter values are given in Table I. The nonlinear differential equation of four degree-of-freedom of drill-string systems is given by eq. (18) where $f_1 = x_1(x_6 + x_7)^2 + \frac{em_b x_8^2 cos(\beta)}{m+m_b} + (\frac{1}{(m+m_b)(e^2 m \times m_b + I_2(m+m_b))})(-(m + m_b)(I_2 + e^2 m_b)x_1(K_r + Ktx_3^2 + K_c\lambda) + em_b(em_b x_1(K_r + K_t x_3^2 + K_c\lambda)cos(\beta)^2 + k_a(m + m_b)(x_2 - x_4)sin(\beta) + eK_t m_b x_1 x_3 cos(\beta)sin(\beta))$, $f_2 = \frac{K_a(-x_2+x_4)+x_1(K_t x_1 x_3 - F_t\lambda)}{I_1}$, $f_3 = [x_1(e^2 m_b(-I_1 K_t(2m + m_b)x_3 + 2K_a m(m + m_b)(x_2 - x_4) + 2m(m + m_b)x_1(-Ktx_1x_3 + F_t\lambda)) + 2I_2(m + m_b)(-I_1 K_t x_3 + (m + m_b)(K_a(x_2 - x_4) + x_1(-K_t x_1 x_3 + F_t\lambda)))) + eI_1 m_b(-2K_a(m + m_b)(x_2 - x_4)cos(\beta) + em_b x_1(-K_t x_3 2cos(\beta) + (K_r + Ktx_3^2 + K_c\lambda)sin(2\beta)))] \times (\frac{1}{2I_1(m+m_b)(e^2 m \times m_b + I_2(m+m_b))x_1}) + \frac{-2(m+m_b)x_5(x_6+x_7)}{(m+m_b)x_1} + \frac{em_b x_8^2 sin(\beta)}{(m+m_b)x_1}$, $f_4 = \frac{K_a(m+m_b)(x_2-x_4)+em_b x_1(K_t x_3 cos(\beta)-(K_r+K_t x_3^2+K_c\lambda)sin(\beta)}{e^2 mm_b + I_2(m+m_b)}$, $d'_1 = (\frac{1}{(m+m_b)(e^2 mm_b + I_2(m+m_b))}) \times (m + m_b)(I_2 + e^2 m_b)(K_r\rho_0 + K_c\lambda\delta) + em_b(em_b(-K_r\rho_0 - K_c\lambda\delta)cos(\beta)^2 + F_t\lambda(0.5dm + 0.5dm_b - em_b cos(\beta))sin(\beta))$ $d'_2 = -\frac{F_t x_1\lambda}{I_1}$, $d'_3 = \frac{1}{I_1(m+m_b)(e^2 mm_b + I_2(m+m_b))x_1}F_t(I_1(e^2(m + 0.5m_b)m_b + I_2(m + m_b)) + (e^2 mm_b(m + m_b) + I_2(m^2 + mm_b + m_b^2))x_1^2)\lambda + eI_1 m_b(0.5eF_t m_b\lambda cos(2\beta) + cos(\beta)(dF_t(-0.5m - 0.5m_b)\lambda + em_b(K_r\rho_0 - K_c\lambda\delta)sin(\beta)))$, $d'_4 = (\frac{1}{e^2 mm_b + I_2(m+m_b)})dF_t(0.5m + 0.5m_b)\lambda - eF_t m_b\lambda cos(\beta) + em_b(K_r\rho_0 + K_c\lambda\delta)sin(\beta)$, $g_1 = \frac{em_b Sin(\beta)}{e^2 mm_b + I_2(m+m_b)}$, $g_2 = 0$, $g_3 = -\frac{em_b cos(\beta)}{I_2 mx_1 + I_2 m_b x_1 + e^2 mm_b x_1)}$, and $g_4 = \frac{(m+m_b)}{e^2 mm_b + I_2(m+m_b)}$, in which $m_{11} = m + m_b$, $m_{12} = m_{13} = 0$, $m_{14} = -em_b sin(\beta)$, $m_{21} = 0$, $m_{22} = I_1 + (m + m_b)\rho^2$, $m_{23} = (m + m_b)\rho^2$, $m_{24} = em_b\rho cos(\beta)$, $m_{31} = 0$, $m_{32} = m_{33} = (m + m_b)\rho$, $m_{34} = em_b cos(\beta)$, $m_{41} = -em_b sin(\beta)$,

