Modified Generalized Way for Optimization Problem

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Abstract: The solution to the problem of circuit optimization is obtained on the basis of a combination of a genetic algorithm (GA) and the idea of generalized optimization, developed earlier for the deterministic case. It is shown that such a GA modification allows one to overcome premature convergence to local minima and to increase the minimization accuracy by several orders of magnitude. In this case, GA forms a set of populations determined by the fitness function, given in different way, depending on the strategy chosen within the framework of the idea of generalized optimization. The way of setting fitness functions as well as the length and structure of chromosomes, are determined by a control vector artificially introduced within the framework of generalized optimization. This vector determines the number of independent variables of the optimization problem and the method for calculating the fitness function. It allows you to build compound strategies that significantly increase the accuracy of the resulting solution. This, in turn, makes it possible to reduce the number of generations required during the operation of the GA and minimize the processor time for solving the problem of circuit optimization.

Keywords: generalized optimization, GA, circuit optimization, control vector, set of strategies

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1. Introduction

One of the major challenges in designing a large system is the excessive computing time required to reach the optimum point in the design process. This problem is important as it has many applications, for example, in the design of VLSI circuits. The design process starts with an initial approximation that is provided by analysis of circuit for the initial point and then the process is continued till adjusting of the system parameters to obtain the necessary performance features defined in the specification. The process of setting parameters is usually based on the optimization procedure. So, the process of design-by-analysis can be realized instead of the difficult problem of synthesis of a complex system. Mathematically, this process is defined as the minimization of a special objective function that includes necessary properties of the designed circuit. It means that any circuit design strategy includes two main blocks: analysis of the mathematical model of the circuit and an optimization procedure that reaches the minimum point of the objective function. The minimum value of this function can ensure that the required circuit characteristics are obtained. The interaction of the circuit analysis block and the optimization procedure block forms the circuit optimization process. Optimization methods for systems of various natures can be divided into two main groups: deterministic optimization algorithms and stochastic search algorithms. Some of the weaknesses of classical deterministic optimization algorithms are the requirement for a good starting point in the parameter space, the difficulty of finding the global minimum, and a long execution time. To overcome these problems some special methods were developed. For example, a method that determines initial point of the optimization process by centering [1], geometric programming methods [2] that guarantee the convergence to a global minimum, but, on the other hand, this require a special formulation of the design equations to which additional difficulties accompany. Other approach based on the idea of space mapping technique [3-4], which achieves a satisfactory solution. This technology successfully used for optimization of microwave systems but there are no experience for solution of other problems.

Some alternative stochastic search algorithms, especially evolutionary computation algorithms, can solve the problem of finding the global minimum and have been developed in recent years. An analysis of various stochastic algorithms for system optimization allowed select some groups: simulated annealing method [5-7], evolutionary computing techniques that produce some different approaches as evolutionary algorithms [8-11] particle swarm optimization (PSO) method, GA, differential evolution, genetic programming. A PSO technique [12-15] is one of the evolutionary algorithms that competes with genetic algorithms. This method has been successfully used to solve electromagnetic problems and to optimize microwave systems.

Separately, we highlight GA that is used to solve nonlinear programming problems both for optimizing systems of various nature [16-22], and, in particular, for optimizing and designing electronic systems [23-24]. GA has been used as an optimization procedure for analog circuits due to the ability to find a satisfactory solution. The disadvantages of these methods include a premature convergence to a local minimum and an increase in computer operation time when setting a sufficiently high accuracy for obtaining the minimum. To prevent this, we propose to use the approach underlying the generalized optimization method defined for the deterministic case of circuit optimization in [25]. In this formulation of the problem, an artificially introduced control vector produces many different optimization strategies and sets a different type of the objective function for each new strategy.

This control vector is introduced into the system of equations describing the optimization process of a certain function, determines the structure of these equations and leads to the emergence of many different optimization strategies that differ in the number of operations and CPU time. The set of different strategies that appear in this case depends on the dimension of the control vector, which, in turn, is determined by the number of circuit nodes. At the same time, this dependence is exponential, i.e. if the number of circuit nodes is M, then the number of different optimization strategies is 2^M. With this approach, each strategy is determined by its objective function, which depends on the structure of the control vector. In this case, the optimization process is generalized and, in fact, is a dynamic controlled system with a control vector.

A detailed analysis of various optimization strategies in the deterministic case showed the prospects and advantages of this approach when solving the problem of reducing the time spent on the optimization and design of electronic systems. It would like to find out the validity of this approach when solving optimization problems by stochastic methods. In this paper, the approach of generalized optimization is included into the implementation of a standard GA. This means that one of the most important steps is the setting of the GA fitness function, which now includes the control vector.

2. GA and Generalized Optimization Approach

The process of circuit optimization can be defined as the problem of minimization of objective function C(X), $X \in R^N$ with additional conditions. It is supposed that the minimum of the objective function C(X) corresponds to achievement of all the necessary design goals of the circuit, and the system of constraints is a mathematical model of the electronic circuit.

