H_∞ based on Type-2 Adaptive Fuzzy Tracking Control Design for PMDC Motor with Dead-Zones

Tzu-Sung, Wu*, Mansour Karkoub, Wen-Shyong Yu, and Ho-Sheng Chen

***Abstract*—In this paper, a type-2 adaptive fuzzy tracking control (T2AFC) via H_∞ control algorithm is developed for permanent magnet DC (PMDC) motor systems with dead-zone nonlinearity, plant uncertainties, and external disturbances. The type-2 fuzzy dynamic model with adaptation capability is used to approximate the PMDC motor system, where the weighting factors of the fuzzy model are obtained from both of the fuzzy inference and online update law. Then, the control law is developed based on Lyapunov criterion and Riccati-inequalities to overcome the nonlinearities and external disturbances such that the uniform ultimate boundedness (UUB) of all signals in the closed loop and H_∞ tracking performance are achieved. The advantage of employing T2AFC is that it can better handle the vagueness or uncertainties inherent in linguistic words by the use of fuzzy membership functions with adaptation to track the the specified reference inputs by linear analytical results instead of estimating non-linear system functions as the system parameters are unknown. Finally, a PMDC motor systems is used as an example to illustrate the validity and confirm the performance of the proposed scheme.**

Keywords–Permanent magnet DC motor systems, type-2 adaptive fuzzy logic system, H_∞ tracking performance, dead-zone.

I. Introduction

PMDC motor systems have been extensively used in control systems applications, e.g. automobile industry (electric vehicle), weak power using battery system (motor of toy), and the electric traction in the multi-machine systems, etc. From the control point of view, PMDC motor systems exhibit excellent control characteristics because of the decoupled nature of the field. Over the past decades, many techniques have been developed for the PMDC motor systems control, some of these methods were based on classical and also intelligent approaches. In [3], the authors proposed a high-order neural network functions (HONNFs) are used for the neuro-fuzzy indirect control of nonlinear dynamical systems, which comprises two interrelated phases: first the identification of the model and second the control of the plant. Also, [4] was used novel motor drive system proposed in this paper will be based on model reference adaptive control (MRAC) structure. A load torque observer using the model reference adaptive system is employed to obtain the better performance from the brushless dc motor in a precision position control [5] and application to chaotic PMDC Motor systems Drives [6]. Hence, there has been a growing interest in studying the problems of stability for non-linear systems, including sliding-Mode control [7, 8] and the authors also proposed an interval type-2 fuzzy neural network (IT2FNN) [9].

Type-2 adaptive fuzzy logic system (T2AFC) have recently been utilized in many control processes due to their ability to model uncertainties [10, 11]. Similar to a type-1 adaptive fuzzy logic system (T1FALS), T2AFC includes a fuzzifier, a rule base, a fuzzy inference engine, an output processor of type-reducer and defuzzifier, and is also characterized by IF-THEN rules, but its antecedent or consequent sets are type-2. The uncertainty in the primary memberships of a type-2 fuzzy set, and consists of a bounded region that is called the footprint of uncertainty (FOU). Furthermore, to simplify the computation, the secondary (MFs) can be set to either zero or one and called interval type-2 secondary MFs. In [12]-[14], the authors proposed an interval type-2 TS fuzzy logic system (IT2TSFLS) for the adaptive tracking control to confront uncertainties in the inference mechanisms, and to design the adaptive laws with compensate the interconnection effects and the reconstruction errors for a permanent magnet synchronous motor (PMSM) [15]. Therefore the T2AFC that are based on type-2 fuzzy sets has the potential to produce a better performance than T1FALS when dealing with uncertainties such as noisy data and changing environments.

Input dead-zone nonlinearity is a common phenomenon that appears in actuators as well as sensors. However, some parameters, e.g., the maximum and the minimum values of dead-zone slopes, have to be known for control design and in order to investigate the key features of the dead-zone in the control problems, it is assumed that the slopes of the dead-zone are the same. As a result, how to deal with these nonlinearities remains a practically challenging problem for the design of closed-loop controller for the PMDC motor systems drives, especially where a high dynamic performance requirement is important in many industrial processes, its presence severely limits system performance, and many scholars try to use different ways to improve the performance of control systems. In [16, 17], the authors proposed a robust adaptive neural network (NN) control based backstepping control [18]-[20] for a class of uncertain multiple-input-multiple-output MIMO nonlinear systems with unknown control coefficient matrices. The success of the approach in [21] proposed a rigorous design procedure with proofs is given that results in a proportional integral (PI) tracking loop with an adaptive fuzzy logic system in the feedforward loop for dead-zone compensation, but also more challenging hard nonlinearities such as the uncertain nonlinear time-delay functions [22].

To attenuate the effects caused by unmodelled dynamics,

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uncertainties and disturbances, we adopt an T2AFC of the PMDC motor systems with dead-zone nonlinearity at the input of a linear plant to achieve H_∞ tracking performance. First, a PMDC motor systems is approximated by the Takagi-Sugeno (T-S) type fuzzy linear models with adaptation capability. In order to reduce the effect of dead-zone nonlinearity, a new model for dead-zone is developed and used in any conventional controllers, uncertainties and, external disturbances such that the H_∞ tracking performance is achieved. The advantage of employing T2AFC is that we can utilise the linguistic information by setting the MFs of fuzzy logical system and the adaptation parameters to force the model uncertainties and plant parameters to track the the specified values for using linear analytical results instead of estimating non-linear system functions as the system parameters are unknown, and the system input is with dead-zone. Thus, a parameter estimation scheme applicable to the T2AFC and controllers for stabilizing nonlinear systems with dead zone nonlinearities in the control input is needed. Based on Lyapunov criterion and Riccati-inequality, some sufficient conditions are derived so that all states of the system are uniformly ultimately bounded (UUB) and the effect of the external disturbance on the tracking error can be attenuated to any prescribed level and consequently an H_∞ tracking control is achieved. Finally, a PMDC motor systems as a simulation example is given to illustrate the validity and effectiveness of the proposed method.

II. Model of the PMDC Motor with Dead-Zone

Let the dynamic model of the permanent magnet DC (PMDC) motor system be described as follows [23]:

$$\begin{cases} \dot{\Lambda} = \omega \\ \dot{\omega} = -\frac{F_d}{J}\omega + \frac{K_t}{J} i_a - \frac{1}{J}\Phi_1(e_a) + \frac{1}{J}\varsigma_1 \\ \dot{i}_a = -\frac{K_E}{L_a}\omega - \frac{R_a}{L_a} i_a + \frac{1}{L_a}\Phi_2(V_a) \end{cases} \tag{1}$$

where Λ, ω, and i_a are the rotational angle, rotational speed, and armature current, respectively, V_a, e_a are the armature voltage and load torque, respectively, and $\Phi_1(e_a)$, $\Phi_2(V_a)$ are the dead-zone nonlinear functions with e_a and V_a as their inputs, respectively. Furthermore, R_a, L_a, K_t, K_E, F_d, J, and ς_1 are the armature resistance, armature inductance, torque constant, back EMF constant, friction coefficient, inertia, and cutting force in machining, respectively. Since PMDC motor system is faced with flux linkage and disturbance torque caused by the variation of the mechanical parameters and external load disturbances, this will deteriorate the steady-state and transient responses, as well as the speed control performance. Let $F_d = F_{dN} + \delta F_d$ and $R_a = R_{aN} + \delta R_a$, where F_{dN}, R_{aN} are the nominal terms and δF_d, δR_a are the mismatched uncertainties for F_d and R_a, respectively. Let $\boldsymbol{x} = [\Lambda\ \omega\ i_a]^\top = [x\ \dot{x}\ \ddot{x}]^\top = [x_1\ x_2\ x_3]^\top$, the torque equation of the system with sector uncertainty of nonlinearities by the dead-zone can be expressed as follows:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} + \begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}\Bigg(\begin{bmatrix}-\frac{F_{dN}}{J}x_{2_i} + \frac{k_t}{J}x_3\\ -\frac{k_E}{L_a}x_2 - \frac{R_{aN}}{L_a}x_3\end{bmatrix} + \begin{bmatrix}-\frac{\delta F_d}{J}x_2\\ -\frac{\delta R_a}{L_a}x_3\end{bmatrix} + \begin{bmatrix}-\frac{1}{J}&0\\0&\frac{1}{L_a}\end{bmatrix}\begin{bmatrix}\Phi_1(e_a)\\ \Phi_2(V_a)\end{bmatrix} + \begin{bmatrix}\frac{1}{J}\varsigma_1\\0\end{bmatrix}\Bigg)$$

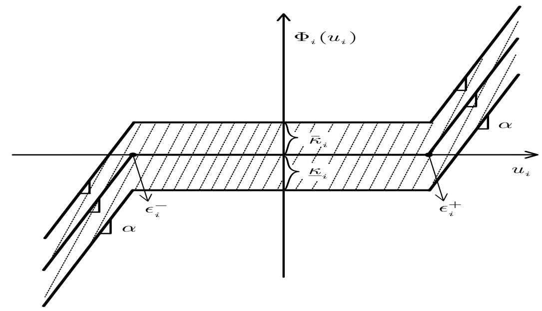

Fig. 1. The dead-zone with uncertainties $\Phi_i(\mathbf{u}_i)$.

