Singular H_∞ finite-time boundedness for a class of uncertain singular systems

MENG LIU, YALI DONG*, XINYUE TANG
School of Mathematical Sciences
Tiangong University
Tianjin 300387
CHINA

Abstract: - This paper is concerned with the problem of observer-based finite-time H_∞ control for a class of uncertain singular systems with norm-bounded uncertainties. We design a suitable observer and a controller to guarantee that the closed-loop is singular H_∞ finite-time bounded. By constructing an appropriate Lyapunov function, and using matrix inequality technique, a sufficient condition for the singular H_∞ finite-time boundedness of the closed-loop system is established. The observer and controller gains are designed based on matrix inequality. Two numerical examples are given to demonstrate the effectiveness of the proposed methods.

Key-Words: - Singular H_∞ finite-time boundedness; uncertain singular system; observer-based feedback controller, norm-bounded uncertainty.

Received: June 19, 2021. Revised: March 15, 2022. Accepted: April 17, 2022. Published: May 7, 2022.

1. Introduction

During the past twenty years, singular systems have been extensively studied and successfully applied to models of many practical systems, such as electrical networks, mechanical systems, economical systems, robotics, and so on [1, 2]. Containing a normal state space system as a special case, singular system form represents a much wider class of systems than its state-space counterpart. So, performance analysis and control design of singular systems are very important research topics, which have attracted extensive attention from researchers [1-6]. In [4], Li et al. considered the finite-time robust guaranteed cost control problem for a class of linear continuous-time singular systems with norm-bounded uncertainties. In [5], Feng et al. studied the exponential stability problems of singular impulsive switched systems. In [6], Zheng et al. addressed the sliding mode control issue for time-delay Markovian jump singular systems.

Uncertainty is frequently encountered in various engineering, biological, chemical systems and economic systems. It is difficult to achieve the ideal result if a system fails to take uncertainty into consideration. Therefore, it is particularly important to take into account the influence of uncertainty on the system in the modeling, analysis and design of generalized control systems. This phenomenon also inspires researchers to study the robustness of uncertain control systems [7-11]. Dong et al. [7] studied the robust exponential stabilization for a class of uncertain neutral neural networks with mixed interval time-varying delays. In [8], Hou et al. given a stability analysis for discrete-time uncertain time-delay systems governed by an infinite-state Markov chain. Recently, Dong et al. [10] investigated observer design for a class of one-sided Lipschitz descriptor systems.

On the other hand, most results of stability are investigated in terms of Lyapunov asymptotic stability, which only focuses on the infinite time interval. However, the problem of the behavior of systems over a fixed finite-time interval in many practical applications also calls for more consideration. So, finite-time boundedness has also received much attention in recent years [12-15]. In [13], Lv et al. studied the problem of finite-time stabilization for a class of uncertain Hamiltonian systems. Event-triggered and guaranteed cost finite-time control for switched systems were considered in [14].

Motivated by the above discussion, this paper investigates the singular H_∞ finite-time boundedness for a class of uncertain singular systems. By using appropriately chosen observer-based feedback controller and Lyapunov function, new sufficient conditions of singular H_∞ finite-time boundedness for uncertain singular systems are established. Then, the design methods of control gain matrix and observer gain matrix are presented. Finally, two numerical examples are given to illustrate the less conservatism and effectiveness.

The paper is organized as follows. Section 2 states the problem formulation and preliminaries. Section 3 presents the main results for singular H_∞ finite-time boundedness for uncertain singular systems and give the design methods of control gain matrix and observer gain matrix by using Lyapunov function method. We present sufficient conditions for the singular H_∞ finite-time boundedness of uncertain singular systems. Two numerical examples are given in Section 4. Finally, the conclusion is given in Section 5.

Notations: A^T and A^{-1} denotes the matrix transpose and inverse of matrix *A*, respectively. A symmetric positive (negative) definite matrix is expressed by $A>0\,(A<0)$. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ stand for Euclidean *n*-space and the set of all $n\times m$ real matrices, respectively. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ be the maximum and minimum eigenvalues for a given matrix *A*. The symbol "$*$" is used to indicate the elements induced by symmetry. $He(A)$ is defined as $A+A^T$ and *I* is the identity matrix with appropriate dimensions. Matrix, if its dimensions are not explicitly stated, is assumed to be compatible for algebraic operations.