A typical formulation of a circuit optimization problem can be defined mathematically as a constrained optimization problem for the objective function $C(X)$. The process to minimize the objective function $C(X)$ is conventionally defined by the following equation:

$$X^{s+1} = \Lambda(X^s), s = 1,2,\ldots, \quad (1)$$

where Λ is the operator of transition from iteration s to iteration s+1. The constraints are determined by the circuit equations and can be described by a system of nonlinear equations:

$$g_j(X) = 0, j = 1,2,\ldots,M. \quad (2)$$

We will declare some of the variables as independent, and the other part as dependent, the value of which is determined from the constraint equations (2): $X = (X', X'')$, $X' \in R^K$ is a vector of independent variables, $X'' \in R^M$ is a vector of dependent variables, K is the number of independent variables, M is the number of the circuit's dependent variables, N is the total number of variables (N=K+M). Traditionally, resistance (conductivity) of resistors are defined as independent variables of optimization procedure (vector X'). Other variables are defined as dependent variables (currents or nodal voltages). However, this partition is conditional, since any variable may be considered independent or dependent.

To calculate the function $C(X)$, it is required to solve a system of nonlinear equations (2) at each step of the optimization process. This approach can be named as traditional strategy of optimization (TSO).

Let us accept the following statement that there is no need to fulfil condition (2) at each step of the optimization procedure, and that it is enough to fulfil it at the final point of the optimization process. We use the approach [25] leading to a generalization of the optimization process. Let's define as independent all the variables included in the vector X'', and previously declared dependent. In this case, the constraint equations (2) can be removed, but to fulfill all the constraints (2), at least at the end point of the optimization process, we introduce a new, generalized, objective function $F(X)$, which can be defined as follows:

$$F(X) = C(X) + \varphi(X), \quad (3)$$

where $\varphi(X)$ is an additional penalty function, the equality of which to zero, at the end point of the optimization process, ensures the fulfilment of conditions (2). This function can be, for example, the following form:

$$\varphi(X) = \sum_{j=1}^{M} g_j^2(X). \quad (4)$$

Generalizing this approach, it is possible to declare independent only a part of the previously dependent variables, for example, Z variables, where $Z \in [0, M]$. In this case, Z equations are removed from system (2), and the formula (4) contains Z terms. This approach generalizes the optimization problem by introducing a special control vector $U=(u_1, u_2,\ldots, u_M)$, that changes the structure of all equations and formulas of the optimization procedure. In this case, system (2) is transformed into the following:

$$(1-u_j)g_j(X) = 0, j = 1,2,\ldots,M, \quad (5)$$

where u_j is the jth component of the control vector $U=(u_1, u_2,\ldots, u_M)$, $u_j \in \Omega$, $\Omega=\{0;1\}$. Formulas (3) and (4) are transformed into the following:

$$F(X,U) = C(X) + \varphi(X,U), \quad (6)$$

$$\varphi(X,U) = \frac{1}{\sigma}\sum_{j=1}^{M} u_j g_j^2(X), \quad (7)$$

where σ is a special adjusting parameter.

Thus, the control vector U allows one to change both the structure of the basic equations of the constraints (5) and the form of the generalized objective function $F(X,U)$. Zero values

of components of the vector U determine the TSO. In this case, the system (2) is solved at each step of the optimization procedure, and the generalized objective function $F(X,U)$ coincides with the function $C(X)$. Further, the penalty function $\varphi(X,U)$ is equal to zero. If some components of the vector U are equal to 1, then the corresponding equations disappear from system (5), but information about them appears in the penalty function and in the function $F(X,U)$. If all components of the vector U are equal to 1, then the optimization is determined by the modified traditional strategy (MTS). This means that system (5) disappears, and the penalty function includes complete information about system (5).

It is also important to note the necessary changes in the optimization procedure. When using the deterministic approach, the optimization procedure is specified by differential (8) or difference (9) equations:

$$\frac{dx_i}{dt} = f_i(X,U), i = 1,2,...,N, \tag{8}$$

$$X^{s+1} = X^s + t_s H^s, \tag{9}$$

where $f_i(X,U)$ or H are determined by a specific optimization method. A change in the components of the control vector U from 0 to 1 corresponds to a transformation of the corresponding dependent component of the vector X into an independent one, leading to a change in the number of independent variables and the number of equations both in the optimization procedure (8) or (9) and in the system of constraints (5). The control vector U defines strategies of the structural basis of generalized optimization. The number of these strategies is 2^M.

Equations (5)-(9) define a set of different strategies, each of which is determined by the corresponding value of the control vector. In this case, as was shown in [26], there are opportunities for a significant (by several orders of magnitude) acceleration of the optimization process due to the different behavior of the trajectories of different strategies and the combination of these strategies in the process of optimization.

