Stability analysis and robust H_∞ filtering for discrete-time nonlinear systems with time-varying delays

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Abstract: In this paper, we investigate the problem of robust H_∞ filter design for a class of discrete-time nonlinear systems. The systems under consider involves time-varying delays and parameters uncertainties. The main objective is to design a linear full-order filter to ensure that the resulting filtering error system is asymptotically stable with a prescribed H_∞ performance level. By constructing an appropriate Lyapunov-Krasovskii functional, some novel sufficient conditions are established to guarantee the filter error dynamics system is robust asymptotically stable with H_∞ performance γ, and the H_∞ filter is designed in term of linear matrix inequalities. Finally, a numerical example is provided to illustrate the efficiency of proposed method.

Key-Words: Filtering; Lyapunov-Krasovskii functional; discrete-time systems; H_∞ performance; time-varying delays

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1 Introduction

Filtering is used to estimate the unavailable state variables of the given system by noise measurement, and has always been an important problem in the field of control signal processing [1-5]. As we know, traditional Kalman filter [6] is an effective method to solve the state estimation, which is widely used in many fields, such as controlling aircraft, aerospace and so on. Kalman filter require system model is linearly and all noise signals and measured values are Gaussian distribution. However, these restrictions are not easy to be satisfied in practical applications. Hence, the H_∞ filtering approach was proposed to overcome these restrictions in [7]. In the past decades, H_∞ filtering has attracted great attention from researchers, and some results have been reported in some literature [8-10]. In [8], the H_∞ filtering problem of stochastic linear systems subject to Markovian jump and multiplicative noise was studied. Zhang et al. [9] discusses the H_∞ filtering for general nonlinear discrete time-varying stochastic systems.

On the other hand, time delay is very common in various practical dynamic systems, and the existence of time delay often leads to poor performance or even instability of the system. In the past decades, the stability and stabilization of time-delay systems have been widely studied by scholars [11-13]. At the same time, it is of great significance to study the H_∞ filtering problem for time delay system, see [14-17] and references therein. In [14], the H_∞ filtering of discrete-time switched singular systems with time-varying delays was researched. H_∞ filtering with uncertainty were studied in [18-20]. In [18], He et al. dealt with the H_∞ filtering for discrete-time systems with polytopic type uncertainties. In [20], Luciano et al. investigated the robust H_∞ filter design with past output measurements for uncertain discrete-time systems.

Motivated by the aforementioned observation, in this paper, we investigated the robust H_∞ filtering for a class of discrete-time nonlinear systems with time-varying delays and norm-bounded uncertainties. By using the Lyapunov Krasovskii functional method, we propose new criteria to guarantee the filtering error dynamics system is robust asymptotically stable with H_∞ performance. Furthermore, the filter gain matrices can be obtained by solving the linear matrix inequality. Finally, a numerical example is provided to verify the validity of the stability criterion.

Notations: Throughout this note, the superscripts T and (-1) mean the transpose of a matrix and the inverse of a matrix, respectively. $P>0(P<0)$ denote a symmetric positive definite matrix P (a symmetric negative definite matrix), $diag\{\cdots\}$ denote a block-diagonal matrix. The symmetric term in a symmetric matrix is denoted by $*$.

2. Problem Formulation

Consider the following systems with time-varying delay.

$$\begin{aligned} x(k+1) &= (A+\Delta A(k))x(k)+B\omega(k)+g(x(k-\tau(k))) \\ &\quad +(A_\tau+\Delta A_\tau(k))x(k-\tau(k))+f(x(k)), \\ y(k) &= (C+\Delta C(k))x(k)+D\omega(k) \\ &\quad +(C_\tau+\Delta C_\tau(k))x(k-\tau(k)), \\ z(k) &= Ex(k)+E_\tau x(k-\tau(k))+F\omega(k), \\ x(\theta) &= \phi(\theta),\quad \theta=\{-\tau_2,-\tau_2+1,\cdots,0\}, \end{aligned} \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the state vector. $y(k)\in\mathbb{R}^m$ is the measurement vector. $\omega(k)\in\mathbb{R}^n$ is the noise signal vector belonging to $l_2[0,+\infty)$ and $z(k)\in\mathbb{R}^p$ is the signal to be estimated. $A,A_\tau,B,C,C_\tau,D,E,E_\tau$ and F are the constant matrices of appropriate dimensions. $\phi(\theta)$ denotes initial function. The delay $\tau(k)$ is a time-varying delay satisfying