2. Problem formulations

Consider the following uncertain singular system

$$\begin{cases} E\dot{x}(t)=(A+\Delta A(t))x(t)+(B+\Delta B(t))u(t)+G\omega(t), \\ z(t)=Fx(t)+D\omega(t), \\ y(t)=Cx(t), \\ Ex(0)=Ex_0 \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state, $u(t)\in\mathbb{R}^m$ is the controlled input, $z(t)\in\mathbb{R}^q$ is the controlled output, $y(t)\in\mathbb{R}^l$ is the measured output, $\omega(t)\in\mathbb{R}^p$ is the exogenous disturbance. Matrix *E* may be singular with $rank(E)=r\le n.$ $A\in\mathbb{R}^{n\times n}, B\in\mathbb{R}^{n\times m}, G\in\mathbb{R}^{n\times p}$, $C\in\mathbb{R}^{l\times n}, D\in\mathbb{R}^{q\times p}, F\in\mathbb{R}^{q\times n}$, are the known constant matrices with appropriate dimension. $\Delta A, \Delta B$ represent the time-varying parametric uncertainties and satisfy

$$\begin{bmatrix}\Delta A & \Delta B\end{bmatrix}=MS(t)\begin{bmatrix}N_a & N_b\end{bmatrix}, \tag{2}$$

where M, N_a, N_b are known as real matrices with appropriate dimension and unknown matrix $S(t)$ satisfies

$$S^T(t)S(t)\le I.$$

Assumption 1. For a given interval $[0,T]$, the exogenous disturbance $\omega(t)\in\mathbb{R}^p$ satisfies

$$\int_0^T \omega^T(t)\omega(t)dt\le d^2, \quad d\ge 0. \tag{3}$$

In this paper, we consider the following observer-based feedback controller

$$E\dot{\hat{x}}(t)=A\hat{x}(t)+Bu(t)+L(y(t)-C\hat{x}(t)), \tag{4}$$

$$u(t)=K\hat{x}(t), \tag{5}$$

where $L\in\mathbb{R}^{n\times l}$ and $K\in\mathbb{R}^{m\times n}$ are the observer and controller gains, respectively, to be determined, $\hat{x}(t)\in\mathbb{R}^n$ is the estimate of the $x(t)$.

Let $e(t)=x(t)-\hat{x}(t)$ be the estimate error. We have that

$$E\dot{e}(t)=(A-LC-\Delta BK)e(t)+(\Delta A+\Delta BK)x(t)+G\omega(t). \tag{6}$$

Substituting (5) into (1) leads to

$$\begin{aligned} E\dot{x}(t)=&(A+BK+\Delta A+\Delta BK)x(t)-(BK \\ &+\Delta BK)e(t)+G\omega(t). \end{aligned} \tag{7}$$

Thus, the closed-loop system can be rewritten as

$$\begin{cases} \bar{E}\dot{\bar{x}}(t)=\bar{A}\bar{x}(t)+\bar{G}\omega(t), \\ z(t)=\bar{F}\bar{x}(t)+\bar{D}\omega(t), \\ E\bar{x}(0)=E\bar{x}_0, \end{cases} \tag{8}$$

where $\bar{x}(t)=\begin{bmatrix}x^T(t) & e^T(t)\end{bmatrix}^T$ and

$$\bar{A}=\begin{bmatrix} A+BK+\Delta A+\Delta BK & -BK-\Delta BK \\ \Delta A+\Delta BK & A-LC-\Delta BK \end{bmatrix},$$

$$\bar{G}=\begin{bmatrix}G\\G\end{bmatrix}, \bar{F}=\begin{bmatrix}F & 0\end{bmatrix}, \bar{E}=\begin{bmatrix}E & 0\\0 & E\end{bmatrix}, \bar{D}=D.$$

In this paper, the following definitions and lemmas play important role in our later proof.

Definition 1. [4] The closed-loop systems (8) is said to be regular in time interval $[0,T]$ if $\det(s\bar{E}-\bar{A})$ is not identically zero for all $t\in[0,T]$.

Definition 2. [4] The closed-loop systems (8) is said to be impulse-free in time interval $[0,T]$ if $\deg(\det(s\bar{E}-\bar{A}))=rank(\bar{E})$ for all $t\in[0,T]$.

Definition 3. [17] The closed-loop system (8) satisfying (3) is said to be singular finite-time boundedness (SFTB) with respect to (c_1,c_2,T,R,d), with $c_1<c_2$ and $R>0$, if the closed-loop system (8) is regular and impulse free in time interval [0, T] and satisfies

$$\bar{x}_0^T\bar{E}^T\bar{R}\bar{E}x_0^T\le c_1 \Rightarrow \bar{x}^T(t)\bar{E}^T\bar{R}\bar{E}x(t)\le c_2, \forall t\in[0,T].$$

Definition 4. [17] The closed-loop system (8) is said to be singular H_∞ finite-time boundedness (SH_∞ FTB) with respect to (c_1,c_2,T,R,γ,d), with $c_1<c_2$ and $R>0$, if system (8) is singular finite-time boundedness with respect to (c_1, c_2, T, R, d) and under the zero-initial condition the following inequality

$$\int_0^T z^T(t)z(t)dt < \gamma^2 \int_0^T \omega^T(t)\omega(t)dt, \tag{9}$$

holds for any non-zero $\omega(t)$ and a scalar $\gamma > 0.$

Lemma 1. [4] The closed-loop system (8) is regular and impulse-free if there exists a scalar $\sigma \geq 0$ and an invertible matrix $\bar{P}$, such that the following conditions hold:

$$\bar{E}^T\bar{P} = \bar{P}^T\bar{E} \geq 0,$$

$$\bar{A}^T\bar{P} + \bar{P}^T\bar{A} < \sigma\bar{E}\bar{P}.$$

Lemma 2. [11] Let M, N, and $S(t)$ be real matrices with appropriate dimensions and $S^T(t)S(t) \leq I.$ For any scalar $\varepsilon > 0,$ the following inequality holds:

$$MS(t)N + N^TS^T(t)M^T \leq \frac{1}{\varepsilon}MM^T + \varepsilon N^TN.$$

Lemma 3. [11] For a scalar ζ and matrices T, Q, U, and W, which is symmetric, the inequality

$$T + W^TQ^T + QW < 0,$$

is satisfied if the following condition holds:

$$\begin{bmatrix} T & \zeta Q + W^TU^T \\ * & -\zeta U - \zeta U^T \end{bmatrix} < 0.$$

The aim of this paper is to design an observer-based robust finite time H_∞ controller such that the closed-loop (8) is singular H_∞ finite-time boundedness.

3. Main results

The following theorem give a sufficient condition which ensure that the closed-loop system (8) is SFTB.

Theorem 1. Given a positive-definite matrix $\bar{R}$ and positive scalars $c_1, T, d, \sigma,$ the closed-loop system (8) is SFTB with respect to $(c_1, c_2, T, R, d),$ if there exist positive scalars $c_2, \eta,$ matrices $\Theta > 0, Q > 0$ and a non-singular matrix $\bar{P}$ such that (10), (11), (12) and (13) hold:

$$\bar{E}^T\bar{P} = \bar{P}^T\bar{E} \geq 0, \tag{10}$$

$$\bar{E}^T\bar{P} = \bar{E}^T\bar{R}^{\frac{1}{2}}\Theta\bar{R}^{\frac{1}{2}}\bar{E}, \tag{11}$$

$$e^{\sigma T}(\lambda_1 c_1 + \lambda_2 d^2) < \lambda_3 c_2, \tag{12}$$

$$\begin{bmatrix} \bar{A}^T\bar{P} + \bar{P}^T\bar{A} - \sigma\bar{E}^T\bar{P} & \bar{P}^T\bar{G} \\ * & Q \end{bmatrix} < 0, \tag{13}$$

where $\lambda_1 = \lambda_{\max}(\Theta), \quad \lambda_2 = \lambda_{\max}(Q), \quad \lambda_3 = \lambda_{\min}(\Theta).$

Proof. From (13), we have

$$\bar{A}^T\bar{P} + \bar{P}^T\bar{A} - \sigma\bar{E}^T\bar{P} < 0. \tag{14}$$

From (13) and (14), and using Lemma 1, we can obtain that system (8) is regular and impulse-free.

Choose a Lyapunov function candidate to be

$$V(t) = \bar{x}^T(t)\bar{E}^T\bar{P}\bar{x}(t), \tag{15}$$

where $\bar{E}^T\bar{P} = \bar{P}^T\bar{E} \geq 0.$ We have

$$\begin{aligned} \dot{V}(t) &= 2\bar{x}^T(t)\bar{P}^T\left[\bar{A}\bar{x}(t) + \bar{G}\omega(t)\right] \\ &= 2\bar{x}^T(t)\bar{P}^T\bar{A}\bar{x}(t) + 2\bar{x}^T(t)\bar{P}^T\bar{G}\omega(t) \\ &= \xi^T\begin{bmatrix} \bar{A}^T\bar{P} + \bar{P}^T\bar{A} & \bar{P}^T\bar{G} \\ * & 0 \end{bmatrix}\xi, \end{aligned}$$

where $\xi = \left[\bar{x}^T(t) \quad \omega^T(t)\right]^T.$

Next, it can be derived that

$$\begin{aligned} &\dot{V}(t) - \sigma V(t) - \omega^T(t)Q\omega(t) \\ &= \xi^T\begin{bmatrix} \bar{A}^T\bar{P} + \bar{P}^T\bar{A} - \sigma\bar{E}\bar{P} & \bar{P}^T\bar{G} \\ * & -Q \end{bmatrix}\xi. \end{aligned} \tag{16}$$

Then, from (13), one has

$$\dot{V}(t) < \sigma V(t) + \omega^T(t)Q\omega(t), \quad t \in [0, T], \tag{17}$$

or

$$\frac{d}{dt}\left(e^{-\sigma t}V(t)\right) < e^{-\sigma t}\omega^T(t)Q\omega(t), \quad t \in [0, T]. \tag{18}$$

Integrating (18) from 0 to t and noting $e^{-\sigma t} \leq 1$, we have

$$\begin{aligned} V(t) &< e^{\sigma t}V(0) + e^{\sigma t}\int_0^t e^{-\sigma s}\omega^T(s)Q\omega(s)ds \\ &\leq e^{\sigma T}\left[V(0) + \int_0^t \omega^T(s)Q\omega(s)ds\right] \\ &\leq e^{\sigma T}\left[\lambda_1 c_1 + \lambda_2 d^2\right], \quad t \in [0, T]. \end{aligned} \tag{19}$$