Let us consider the application of the idea of generalized optimization in the case of using a GA as the main optimization procedure. Instead of using equation (8) or (9), the optimization procedure was carried out on the basis of a GA. Let us consider the classic version of GA [27], in which the selection of chromosomes is carried out by a tournament method and two main genetic operators are used: crossover and mutation. Variants with one-, two-, and four-point crossover operators with a probability of 0.95 and mutation operators with a probability of 0.05 to 0.1 were analyzed. Let's define NN as the number of chromosomes in a population, and $\mathbf{X}$ is a special matrix with N rows and NN columns, provided that each column corresponds a specific value of the vector X.

Let's define the fitness function according to the following generalized formula:

$$P(\mathbf{X},U) = 1/F(\mathbf{X},U). \tag{10}$$

Taking into account the concepts of generalized optimization, the structure of GA can be represented in Fig. 1. A new element of this algorithm is the control vector U, which provides the implementation of various GA variants with different objective functions (fitness functions in GA terminology). Thus, the fitness function also depends on the vector U.

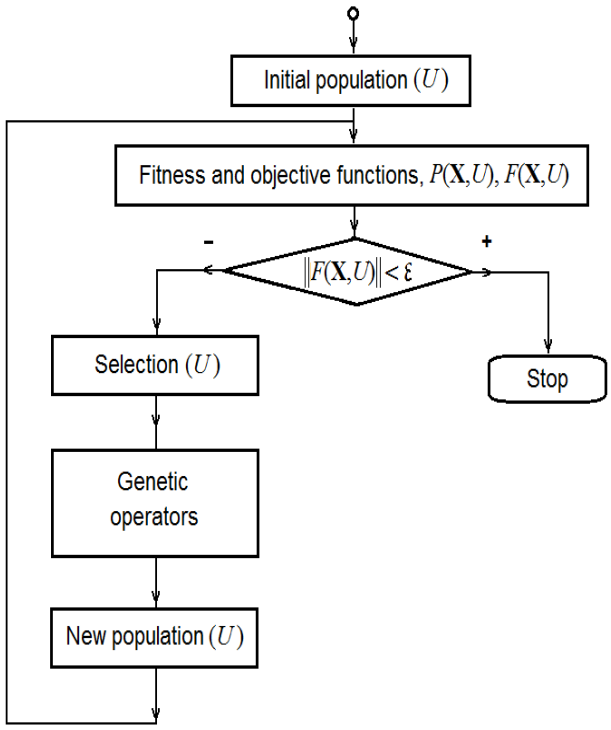


Fig. 1. Modified GA flowchart.

The presence of the control vector U is reflected in the corresponding blocks, since in these blocks either the fitness function is calculated or it is used. The presence of the control vector U in the blocks of the diagram ensures the determination and change of the structure of both the initial generation of chromosomes and the current generations during the operation of the algorithm. For this stochastic algorithm, we can also introduce a vector X consisting of N components, where each component is calculated as the arithmetic mean of all values of this component in the generation:

$$x_i = \frac{1}{NN}\sum_{j=1}^{NN} x_{ij}, \tag{11}$$

where x_{ij} is the element of matrix $\mathbf{X}$.

For the analyzed examples, the length of chromosomes (L) in GA varied from 20 to 80 for each of the variables, and the number of chromosomes (NN) in the population varied from 40 to 400 depending on the length of the chromosomes.

3. Results

3.1 Example 1

Minimize $C(X)$

$$C(X) = -4x_1^2 + x_2^2 + x_1^3, \quad (12)$$

Subject to:

$$(x_1 - 2)^2 + (x_2 - 1)^2 = 0. \quad (13)$$

In this example, parameter M=1 because there is only one constraint equation (13). Define variable x_1 as independent, and variable x_2 as dependent, the value of which is determined from equation (14). Based on the generalized approach, equation (13) is transformed into the following equation:

$$(1-u)\{(x_1 - 2)^2 + (x_2 - 1)^2\} = 0, \quad (14)$$

where u is a control vector consisting, in this particular case, of one component. Consider two basic strategies: the TSO with control vector u = 0 and the MTS with control vector u = 1. Here, we analyse the results of optimization by means of a GA for these strategies. However, it was shown that in the case of a deterministic optimization process, a combination of several strategies can reduce both the number of steps in the optimization procedure and the computing time of the optimization process.

Table I shows the number of generations required to achieve the minimum of the function $C(X)$ with accuracy δ for the strategies TSO (U=(0)), MTS (U=(1)) and for the third, combined strategy determined by the control vector (1),(0) with one switching point Sp = 2 between strategies (1) and (0). That is, the first two iterations correspond to strategy (1) and the next ones correspond to strategy (0).