$$= \mathbf{A}_i \boldsymbol{x}_i + \mathbf{B}_i\Big(\boldsymbol{f}_i + \Delta\boldsymbol{f}_i + \mathbf{G}_i\boldsymbol{\Phi}_i(\mathbf{u}_i) + \boldsymbol{d}_i\Big) \tag{2}$$

where

$$\mathbf{u}_i = \begin{bmatrix}e_a\\V_a\end{bmatrix} = \begin{bmatrix}u_{1_i}\\u_{2_i}\end{bmatrix},\ \mathbf{A}_i = \begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix},\ \mathbf{B}_i = \begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix},$$

$$\boldsymbol{f}_i = \begin{bmatrix}-\frac{F_{dN}}{J}x_2 + \frac{k_t}{J}x_3\\ -\frac{k_E}{L_a}x_2 - \frac{R_{aN}}{L_a}x_3\end{bmatrix} = \begin{bmatrix}f_{1_i}\\ f_{2_i}\end{bmatrix},\ \Delta\boldsymbol{f}_i = \begin{bmatrix}-\frac{\delta F_d}{J}x_2\\ -\frac{\delta R_a}{L_a}x_3\end{bmatrix} = \begin{bmatrix}\Delta f_{1_i}\\ \Delta f_{2_i}\end{bmatrix},$$

$$\mathbf{G}_i = \begin{bmatrix}-\frac{1}{J}&0\\0&\frac{1}{L_a}\end{bmatrix} = \begin{bmatrix}g_{1_i}&0\\0&g_{2_i}\end{bmatrix},\ \boldsymbol{d}_i = \begin{bmatrix}\frac{1}{J}\varsigma_1\\0\end{bmatrix} = \begin{bmatrix}d_{1_i}\\d_{2_i}\end{bmatrix},$$

$$\boldsymbol{\Phi}_i(\mathbf{u}_i) = \begin{bmatrix}\Phi_1(e_a)\\ \Phi_2(V_a)\end{bmatrix} \equiv \begin{bmatrix}\Phi_1(u_1)\\ \Phi_2(u_2)\end{bmatrix},$$

and $\boldsymbol{d}_i \in \mathbb{R}^2$ is the bounded external disturbance. It is assumed that $\boldsymbol{f}_i$, $\mathbf{G}_i$ are unknown smooth functions, and $\Delta\boldsymbol{f}_i$ is the uncertainty of the system. Let $\mathbf{A}_i = diag\{\mathbf{A}_{1_i}, \mathbf{A}_{2_i}\}$ and $\mathbf{B}_i = diag\{\mathbf{B}_{1_i}, \mathbf{B}_{2_i}\}$, where $\mathbf{A}_{1_i} = \begin{bmatrix}0&1\\0&0\end{bmatrix}$, $\mathbf{A}_{2_i} = 0$, $\mathbf{B}_{1_i} = \begin{bmatrix}0\\1\end{bmatrix}$, and $\mathbf{B}_{2_i} = 1$.

The nonlinear continuous functions $\Phi_i(u_i)$ with input u_i, $i = 1, 2$ can be intervalized shown in Fig. 1 as follows:

$$\Phi_i(u_i) = \begin{cases} \bar{\Phi}_i(u_i) = \begin{cases} \bar{\alpha}_i(u_i - \epsilon_i^+) + \bar{\kappa}_i,\ u_i \geq \epsilon_i^+ \text{ and } \bar{\Phi}_i(u_i) \geq \bar{\kappa}_i \\ \bar{\kappa}_i, \quad \epsilon_i^- < u_i < \epsilon_i^+ \text{ and } 0 < \bar{\Phi}_i(u_i) < \bar{\kappa}_i \\ \bar{\alpha}_i(u_i - \epsilon_i^-) + \bar{\kappa}_i,\ u_i \leq \epsilon_i^- \text{ and } \bar{\Phi}_i(u_i) \leq \bar{\kappa}_i \end{cases} \\ \Phi_i'(u_i) = \begin{cases} \alpha_i'(u_i - \epsilon_i^+), & u_i \geq \epsilon_i^+ \\ 0, & \epsilon_i^- < u_i < \epsilon_i^+ \\ \alpha_i'(u_i - \epsilon_i^-), & u_i \leq \epsilon_i^- \end{cases} \\ \underline{\Phi}_i(u_i) = \begin{cases} \underline{\alpha}_i(u_i - \epsilon_i^+) + \underline{\kappa}_i,\ u_i \geq \epsilon_i^+ \text{ and } \underline{\Phi}_i(u_i) \geq \underline{\kappa}_i \\ \underline{\kappa}_i, \quad \epsilon_i^- < u_i < \epsilon_i^+ \text{ and } \underline{\kappa}_i < \underline{\Phi}_i(u_i) < 0 \\ \underline{\alpha}_i(u_i - \epsilon_i^-) + \underline{\kappa}_i,\ u_i \leq \epsilon_i^- \text{ and } \underline{\Phi}_i(u_i) \leq \underline{\kappa}_i \end{cases} \end{cases} \tag{3}$$

where $\Phi_i'(u_i)$, $\underline{\Phi}_i(u_i)$, and $\bar{\Phi}_i(u_i)$ are the median, lower, and upper bounds, respectively, ϵ_i^-, ϵ_i^+, $\underline{\kappa}_i$, and $\bar{\kappa}_i$ are the dead zone envelope and are constants, and the parameters α_i', $\underline{\alpha}_i$, and $\bar{\alpha}_i$ stand for the median, lower, and upper slope of the dead-zone characteristic. In order to obtain the key features of the dead-zone in the control problems, we have the following assumptions:

Assumption 1: The dead-zone outputs $\Phi_i(u_i)$, $i = 1, 2$, are not available for measurement.

Assumption 2: The dead-zone slopes in positive and negative region are the same, i.e. $\alpha_i = \alpha_i' = \underline{\alpha}_i = \bar{\alpha}_i$, $i = 1, 2$.

Assumption 3: There exist known constants $\epsilon^-_{i_{max}}$, $\epsilon^-_{i_{min}}$, $\epsilon^+_{i_{max}}$, $\epsilon^+_{i_{min}}$, $\underline{\kappa}_{i_{max}}$, $\underline{\kappa}_{i_{min}}$, $\bar{\kappa}_{i_{max}}$, $\bar{\kappa}_{i_{min}}$, $\alpha'_{i_{max}}$, $\alpha'_{i_{min}}$, $\underline{\alpha}_{i_{max}}$, $\underline{\alpha}_{i_{min}}$, $\bar{\alpha}_{i_{max}}$, and $\bar{\alpha}_{i_{min}}$ such that the unknown dead-zone parameters ϵ^-_i, ϵ^+_i, $\underline{\kappa}_i$, $\bar{\kappa}_i$, α'_i, $\underline{\alpha}_i$, and $\bar{\alpha}_i$, $i = 1, 2$, satisfy $\epsilon^-_i \in \{\epsilon^-_{i_{min}}, \epsilon^-_{i_{max}}\}$, $\epsilon^+_i \in \{\epsilon^+_{i_{min}}, \epsilon^+_{i_{max}}\}$, $\underline{\kappa}_i \in \{\underline{\kappa}_{i_{min}}, \underline{\kappa}_{i_{max}}\}$, $\bar{\kappa}_i \in \{\bar{\kappa}_{i_{min}}, \bar{\kappa}_{i_{max}}\}$, $\alpha'_i \in \{\alpha'_{i_{min}}, \alpha'_{i_{max}}\}$, $\underline{\alpha}_i \in \{\underline{\alpha}_{i_{min}}, \underline{\alpha}_{i_{max}}\}$, and $\bar{\alpha}_i \in \{\bar{\alpha}_{i_{min}}, \bar{\alpha}_{i_{max}}\}$, for which $\alpha'_{i_{min}}$, $\underline{\alpha}_{i_{min}}$, $\bar{\alpha}_{i_{min}} \neq 0$.

Based on the above assumptions, the expression (3) can be represented as

$$\Phi_i(u_i) = \alpha_i u_i + \psi_{1_i}(u_i) \tag{4}$$

where

$$\psi_{1_i}(u_i) = \begin{cases} -\alpha_i \epsilon^+_i + \bar{\kappa}_i, & u_i \geq \epsilon^+_i \text{ and } \Phi_i(u_i) \geq \bar{\kappa}_i \\ -\alpha_i u_i + \kappa_i, & \epsilon^-_i < u_i < \epsilon^+_i \text{ and } \underline{\kappa}_i < \Phi_i(u_i) < \bar{\kappa}_i \\ -\alpha_i \epsilon^-_i + \underline{\kappa}_i, & u_i \leq \epsilon^-_i \text{ and } \Phi_i(u_i) \leq \underline{\kappa}_i \end{cases}$$

Since T2AFLS and dead zone property, it is seen that $\psi_{1_i}(u_i)$ can be bounded to satisfy $|\psi_{1_i}(u_i)| \leq \rho_i$, and ρ_i is positive constants which can be chosen from $\rho_i \in max\{\alpha_{i_{max}}\epsilon^+_{i_{max}} + \bar{\kappa}_{i_{max}}, \alpha_{i_{max}}\epsilon^-_{i_{min}} + \underline{\kappa}_{i_{min}}\}$, where $\epsilon^-_{i_{min}}$ is a negative value.

III. TYPE-2 ADAPTIVE FUZZY CONTROL DESIGN

A type-2 fuzzy logic system (T2FLS) is very similar to a type-1 fuzzy logic system (T1FLS), the major structural difference is that the defuzzifier block of a T1FLS is replaced by the output processing block in a T2FLS which consists of type reduction followed by defuzzification. Consider a T2FLS with inputs x_1, x_2, x_3 and outputs y_{f_i}, y_{g_i}, $y_{h_{f_i}}$ $(i = 1, 2)$, which the rule base consists of a collection of IF-THEN rules as in the T1FLS case. Let us consider the l-th rule as follows:

$R^l_{f_k}$:IF x_1 is $F^l_{f_1}$AND x_2 is $F^l_{f_2}$ AND x_3 is $F^l_{f_3}$ THEN y_{f_i} is $G^l_{f_i}$
$R^l_{g_k}$:IF x_1 is $F^l_{g_1}$AND x_2 is $F^l_{g_2}$ AND x_3 is $F^l_{g_3}$ THEN y_{g_i} is $G^l_{g_i}$
$R^l_{h_{f_k}}$:IF x_1 is $F^l_{h_{f_1}}$ AND x_2 is $F^l_{h_{f_2}}$ AND x_3 is $F^l_{h_{f_3}}$
THEN $y_{h_{f_i}}$ is $G^l_{h_{f_i}}$

where the T2 input fuzzy sets and the T2 output fuzzy set are denoted by $F^l_{f_p}$, $F^l_{g_p}$, $F^l_{h_{f_p}}$ $(p = 1, 2, 3, l = 1, \cdots, M)$ and $G^l_{f_i}$, $G^l_{g_i}$, $G^l_{h_{f_i}}$, respectively, and their membership functions are $\mu_{F^l_{f_p}}(x_p)$, $\mu_{F^l_{g_p}}(x_p)$, $\mu_{F^l_{h_{f_p}}}(x_p)$ and $\mu_{G^l_{f_i}}(y_{f_i})$, $\mu_{G^l_{g_i}}(y_{g_i})$, $\mu_{G^l_{h_{f_i}}}(y_{h_{f_i}})(i = 1, 2)$, respectively, for which M is the total number of the T2 fuzzy rules. The bounds of footprint of uncertainty (FOU) can be divided into two T1 membership functions $\bar{\mu}_{G^l_{f_i}}(y_{f_i})$, $\bar{\mu}_{G^l_{g_i}}(y_{g_i})$, $\bar{\mu}_{G^l_{h_{f_i}}}(y_{h_{f_i}})$ and $\underline{\mu}_{G^l_{f_i}}(y_{f_i})$, $\underline{\mu}_{G^l_{g_i}}(y_{g_i})$, $\underline{\mu}_{G^l_{h_{f_i}}}(y_{h_{f_i}})$ are upper and lower membership functions, respectively. The fuzzy inference engine is decision making logic which uses the fuzzy rules to determine the mapping from the input T2 fuzzy sets to the output T2 fuzzy sets. The firing interval of the l-th rule for $f^l_{f_i} \in G^l_{f_i}$, $g^l_{g_i} \in G^l_{g_i}$, and $h^l_{h_{f_i}} \in G^l_{h_{f_i}}$ in (20) are interval T2 fuzzy sets and not a crisp values, and $f^l_{f_i} = [\underline{f}^l_{f_i} \ \bar{f}^l_{f_i}]$, $g^l_{g_i} = [\underline{g}^l_{g_i} \ \bar{g}^l_{g_i}]$, and $h^l_{h_{f_i}} = [\underline{h}^l_{h_{f_i}} \ \bar{h}^l_{h_{f_i}}]$ are determined by its left-most and right-most points $\underline{f}^l_{f_i}$, $\underline{g}^l_{g_i}$, $\underline{h}^l_{h_{f_i}}$ and $\bar{f}^l_{f_i}$, $\bar{g}^l_{g_i}$, $\bar{h}^l_{h_{f_i}}$, respectively. With the use of the singleton fuzzifer and product inference, we can be obtained as:

$$\begin{cases} \bar{f}^l_{f_i} = \bar{\mu}_{G^l_{f_i}}(\hat{y}_{f_i}) \times \Pi^3_{p=1}\bar{\mu}_{F^l_{f_p}}(x_p) \\ \underline{f}^l_{f_i} = \underline{\mu}_{G^l_{f_i}}(\hat{y}_{f_i}) \times \Pi^3_{p=1}\underline{\mu}_{F^l_{f_p}}(x_p) \\ \bar{g}^l_{g_i} = \bar{\mu}_{G^l_{g_i}}(\hat{y}_{g_i}) \times \Pi^3_{p=1}\bar{\mu}_{F^l_{g_p}}(x_p) \\ \underline{g}^l_{g_i} = \underline{\mu}_{G^l_{g_i}}(\hat{y}_{g_i}) \times \Pi^3_{p=1}\underline{\mu}_{F^l_{g_p}}(x_p) \\ \bar{h}^l_{h_{f_i}} = \bar{\mu}_{G^l_{h_{f_i}}}(\hat{y}_{h_{f_i}}) \times \Pi^3_{p=1}\bar{\mu}_{F^l_{h_{f_p}}}(x_p) \\ \underline{h}^l_{h_{f_i}} = \underline{\mu}_{G^l_{h_{f_i}}}(\hat{y}_{h_{f_i}}) \times \Pi^3_{p=1}\underline{\mu}_{F^l_{h_{f_p}}}(x_p) \end{cases} \tag{5}$$

Then, we adopt the Gaussian type to simply present fuzzy set membership functions of the T2FAC for x_p as follows

$$\bar{\mu}_{F^l_{f_p}}(x_p) = e^{-(\frac{x_p - m_{f_p}}{\bar{\sigma}_{f_p}})^2}, \ \underline{\mu}_{F^l_{f_p}}(x_p) = a_{f_p} e^{-(\frac{x_p - m_{f_p}}{\underline{\sigma}_{f_p}})^2} \tag{6}$$

$$\bar{\mu}_{F^l_{g_p}}(x_p) = e^{-(\frac{x_p - m_{g_p}}{\bar{\sigma}_{g_p}})^2}, \ \underline{\mu}_{F^l_{g_p}}(x_p) = a_{g_p} e^{-(\frac{x_p - m_{g_p}}{\underline{\sigma}_{g_p}})^2} \tag{7}$$

$$\bar{\mu}_{F^l_{h_{f_p}}}(x_p) = e^{-(\frac{x_p - m_{h_{f_p}}}{\bar{\sigma}_{h_{f_p}}})^2}, \ \underline{\mu}_{F^l_{h_{f_p}}}(x_p) = a_{h_{f_p}} e^{-(\frac{x_p - m_{h_{f_p}}}{\underline{\sigma}_{h_{f_p}}})^2} \tag{8}$$

where $\underline{\sigma}_{f_p}$, $\underline{\sigma}_{g_p}$, $\underline{\sigma}_{h_{f_p}}$ and $\bar{\sigma}_{f_p}$, $\bar{\sigma}_{g_p}$, $\bar{\sigma}_{h_{f_p}}$ are fixed standard deviations of the lower and upper membership functions which characterize the shape of $\mu_{F^l_{f_p}}(x_p)$, $\mu_{F^l_{g_p}}(x_p)$, $\mu_{F^l_{h_{f_p}}}(x_p)$, respectively, a_{f_p}, a_{g_p}, $a_{h_{f_p}}$ are the FOU width coefficient which defines the FOU width between the upper membership function and the lower membership function, and m_{f_p}, m_{g_p}, $m_{h_{f_p}}$ are the means of the T2FS. These parameters are subject to $0 < a_{f_p} < 1$, $0 < a_{g_p} < 1$, $0 < a_{h_{f_p}} < 1$, and $\bar{\sigma}_{f_p} < \underline{\sigma}_{f_p}$, $\bar{\sigma}_{g_p} < \underline{\sigma}_{g_p}$, $\bar{\sigma}_{h_{f_p}} < \underline{\sigma}_{h_{f_p}}$. For the consequent fuzzy set membership function $\mu_{G^l_{f_i}}(\hat{y}_{f_i})$, $\mu_{G^l_{g_i}}(\hat{y}_{g_i})$, and $\mu_{G^l_{h_{f_i}}}(\hat{y}_{h_{f_i}})$, because $\hat{y}^l_{f_i}$, $\hat{y}^l_{g_i}$, and $\hat{y}^l_{h_{f_i}}$ are the point at which $\mu_{G^l_{f_i}}(y_{f_i})$, $\mu_{G^l_{g_i}}(y_{g_i})$, and $\mu_{G^l_{h_{f_i}}}(y_{h_{f_i}})$, respectively, achieves its maximum values, without loss of generality, we assume that $\bar{\mu}_{G^l_{f_i}}(\hat{y}_{f_i}) = 1$, $\bar{\mu}_{G^l_{g_i}}(\hat{y}_{g_i}) = 1$, and $\bar{\mu}_{G^l_{h_{f_i}}}(\hat{y}_{h_{f_i}}) = 1$. So we simplify the membership functions as

$$\bar{\mu}_{G^l_{f_i}}(\hat{y}_{f_i}) = 1 \ , \underline{\mu}_{G^l_{f_i}}(\hat{y}_{f_i}) = b_{f_i} \tag{9}$$

$$\bar{\mu}_{G^l_{g_i}}(\hat{y}_{g_i}) = 1 \ , \underline{\mu}_{G^l_{g_i}}(\hat{y}_{g_i}) = b_{g_i} \tag{10}$$

$$\bar{\mu}_{G^l_{h_{f_i}}}(\hat{y}_{h_{f_i}}) = 1 \ , \underline{\mu}_{G^l_{h_{f_i}}}(\hat{y}_{h_{f_i}}) = b_{h_{f_i}} \tag{11}$$

where $0 < b_{f_i}$, b_{g_i}, $b_{h_{f_i}} < 1$ are he FOU width coefficient for the consequence fuzzy set membership functions. Thus, we can obtain as

$$\bar{f}^l_{f_i} = 1 \times \Pi^3_{p=1}\bar{\mu}_{F^l_{f_p}}(x_p), \ \underline{f}^l_{f_i} = b_{f_i}\Pi^3_{p=1}\underline{\mu}_{F^l_{f_p}}(x_p) \tag{12}$$

$$\bar{g}^l_{g_i} = 1 \times \Pi^3_{p=1}\bar{\mu}_{F^l_{g_p}}(x_p), \ \underline{g}^l_{g_i} = b_{g_i}\Pi^3_{p=1}\underline{\mu}_{F^l_{g_p}}(x_p) \tag{13}$$

$$\bar{h}^l_{h_{f_i}} = 1 \times \Pi^3_{p=1}\bar{\mu}_{F^l_{h_{f_p}}}(x_p), \ \underline{h}^l_{h_{f_i}} = b_{h_{f_i}}\Pi^3_{p=1}\underline{\mu}_{F^l_{h_{f_p}}}(x_p) \tag{14}$$

To obtain a crisp output from the T2FLS $f^l_{f_i}$, $g^l_{g_i}$, and $h^l_{h_{f_i}}$, we must defuzzify the type-reduced set. Since this type-reduced set is an interval set, therefore, the defuzzified output of $f^l_{f_i}$, $g^l_{g_i}$, and $h^l_{h_{f_i}}$ will be the average of $\bar{f}^l_{f_i}$, $\underline{f}^l_{f_i}$, $\bar{g}^l_{g_i}$, $\underline{g}^l_{g_i}$, and $\bar{h}^l_{h_{f_i}}$, $\underline{h}^l_{h_{f_i}}$ as follows:

$$f^l_{f_i} = \frac{1}{2}(\bar{f}^l_{f_i} + \underline{f}^l_{f_i}) \tag{15}$$

$$g^l_{g_i} = \frac{1}{2}(\bar{g}^l_{g_i} + \underline{g}^l_{g_i}) \tag{16}$$

$$h^l_{h_{f_i}} = \frac{1}{2}(\bar{h}^l_{h_{f_i}} + \underline{h}^l_{h_{f_i}}) \tag{17}$$