On the anther hand, by (11), we have

$$\begin{aligned} V(t) &= \bar{x}^T(t)\bar{E}^T\bar{P}\bar{x}(t) \\ &= \bar{x}^T(t)\bar{E}^T\bar{R}^{\frac{1}{2}}\Theta\bar{R}^{\frac{1}{2}}\bar{E}\bar{x}(t) \\ &\geq \lambda_3\bar{x}^T(t)\bar{E}^T\bar{R}\bar{x}(t). \end{aligned} \tag{20}$$

Combining (12), (19) and (20), it can be derived that

$$\bar{x}^T(t)\bar{E}^T\bar{R}\bar{x}(t) < e^{\sigma T}\frac{\lambda_1 c_1 + \lambda_2 d^2}{\lambda_3} < c_2, \quad \forall t \in [0, T].$$

This completes the proof.

The following theorem gives a sufficient condition which ensure that the closed-loop system (8) is SH_∞ FTB.

Theorem 2. Given a positive-definite matrix $\bar{R}$ and positive scalars $c_1, T, d, \sigma,$ the closed-loop system (8) is SH_∞ FTB with respect to $(c_1, c_2, T, \bar{R}, \gamma, d)$ with $\gamma = \sqrt{\lambda_2 e^{\sigma T}}$, if there exist positive scalars $c_2, \eta,$ matrices $\Theta > 0, Q > 0$ and a non-singular matrix $\bar{P}$ such that (10), (11), (12) and (21) hold:

$$\Omega_1 = \begin{bmatrix} \bar{A}^T\bar{P} + \bar{P}^T\bar{A} + \bar{F}^T\bar{F} - \sigma\bar{E}^T\bar{P} & \bar{P}^T\bar{G} + \bar{F}^T\bar{D} \\ * & -Q + \bar{D}^T\bar{D}^T \end{bmatrix} < 0. \tag{21}$$

Proof. Select the same Lyapunov function candidate as one in (15). Let

$$J = \dot{V}(t) - \sigma V(t) - \omega^T(t)Q\omega(t) + z^T(t)z(t).$$

Then, we have

$$J = \xi^T \begin{bmatrix} \bar{A}^T\bar{P} + \bar{P}^T\bar{A} - \sigma\bar{E}\bar{P} & \bar{P}^T\bar{G} \\ * & -Q \end{bmatrix}\xi$$
$$+ (\bar{F}\bar{x}(t) + \bar{D}\omega(t))^T(\bar{F}\bar{x}(t) + \bar{D}\omega(t))$$
$$= \xi^T \begin{bmatrix} \bar{A}^T\bar{P} + \bar{P}^T\bar{A} + \bar{F}^T\bar{F} - \sigma\bar{E}^T\bar{P} & \bar{P}^T\bar{G} + \bar{F}^T\bar{D} \\ * & -Q + \bar{D}^T\bar{D} \end{bmatrix}\xi.$$

From (21), we can obtain that

$$J < 0, \quad t \in [0, T].$$

Thus, we get

$$\dot{V}(t) - \sigma V(t) < \omega^T(t)Q\omega(t) - z^T(t)z(t). \tag{22}$$

Furthermore, (22) can be rewritten as

$$\frac{d}{dt}\left(e^{-\sigma t}V(t)\right) < e^{-\sigma t}\left[\omega^T(t)Q\omega(t) - z^T(t)z(t)\right]. \tag{23}$$

Next, integrating (23) from 0 to T gives, under the zero-initial condition, we can obtain that

$$V(T) < e^{\sigma T}\int_0^T e^{-\sigma s}\left(\omega^T(s)Q\omega(s) - z^T(s)z(s)\right)ds. \tag{24}$$

Noting $e^{-\sigma T} \le e^{-\sigma t} \le 1$, and $V(T) \ge 0$, it follows that

$$V(T) < e^{\sigma T}\int_0^T \lambda_2\omega^T(s)\omega(s)ds - e^{\sigma T}\int_0^T e^{-\sigma s}z^T(s)z(s)ds,$$
$$0 < e^{\sigma T}\int_0^T \lambda_2\omega^T(s)\omega(s)ds - \int_0^T z^T(s)z(s)ds,$$
$$\int_0^T z^T(s)z(s)ds < \lambda_2 e^{\sigma T}\int_0^T \omega^T(s)\omega(s)ds,$$
$$\int_0^T z^T(s)z(s)ds < \left(\sqrt[2]{\lambda_2 e^{\sigma T}}\right)^2\int_0^T \omega^T(s)\omega(s)ds,$$

Hence, by Definition 4, the closed-loop system (8) is S H_∞ FTB with $\gamma = \sqrt{\lambda_2 e^{\sigma T}}$. This completes the proof.