TABLE I. NUMBER OF GENERATIONS G FOR STRATEGIES (0), (1) AND COMBINED STRATEGY (1)(0) WITH SWITCH POINT SP FOR DIFFERENT PRECISION δ

Precision δ	Control vector (0)	Control vector (1)	Control vector (1)(0) Sp=2
10^{-1}	15	45	17
10^{-2}	18	51	18
10^{-3}	21	61	20
10^{-4}	29	74	25
10^{-5}	70	83	40
10^{-6}	-	87	47
10^{-7}	-	90	49
10^{-8}	-	90	68
10^{-9}	-	102	68
10^{-10}	-	114	69

It can be seen that when using the TSO, the number of generations is less than for MTS up to a certain level of accuracy (10^{-5}). If the required error is reduced to 10^{-6} or less, no solution based on the traditional strategy is found. The MTS with control vector (1) finds a solution up to an accuracy of 10^{-10}. At the same time, a combined strategy consisting of two, (1) and (0) with a switching point between them Sp = 2, also finds a solution to the problem with no errors up to an accuracy of 10^{-10} and, importantly, for a lower number of generations. The final result for a combined strategy clearly depends on the switching point from one strategy to another. Table II shows the dependence of the number of generations on the switching point for a given error $\delta = 10^{-5}$.

TABLE II. NUMBER OF GENERATIONS G FOR COMBINED STRATEGY (1)(0) FOR DIFFERENT SWITCHING POINT SP.

Switch point Sp	2	3	4	5	6	7	8	9	10
G	40	39	42	49	43	53	40	72	74

This shows that the switching point affects the number of generations required to achieve the required accuracy. The minimum value for this example corresponds to the switching point Sp = 3. Fig. 2 shows the dependence of the minimized function F on the number of generations G for three strategies corresponding to three variants of calculating the fitness function.

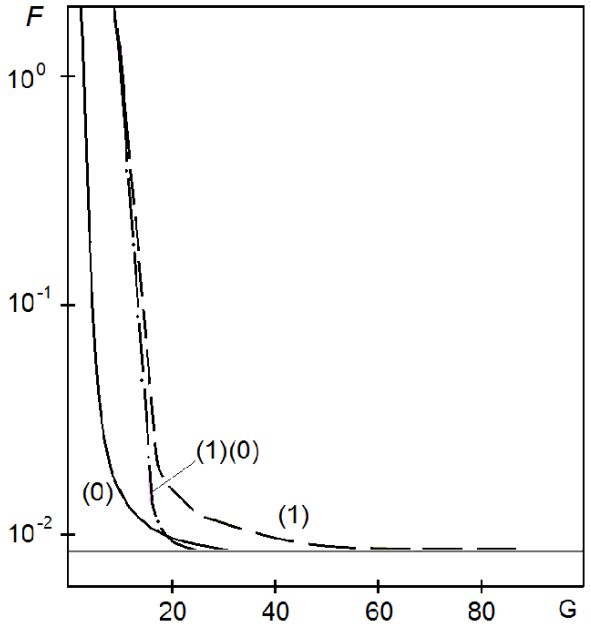


Fig. 2. Minimized function F for strategies (0), (1) and (1)(0) on the number of generations G.

It can be seen that the best strategy for calculating the fitness function is the composite strategy (1)(0), which after the 18th generation solves the problem in the best way compared to other strategies.

3.2 Example 2

We need to optimize the nonlinear circuit shown in Fig. 3.

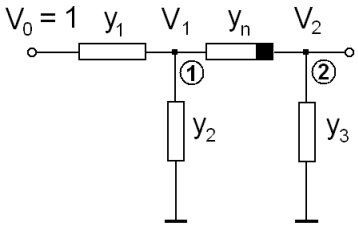


Fig. 3. Two-node nonlinear passive circuit.

Consider a simple nonlinear voltage divider circuit. A nonlinear element has the following dependency: $y_n=a+b(V_1-V_2)^2$. The admittances y_1, y_2, y_3 are positive and compose a set of independent circuit parameters (K=3). The node voltages V_1, V_2 are the dependent parameters (M=2). Vector X consists of the following five components: (x_1, x_2, x_3, x_4, x_5): $x_1^2=y_1$, $x_2^2=y_2$, $x_3^2=y_3$, $x_4=V_1$, $x_5=V_2$. By defining the components x_1, x_2, x_3 using the above formulas, we can automatically obtain positive values of the conductance, which eliminates the issue of positive definiteness for each conductance and allows us to perform optimization in the full space of the values of these variables without any restrictions.

The model of this circuit includes two equations corresponding to Kirchhoff's laws. The objective function $C(X)$ is determined by the formula $C(X)=(x_5-m_1)^2+((x_4-x_5)-m_2)^2$, where m_1 and m_2 are predetermined values of the divider voltages. This circuit is characterised by two (M=2) dependent parameters (x_4, x_5), and three (K=3) independent parameters (x_1, x_2, x_3). The control vector has the next structure: $U=(u_1,u_2)$. The structural basis of the various strategies includes four strategies with the following control vectors: (00), (01), (10), and (11). The mathematical model of the circuit is determined by the following equations:

$$g_1(X)\equiv(1-x_4)x_1^2-(x_4-x_5)(a+b(x_4-x_5)^2)-x_4x_2^2=0$$
$$g_2(X)\equiv(x_4-x_5)(a+b(x_4-x_5)^2)-x_5x_2^2=0 \quad (15)$$