$$\tau_1\le\tau(k)\le\tau_2, \tag{2}$$

where τ_1 and τ_2 are known positive integer. The parametric uncertainties $\Delta A(k),\Delta A_\tau(k),\Delta C(k)$ and $\Delta C_\tau(k)$ are unknown matrices satisfying the following conditions

$$\begin{aligned} \left[\Delta A(k)\quad \Delta A_\tau(k)\right] &= M_1H(k)\left[N_1\quad N_2\right], \\ \left[\Delta C(k)\quad \Delta C_\tau(k)\right] &= M_2\bar{H}(k)\left[N_3\quad N_4\right], \end{aligned} \tag{3}$$

where M_1,M_2,N_1,N_2,N_3 and N_4 are some given constant matrices with appropriate dimensions, $H(k),\bar{H}(k)$ is an unknown matrix satisfying

$$H^T(k)H(k)\le I,\quad \bar{H}^T(k)\bar{H}(k)\le I. \tag{4}$$

The function $f(x(k)),g(x(k-\tau(k)))$ are nonlinear with $f(0)=0,g(0)=0$ and satisfy the following Lipschitz condition for all $x,\hat{x}\in\mathbb{R}^n$:

$$\begin{aligned} &\|f(x)-f(\hat{x})\|\le\|L_1(x-\hat{x})\|, \\ &\|g(x(k-\tau(k)))-g(\hat{x}(k-\tau(k)))\| \\ &\le\|L_2(x(k-\tau(k))-\hat{x}(k-\tau(k)))\|, \end{aligned} \tag{5}$$

where L_1,L_2 are known constant matrices.

Consider a linear full-order filter described by

$$\begin{aligned} \hat{x}(k+1) &= A_F\hat{x}(k)+B_Fy(k),\quad \hat{x}(0)=0, \\ \hat{z}(k) &= C_F\hat{x}(k)+D_F\,y(k), \end{aligned} \tag{6}$$

where $\hat{x}(k)$ is the filter state vector, $\hat{z}(k)$ is the estimated vector of $z(k)$. A_F,B_F,C_F and D_F are the filter parameters to be determined.

Define the estimation error $e(k)=z(k)-\hat{z}(k)$ and the augmented state vector $\eta(k)=[x^T(k)\ \ \hat{x}^T(k)]^T$.

Then, the augmented system from (1) and the filter (6) can be described as

$$\begin{aligned} \eta(k+1) &= \bar{A}\eta(k)+\bar{A}_\tau K\eta(k-\tau(k))+\bar{I}f(x(k)) \\ &\quad +\bar{I}g(x(k-\tau(k)))+\bar{B}\omega(k), \\ e(k) &= \bar{E}\eta(k)+\bar{E}_\tau K\eta(k-\tau(k))+\bar{F}\omega(k), \\ \eta(\theta) &= [\phi^T(\theta)\ \ 0]^T,\theta=-\tau_2,-\tau_2+1,\cdots,0, \end{aligned} \tag{7}$$

where

$$\bar{A}=\begin{bmatrix} A+\Delta A(k) & 0 \\ B_FC+B_F\Delta C(k) & A_F \end{bmatrix},\quad \bar{I}=\begin{bmatrix} I \\ 0 \end{bmatrix},$$

$$\bar{A}_\tau=\begin{bmatrix} A_\tau+\Delta A_\tau(k) \\ B_FC_\tau+B_F\Delta C_\tau(k) \end{bmatrix},\quad \bar{B}=\begin{bmatrix} B \\ B_FD \end{bmatrix},$$

$$\bar{E}=[E-D_FC-D_F\Delta C(k)\quad -C_F],\quad \bar{F}=F-D_FD,$$

$$\bar{E}_\tau=E_\tau-D_FC_\tau-D_F\Delta C_\tau(k),\quad K=[I\quad 0].$$

The aim of this paper is to design a filter such that the following conditions are satisfied:

(1) the system (7) with $\omega(k)=0$ is asymptotically stable; and

(2) the H_∞ performance

$$\|e(k)\|_2\le\gamma\|\omega(k)\|_2 \tag{8}$$

is guaranteed under zero-initial conditions for all nonzero $\omega(k)\in l_2[0,+\infty)$ and a prescribed positive number γ.