Theorem 3. Given a positive-definite matrix R and positive scalars $c_1, T, d, \sigma, \zeta,$ the closed-loop system (8) is S H_∞ FTB with respect to $(c_1, c_2, T, R, \gamma, d)$ with $\gamma = \sqrt{\lambda_2 e^{\sigma T}}$, if there exist positive scalars $c_2, \eta, \kappa,$ and matrices $Z > 0,\ Q > 0, Y, H\ \ U, V$ such that (25), (26), (27) and (28) hold:

$$R < Z < \kappa R, \tag{25}$$

$$e^{\sigma T}\left(\kappa c_1 + \lambda_2 d^2\right) < c_2, \tag{26}$$

$$\begin{bmatrix} I & I - E^T Z^T - H^T E_\perp \\ * & I \end{bmatrix} > 0, \tag{27}$$

$$\Pi = \begin{bmatrix} \Pi_{11} & -BV & \Pi_{13} & \Pi_{14} & \Pi_{15} & \Pi_{16} \\ * & \Pi_{22} & \Pi_{23} & \Pi_{14} & \Pi_{25} & -V^T \\ * & * & \Pi_{33} & 0 & 0 & 0 \\ * & * & * & -\eta I & 0 & 0 \\ * & * & * & * & -\eta I & \Pi_{56} \\ * & * & * & * & * & \Pi_{66} \end{bmatrix} < 0, \tag{28}$$

where

$$\Pi_{11} = He(A^T ZE + A^T E_\perp^T H + BV) - \sigma E^T ZE + F^T F,$$
$$\Pi_{13} = ZEG + E_\perp^T HG + F^T D,$$
$$\Pi_{14} = \eta E^T Z^T M + \eta H^T E_\perp M,$$
$$\Pi_{15} = N_a^T + V^T B^T,$$
$$\Pi_{16} = \zeta(E^T Z^T B + H^T E_\perp B - BU) + V^T,$$
$$\Pi_{22} = He\left\{A^T ZE + A^T E_\perp^T H - YC\right\} - \sigma E^T ZE,$$
$$\Pi_{23} = ZEG + E_\perp^T HG,$$
$$\Pi_{25} = -V^T B^T,$$
$$\Pi_{33} = -Q + D^T D,$$
$$\Pi_{56} = \zeta(N_b - BU),$$
$$\Pi_{66} = -\zeta U - \zeta U^T.$$

and $E_\perp \in \mathbb{R}^{(n-r)\times n}$ is the orthogonal complement of E such that $E_\perp E = 0$ and $rank\left(E_\perp\right) = n - r$. Furthermore, the observer gain matrix L, controller gain matrix K, are computed as

$$K = U^{-1}V, \quad L = (E^T Z^T + H^T E_\perp)^{-1}Y. \tag{29}$$

Proof. Let $Y = P^T L, \bar{P} = diag\{P, P\}, \bar{R} = diag\{R, R\}$. (21) can be written as

$$\Omega_1 = \begin{bmatrix} \Omega_{11} & \Omega_{12} & P^T G + F^T D \\ * & \Omega_{22} & P^T G \\ * & * & -Q + D^T D \end{bmatrix} < 0, \tag{30}$$

where

$$\Omega_{11} = He\left\{A^T P + P^T BK + \Delta A^T P + P^T \Delta BK\right\}$$
$$- \sigma E^T P + F^T F,$$
$$\Omega_{12} = -P^T BK - P^T \Delta BK + \Delta A^T P + K^T \Delta B^T P,$$
$$\Omega_{22} = He\left\{A^T P - YC - K^T \Delta B^T P\right\} - \sigma E^T P.$$

Further segregating the (30) for uncertain and known terms, yields

$$\Omega_1 = \bar{\Omega} + \begin{bmatrix} \Delta_{11} & \Delta_{12} & 0 \\ * & -K^T \Delta B^T P - P^T \Delta BK & 0 \\ * & * & 0 \end{bmatrix} < 0, \tag{31}$$

where

$$\bar{\Omega} = \begin{bmatrix} \bar{\Omega}_{11} & -P^T BK & P^T G + F^T D \\ * & \bar{\Omega}_{22} & P^T G \\ * & * & -Q + D^T D \end{bmatrix},$$

$\bar{\Omega}_{11} = A^T P + P^T BK + P^T A + K^T B^T P - \sigma E^T P + F^T F,$

$\bar{\Omega}_{22} = A^T P - YC + PA^T - C^T Y^T - \sigma E^T P,$

$\Delta_{11} = \Delta A^T P + P^T \Delta BK + P^T \Delta A + K^T \Delta B^T P,$

$\Delta_{12} = -P^T \Delta BK + \Delta A^T P + K^T \Delta B^T P.$

From (2), (31) can be rewriter as:

$$\Omega_1 = \bar{\Omega} + He\left\{ \begin{bmatrix} P^T M \\ P^T M \\ 0 \end{bmatrix} S(t) \begin{bmatrix} N_a + N_b K & -N_b K & 0 \end{bmatrix} \right\} < 0. \quad (32)$$

By Lemma 2, for $\eta > 0,$ we have that $\Omega_1 < 0$ if

$$\bar{\Omega} + \eta M M^T + \eta^{-1} N^T N < 0, \quad (33)$$

where

$M = \begin{bmatrix} MP^T & MP^T & 0 \end{bmatrix}^T, N = \begin{bmatrix} N_a + N_b K & -N_b K & 0 \end{bmatrix}.$