It is from the solution of system (15) that the values of the dependent variables x_4, x_5 can be determined and then the value of the objective function $C(X)$ can be calculated. In the case of the transformation of the two dependent variables x_4, x_5 (or at least one of them) into independent ones, it is necessary to form a generalized objective function $F(X,U)$ according to the following formula:

$$F(X,U)=C(X)+(u_1g_1^2(X)+u_2g_2^2(X))/\sigma. \quad (16)$$

Consider the optimization problem for the circuit shown in Fig. 1. Let a = 1, b = 1, m_1=0.2, and m_2=0.25. For the example analyzed, the length L of the chromosomes varied from 20 to 80 for each of the five variables, and the number NN of chromosomes in the population varied from 60 to 320. Variable limits are specified in a normalized form; for variables x_1, x_2, x_3, they were set from 10^{-5} to 2.0, and for variables x_4, x_5 from 10^{-3} to 1.0. Matrix **X** has five rows (N = 5) and NN columns. At the same time, we introduce a vector X consisting of five components, each of which is calculated by formula (11). This vector is not directly involved in the calculations, but serves as an informative object for constructing averaged trajectories of the optimization process. In this case, we can follow the evolution of the mean values and build graphs of the trajectories of the optimization process based on the GA in N dimensions or different projections of these trajectories. The calculations for each trajectory continued until the required accuracy δ for the generalized objective function $F(X,U)$ was achieved. For a given accuracy $\delta = 3\cdot10^{-5}$, two projections of the four trajectories for strategies (00), (01), (10), and (11) are shown in Fig. 4. Strategy (00) corresponds to TSO, and strategy (11) corresponds to MTS. These dependences are obtained by averaging the stochastic results of the GA, in contrast to the trajectories obtained by direct integration of differential equations in the analytical approach [25]. However, they reflect the behavior of the optimization trajectory. All strategies start at one point S and end at approximately one point F, but their behavior during optimization process are very different.

It is important to note that the required accuracy of minimising the objective function $F(X,U)$ has a significant impact on the optimization process and its characteristics. Each of the four strategies has its own convergence accuracy. Table III shows the results reflecting the potential accuracy ε of the optimization process that each strategy can achieve and the number of generations required to obtain a solution with precision $\delta = 10^{-5}$.

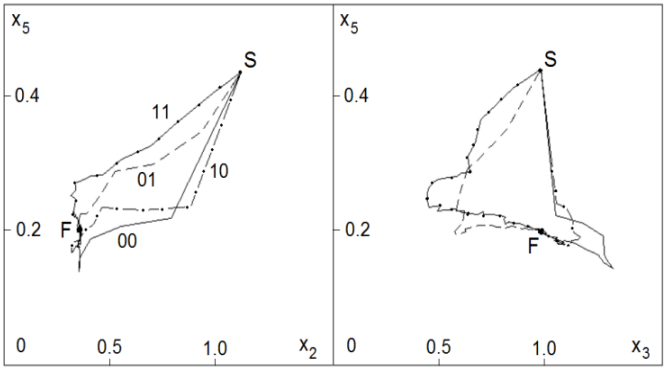


Fig. 4. x_2-x_5 and x_3-x_5 projections for four optimization strategies.

TABLE III. POTENTIAL ACCURACY ε AND NUMBER OF GENERATIONS

N	Control vector	Potential accuracy ε	Number of generations for $\delta=10^{-5}$
1	(00)	$1.69\cdot10^{-5}$	No solution
2	(01)	$2.04\cdot10^{-5}$	No solutiion
3	(10)	$6.05\cdot10^{-6}$	77

4	(11)	$2.87 \cdot 10^{-5}$	No solution

Table III reveals that for an error of $1.6 \cdot 10^{-5}$ or less in obtaining the solution, only one of the strategies, namely strategy (10), will allow solving the problem. In this case, 77 generations are required. Other strategies, including the traditional strategy (00), cannot find solutions for any number of steps in the optimization procedure. The presence of various strategies determined by the control vector U allows one to formulate the problem of constructing a complex optimization strategy consisting of several different strategies. It is possible to propose the structure of a composite strategy consisting of two, three, or more different strategies, where each composite strategy is determined by the control vector U. In this case, it is important to obtain the optimal position of the switching points from one strategy to another, which ensures a decrease in the parameter ε, i.e., an increase in the accuracy of solving the optimization problem. Table IV shows the results of some composite strategies with optimal switching point Sp that can significantly improve the accuracy of solving the problem. This table contains data for six composite strategies, each of which consists of two strategies in Table III. As can be seen, the accuracy of the solution was improved by 2 to 3 orders of magnitude. The table shows that a decrease in the required error leads at the same time to a slight increase in the number of required populations. We have seen that only one strategy from Table III can solve the problem with an accuracy of 10^{-5}.