By the (12)-(17), we obtain

$$\hat{f}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{f_i}) = \frac{\sum_{l=1}^M f^l_{f_i}\hat{y}^l_{f_i}}{\sum_{l=1}^M f^l_{f_i}} = \boldsymbol{\xi}^\top_{f_i}(\boldsymbol{x})\hat{\boldsymbol{\theta}}_{f_i} \tag{18}$$

$$\hat{g}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{g_i}) = \frac{\sum_{l=1}^M g^l_{g_i}\hat{y}^l_{g_i}}{\sum_{l=1}^M g^l_{g_i}} = \boldsymbol{\xi}^\top_{g_i}(\boldsymbol{x})\hat{\boldsymbol{\theta}}_{g_i} \tag{19}$$

$$\hat{h}_{f_i}(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{h_{f_i}}) = \frac{\sum_{l=1}^M h^l_{h_{f_i}}\hat{y}^l_{h_{f_i}}}{\sum_{l=1}^M h^l_{h_{f_i}}} = \boldsymbol{\xi}^\top_{h_{f_i}}(\boldsymbol{x})\hat{\boldsymbol{\theta}}_{h_{f_i}} \tag{20}$$

where $\hat{\boldsymbol{\theta}}_{f_i} = [\hat{y}^l_{f_i}\cdots\hat{y}^M_{f_i}]^\top$, $\hat{\boldsymbol{\theta}}_{g_i} = [\hat{y}^l_{g_i}\cdots\hat{y}^M_{g_i}]^\top$, $\hat{\boldsymbol{\theta}}_{h_{f_i}} = [\hat{y}^l_{h_{f_i}}\cdots\hat{y}^M_{h_{f_i}}]^\top$ are adaptive parameter vectors, $\boldsymbol{\xi}_{f_i}(\boldsymbol{x}) = [\xi^1_{f_i}(\boldsymbol{x})\cdots\xi^M_{f_i}(\boldsymbol{x})]^\top$, $\boldsymbol{\xi}_{g_i}(\boldsymbol{x}) = [\xi^1_{g_i}(\boldsymbol{x})\cdots\xi^M_{g_i}(\boldsymbol{x})]^\top$, $\boldsymbol{\xi}_{h_{f_i}}(\boldsymbol{x}) = [\xi^1_{h_{f_i}}(\boldsymbol{x})\cdots\xi^M_{h_{f_i}}(\boldsymbol{x})]^\top$ are T2 fuzzy regressive vectors, $\hat{y}^l_{f_i}$, $\hat{y}^l_{g_i}$, $\hat{y}^l_{h_{f_i}}$ are points of the T2 fuzzy system output variables y_{f_i}, y_{g_i}, $y_{h_{f_i}}$ at which $\mu_{G^l_{f_i}}(y_{f_i})$, $\mu_{G^l_{g_i}}(y_{g_i})$, $\mu_{G^l_{h_{f_i}}}(y_{h_{f_i}})$ achieves its maximum value, respectively, $f^l_{f_i}$, $g^l_{g_i}$, $h^l_{h_{f_i}}$ are the firing interval of the l-th rule, and $\xi^l_{f_i}(\boldsymbol{x}) = \frac{f^l_{f_i}}{\sum_{l=1}^M f^l_{f_i}}$, $\xi^l_{g_i}(\boldsymbol{x}) = \frac{g^l_{g_i}}{\sum_{l=1}^M g^l_{g_i}}$, $\xi^l_{h_{f_i}}(\boldsymbol{x}) = \frac{h^l_{h_{f_i}}}{\sum_{l=1}^M h^l_{h_{f_i}}}$.

Let the reference signal $\boldsymbol{x}_m = [x_{m_1}\ x_{m_2}\ x_{m_3}]^\top$ and $\bar{\boldsymbol{e}} = [x_{m_1} - x_1\ x_{m_2} - x_2\ x_{m_3} - x_3] = [e_1\ e_2\ e_3]^\top \in \mathbb{R}^3$. If the functions $\boldsymbol{f}_i$ and $\mathbf{G}_i$ in (2) are known, then the control law can be given by

$$\mathbf{u}^*_i = (\alpha_i\mathbf{G}_i(\boldsymbol{x}_i))^{-1}(-\boldsymbol{f}_i(\boldsymbol{x}_i) - \Delta\boldsymbol{f}_i(\boldsymbol{x}_i) + \boldsymbol{x}^{(r)}_{m_i} - \mathbf{K}^\top_{a_i}\bar{\boldsymbol{e}}_i) \tag{21}$$

for $i = 1, 2, 3,\ r = 1, 2$, where $\boldsymbol{x}^{(r)}_{m_i} = [x_{m_2}\ x_{m_3}]^\top$, $\bar{\boldsymbol{e}}_1[e_1\ e_2]^\top$ and $\bar{\boldsymbol{e}}_2 = e_3$, $\mathbf{K}^\top_a = \begin{bmatrix} k_{a_1} & k_{a_2} & k_{a_3} \\ k_{a_4} & k_{a_5} & k_{a_6}\end{bmatrix}$, $\mathbf{k}^\top_{a_1} = [\ k_{a_1}\ \ k_{a_2}\ ]$, $\mathbf{k}^\top_{a_2} = k_{a_6}$. In this situation, to approximation f_i, g_i by fuzzy logic systems, f_i, g_i, and Δf_i in (21) can be replaced by the fuzzy logic systems $\hat{f}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{f_i})$, $\hat{g}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{g_i})$, and $\hat{h}_{f_i}(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{h_{f_i}})$, respectively. Based on the given plant (2) and dead-zone models under the assumptions 1-3 are available for measurement, the proposed T2AFC can be given as

$$\begin{aligned} u_i = &(\alpha_i\hat{g}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{g_i}))^{-1}(-\hat{f}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{f_i}) + z_i - \mathbf{k}^\top_{a_i}\bar{\boldsymbol{e}}_i \\ &+ u_{h_{f_i}} + u_{w_i} + u_{a_i} + u_{eq_i}),\ i = 1, 2 \end{aligned} \tag{22}$$

where $u_{h_{f_i}}$, u_{w_i}, u_{a_i}, and u_{eq_i} are compensator controllers for uncertainties, approximation error, H_∞ robust control to attenuate the effect on system, and compensation of the adaptive dead-zone respectively. Thus, from (2) and (4), the tracking error dynamic equation can be expressed as

$$\begin{aligned} \dot{\bar{\boldsymbol{e}}}_i = &\boldsymbol{a}_{m_i}\bar{\boldsymbol{e}}_i + \boldsymbol{b}_i\Big(\hat{f}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{f_i}) - f_i(\boldsymbol{x}) - \hat{f}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{f_i}) \\ &+ \alpha_i(\hat{g}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{g_i}) - g_i(\boldsymbol{x}))u_i - \alpha_i\hat{g}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{g_i})u_i - \Delta f_i(\boldsymbol{x}) - d_i \\ &- \mathbf{k}^\top_{a_i}\bar{\boldsymbol{e}}_i + z_i - g_i\psi_{1_i}(u_i)\Big) \end{aligned} \tag{23}$$

where,

$$\mathbf{A}_m = \begin{bmatrix} 0 & 1 & 0 \\ k_{a_1} & k_{a_2} & k_{a_3} \\ k_{a_4} & k_{a_5} & k_{a_6}\end{bmatrix},\ \boldsymbol{a}_{m_1} = \begin{bmatrix} 0 & 1 \\ k_{a_1} & k_{a_2}\end{bmatrix},\ \boldsymbol{a}_{m_2} = k_{a_6},$$

the feedback gain matrix $\mathbf{k}^\top_{a_i}$ is to make the eigenvalues of the matrix $\boldsymbol{a}_{m_i} \triangleq \boldsymbol{a}_i - \boldsymbol{b}_i\mathbf{k}^\top_{a_i}$ are with negative real part and substitute (22) into (23) the tracking error vector as

$$\begin{aligned} \dot{\bar{\boldsymbol{e}}}_i = &\boldsymbol{a}_{m_i}\bar{\boldsymbol{e}}_i + \boldsymbol{b}_i\Big(\hat{f}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{f_i}) - f_i(\boldsymbol{x}) + \alpha_i(\hat{g}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{g_i}) - g_i(\boldsymbol{x}))u_i \\ &- \Delta f_i(\boldsymbol{x}) - d_i - u_{h_{f_i}} - u_{w_i} - u_{a_i} - u_{eq_i} - g_i\psi_{1_i}(u_i)\Big) \end{aligned} \tag{24}$$

The control objective is to make the fuzzy controller u_i and some adaptive laws for adjusting the parameter vectors $\hat{\boldsymbol{\theta}}_{f_i}$, $\hat{\boldsymbol{\theta}}_{g_i}$, and $\hat{\boldsymbol{\theta}}_{h_{f_i}}$ such that the following conditions and assumptions are met:

Assumption 4: *The $g_i \neq 0$ and f_i, g_i are bounded for all $\boldsymbol{x} \in \mathbb{R}^3$, and the plant is feedback linearizable by static state feedback.*

Assumption 5: *$|\Delta f_i(\boldsymbol{x})| \leq h_{f_i}(\boldsymbol{x})$, where $h_{f_i}(\boldsymbol{x})$ is an unknown continuous function and can be estimated by an adaptive law in the latter.*

We design an adaptive fuzzy controller u_i to control the plant output to follow a reference trajectory $\boldsymbol{x}_m$ and $\hat{f}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{f_i})$, $\hat{g}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{g_i})$, $\hat{h}_{f_i}(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{h_{f_i}})$ are three T2 fuzzy universal approximators to $f_i(\boldsymbol{x})$, $g_i(\boldsymbol{x})$, $h_{f_i}(\boldsymbol{x})$, respectively.