Lemma 1. ([21]) For any $\varepsilon>0$, and known real matrices Ξ_1,Ξ_2 and $H(k)$ of appropriate dimensions, the inequality

$$\Xi_1H(k)\Xi_2+(\Xi_1H(k)\Xi_2)^T\le\varepsilon^{-1}\Xi_1\Xi_1^T+\varepsilon\Xi_2^T\Xi_2, \tag{9}$$

holds, where $H(k)$ is a time-varying uncertain matrix fulfilling $H^T(k)H(k)\le I$.

3. Main Results

In this section, sufficient conditions of stability are derived for system (7).

Theorem 1. Given integers $\tau_2\ge\tau_1>0$ and a scalar $\gamma>0$, the system (7) is robust asymptotically stable for $\omega(k)=0$ and also satisfies $\|e(k)\|_2 \le \gamma \|\omega(k)\|_2$ under zero-initial conditions for all nonzero $\omega(k) \in l_2[0,+\infty)$, if there exist positive scalars $\alpha_1, \alpha_2, \varepsilon_1, \varepsilon_2$, symmetric positive definite matrices P_1, P_2, Q, R, W, and any matrices X, Y, C_F, D_F such that the following LMI holds:

$$\begin{bmatrix} \Sigma_{11} & \bar{\Sigma}_{12} \\ * & \Sigma_{22} \end{bmatrix} < 0, \tag{10}$$

where

$$\Sigma_{11} = \begin{bmatrix} \Sigma_1 & 0 & \Sigma_2 & 0 & 0 & 0 & 0 & 0 \\ * & -P_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Sigma_3 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q & 0 & 0 & 0 & 0 \\ * & * & * & * & -R & 0 & 0 & 0 \\ * & * & * & * & * & -\alpha_1 I & 0 & 0 \\ * & * & * & * & * & * & -\alpha_1 I & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\bar{\Sigma}_{12} = \begin{bmatrix} A^T P_1 & C^T Y^T & E^T - C^T D_F^T & 0 & 0 \\ 0 & X^T & -C_F^T & 0 & 0 \\ A_\tau^T P_1 & C_\tau^T Y^T & E_\tau^T - C_\tau^T D_F^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ P_1 & 0 & 0 & 0 & 0 \\ P_1 & 0 & 0 & 0 & 0 \\ B^T P_1 & D^T Y^T & F^T - D^T D_F^T & 0 & 0 \end{bmatrix},$$

$$\Sigma_{22} = \begin{bmatrix} -P_1 & 0 & 0 & P_1 M_1 & 0 \\ * & -P_2 & 0 & 0 & Y M_2 \\ * & * & -I & 0 & -D_F M_2 \\ * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & -\varepsilon_2 I \end{bmatrix}$$

$\Sigma_1 = -P_1 + Q + R + \bar{\tau} W + \alpha_1 L_1^T L_1 + \varepsilon_1 N_1^T N_1 + \varepsilon_2 N_3^T N_3$,

$\Sigma_2 = \varepsilon_1 N_1^T N_2 + \varepsilon_2 N_3^T N_4$,

$\Sigma_3 = -W + \alpha_2 L_2^T L_2 + \varepsilon_1 N_2^T N_2 + \varepsilon_2 N_4^T N_4$.

Moreover, the filter gains can be obtained by

$$A_F = P_2^{-1} X, B_F = P_2^{-1} Y.$$

Proof. Choose the following Lyapunov-Krasovskii functional

$$V(k) = V_1(k) + V_2(k) + V_3(k), \tag{11}$$

where

$$V_1(k) = x^T(k) P_1 x(k) + \hat{x}^T(k) P_2 \hat{x}(k),$$
$$V_2(k) = \sum_{i=k-\tau_1}^{k-1} x^T(i) Q x(i) + \sum_{i=k-\tau_2}^{k-1} x^T(i) R x(i),$$
$$V_3(k) = \sum_{i=-\tau_2+1}^{-\tau_1+1} \sum_{j=k+i-1}^{k-1} x^T(i) W x(i).$$