By using Schur complement, (33) holds if and only if

$$\breve{\Omega} = \begin{bmatrix} \bar{\Omega}_{11} & -P^T BK & \breve{\Omega}_{13} & \eta P^T M & N_a^T + K^T N_b^T \\ * & \bar{\Omega}_{22} & P^T G & \eta P^T M & -K^T N_b^T \\ * & * & \breve{\Omega}_{33} & 0 & 0 \\ * & * & * & -\eta I & 0 \\ * & * & * & * & -\eta I \end{bmatrix} < 0. \quad (34)$$

where $\breve{\Omega}_{13} = P^T G + F^T D, \quad \breve{\Omega}_{33} = -Q + D^T D.$

Now, we introduce a nonsingular matrix U and set $K = U^{-1} V,$ thus, we can establish that

$$P^T BK = (P^T B - BU) U^{-1} V + BV,$$

$$N_b K = (N_b - BU) U^{-1} V + BV,$$

so, (34) can be written as

$$\breve{\Omega} = \begin{bmatrix} \tilde{\Omega}_{11} & -BV & \breve{\Omega}_{13} & \eta P^T M & N_a^T + V^T B^T \\ * & \bar{\Omega}_{22} & PG & \eta P^T M & -V^T B^T \\ * & * & \breve{\Omega}_{33} & 0 & 0 \\ * & * & * & -\eta I & 0 \\ * & * & * & * & -\eta I \end{bmatrix}$$

$$+ He\left\{ \begin{bmatrix} P^T B - BU \\ 0 \\ 0 \\ 0 \\ N_b - BU \end{bmatrix} U^{-1} \begin{bmatrix} V & -V & 0 & 0 & 0 \end{bmatrix} \right\} < 0,$$

where $\tilde{\Omega}_{11} = A^T P + BV + P^T A + V^T B^T - \sigma E^T P + F^T F.$

By the Lemma 3, we have that (34) holds if

$$\Omega_2 = \begin{bmatrix} \tilde{\Omega}_{11} & -BV & \breve{\Omega}_{13} & \eta P^T M & \Pi_{15} & \breve{\Omega}_{16} \\ * & \bar{\Omega}_{22} & PG & \eta P^T M & \Pi_{25} & -V^T \\ * & * & \breve{\Omega}_{13} & 0 & 0 & 0 \\ * & * & * & -\eta I & 0 & 0 \\ * & * & * & * & -\eta I & \Pi_{56} \\ * & * & * & * & * & \Pi_{66} \end{bmatrix} < 0, \quad (35)$$

where $\breve{\Omega}_{16} = \zeta(P^T B - BU) + V^T.$

We can see that (10) is not a strict LMI, we can convert it into a strict LMI by

$$P = ZE + E_{\perp}^T H. \quad (36)$$

Thus, we have

$$0 \le E^T P = P^T E = E^T Z E,$$

it is equivalent to $Z > 0.$

In addition, substituting (36) into (35) and applying Schur complement yields (28). Thus, we can conclude that (28) and (29) can guarantee (21). Setting

$$\tilde{P} = R^{-\frac{1}{2}} Z R^{-\frac{1}{2}}, \quad \Theta = diag\left\{ \tilde{P}, \tilde{P} \right\}, \quad (37)$$

then, with the help of (38), we know that (12) is satisfied. By (25), we have $I < \tilde{P} < \kappa I,$ which can further yield $\lambda_1 < \kappa$ and $\lambda_3 > 1.$ Along with (26), we have

$$e^{\sigma T}(\lambda_1 c_1 + \lambda_2 d^2) < e^{\sigma T}(\kappa c_1 + \lambda_2 d^2) < c_2 < \lambda_3 c_2,$$

so, (12) is guaranteed by (25) and (26). To ensure the matrix P invertible, we require the following

$$\begin{bmatrix} I & I - P^T \\ * & I \end{bmatrix} > 0, \quad (38)$$

Substituting (36) into (38), we can obtain (27) and matrix L in (29) is solvable. This completes the proof.

Remark 1. We can see that (28) is not a strict LMI, we can solve this problem by giving a value of η in advance. By this way, we can solve linear matrix inequalities (28) through MATLAB toolbox.

4. Numerical examples

In this section, two numerical examples are provided to demonstrate the effectiveness of the proposed method.