Define the optimal parameter vectors $\boldsymbol{\theta}^*_{f_i}$, $\boldsymbol{\theta}^*_{g_i}$, and $\boldsymbol{\theta}^*_{h_{f_i}}$ as follows:

$$\begin{cases} \boldsymbol{\theta}^*_{f_i} = \arg\min_{\hat{\boldsymbol{\theta}}_{f_i} \in \boldsymbol{\Omega}_{f_i}} \{\sup_{\boldsymbol{x}\in\mathbb{R}^3} \|f_i(\boldsymbol{x}) - \hat{f}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{f_i})\|\} \\ \boldsymbol{\theta}^*_{g_i} = \arg\min_{\hat{\boldsymbol{\theta}}_{g_i} \in \boldsymbol{\Omega}_{g_i}} \{\sup_{\boldsymbol{x}\in\mathbb{R}^3} \|g_i - \hat{g}_i(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{g_i})\|\} \\ \boldsymbol{\theta}^*_{h_{f_i}} = \arg\min_{\hat{\boldsymbol{\theta}}_{h_{f_i}} \in \boldsymbol{\Omega}_{h_{f_i}}} \{\sup_{\boldsymbol{x}\in\mathbb{R}^3} \|h_{f_i}(\boldsymbol{x}) - \hat{h}_{f_i}(\boldsymbol{x}, \hat{\boldsymbol{\theta}}_{h_{f_i}})\|\} \end{cases} \tag{25}$$

where $\boldsymbol{\Omega}_{f_i} = \{\hat{\boldsymbol{\theta}}_{f_i} : \|\hat{\boldsymbol{\theta}}_{f_i}\| \leq M_{f_i}\}$, $\boldsymbol{\Omega}_{g_i} = \{\hat{\boldsymbol{\theta}}_{g_i} : \|\hat{\boldsymbol{\theta}}_{g_i}\| \leq M_{g_i}\}$, and $\boldsymbol{\Omega}_{h_{f_i}} = \{\hat{\boldsymbol{\theta}}_{h_{f_i}} : \|\hat{\boldsymbol{\theta}}_{h_{f_i}}\| \leq M_{h_{f_i}}\}$ are proper closed sets, also the parameter estimation errors are defined as $\tilde{\boldsymbol{\theta}}_{f_i} = \hat{\boldsymbol{\theta}}_{f_i} - \boldsymbol{\theta}^*_{f_i}$, $\tilde{\boldsymbol{\theta}}_{g_i} = \hat{\boldsymbol{\theta}}_{g_i} - \boldsymbol{\theta}^*_{g_i}$, and $\tilde{\boldsymbol{\theta}}_{h_{f_i}} = \hat{\boldsymbol{\theta}}_{h_{f_i}} - \boldsymbol{\theta}^*_{h_{f_i}}$. We should note that perfect matching of the unknown functions is almost impossible even with the optimal parameter vectors, the optimal matching error between fuzzy logic approximation model and the plant is denoted by

$$\begin{cases} \omega_{1_i} = \hat{f}_i(\boldsymbol{x}, \boldsymbol{\theta}^*_{f_i}) - f_i(\boldsymbol{x}) \\ \omega_{2_i} = \hat{h}_{f_i}(\boldsymbol{x}, \boldsymbol{\theta}^*_{h_{f_i}}) - h_{f_i}(\boldsymbol{x}) \end{cases} \tag{26}$$

as the minimum approximation errors, which correspond to approximation errors obtained when optimal parameters are used. Based on the approximation theory, the approximation errors are assumed as bounded

$$|\omega_{1_i} + \omega_{2_i}| \leq \omega_i \tag{27}$$

For the reduction of the learning, the unknown approximation error (27) and its corresponding learning error is defined as follows:

$$\tilde{\omega}_i = \omega_i - \hat{\omega}_i \tag{28}$$

where $\hat{\omega}_i$ is the learning of ω_i. Then, the adaptive control laws can be obtained as

$$\dot{\tilde{\boldsymbol{\theta}}}_{f_i} = -\gamma_{f_i}\boldsymbol{\xi}_{f_i}(\boldsymbol{x})\boldsymbol{b}_i^\top \boldsymbol{p}_i\bar{\boldsymbol{e}}_i \quad (29)$$

$$\dot{\tilde{\boldsymbol{\theta}}}_{g_i} = -\gamma_{g_i}\boldsymbol{\xi}_{g_i}(\boldsymbol{x})\boldsymbol{b}_i^\top \boldsymbol{p}_i\bar{\boldsymbol{e}}_i u_i \quad (30)$$

$$\dot{\tilde{\boldsymbol{\theta}}}_{h_{f_i}} = \gamma_{h_{f_i}}\boldsymbol{\xi}_{h_{f_i}}(\boldsymbol{x})|\boldsymbol{b}_i^\top \boldsymbol{p}_i\bar{\boldsymbol{e}}_i| \quad (31)$$

$$\dot{\hat{\omega}}_i = -\gamma_{\omega_i}|\boldsymbol{b}_i^\top \boldsymbol{p}_i\bar{\boldsymbol{e}}_i| \quad (32)$$

where γ_{f_i}, γ_{g_i}, $\gamma_{h_{f_i}}$, γ_{ω_i}, are adaption rates, and $\boldsymbol{p}_i = \boldsymbol{p}_i^\top > 0$ is the positive solution of the Riccati-like equation described later. Let the T2AFC law can be given in (22) with

$$u_{a_i} = -\frac{1}{\gamma_i}\boldsymbol{b}_i^\top \boldsymbol{p}_i\bar{\boldsymbol{e}}_i \quad (33)$$

$$u_{h_{f_i}} = \frac{|\bar{\boldsymbol{e}}_i^\top \boldsymbol{p}_i\boldsymbol{b}_i|}{\bar{\boldsymbol{e}}_i^\top \boldsymbol{p}_i\boldsymbol{b}_i}\hat{h}_{f_i}(\boldsymbol{x},\hat{\boldsymbol{\theta}}_{h_{f_i}}) \quad (34)$$

$$u_{w_i} = \frac{|\bar{\boldsymbol{e}}_i^\top \boldsymbol{p}_i\boldsymbol{b}_i|}{\bar{\boldsymbol{e}}_i^\top \boldsymbol{p}_i\boldsymbol{b}_i}\hat{\omega}_i \quad (35)$$

$$u_{eq_i} = \frac{|\bar{\boldsymbol{e}}_i^\top \boldsymbol{p}_i\boldsymbol{b}_i|}{\bar{\boldsymbol{e}}_i^\top \boldsymbol{p}_i\boldsymbol{b}_i}\zeta_i \quad (36)$$

where $\zeta_i \geq |\alpha_i(\hat{g}_i(\boldsymbol{x},\boldsymbol{\theta}^*_{g_i}) - g_i(\boldsymbol{x}))u_i - g_i(\boldsymbol{x})(-\alpha_i\epsilon_i + \kappa_i)|$, $\gamma_i > 0$ is a gain parameter to be designed, $\boldsymbol{p}_i = \boldsymbol{p}_i^\top > 0$ is a symmetric positive definite matrix satisfying the following Riccati-like equation:

$$\boldsymbol{a}_{m_i}^\top \boldsymbol{p}_i + \boldsymbol{p}_i\boldsymbol{a}_{m_i} + \mathbf{Q}_i - \frac{1}{\gamma_i}\boldsymbol{p}_i\boldsymbol{b}_i\boldsymbol{b}_i^\top \boldsymbol{p}_i + \frac{1}{\eta_i^2}\boldsymbol{p}_i\boldsymbol{b}_i\boldsymbol{b}_i^\top \boldsymbol{p}_i \leq 0 \quad (37)$$

for which $\eta_i > 0$ is attenuation level parameters for coping with the error $d_i(t)$ and $\mathbf{Q}_i = \mathbf{Q}_i^\top \geq 0$ is a prescribed weighting matrix. Then, for any $t \geq 0$, $\bar{\boldsymbol{e}}_i$, $\tilde{\boldsymbol{\theta}}_{f_i}$, $\tilde{\boldsymbol{\theta}}_{g_i}$, $\tilde{\boldsymbol{\theta}}_{h_{f_i}}$, and $\tilde{\omega}_i$ are uniformly ultimately bounded (UUB), and the H_∞ tracking performance within a prescribed value η_i^2 for the overall system satisfies the following relationship:

$$J = \int_0^{t_f}\bar{\boldsymbol{e}}_i^\top(t)\mathbf{Q}_i\bar{\boldsymbol{e}}_i(t)dt \leq \bar{\boldsymbol{e}}_i^\top(0)\boldsymbol{p}_i\bar{\boldsymbol{e}}_i(0)dt + \frac{1}{\gamma_{f_i}}\tilde{\boldsymbol{\theta}}_{f_i}^\top(0)\tilde{\boldsymbol{\theta}}_{f_i}(0) + \frac{1}{\gamma_{g_i}}\tilde{\boldsymbol{\theta}}_{g_i}^\top(0)\tilde{\boldsymbol{\theta}}_{g_i}(0) + \frac{1}{\gamma_{h_{f_i}}}\tilde{\boldsymbol{\theta}}_{h_{f_i}}^\top(0)\tilde{\boldsymbol{\theta}}_{h_{f_i}}(0) + \frac{1}{\gamma_{\omega_i}}\tilde{\omega}_i^2(0) + \eta_i^2\int_0^{t_f}d_i^2dt \quad (38)$$

IV. Analysis of System Stability

Proof: The following Lyapunov function candidate is considered

$$V_i = \frac{1}{2\alpha_i}\Big(\bar{\boldsymbol{e}}_i^\top \boldsymbol{p}_i\bar{\boldsymbol{e}}_i + \frac{1}{\gamma_{f_i}}\tilde{\boldsymbol{\theta}}_{f_i}^\top\tilde{\boldsymbol{\theta}}_{f_i} + \frac{\alpha_i}{\gamma_{g_i}}\tilde{\boldsymbol{\theta}}_{g_i}^\top\tilde{\boldsymbol{\theta}}_{g_i} + \frac{1}{\gamma_{h_{f_i}}}\tilde{\boldsymbol{\theta}}_{h_{f_i}}^\top\tilde{\boldsymbol{\theta}}_{h_{f_i}} + \frac{1}{\gamma_{\omega_i}}\tilde{\omega}_i^2\Big) \quad (39)$$

Taking time derivative of in (39), we have

$$\dot{V}_i = \frac{1}{\alpha_i}\dot{\bar{\boldsymbol{e}}}_i^\top \boldsymbol{p}_i\bar{\boldsymbol{e}}_i + \frac{1}{\alpha_i\gamma_{f_i}}\tilde{\boldsymbol{\theta}}_{f_i}^\top\dot{\tilde{\boldsymbol{\theta}}}_{f_i} + \frac{1}{\gamma_{g_i}}\tilde{\boldsymbol{\theta}}_{g_i}^\top\dot{\tilde{\boldsymbol{\theta}}}_{g_i} + \frac{1}{\alpha_i\gamma_{h_{f_i}}}\tilde{\boldsymbol{\theta}}_{h_{f_i}}^\top\dot{\tilde{\boldsymbol{\theta}}}_{h_{f_i}} + \frac{1}{\alpha_i\gamma_{\omega_i}}\tilde{\omega}_i\dot{\tilde{\omega}}_i$$