Defined $\Delta V(k) = V(k+1) - V(k)$. Then, we have

$$\begin{aligned} \Delta V_1(k) &= x^T(k+1) P_1 x(k+1) - x^T(k) P_1 x(k) \\ &\quad + \hat{x}^T(k+1) P_2 \hat{x}(k+1) - \hat{x}^T(k) P_2 \hat{x}(k) \\ &= \xi^T(k)(\Phi_1^T P_1 \Phi_1 + \Phi_2^T P_2 \Phi_2)\xi(k) \\ &\quad - x^T(k) P_1 x(k) - \hat{x}^T(k) P_2 \hat{x}(k). \end{aligned} \tag{12}$$

where

$$\xi(k) = \left[x^T(k), \hat{x}^T(k), x^T(k-\tau(k)), x^T(k-\tau_1), x^T(k-\tau_2), f^T(x(k)), g^T(x(k-\tau(k))), \omega^T(k) \right]^T,$$

$$\Phi_1 = \left[A + \Delta A(k) \;\; 0 \;\; A_\tau + \Delta A_\tau(k) \;\; 0 \;\; 0 \;\; I \;\; I \;\; B \right],$$

$$\Phi_2 = \left[B_F C + B_F \Delta C(k) \;\; A_F \;\; B_F C_\tau + B_F \Delta C_\tau(k) \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; B_F D \right].$$

In addition, we have

$$\begin{aligned} \Delta V_2(k) &= x^T(k) Q x(k) - x^T(k-\tau_1) Q x^T(k-\tau_1) \\ &\quad + x^T(k) R x(k) - x^T(k-\tau_2) R x^T(k-\tau_2). \end{aligned} \tag{13}$$

$$\begin{aligned} \Delta V_3(k) &= \bar{\tau} x^T(k) W x(k) - \sum_{i=k-\tau_2}^{k-\tau_1} x^T(i) W x(i) \\ &\le \bar{\tau} x^T(k) W x(k) - x^T(k-\tau(k)) W x(k-\tau(k)), \end{aligned} \tag{14}$$

where $\bar{\tau} = \tau_2 - \tau_1 + 1$.

For any positive scalars α_1, α_2, it follows from (5) that

$$\begin{aligned} &\alpha_1 (x^T(k) L_1^T L_1 x(k) - f^T(x(k)) f(x(k))) \ge 0, \\ &\alpha_2 (x^T(k-\tau(k)) L_2^T L_2 x^T(k-\tau(k)) \\ &\quad - g^T(x(k(k-\tau(k)))) g(x(k(k-\tau(k))))) \ge 0. \end{aligned} \tag{15}$$

For any nonzero $\omega(k) \in l_2[0,+\infty)$, let

$$J = \sum_{k=0}^{\infty} \left[e^T(k) e(k) - \gamma^2 \omega^T(k) \omega(k) \right].$$

And under zero initial condition $V(0) = 0, V(\infty) > 0$, and $\omega(k) \ne 0$, one obtains

$$J \le \sum_{k=0}^{\infty} \left[e^T(k) e(k) - \gamma^2 \omega^T(k) \omega(k) + \Delta V(k) \right].$$

According to (7) and (12)-(15), we have

$$\begin{aligned}
&e^T(k)e(k)-\gamma^2\omega^T(k)\omega(k)+\Delta V(k)\\
&\le \xi^T(k)(\Phi_1^T P_1\Phi_1+\Phi_2^T P_2\Phi_2+\Phi_3^T\Phi_3)\xi(k)+x^T(k)\\
&\times[-P_1+Q+R+\bar{\tau}W+\alpha_1 L_1^T L_1)x(k)-\hat{x}^T(k)P_2\hat{x}(k)\\
&-x^T(k-\tau_1)Qx(k-\tau_1)-x^T(k-\tau_2)Rx(k-\tau_2)\\
&-x^T(k-\tau(k))(W-\alpha_2 L_2^T L_2)x(k-\tau(k))\\
&-\alpha_1 f^T(x(k))f(x(k)))-\alpha_2 g^T(x(k(k-\tau(k))))\\
&\times g(x(k(k-\tau(k))))-\gamma^2\omega^T(k)\omega(k)\\
&=\xi^T(k)\bar{\Theta}\xi(k),
\end{aligned}$$