Example 1. Consider the uncertain linear singular system (1) with

$$E=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\quad A=\begin{bmatrix}-1 & 1\\ -3 & -6\end{bmatrix},\quad B=\begin{bmatrix}1 & -1\\ 3 & -5\end{bmatrix},$$
$$G=\begin{bmatrix}-0.5 & 0.3\\ 0.2 & 0.1\end{bmatrix}, F=\begin{bmatrix}0 & 1\\ 1 & 2\end{bmatrix}, D=\begin{bmatrix}0.07 & 0.1\\ 0.05 & 0.3\end{bmatrix},$$
$$C=\begin{bmatrix}0.5 & 0\\ 1.2 & 1\end{bmatrix}, M=\begin{bmatrix}1 & 1\\ -1 & 2\end{bmatrix}, N_a=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},$$
$$N_b=\begin{bmatrix}-0.01 & 1\\ -0.1 & 0.01\end{bmatrix},\quad R=I.$$

Take $c_1=1, \sigma=0.01, T=5, d=0.5, \zeta=1.$ By using Matlab LMI control Toolbox to solve inequalities (26) - (28), we obtain that

$$Q=\begin{bmatrix}3.4200 & 0.2413\\ 0.2413 & 3.6391\end{bmatrix},\ Z=\begin{bmatrix}0.1315 & 6.3226\\ -6.3226 & 2.0310\end{bmatrix},$$
$$Y=\begin{bmatrix}2.3678 & -0.9949\\ -0.9949 & 3.6223\end{bmatrix},\ H=\begin{bmatrix}0 & 0.0345\\ 0.0345 & 0.1897\end{bmatrix},$$
$$U=\begin{bmatrix}34.5969 & 21.1597\\ 21.1597 & 13.3006\end{bmatrix},\ V=\begin{bmatrix}-14.2328 & -8.2772\\ -8.2772 & 2.9724\end{bmatrix},$$

and $\kappa=5.2192, \eta=10, c_2=6.4841.$

According to Theorem 3, the system (8) is H_∞ FTB with respect to $(c_1, c_2, T, R, \gamma, d)$ and the H_∞ performance index (10) satisfied with $\gamma=1.9973.$ The observer gain matrix and controller gain matrix are

$$K=\begin{bmatrix}-1.1397 & -3.7987\\ 1.1909 & 5.8199\end{bmatrix},\quad L=\begin{bmatrix}1.9381 & -1.2576\\ -0.5245 & 1.9097\end{bmatrix}.$$

Example 2. Consider the uncertain linear singular system (1) with

$$E=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}, A=\begin{bmatrix}-1 & 0 & 7\\ -3 & -6 & 2\\ 6 & 8 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 2 & 4\\ 2 & 0 & 3\\ 5 & 3 & 4\end{bmatrix},$$
$$G=\begin{bmatrix}5 & 4 & 6\\ -3 & 1 & 4\\ 2 & 8 & 7\end{bmatrix}, F=\begin{bmatrix}0 & 0 & 1\\ 3 & 5 & 7\\ 1 & 4 & 3\end{bmatrix}, D=\begin{bmatrix}7 & 1 & 5\\ 5 & 3 & 1\\ 5 & 3 & 8\end{bmatrix},$$
$$C=\begin{bmatrix}0.5 & 0 & 0.2\\ 1.2 & 1 & 0.9\\ 0.1 & 2.1 & 0.3\end{bmatrix}, M=\begin{bmatrix}0.1 & 0.1 & 0.2\\ -0.1 & 0.2 & 0.1\\ 0.2 & 0.4 & 0.6\end{bmatrix},$$
$$N_a=\begin{bmatrix}0.1 & 0 & 0\\ 0 & 0.1 & 0\\ 0 & 0 & 0.1\end{bmatrix}, N_b=\begin{bmatrix}-0.01 & 1 & 0.2\\ -0.1 & 0.01 & 0.4\\ 0.5 & 0.7 & 0.3\end{bmatrix},$$
$$R=\begin{bmatrix}1.2 & 0 & 0\\ 0 & 1.8 & 0\\ 0 & 0 & 1.6\end{bmatrix},$$

Take $c_1=2, \sigma=0.01, T=5, d=1.2, \zeta=20.$ By using Matlab LMI control Toolbox to solve inequalities (26) - (28), we obtain that

$$Q=\begin{bmatrix}4.1716 & 1.0150 & 1.2286\\ 1.0150 & 3.2450 & 0.7180\\ 1.2286 & 0.7180 & 3.4683\end{bmatrix},$$
$$Z=\begin{bmatrix}0.7131 & -0.6757 & -0.0292\\ -0.6757 & 1.2530 & 0.0422\\ -0.0292 & 0.0422 & 2.2644\end{bmatrix},$$
$$Y=\begin{bmatrix}2.2853 & 0.3163 & -2.1351\\ 0.3163 & 0.5833 & 1.9500\\ -2.1351 & 1.9500 & -0.5382\end{bmatrix},$$
$$H=\begin{bmatrix}0 & 0 & -0.0239\\ 0 & 0 & -0.2113\\ -0.0239 & -0.2113 & 0.1557\end{bmatrix},$$
$$U=\begin{bmatrix}0.5079 & -0.3112 & -0.1663\\ -0.3112 & 1.4455 & -0.3268\\ -0.1663 & -0.3268 & 0.4159\end{bmatrix},$$
$$V=\begin{bmatrix}13.4375 & -8.4193 & -9.7884\\ -8.4193 & 30.2443 & -11.9876\\ -9.7884 & -11.9876 & 7.0116\end{bmatrix},$$