The above equation along the trajectory of (24) is given by:

$$\begin{aligned}\dot{V}_i = &\frac{1}{2\alpha_i}\Big(\boldsymbol{a}_{m_i}\bar{\boldsymbol{e}}_i + \boldsymbol{b}_i(\hat{f}_i(\boldsymbol{x},\hat{\boldsymbol{\theta}}_{f_i}) - f_i(\boldsymbol{x}) + \alpha_i(\hat{g}_i(\boldsymbol{x},\hat{\boldsymbol{\theta}}_{g_i}) - g_i(\boldsymbol{x}))u_i\\ &-\Delta f_i(\boldsymbol{x}) - d_i - u_{h_{f_i}} - u_{w_i} - u_{a_i} - u_{eq_i} - g_i\psi_{1_i}(u_i))\Big)^\top \boldsymbol{p}_i\bar{\boldsymbol{e}}_i\\ &+\frac{1}{2\alpha_i}\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\Big(\boldsymbol{a}_{m_i}\bar{\boldsymbol{e}}_i + \boldsymbol{b}_i(\hat{f}_i(\boldsymbol{x},\hat{\boldsymbol{\theta}}_{f_i}) - f_i(\boldsymbol{x}) + \alpha_i(\hat{g}_i(\boldsymbol{x},\hat{\boldsymbol{\theta}}_{g_i})\\ &-g_i(\boldsymbol{x}))u_i - \Delta f_i(\boldsymbol{x}) - d_i - u_{h_{f_i}} - u_{w_i} - u_{a_i} - u_{eq_i} - g_i\psi_{1_i}(u_i))\Big)\\ &+\frac{1}{\alpha_i\gamma_{f_i}}\tilde{\boldsymbol{\theta}}_{f_i}^\top\dot{\tilde{\boldsymbol{\theta}}}_{f_i} + \frac{1}{\gamma_{g_i}}\tilde{\boldsymbol{\theta}}_{g_i}^\top\dot{\tilde{\boldsymbol{\theta}}}_{g_i} + \frac{1}{\alpha_i\gamma_{h_{f_i}}}\tilde{\boldsymbol{\theta}}_{h_{f_i}}^\top\dot{\tilde{\boldsymbol{\theta}}}_{h_{f_i}} + \frac{1}{\alpha_i\gamma_{\omega_i}}\tilde{\omega}_i\dot{\tilde{\omega}}_i\end{aligned}$$

By the fact $\dot{\tilde{\boldsymbol{\theta}}}_{f_i} = \dot{\hat{\boldsymbol{\theta}}}_{f_i}$, $\dot{\tilde{\boldsymbol{\theta}}}_{g_i} = \dot{\hat{\boldsymbol{\theta}}}_{g_i}$, $\dot{\tilde{\boldsymbol{\theta}}}_{h_{f_i}} = \dot{\hat{\boldsymbol{\theta}}}_{h_{f_i}}$, and $\dot{\tilde{\omega}}_i = \dot{\hat{\omega}}_i$, the above equation becomes

$$\begin{aligned}\dot{V}_i = &\frac{1}{2\alpha_i}\bar{\boldsymbol{e}}_i^\top\boldsymbol{a}_{m_i}^\top\boldsymbol{p}_i\bar{\boldsymbol{e}}_i + \frac{1}{2\alpha_i}\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{a}_{m_i}\bar{\boldsymbol{e}}_i - \frac{1}{2\alpha_i\gamma_i}\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i\boldsymbol{b}_i^\top\boldsymbol{p}_i\bar{\boldsymbol{e}}_i\\ &-\frac{1}{2\alpha_i\gamma_i}\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i\boldsymbol{b}_i^\top\boldsymbol{p}_i\bar{\boldsymbol{e}}_i - \frac{1}{2\alpha_i}d_i^\top\boldsymbol{b}_i^\top\boldsymbol{p}_i\bar{\boldsymbol{e}}_i - \frac{1}{2\alpha_i}\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i d_i\\ &+\frac{1}{\alpha_i}\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i(\hat{f}_i(\boldsymbol{x},\hat{\boldsymbol{\theta}}_{f_i}) - f_i(\boldsymbol{x}) + \alpha_i(\hat{g}_i(\boldsymbol{x},\hat{\boldsymbol{\theta}}_{g_i}) - g_i(\boldsymbol{x}))u_i\\ &-\Delta f_i(\boldsymbol{x}) - u_{h_{f_i}} - u_{w_i} - u_{eq_i} - g_i\psi_{1_i}(u_i))\\ &+\frac{1}{\alpha_i\gamma_{f_i}}\tilde{\boldsymbol{\theta}}_{f_i}^\top\dot{\tilde{\boldsymbol{\theta}}}_{f_i} + \frac{1}{\gamma_{g_i}}\tilde{\boldsymbol{\theta}}_{g_i}^\top\dot{\tilde{\boldsymbol{\theta}}}_{g_i} + \frac{1}{\alpha_i\gamma_{h_{f_i}}}\tilde{\boldsymbol{\theta}}_{h_{f_i}}^\top\dot{\tilde{\boldsymbol{\theta}}}_{h_{f_i}} + \frac{1}{\alpha_i\gamma_{\omega_i}}\tilde{\omega}_i\dot{\tilde{\omega}}_i\\ \leq &-\frac{1}{2\alpha_i}\bar{\boldsymbol{e}}_i^\top\mathbf{Q}_i\bar{\boldsymbol{e}}_i + \frac{1}{2\alpha_i}\eta_i^2 d_i^2 + \frac{1}{\alpha_i}\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i\boldsymbol{\xi}_{f_i}^\top(\boldsymbol{x})\tilde{\boldsymbol{\theta}}_{f_i} + \frac{1}{\alpha_i\gamma_{f_i}}\tilde{\boldsymbol{\theta}}_{f_i}^\top\dot{\hat{\boldsymbol{\theta}}}_{f_i}\\ &+\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i\boldsymbol{\xi}_{g_i}^\top(\boldsymbol{x})\tilde{\boldsymbol{\theta}}_{g_i}u_i + \frac{1}{\gamma_{g_i}}\tilde{\boldsymbol{\theta}}_{g_i}^\top\dot{\hat{\boldsymbol{\theta}}}_{g_i} - \frac{1}{\alpha_i}|\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i|\boldsymbol{\xi}_{h_{f_i}}^\top(\boldsymbol{x})\tilde{\boldsymbol{\theta}}_{h_{f_i}}\\ &+\frac{1}{\alpha_i\gamma_{h_{f_i}}}\tilde{\boldsymbol{\theta}}_{h_{f_i}}^\top\dot{\hat{\boldsymbol{\theta}}}_{h_{f_i}} + \frac{1}{|\alpha_i|}|\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i|\tilde{\omega}_i + \frac{1}{\alpha_i\gamma_{\omega_i}}\tilde{\omega}_i\dot{\hat{\omega}}_i + \frac{1}{|\alpha_i|}|\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i|\hat{\omega}_i\\ &-\frac{1}{\alpha_i}\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i u_{w_i} + \frac{1}{\alpha_i}|\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i|\hat{h}_{f_i}(\boldsymbol{x},\hat{\boldsymbol{\theta}}_{h_{f_i}}) - \frac{1}{\alpha_i}\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i u_{h_{f_i}}\\ &+|\bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i|\zeta_i - \bar{\boldsymbol{e}}_i^\top\boldsymbol{p}_i\boldsymbol{b}_i u_{eq_i}\end{aligned}$$

Hence, applying the adaptive control laws (29)-(32), the T2AFC law in (22), Form the Riccati-like equation (37), we can obtain

$$\dot{V}_i \leq -\frac{1}{2\alpha_i}\bar{\boldsymbol{e}}_i^\top\mathbf{Q}_i\bar{\boldsymbol{e}}_i + \frac{1}{2\alpha_i}\eta_i^2 d_i^2 \quad (40)$$

where $\lambda_{i_{min}}(\mathbf{Q}_i)$ denotes the minimal eigenvalue of $\mathbf{Q}_i$. Therefore, whenever

$$||\bar{\boldsymbol{e}}_i|| \geq \frac{\eta\sqrt{d_i^2(t)}}{\lambda_{i_{min}}(\mathbf{Q}_i)} \quad (41)$$

we have $\dot{V}_i \leq 0$. Therefore, the overall system satisfies the following relationship:

$$\begin{aligned}&\int_0^{t_f}\bar{\boldsymbol{e}}_i^\top(t)\mathbf{Q}_i\bar{\boldsymbol{e}}_i(t)dt \leq \bar{\boldsymbol{e}}_i^\top(0)\boldsymbol{p}_i\bar{\boldsymbol{e}}_i(0)dt\\ &+\frac{1}{\gamma_{f_i}}\tilde{\boldsymbol{\theta}}_{f_i}^\top(0)\tilde{\boldsymbol{\theta}}_{f_i}(0) + \frac{1}{\gamma_{g_i}}\tilde{\boldsymbol{\theta}}_{g_i}^\top(0)\tilde{\boldsymbol{\theta}}_{g_i}(0) + \frac{1}{\gamma_{h_{f_i}}}\tilde{\boldsymbol{\theta}}_{h_{f_i}}^\top(0)\tilde{\boldsymbol{\theta}}_{h_{f_i}}(0)\\ &+\frac{1}{\gamma_{\omega_i}}\tilde{\omega}_i^2(0) + \eta_i^2\int_0^{t_f}d_i^2dt.\end{aligned} \quad (42)$$

In light of Lyapunov stability theory to the retarded functional differential equation [24, 26], $\boldsymbol{e}$ and $\boldsymbol{x}_m$ as well as the parameter estimation errors $\tilde{f}_i(\boldsymbol{x},\hat{\boldsymbol{\theta}}_{f_i})$, $\tilde{g}_i(\boldsymbol{x},\hat{\boldsymbol{\theta}}_{g_i})$, $\tilde{h}_{f_i}(\boldsymbol{x},\hat{\boldsymbol{\theta}}_{h_{f_i}})$, and $\hat{\omega}_i$ are guaranteed to be uniformly ultimately bounded (UUB) for all realizations of uncertainties [25]. This completes the proof. ■