where

$$\bar{\Theta}=\Theta_{11}+\Phi_1^T P_1\Phi_1+\Phi_2^T P_2\Phi_2+\Phi_3^T\Phi_3,$$

$$\Theta_{11}=diag\{-P_1+Q+R+\bar{\tau}W+\alpha_1 L_1^T L_1,-P_2,\ -W+\alpha_2 L_2^T L_2,-Q,-R,-\alpha_1 I,-\alpha_2 I,-\gamma^2 I\},$$

$$\Phi_3=[\bar{E}\quad \bar{E}_\tau\quad 0\quad 0\quad 0\quad 0\quad \bar{F}].$$

Using Schur complement, $\bar{\Theta}<0$ if and only if

$$\Theta=\begin{bmatrix}\Theta_{11} & \Theta_{12}\\ * & \Theta_{22}\end{bmatrix}<0, \tag{16}$$

where

$$\Theta_{12}=\begin{bmatrix}\Xi_1 & \Xi_2 & \Xi_3\\ 0 & A_F^T P_2 & -C_F^T\\ \Xi_4 & \Xi_5 & \Xi_6\\ 0&0&0\\ 0&0&0\\ P_1&0&0\\ P_1&0&0\\ B^T P_1 & D^T Y^T & F^T-D^T D_F^T\end{bmatrix},$$

$$\Theta_{22}=diag\{-P_1,-P_2,-I\},$$

$$\Xi_1=A^T P_1+\Delta A^T P_1,$$

$$\Xi_2=C^T Y^T+\Delta C^T(k)Y^T,$$

$$\Xi_3=E^T-C^T D_F^T-\Delta C^T(k)D_F^T,$$

$$\Xi_4=A_\tau^T P_1+\Delta A_\tau^T P_1,$$

$$\Xi_5=C_\tau^T Y^T+\Delta C_\tau^T(k)Y^T,\ Y=P_2 B_F,$$

$$\Xi_6=E_\tau^T-C_\tau^T D_F^T-\Delta C_\tau^T(k)D_F^T.$$

On the other hand, considering the uncertain terms in (16), Θ can be rewritten

$$\Theta=\tilde{\Theta}+\hat{\Theta},$$

where

$$\tilde{\Theta}=\begin{bmatrix}\Theta_{11} & \tilde{\Theta}_{12}\\ * & \Theta_{22}\end{bmatrix},\hat{\Theta}=\begin{bmatrix}0 & \hat{\Theta}_{12}\\ * & 0\end{bmatrix},$$

$$\tilde{\Theta}_{12}=\begin{bmatrix}A^T P_1 & C^T Y^T & E^T-C^T D_F^T\\ 0 & A_F^T P_2 & -C_F^T\\ A_\tau^T P_1 & C_\tau^T Y^T & E_\tau^T-C_\tau^T D_F^T\\ 0&0&0\\ 0&0&0\\ P_1&0&0\\ P_1&0&0\\ B^T P_1 & D^T Y^T & F^T-D^T D_F^T\end{bmatrix},$$

$$\hat{\Theta}_{12}=\begin{bmatrix}\Delta A^T P_1 & \Delta C^T(k)Y^T & -\Delta C^T(k)D_F^T\\ 0&0&0\\ \Delta A_\tau^T P_1 & \Delta C_\tau^T(k)Y^T & -\Delta C_\tau^T(k)D_F^T\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\end{bmatrix}.$$

Let

$$\Upsilon_1^T=\left[0\,0\,0\,0\,0\,0\,0\,0\,(P_1M_1)^T\ 0\,0\right],$$

$$\Upsilon_2=\left[N_1\ 0\ N_2\ 0\,0\,0\,0\,0\right],$$

$$\Upsilon_3^T=\left[0\,0\,0\,0\,0\,0\,0\,0\,0\,(YM_2)^T\ (-D_F M_2)^T\right],$$

$$\Upsilon_4=\left[N_3\ 0\ \ N_4\ 0\,0\,0\,0\,0\,0\,0\,0\right].$$

According to (3) and using the Lemma 1, $\hat{\Theta}$ can be rewritten as

$$\begin{aligned}
\hat{\Theta}&=\Upsilon_1 H(k)\Upsilon_2+(\Upsilon_1 H(k)\Upsilon_2)^T+\Upsilon_3\bar{H}(k)\Upsilon_4\\
&+(\Upsilon_3\bar{H}(k)\Upsilon_4)^T\\
&\le \varepsilon_1^{-1}\Upsilon_1\Upsilon_1^T+\varepsilon_1\Upsilon_2^T\Upsilon_2+\varepsilon_2^{-1}\Upsilon_3\Upsilon_3^T+\varepsilon_2\Upsilon_4^T\Upsilon_4.
\end{aligned}$$