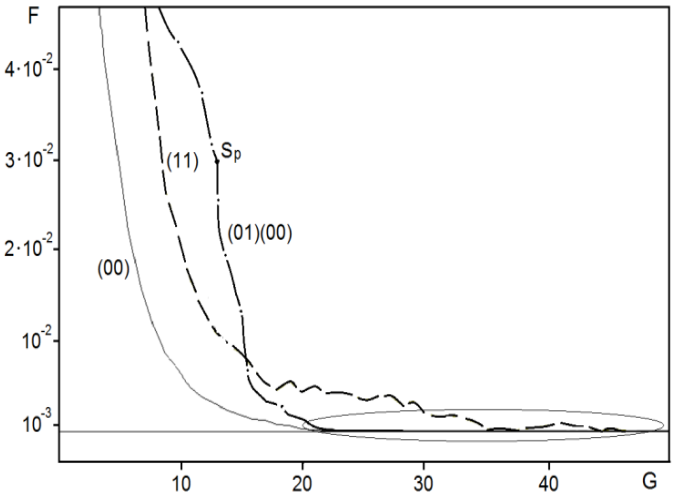
TABLE IV. POTENTIAL ACCURACY ε AND NUMBER OF GENERATIONS FOR COMPLEX SINGLE SWITCHING POINT STRATEGIES

N	Control vector	Sp	ε	Number of generations G for various precision δ			
				10^{-5}	10^{-6}	10^{-7}	$4 \cdot 10^{-8}$
1	(01) (00)	13	$3.94 \cdot 10^{-8}$	31	35	44	51
2	(10) (00)	2	$3.94 \cdot 10^{-8}$	32	38	44	49
3	(11) (00)	8	$3.94 \cdot 10^{-8}$	31	38	44	57
4	(00) (01)	20	10^{-7}	34	46	62	-
5	(00) (10)	16	$6.75 \cdot 10^{-7}$	35	57	-	-
6	(00) (11)	20	$7.22 \cdot 10^{-7}$	44	72	-	-

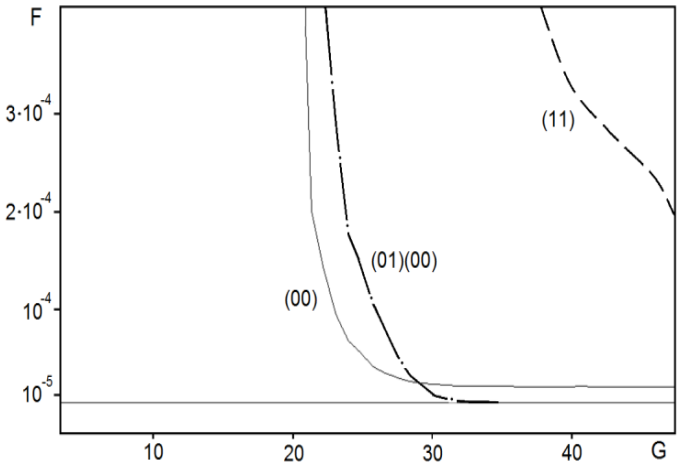
However, the results presented in Table IV show that complex strategies allow solving the problem with much more stringent requirements for the accuracy of the solution obtained. This table provides data on the potential accuracy achievable for various compound strategies at the optimum position of switching point. The results of the optimization process are also presented in the form of the required number of GA populations at which the required accuracy δ is achieved. The composite strategies allow solving the optimization problem with a significantly higher accuracy than the original strategies of Table III. Some of these strategies can solve the problem up to $4 \cdot 10^{-8}$ accuracy. In this case, as can be seen from Table IV, number of generations increases insignificantly with an increase in the required accuracy of solving the problem. This happens until the potential strategy error exceeds the required precision for solving the problem.

Fig. 5 shows the dependence of the minimized function F on the number of generations G for three strategies corresponding to three variants of calculating the fitness function for precision $\delta = 10^{-6}$.

These dependences are plotted for two scales - large (Fig. 5(a)) and small, which corresponds to the inner region of the ellipse in the first figure (Fig. 5(b)). The composite strategy includes two strategies (01) and (00) with the switching point between them Sp=13. Simple strategies (00) and (11) do not achieve the required accuracy $\delta = 10^{-6}$, but the composite strategy solves the problem in 35 generations.



(a) Large scale



(b) Small scale

Fig. 5. Minimized function F for strategies (00), (11) and (01)(00) on the number of generations G.

The results of the analysis of compound strategies with two switching points are presented in Table V. This table shows similar results for compound strategies with three parts. Again, a significant improvement in the accuracy of the solution by 2 to 3 orders of magnitude was obtained in comparison with the strategies in Table III. In this case, the possible number of compound strategies that allow solving the problem with high accuracy also increases.