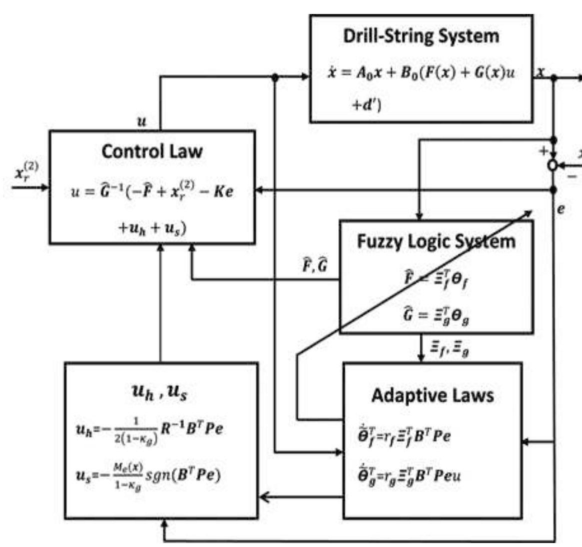

Fig. 5. The flow chart of the overall VS adaptive fuzzy control system.

$m_{42} = m_{43} = em_b\rho cos(\beta)$, and $m_{44} = I_2 + m_b e^2$. $c_{11} = 0$, $c_{12} = -(m + m_b)\rho(\dot{\theta} + \dot{\varphi})$, $c_{13} = -(m + m_b)\rho(\dot{\theta} + \dot{\varphi})$, $c_{14} = -em_b\dot{\alpha}cos(\beta)$, $c_{21} = 0$, $c_{22} = 2(m + m_b)\rho\dot{\rho}$, $c_{23} = 2(m + m_b)\rho\dot{\rho}$, $c_{24} = -em_b\rho\dot{\alpha}sin(\beta)$, $c_{31} = 0$, $c_{32} = c_{33} = 2(m + m_b)\dot{\rho}$, $c_{34} = -em_b\dot{\alpha}sin(\beta)$, $c_{41} = 2em_b(\dot{\theta} + \dot{\varphi})cos(\beta)$, $c_{42} = c_{43} = em_b\rho(\dot{\theta} + \dot{\varphi})sin(\beta)$, and $c_{44} = 0$. $w_{11} = \lambda K_p + K_r$, $w_{12} = 0$, $w_{13} = K_t\rho\varphi$, $w_{14} = 0$, $w_{21} = \lambda F_t$, $w_{22} = K_{tor}$, $w_{23} = 0$, $w_{24} = -K_{tor}$, $w_{31} = w_{32} = w_{34} = 0$, $w_{33} = K_t\rho$, $w_{41} = w_{43} = 0$, $w_{42} = -K_{tor}$, and $w_{44} = K_{tor}$, $d_1 = \lambda K_p\delta + K_r\rho_0$, $d_2 = 0$, $d_3 = \lambda F_t$, $d_4 = \lambda F_t R$. The idea here that, we hope stator and rotor of drill-string system have the same velocity (x_{r_6}=x_{r_8}=5 rpm). Therefore, there are have the same rotate angle of stator and rotor (x_{r_2}=x_{r_4}=$\frac{1}{6}\pi t$). Meanwhile, we do not want to produce radial or lateral displacement and velocity of rotor (x_{r_1}=0.019 and x_{r_5}=0), rotation displacement and velocity due to bending (x_{r_3}=x_{r_7}=0).

The design procedure of the adaptive fuzzy control is synthesize as follows:

1). For high penalty on initial parameter errors, the initial conditions are chosen as $\mathbf{\Theta}_f(0)$=0.03$\mathbf{I}_{400\times1}$, $\mathbf{\Theta}_g(0)$=0.01$\mathbf{I}_{400\times1}$, $x_1(0)$=0.019, and $x_2(0)$=$x_3(0)$=$x_4(0)$=$x_5(0)$=$x_6(0)$=$x_7(0)$=$x_8(0)$=0. Select design parameters as γ_f=9, γ_g=0.1. The bounds of the uncertainties are $\kappa_f = 0.04$ and $\kappa_g = 0.03$.