(16) holds if

$$\tilde{\Theta}+\varepsilon_1^{-1}\Upsilon_1^T\Upsilon_1+\varepsilon_1\Upsilon_2^T\Upsilon_2+\varepsilon_2^{-1}\Upsilon_3^T\Upsilon_3+\varepsilon_2\Upsilon_4^T\Upsilon_4<0. \tag{17}$$

Using Schur complement, (17) holds if and only if

$$\begin{bmatrix}\Sigma_{11} & \Sigma_{12}\\ * & \Sigma_{22}\end{bmatrix}<0, \tag{18}$$

where

$$\Sigma_{11}=\begin{bmatrix}\Sigma_1 & 0 & \Sigma_2 & 0 & 0 & 0 & 0 & 0\\ * & -P_2 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \Sigma_3 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -Q & 0 & 0 & 0 & 0\\ * & * & * & * & -R & 0 & 0 & 0\\ * & * & * & * & * & -\alpha_1 I & 0 & 0\\ * & * & * & * & * & * & -\alpha_1 I & 0\\ * & * & * & * & * & * & * & -\gamma^2 I\end{bmatrix},$$

$$\Sigma_{12}=\begin{bmatrix} A^TP_1 & C^TY^T & E^T-C^TD_F^T & 0 & 0\\ 0 & A_F^TP_2 & -C_F^T & 0 & 0\\ A_\tau^TP_1 & C_\tau^TY^T & E_\tau^T-C_\tau^TD_F^T & 0 & 0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ P_1&0&0&0&0\\ P_1&0&0&0&0\\ B^TP_1 & D^TY^T & F^T-D^TD_F^T & 0 & 0\end{bmatrix},$$

$$\Sigma_{22}=\begin{bmatrix} -P_1 & 0 & 0 & P_1M_1 & 0\\ * & -P_1 & 0 & 0 & YM_1\\ * & * & -I & 0 & -D_FM_2\\ * & * & * & -\varepsilon_1 I & 0\\ * & * & * & * & -\varepsilon_2 I\end{bmatrix},$$

$\Sigma_1=-P_1+Q+R+\bar{\tau}W+\alpha_1L_1^TL_1+\varepsilon_1N_1^T\ N_1+\varepsilon_2N_3^TN_3,$

$\Sigma_2=\varepsilon_1N_1^T\ N_2+\varepsilon_2N_3^TN_4,$

$\Sigma_3=-W+\alpha_2L_2^TL_2+\varepsilon_1N_2^T\ N_2+\varepsilon_2N_4^TN_4.$

Let $X=P_2A_F, Y=P_2B_F$. we obtain (18) holds if the LMI (10) is satisfied. Then we have

$$e^T(k)e(k)-\gamma^2\omega^T(k)\omega(k)+\Delta V(k)<0. \quad (19)$$

So we have $J<0$. This ensures that (8) holds under zero-initial conditions for all nonzero $\omega(k)\in l_2[0,+\infty)$, and a prescribed $\gamma>0$.

On the other hand, when $\omega(k)=0$, from (19), it is easy to see that $\Delta V(k)<0$, which implies the system (7) is asymptoticly stable. This completes the proof.