TABLE V. POTENTIAL ACCURACY ϵ AND NUMBER OF GENERATIONS FOR COMPLEX STRATEGIES WITH TWO SWITCHING POINTS SP1, SP2

N	Control vector	Sp1, Sp2	ε	Number of generations for various precision δ			
				10^{-5}	10^{-6}	10^{-7}	$4\cdot 10^{-8}$
1	(00)(01)(00)	2, 4	$3.94\cdot10^{-8}$	30	38	45	57
2	(00)(10)(00)	4, 6	$7.53\cdot10^{-7}$	32	42	-	-
3	(00)(11)(00)	4, 11	$7.53\cdot10^{-7}$	33	44	-	-
4	(00)(11)(10)	15, 16	$9.03\cdot10^{-8}$	34	40	51	-
5	(01)(11)(00)	3, 6	$3.94\cdot10^{-8}$	30	37	41	48
6	(10)(11)(00)	3, 4	$3.94\cdot10^{-8}$	28	36	45	51
7	(11)(01)(00)	3, 4	$3.94\cdot10^{-8}$	29	34	40	45
8	(11)(10)(00)	4, 9	$7.53\cdot10^{-7}$	29	42	-	-
9	(11)(00)(01)	2, 11	$6.84\cdot10^{-8}$	37	49	65	-
10	(01)(10)(00)	4, 7	$3.94\cdot10^{-8}$	29	35	42	53

The obtained result shows that the change in the structure of the fitness function in the course of the optimization algorithm allows us to bypass local minima and overcome premature convergence. Such an improvement in the accuracy of the solution leads to a significant reduction in the number of GA generations needed to obtain the required accuracy of the solution to the optimization problem.

A. Example 3

Let's analyze the optimization process of the nonlinear circuit shown in Fig. 6.

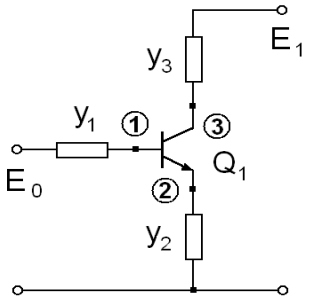


Fig. 6. Single-stage amplifier.

The conductivities y_1, y_2, y_3 are positive and compose the set of non-dependent parameters of the circuit (K=3). Nodal voltages V_1, V_2, V_3 for nodes 1, 2 and 3 are the dependent parameters (M=3). Let's define a vector of variables $X \in R^6$, including six components (x_1, x_2, x_3, x_4, x_5, x_6): $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$, $x_6 = V_3$. A static Ebers-Moll model of transistor was used [28].

The objective function $C(X)$ of the optimization process was determined as the sum of the squares of the differences between the previously specified and current values of the nodal voltages:

$$C(X)=\sum_{i=1}^{M}(x_{K+i}-V_{i0})^2 \tag{17}$$

where V_{10}, V_{20}, V_{30} are the before-defined values of nodal voltages.

The circuit model is defined by Kirchhoff's law as:

$$\begin{aligned} g_1(X) &\equiv I_B-(E_0-x_4)x_1^2=0,\\ g_2(X) &\equiv I_E-x_2^2x_5=0,\\ g_3(X) &\equiv I_C-(E_1-x_6)x_3^2=0, \end{aligned} \tag{18}$$

where I_B, I_E, I_C – are the base, emitter and collector currents, respectively. This system serves as a system of constraints for minimizing the objective function $C(X)$. The control vector includes three components $U=(u_1,u_2,u_3)$. Using the generalized approach, we transform system (18) into system (19).

$$(1-u_j)g_j(X)=0,\ j=1,2,3. \tag{19}$$

The generalized objective function is defined by the following formula:

$$F(X,U)=C(X)+\frac{1}{\sigma}\sum_{j=1}^{3}u_j g_j^2(X)\cdot \tag{20}$$

Table VI shows the number of generations required to achieve the minimum of the function F with accuracy δ for

three strategies. The first two are TSO with control vector U = (0,0,0) and MTS with control vector U = (1,1,1). The third strategy is a combined strategy defined by the control vector (000)(111) with one switching point Sp = 9. That is, the first nine iterations correspond to the traditional strategy (000), and the next ones correspond to the strategy (111).

It can be seen that when using the traditional strategy, the required number of generations is much larger than in the case of the modified traditional strategy and the combined strategy to achieve the same accuracy. In addition, the traditional strategy does not provide a good accuracy of achieving the minimum of the objective function. When the required error is 10^{-4} or less, no solution based on the traditional strategy is found. At the same time, the modified strategy (111) and the combined strategy, consisting of two strategies (000) and (111), find a solution with an accuracy of $5 \cdot 10^{-11}$. Note that the combined strategy finds a solution to the problem in a significantly fewer generations.

TABLE VI. NUMBER OF GENERATIONS G FOR STRATEGIES (000), (111) AND (000)(111) FOR DIFFERENT PRECISION δ

Precision δ	Control vector (000)	Control vector (111)	Control vector (000)(111) Sp=9
10^{-1}	137	36	27
10^{-2}	20706	47	32
10^{-3}	348514	63	38
10^{-4}	-	77	42
10^{-5}	-	88	49
10^{-6}	-	102	59
10^{-7}	-	109	69
10^{-8}	-	123	76
10^{-9}	-	144	89
10^{-10}	-	172	99
$5 \cdot 10^{-11}$	-	210	108
10^{-11}	-	-	-

It can be seen that the gain in the number of iterations (number of generations) for the combined strategy and MTS is three to four orders of magnitude compared to the traditional one, if the traditional strategy as a whole is able to find a solution. The reduction in CPU time is even greater, since the time of one iteration of the modified strategy is much less than the traditional one.