2). Select the feedback gain $\boldsymbol{K}$=[$\boldsymbol{k}_1$ $\boldsymbol{k}_2$],

$$\boldsymbol{k}_1 = \mathbf{I}_{4\times4} \times [50.5\ \ 45.3\ \ 40.7\ \ 65.3]^\top$$
$$\boldsymbol{k}_2 = \mathbf{I}_{4\times4} \times [10.1\ \ 9\ \ 8.1\ \ 13.1]^\top$$

such that the characteristic polynomials of $\boldsymbol{A} = \boldsymbol{A}_0 - \boldsymbol{BK}$ is Hurwitz. We choose the $\rho = 0.1$, $\boldsymbol{R} = 0.1\rho^2\boldsymbol{I}_{4\times4}$, the weighting matrix $\boldsymbol{Q} = 0.01\mathbf{I}_{8\times8}$. Then, the positive definite symmetric matrix $\boldsymbol{P}$ can be obtained as

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{p}_1\mathbf{I}_{4\times4} & \boldsymbol{p}_3\mathbf{I}_{4\times4} \\ \boldsymbol{p}_2\mathbf{I}_{4\times4} & \boldsymbol{p}_4\mathbf{I}_{4\times4} \end{bmatrix}$$

where

$$\boldsymbol{p}_1 = [4.1447\ \ 3.7940\ \ 3.4977\ \ 5.2817]$$
$$\boldsymbol{p}_2 = [1.2026\ \ 1.0846\ \ 0.9781\ \ 1.5049]$$
$$\boldsymbol{p}_3 = [1.2026\ \ 1.0846\ \ 0.9781\ \ 1.5049]$$
$$\boldsymbol{p}_4 = [2.2219\ \ 2.1914\ \ 2.1610\ \ 1.2659]$$

3). In order to approximate the unknown nonlinear functions $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$, select five membership functions $\mu_{C_i^k}$, $i = 1, \ldots, 8$. At the beginning, the conventional fuzzy design needs all possible combinations generating 100 rules. However, x_1, x_2, x_3 and x_4 are dependent on displacement, and x_5, x_6, e_7 and x_8 are dependent on velocity. Therefore, we can simplify the fuzzy rules for states x_1, and x_3, for states x_2, and x_4, for states x_5, and x_7, and for states x_6, and x_8, respectively, as follows

R_{ab}^k : IF x_1 is E_1^k and x_3 is E_3^k THEN w is E_{ab}^k
R_{ab}^k : IF x_2 is E_2^k and x_4 is E_4^k THEN w is E_{ab}^k
R_{ab}^k : IF x_5 is E_5^k and x_7 is E_7^k THEN w is E_{ab}^k
R_{ab}^k : IF x_6 is E_6^k and x_8 is E_8^k THEN w is E_{ab}^k

in which, $k = 1, 2, \ldots, 25$, and $ab = 1, 2, \ldots, 5$. The range of membership functions of E_1 is 0.0185 to 0.0195, the range of membership functions of E_2 and E_4 are 0 to 2π, the range of membership functions of E_3 is -1 to 1, the range of membership functions of E_5 and E_7 are -0.5 to 0.5, and the range of membership functions of E_6 and E_8 are -1 to 1.

4). The above parameters be substituting (38) into (36) and (37). Apply the control force (33) into the drill-string system. Then, the adaptive law (39) and (40) can be computed.

The simulation results are shown in Figs. 6-11. Figs. 6 and 9 correspond to the state variable values time histories of the tangential component, Figs. 7 and 10 correspond to the velocity time histories of the tangential component, and Figs. 8 and 11 correspond to the radial motion phase portrait projections. It is seen that from the figure Figs. 6-8 are shown that the drill-string is driven by a fixed external torque. Because there is no controller in the system, the flexible part of drill-string appear a substantial swing. Comparing the results of Figs. 8 and 11, it is clear that the results of Figs. 11 show that the rotor stays closer to the radial center in the same simulation time. It is seen that tangential velocity jumps in Figs. 7 and 10, which is attributed to the impact between the rotor and the outer shell. In addition, the duration of jumping also increases denoting drill-string uncontrolled in Fig. 7. The results presented from Figs. 9-11 demonstrate that the proposed scheme indeed improves the system performances including convergence of the estimations and tracking errors such that accomplish flexible part of drill-string swing suppression and ensure the rotation of a drill-string keeping close to the radial center.

V. CONCLUSION

In this paper, a adaptive fuzzy control scheme with different cases of stick-slip interactions between the drill-string and the outer shell of a drill-string systems has been developed based on the H_∞ tracking design technique. The proposed control scheme can elimate the affect of the external disturbances and fuzzy approximation errors such that the states of drill-string system can approach to the desired reference signal exactly. It can be concluded from the simulation results that the proposed scheme is effective to reduce the vibration phenomenon of drill-string hitting with outer shell due to the drill-string bending and ensure drill-string can close to the radial center.