and $\kappa=3.9423, \eta=0.1, c_2=7.$

According to Theorem 3, the system (8) is H_∞ FTB with respect to $(c_1, c_2, T, R, \gamma, d)$ and the H_∞ performance index (10) satisfied with $\gamma=2.4525.$ The observer gain matrix and controller gain matrix are

$$L=\begin{bmatrix}0.1366 & 0.6996 & -0.4539\\ -0.0861 & 0.5928 & -0.1358\\ -1.3714 & 1.2526 & -0.3457\end{bmatrix}.$$
$$K=\begin{bmatrix}12.7591 & -19.7435 & -33.8923\\ -8.8089 & 10.1807 & -18.0471\\ -25.3543 & -28.7190 & -10.8714\end{bmatrix},$$

5. Conclusion

This paper addresses the problem of the singular H_∞ finite-time boundedness for a class of uncertain singular systems. The observer-based feedback controller is designed. We propose a new criterion of singular H_∞ finite-time boundedness for uncertain singular systems, and present the design methods of control gain matrix and observer gain matrix. Finally, two numerical examples are given to illustrate the less conservatism and effectiveness.

References:

[1] Z. Jin, Q. Zhang, X. Meng, The stability analysis and control of uncertain singular biological economic system with invasion of alien species based on sliding mode control, Journal of Intelligent and Fuzzy Systems, Vol. 34, No. 6, 2018, pp. 1-13.

[2] F. Lei, Y. Ma, Passive control for singular time-delay system with actuator saturation, Applied mathematics and computation, Vol. 289, 2016, pp. 181-193.

[3] G. Zheng, D. Boutat, H. Wang, Observer design for a class of nonlinear singular systems, 2016 IEEE 55th Conference on Decision and Control, 2016, pp. 3910-3914.

[4] B. Li, S. Wo, J. Zhao, X. Ren, Finite-time guaranteed cost controller design for uncertain linear continuous-time singular systems, Transactions of the Institute of Measurement and Control, 2019, pp.1-9.

[5] G. Feng, J. Cao, Stability analysis of impulsive switched singular systems, IET Control Theory Appl, Vol. 9, 2015, pp. 863-870.

[6] M. Zheng, S. Yang, L. Li, Sliding mode control for fuzzy Markovian jump singular system with time-varying delay, International Journal of Control, Automation and Systems, Vol. 17, 2019, pp. 1677-1686.

[7] Y. Dong, L. Guo, J. Hao, Robust exponential stabilization for uncertain neutral neural networks with interval time-varying delays by periodically intermittent control, Neural Computing and Applications, Vol. 32, 2020, pp. 2651-2664.

[8] T. Hou, T. Liu, F. Deng, Stability for discrete-time uncertain systems with infinite Markov jump and time-delay, Science China Information Sciences, Vol. 64, 2021, pp. 152202.

[9] F. Wang, G. Lai, Fixed-time control design for nonlinear uncertain systems via adaptive method, Systems and Control Letters, Vol. 140, 2020, pp. 104704.

[10] Y. Dong, J. Hao, S. Mei, X. Deng, Observer design for one-sided Lipschitz uncertain descriptor systems with time-varying delay and nonlinear uncertainties, Circuits, Systems, and Signal Processing, Vol. 40, 2021, pp. 4779-4798.

[11] E. Badreddine, E. Hicham, H. Abdelaziz, E. Ahmed, E. Tissir, New approach to robust observer-based control of one-sided Lipschitz non-linear systems, IET Control Theory Appl, Vol. 13, 2019, pp. 333-342.

[12] Y. Qi, M. Cao, Finite-time boundedness and stabilisation of switched linear systems using event-triggered controllers, IET Control Theory Appl, Vol. 11, 2017, pp. 3240-3248.

[13] X. Lv, Y. Niu, J. Song, Finite-time boundedness of uncertain Hamiltonian systems via sliding mode control approach, Nonlinear Dyn, Vol. 104, 2021, pp. 497-507.

[14] M. Liu, B. Wu, Y. Wang, L. Liu, Dynamic output feedback control and guaranteed cost finite-time boundedness for switched linear systems. Circuits, Systems, and Signal Processing, Vol. 41, 2021, No. 5, pp. 2653-2668.

[15] Y. Ruan, T. Huang, Finite-time control for nonlinear systems with time-varying delay and exogenous disturbance, Symmetry, Vol. 12, 2020, pp.447.

[16] A. Zulfiqar, M. Rehan, M. Abid, Observer design for one-sided Lipschitz descriptor systems, Applied Mathematical Modelling, Vol. 40, 2016 pp. 2301-2311.

[17] Y. Zhang, C. Liu, X. Mu, Robust finite-time stabilization of uncertain singular Markovian jump systems, Applied Mathematical Modelling, Vol. 36, 2012, pp. 5109-5121.

[18] X. Chang, L. Zhang, J. Park, Robust static output feedback H_∞ control for uncertain fuzzy systems, Vol. 273, 2015, pp. 87-104.