It is clear that the final result of the combined strategy depends on the switching point from one strategy to another. Table VII shows the dependence of the number of generations on the switching point for a given error $\delta = 10^{-5}$. It can be seen that the switching point significantly affects the number of generations required to achieve the necessary accuracy. The minimum value for this combined strategy corresponds to the switching point Sp = 9.

TABLE VII. NUMBER OF GENERATIONS G FOR COMBINED STRATEGIES (000)(111) FOR DIFFERENT SWITCHING POINT SP

Control vector (000),(111)	2	3	4	8	9	10	11	12	13
G	67	56	96	53	49	52	99	57	74

Table VIII contains information for the three considered strategies, summarizing their comparative characteristics when achieving an accuracy of 10^{-2} and 10^{-3} for the minimized objective function. The numerical values of the number of generations, the CPU time of all strategies, as well as the comparative gain for the MTS and for the combined strategy, both in the number of generations and in the CPU time compared to TSO, are given. Note that TSO does not allow finding a solution to the problem with the required error less than 10^{-3}. It can be seen that both the MTS and the combined strategy provide a large gain over TSO.

With an error of 10^{-2}, the gain in terms of the number of generations is more than two orders of magnitude, and in terms of CPU time, more than three orders of magnitude. With a given error of 10^{-3}, the gain in terms of the number of generations is 3-4 orders of magnitude, and in terms of CPU time, it is almost five orders of magnitude.

TABLE VIII. GENERALIZED COMPARATIVE CHARACTERISTICS FOR THREE DIFFERENT STRATEGIES

Precision δ	Control vector	(000)	(111)	(000)(111) Sp=9
10^{-2}	Number of generations	20706	47	32
	Gain in the number of generations		440	647
	CPU time (s)	1178.15	0.266	0.244
	Time gain		4429	4828
10^{-3}	Number of generations	348514	63	38
	Gain in the number of generations		5532	9171
	CPU time (s)	20518.2	0.353	0.276
	Time gain		58124	74340

The information presented in this table is the main practical result of the work. It can be stated that the use of a generalized approach that changes the structure of the vector of basic variables X and the shape of the fitness function makes it possible to overcome the problem of the GA's premature convergence to a local minimum. In this case new strategies appear that can substantially increase the accuracy of solving the problem and significantly speed up the optimization procedure.

In Fig. 7 shows the dependence of the function F to be minimized on the number of generations G for the three analyzed strategies at a given accuracy of 10^{-5}.

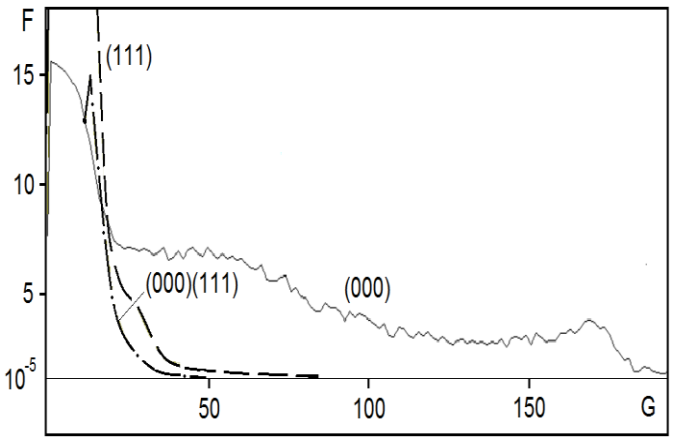


Fig. 7. Minimized function *F* for strategies (000), (111) and (000)(111) on the number of generations G.

Traditional strategy of optimization cannot find a solution to the problem with the required accuracy. On the contrary, it is obvious that the modified traditional strategy and the combined strategy find a solution to the task rather quickly. Thus, we can conclude that new optimization strategies that appear within the framework of the presented generalized approach have good prospects for improving the optimization process of electronic circuits.

4. Conclusion

A generalized approach in terms of control theory to solving the problem of optimizing electronic circuits using deterministic optimization methods was developed earlier. The obtained algorithms have shown high efficiency in comparison with the traditional approach in terms of both accuracy and speed.

This paper demonstrates the possibility of embedding the idea of generalized optimization into the body of stochastic optimization methods. It was shown that this approach can be built into GA, which leads to the formation of a set of different optimization strategies and a significant improvement in the main characteristics of GA.

The studied examples demonstrate the practical implementation of a modified GA based on a generalized approach for solving the problem of optimizing electronic circuits. The emerging new optimization strategies make it possible to increase the accuracy of the problem solution by several orders of magnitude. It should also be emphasized that the real gain of these strategies in CPU time compared to the traditional approach is much higher than the gain in the number of GA populations. This is due to the fact that the processor time for evaluating the fitness function for new strategies is much less than in the traditional case.

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