V. Simulations

In this section, a PMDC motor system in Fig. 3 with sector uncertainty of nonlinearities by the dead-zone is used to illustrates the effectiveness of the proposed scheme. The PMDC motor dynamic is given by (1) where the parameters are as follows: $R_{aN} = 1.1\Omega$, $R_{a\Delta} = 1.1\Omega$, $L_a = 0.4mH$, $k_t = 0.05Nm/A$, $k_E = 0.05V/rad/s$, $F_{dN} = 1.0 \times 10^{-4}Nm/rad/s$, $F_{d\Delta} = 1.0 \times 10^{-4}Nm/rad/s$, $J = 1.0338 \times 10^{-5}kgm^2$, and $\varsigma_1 = 0.175$. The external disturbance is $d_1 = (1/J)\varsigma$, $\mathbf{\Phi}(\mathbf{u}) = [\Phi_1(u_1)\ \ \Phi_2(u_2)]^\top$ are the load torque and armature voltage with dead zone (22), respectively. Based on the design procedure illustrated previously, the T2AFC design is synthesize as follows:

Step 1: For high penalty on initial parameter errors, select design parameters as: the adaptive control laws (29) and (30), we choose $\gamma_f = 1$, $\gamma_{f_\Delta} = 0.1$, $\gamma_{g_1} = 0.1$, $\gamma_{g_2} = 0.1$, and $\gamma_\varepsilon = 0.5$. The initial conditions are chosen as $\boldsymbol{x}(0) = [x_1(0)\ \ x_2(0)\ \ x_3(0)]^\top = [-\pi/3\ \ 1\ \ 1]^\top$ are

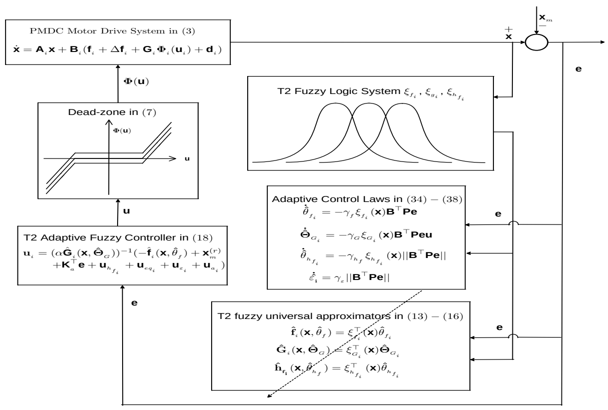

Fig. 2. Overall closed control system of the type-2 adaptive fuzzy H_∞ tracking control with dead-zone.

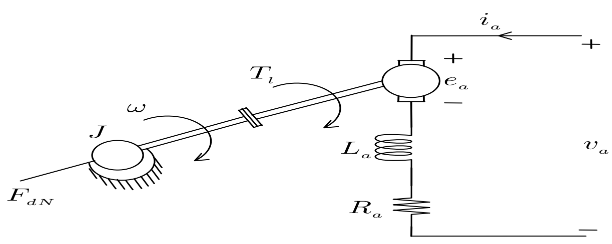

Fig. 3. PMDC motor system [23].

the rotational angle, rotational speed, and armature current, respectively, the control objective is to make x_1, x_2, x_3 follow the reference trajectories $\boldsymbol{x}_m(0) = [x_{m_1}(0)\ x_{m_2}(0)\ x_{m_3}(0)]^\top = [\frac{\pi}{2}sin(t)\ \frac{\pi}{2}cos(t)\ -\frac{\pi}{6}sin(t)]^\top$, $\theta_{f_1} = 0.5$, $\theta_{f_2} = 0.5$, $\theta_{h_{f_1}} = 1.5$, $\theta_{h_{f_2}} = 1.5$, $\theta_{g_{11}} = 0.5$, $\theta_{g_{22}} = 0.5$, and step size is $h = 0.01$. According to the boundary conditions of the PMDC motor system, the bounds $h_{f_1} = 15$, $h_{f_2} = 25$, $M_f = 100$, $M_{h_f} = 800$, and $M_g = 100$.

Step 2: In the simulation, the beam is actuated by a PMDC motor system with sector uncertainty of nonlinearities by the dead-zone being $\epsilon_1^+ = 0.8$, $\epsilon_1^- = -0.8$, $\epsilon_2^+ = 0.5$, $\epsilon_2^- = -0.5$, $\bar{\kappa}_1 = 0.8$, $\underline{\kappa}_1 = -0.8$, $\bar{\kappa}_2 = 0.5$, $\underline{\kappa}_2 = -0.5$, and $\alpha' = \underline{\alpha} = \bar{\alpha} = \alpha = 1$. And the bounds of them are chosen as $\alpha_{max} = 1.25$, $\alpha_{min} = 0.85$, $\epsilon_{1_{imax}}^+ = 0.9$, $\epsilon_{1_{imin}}^- = -0.9$, $\epsilon_{2_{imax}}^+ = 0.9$, $\epsilon_{2_{imin}}^- = -0.9$, $\bar{\kappa}_{1_{imax}} = 0.9$, $\underline{\kappa}_{1_{imin}} = -0.9$, $\bar{\kappa}_{2_{imax}} = 0.9$, $\underline{\kappa}_{2_{imin}} = -0.9$, $\rho_1 = max\{\alpha_{max}\epsilon_{1_{max}}^+ + \bar{\kappa}_{1_{max}}, \alpha_{max}\epsilon_{1_{min}}^- + \underline{\kappa}_{1_{min}}\}$, and $\rho_2 = max\{\alpha_{max}\epsilon_{2_{max}}^+ + \bar{\kappa}_{2_{max}}, \alpha_{max}\epsilon_{2_{min}}^- + \underline{\kappa}_{2_{min}}\}$

Step 3: Select the feedback gain $\mathbf{K}^\top = \begin{bmatrix} -100 & -15 & 55 \\ 15 & 15 & 0.55 \end{bmatrix}$ such that all the roots of the characteristic polynomials of $\mathbf{A}_m = \mathbf{A} - \mathbf{B}\mathbf{K}^\top$ are with negative real part. We choose the prescribed attenuation levels $\eta = 0.5$, $\eta = 0.7$, and $\eta = 0.9$ for performance comparisons, the weighting matrix $\mathbf{Q} = 0.009\mathbf{I}_{3X3}$, and $\gamma = 0.8$. Then, the positive definite symmetric matrices $\boldsymbol{p}_i$ for $\eta = 0.5$ can be obtained as

$$\boldsymbol{p}_i = \begin{bmatrix} 12.4554 & 0.3060 & 1.6390 \\ 0.3060 & 0.3675 & 0.1057 \\ 1.6390 & 0.1057 & 1.5450 \end{bmatrix}$$

for $\eta = 0.7$

$$\boldsymbol{p}_i = \begin{bmatrix} 11.7080 & 0.3758 & 2.0491 \\ 0.3758 & 0.4159 & 0.0980 \\ 2.0491 & 0.0980 & 1.6794 \end{bmatrix}$$

for $\eta = 0.9$

$$\boldsymbol{p}_i = \begin{bmatrix} 4.5103 & 0.1127 & 0.6535 \\ 0.1127 & 0.1359 & 0.0279 \\ 0.6535 & 0.0279 & 0.5354 \end{bmatrix}$$

Step 4: In order to approximate the unknown nonlinear functions $\mathbf{F}_N(\boldsymbol{x})$, $\mathbf{F}_\Delta(\boldsymbol{x})$, and $\mathbf{G}(\boldsymbol{x})$, select $\bar{\mu}_{C^\ell_{f_{Ni}}}(\hat{y}_{f_{Ni}})$, $\bar{\mu}_{C^\ell_{h_{f_i}}}(\hat{y}_{h_{f_i}})$, $\bar{\mu}_{C^\ell_{g_{ii}}}(\hat{y}_{g_{ii}})$, and $\underline{\mu}_{C^\ell_{f_{Ni}}}(\hat{y}_{f_{Ni}})$, $\underline{\mu}_{C^\ell_{h_{f_i}}}(\hat{y}_{h_{f_i}})$, $\underline{\mu}_{C^\ell_{g_{ii}}}(\hat{y}_{g_{ii}})$ in (6)-(8) are upper and lower membership functions, respectively, where these parameters are subject to $\underline{\sigma}_{f_p} = \underline{\sigma}_{g_p} = \underline{\sigma}_{h_{f_p}} = \pi/24$ and $\bar{\sigma}_{f_p} = \bar{\sigma}_{g_p} = \bar{\sigma}_{h_{f_p}} = \pi/18$ are fixed standard deviations of the lower and upper membership functions which characterize the shape of $\mu_{F^l_{f_p}}(x_p)$, $\mu_{F^l_{g_p}}(x_p)$, $\mu_{F^l_{h_{f_p}}}(x_p)$, respectively, $a_{f_{N1}} = a_{f_{N2}} = a_{f_{N3}} = a_{h_{f_1}} = a_{h_{f_2}} = a_{h_{f_3}} = a_{g_{11}} = a_{g_{22}} = a_{g_{33}} = 0.1$ are the FOU width coefficient which defines the FOU width between the upper membership function and the lower membership function, $m_{f_p} = m_{g_p} = m_{h_{f_p}} = [-\pi/6, +\pi/6]$ are the means of the T2FS, and we select the FOU width coefficients for the consequence fuzzy set membership functions as $b_{f_{N1}} = b_{f_{N2}} = b_{h_{f_1}} = b_{h_{f_2}} = b_{g_1} = b_{g_2} = 0.1$. The mathematical mean of the defuzzified output are $f^\ell_{f_{N1}} = \frac{1}{2}(\bar{f}^\ell_{f_{N1}} + \underline{f}^\ell_{f_{N1}})$, $f^\ell_{f_{N2}} = \frac{1}{2}(\bar{f}^\ell_{f_{N2}} + \underline{f}^\ell_{f_{N2}})$, $h^\ell_{h_{f_1}} = \frac{1}{2}(\bar{h}^\ell_{h_{f_1}} + \underline{h}^\ell_{h_{f_1}})$, $h^\ell_{h_{f_2}} = \frac{1}{2}(\bar{h}^\ell_{h_{f_2}} + \underline{h}^\ell_{h_{f_2}})$, $g^\ell_{g_{11}} = \frac{1}{2}(\bar{g}^\ell_{g_{11}} + \underline{g}^\ell_{g_{11}})$, and $g^\ell_{g_{22}} = \frac{1}{2}(\bar{g}^\ell_{g_{22}} + \underline{g}^\ell_{g_{22}})$, $l = 1, \cdots, M$.