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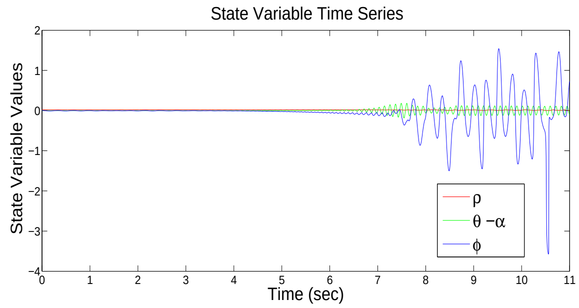

Fig. 6. State variable values and time histories (without controller).

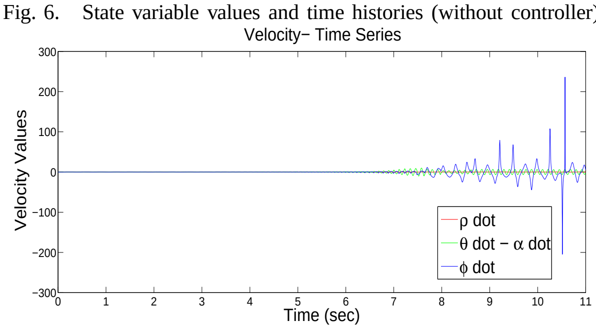

Fig. 7. Velocity and time histories (without controller).

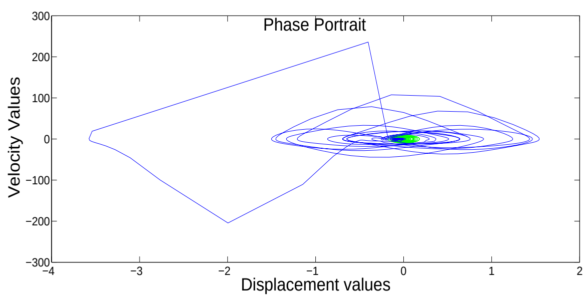

Fig. 8. Phase portraits and time histories (without controller).

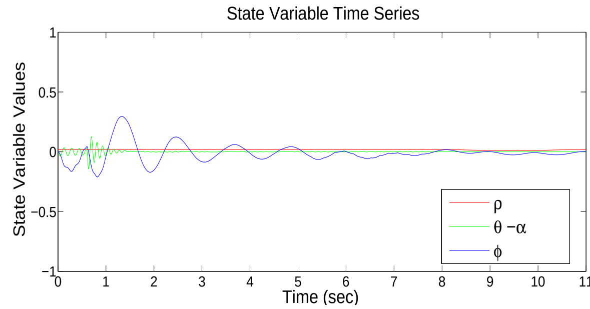

Fig. 9. State variable values and time histories (with controller).

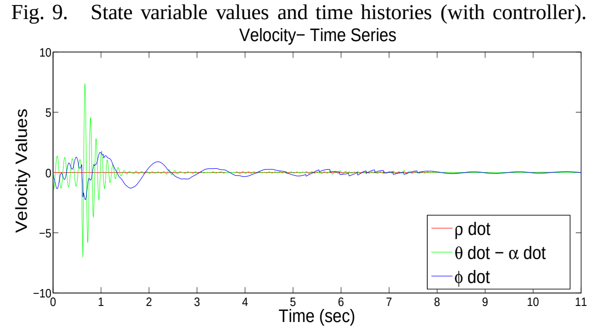

Fig. 10. Velocity and time histories (with controller).

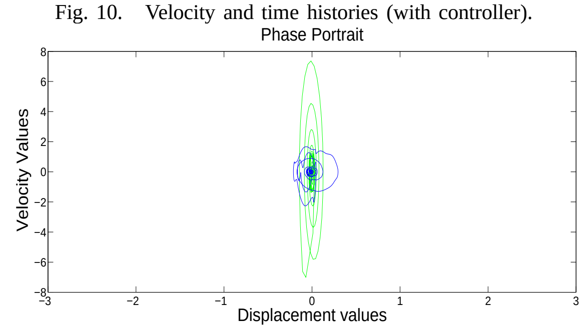

Fig. 11. Phase portraits and time histories (with controller).

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