Step 5: Apply the control force in (42) to the PMDC motor system. Then, computed the adaptive laws in (29)-(32) to adjust parameter $\boldsymbol{\Theta}_f$, $\boldsymbol{\Theta}_{f_\Delta}$, $\boldsymbol{\Theta}_g$, and $\hat{\omega}_i$

According to Riccati-inequality equation (37), we choose the prescribed attenuation levels $\eta = 0.5$, $\eta = 0.7$, and $\eta = 0.9$ for performance comparisons. To illustrate the efficacy of the H_∞ performances for $\eta = 0.5$, $\eta = 0.7$, and $\eta = 0.9$ (see Figs. 4-6). Fig. 4 shows the trajectories of the rotational angle, x_1. The responses of the rotational speed, x_2, are depicted in Fig. 5. Fig. 6 show the armature current, x_3. x_1 converges to the neighborhood of sinusoidal function in about $3s$ for $\eta = 0.5$ as shown in Fig. 4. Fig. 5 shows that the trajectories of x_2 also converge to the neighborhood of cosine function in about 2s for $\eta = 0.5$. It is seen that x_3 converges to the vicinity of sinusoidal function converge to the neighborhood of sinusoidal function in about 6s for $\eta = 0.5$ as shown Fig. 6. It should be mentioned that good performance was achieved when using the proposed control scheme (22) as in Figs. 7 and 8 and show that the control inputs u_1 and u_2 eventually drop down to zero and reduce the tracking error $e_1(t)$, $e_2(t)$, and $e_3(t)$ to very small values (see Figs. 11-13). It is seen that the H_∞ tracking performance is better in the vaule of $\eta = 0.5$ then in the other two vaules, i.e., $\eta = 0.7$ and $\eta = 0.9$. The desired and actual trajectories are almost identical in the vaule of $\eta = 0.5$ and the proposed T2AFC guarantees both the stability and good tracking performance of the PMDC motor system. Further, to further analyse the efficiency and feasibility of the H_∞ control scheme, we define the performance index, STE, as follows:

$$STE = \sum |x_{m_p}(t) - x_p(t)|, \text{ for } p = 1, 2, 3$$

where t is the time from $0 - 50s$, and the sampling rate is $0.01s$. From Table 1, the STE values for $\eta = 0.5$, $\eta = 0.7$ and $\eta = 0.9$ are e_1=208.8817, 247.5688, and 296.5253, e_2=53.59040, 107.0801, and 156.8613, e_3=882.8061, 885.6963, and 885.8151, respectively. It

TABLE I
COMPARISONS OF STE FOR DIFFERENT ATTENUATION LEVELS ρ

	STE for $x_{m_1} - x_1$	STE for $x_{m_2} - x_2$	STE for $x_{m_3} - x_3$
$\eta = 0.5$	208.8817	53.59040	882.8061
$\eta = 0.7$	247.5688	107.0801	885.6963
$\eta = 0.9$	296.5253	156.8613	885.8151

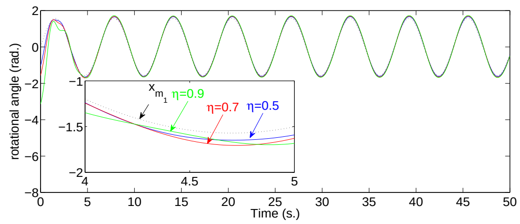

Fig. 4. The trajectories of $x_1(t)$ and $x_{m_1}(t)$ for three attenuation levels $\eta = 0.5$, $\eta = 0.7$, and $\eta = 0.9$.

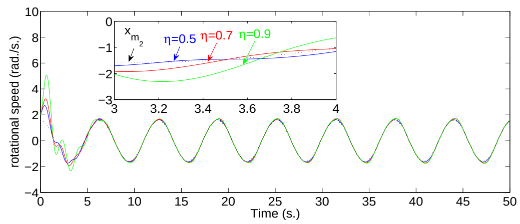

Fig. 5. The trajectories of $x_2(t)$ and $x_{m_2}(t)$ for three attenuation levels $\eta = 0.5$, $\eta = 0.7$, and $\eta = 0.9$.

is seen that the smaller η is the better the performance of the system when it comes to steady state error. In other words, the performance of the H_∞ control scheme is better when using smaller η for T2AFC weights of the PMDC motor system. However, the actual states track the reference trajectory closely, and thus, the controller successfully controls the rotational angle, rotational speed, and armature current of the PMDC motor system and achieve satisfactory H_∞ tracking performance in the presence of the deadzone and external disturbances.

VI. CONCLUSION

In this paper, we have proposed the T2AFC control scheme for PMDC motor systems with dead-zone nonlinearity at the input of a linear plant to achieve H_∞ tracking performance. First, a PMDC motor systems is approximated by the Takagi-Sugeno (T-S)-type fuzzy linear models with adaptation capability. In order to reduce the effect of dead-zone nonlinearity, a new model for dead-zone is developed and used in any conventional controllers, uncertainties and external disturbances such that the H_∞ tracking performance is achieved. The advantage of employing T2AFC is that we can utilise the linguistic information by setting the MFs of fuzzy logical system and the adaptation parameters to force the model uncertainties and plant parameters to track the the specified values for using linear analytical results instead of estimating non-linear system functions as the system parameters are unknown, and the system input is with dead-zone. Thus, a parameter estimation scheme applicable to the T2AFC and controllers for stabilizing nonlinear systems with dead zone nonlinearities in the control input is needed. Based on Lyapunov criterion and Riccati-inequality, some sufficient conditions are derived so that all states of the system are UUB and the effect of the external disturbance on the tracking error can be attenuated to any prescribed

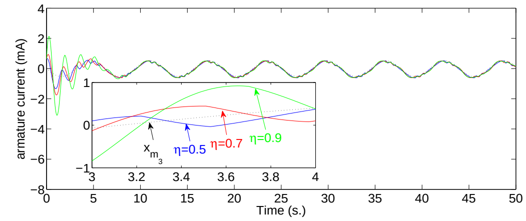

Fig. 6. The trajectories of $x_3(t)$ and $x_{m_3}(t)$ for three attenuation levels $\eta = 0.5$, $\eta = 0.7$, and $\eta = 0.9$.

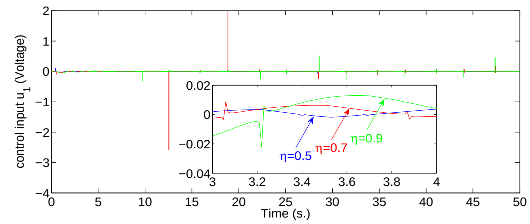

Fig. 7. The trajectories of control input u_1 with a dead-zone for three attenuation levels $\eta = 0.5$, $\eta = 0.7$, and $\eta = 0.9$.

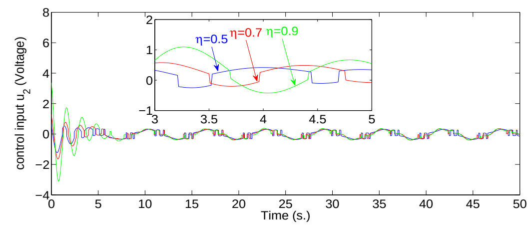

Fig. 8. The trajectories of control input u_2 with a dead-zone for three attenuation levels $\eta = 0.5$, $\eta = 0.7$, and $\eta = 0.9$.

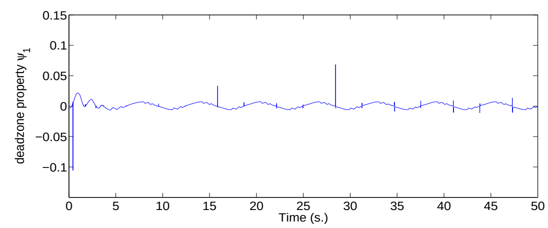

Fig. 9. The trajectories of control input u_1 with a dead-zone.

level and consequently an H_∞ tracking control is achieved. Finally, a PMDC motor systems as a simulation example and some comparisons are given to illustrate the validity and effectiveness of the proposed method.

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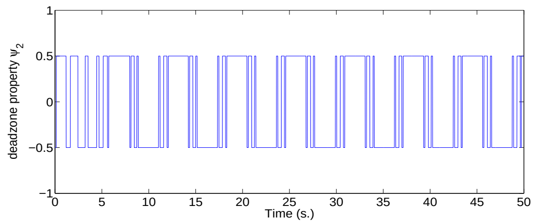

Fig. 10. The trajectories of control input u_2 with a dead-zone.

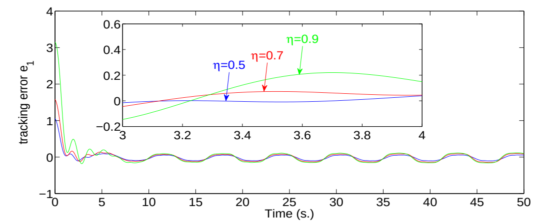

Fig. 11. The trajectory of the tracking errors e_1 for three attenuation levels $\eta = 0.5$, $\eta = 0.7$, and $\eta = 0.9$.

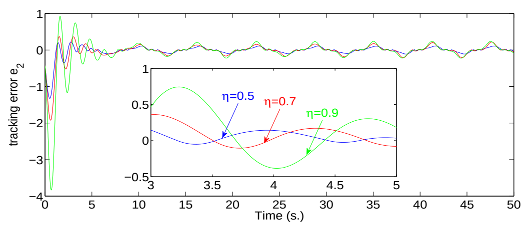

Fig. 12. The trajectory of the tracking errors e_2 for three attenuation levels $\eta = 0.5$, $\eta = 0.7$, and $\eta = 0.9$.

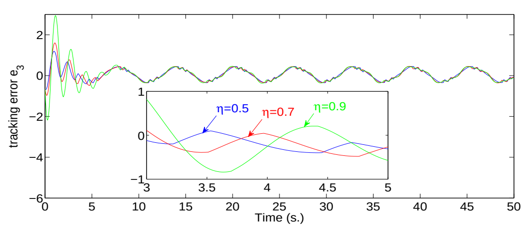

Fig. 13. The trajectory of the tracking errors e_3 for three attenuation levels $\eta = 0.5$, $\eta = 0.7$, and $\eta = 0.9$.

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