When $\tau(k)=\tau$, consider the following system

$$\begin{aligned}\eta(k+1)&=\bar{A}\eta(k)+\bar{A}_\tau K\eta(k-\tau)+\bar{I}f(x(k))\\&\quad+\bar{I}g(x(k-\tau))+\bar{B}\omega(k),\\ e(k)&=\bar{E}\eta(k)+\bar{E}_\tau K\eta(k-\tau)+\bar{F}\omega(k),\\ \eta(\theta)&=[\phi^T(\theta)\ \ 0]^T.\end{aligned} \quad (20)$$

Corollary 1. Given positive integer τ and a scalar $\gamma>0$, the system (20) is robust asymptotically stable for $\omega(k)=0$ and also satisfies $\|e(k)\|_2\le\gamma\|\omega(k)\|_2$ under zero-initial conditions for all nonzero $\omega(k)\in l_2[0,+\infty)$, if there exist positive scalars $\alpha_1,\alpha_2,\varepsilon_1,\varepsilon_2$, symmetric positive definite matrices P_1,P_2,Q, and any matrices X,Y,C_F,D_F such that the following LMI holds:

$$\begin{bmatrix}\underline{\Sigma}_{11} & \underline{\Sigma}_{12}\\ * & \Sigma_{22}\end{bmatrix}<0, \quad (21)$$

where

$$\underline{\Sigma}_{11}=\begin{bmatrix}\underline{\Sigma}_1 & \Sigma_2 & 0&0&0&0\\ * & -P_2 & 0&0&0&0\\ * & * & \underline{\Sigma}_3 & 0&0&0\\ *&*&*&-\alpha_1 I&0&0\\ *&*&*&*&-\alpha_1 I&0\\ *&*&*&*&*&-\gamma^2 I\end{bmatrix},$$

$$\underline{\Sigma}_{12}=\begin{bmatrix} A^TP_1 & C^TY^T & E^T-C^TD_F^T & 0 & 0\\ 0 & X^T & -C_F^T & 0 & 0\\ A_\tau^TP_1 & C_\tau^TY^T & E_\tau^T-C_\tau^TD_F^T & 0 & 0\\ P_1&0&0&0&0\\ P_1&0&0&0&0\\ B^TP_1 & D^TY^T & F^T-D^TD_F^T & 0 & 0\end{bmatrix},,$$

$\underline{\Sigma}_1=-P_1+Q+\alpha_1L_1^TL_1+\varepsilon_1N_1^T\ N_1+\varepsilon_2N_3^TN_3,$

$\underline{\Sigma}_3=-Q+\alpha_2L_2^TL_2+\varepsilon_1N_2^T\ N_2+\varepsilon_2N_4^TN_4.$

Moreover, the filter gains can be obtained by

$$A_F=P_2^{-1}X, B_F=P_2^{-1}Y.$$

Proof Choose the following Lyapunov-Krasovskii functional

$$V(k)=x^T(k)P_1x(k)+\hat{x}^T(k)P_2\hat{x}(k)+\sum_{i=k-\tau}^{k-1}x^T(i)Qx(i) \quad (22)$$

The proof is similar to theorem 1, so it is omitted

When $\Delta A(k)=\Delta A_\tau(k)=0, \Delta C(k)=\Delta C_\tau(k)=0,$ (7) can be rewritten as

$$\begin{aligned}\eta(k+1)&=\underline{A}\eta(k)+\underline{A}_\tau K\eta(k-\tau(k))+\bar{I}f(x(k))\\&\quad+\bar{I}g(x(k-\tau(k)))+\bar{B}\omega(k),\\ e(k)&=\underline{E}\eta(k)+\underline{E}_\tau K\eta(k-\tau(k))+\bar{F}\omega(k),\\ \zeta(k)&=[\phi^T(\theta)\ \ 0]^T,\end{aligned} \quad (23)$$

where

$$\underline{A}=\begin{bmatrix}A & 0\\ B_FC & A_F\end{bmatrix}, \underline{A}_\tau=\begin{bmatrix}A_\tau\\ B_FC_\tau\end{bmatrix},$$

$$\underline{E}=[E-D_FC\quad -C_F], \underline{E}_\tau=E_\tau-D_FC_\tau.$$

Corollary 2. Given integers $\tau_2\ge\tau_1>0$ and a scalar $\gamma>0$, the system (23) is asymptotically stable for $\omega(k)=0$ and also satisfies $\|e(k)\|_2\le\gamma\|\omega(k)\|_2$ under zero-initial conditions for all nonzero $\omega(k)\in l_2[0,+\infty)$, if there exist positive scalars α_1,α_2, symmetric positive definite matrices P_1,P_2,Q,R,W, and any matrices X,Y,C_F,D_F such that the following LMI holds:

$$\begin{bmatrix}\tilde{\Sigma}_{11} & \tilde{\Sigma}_{12}\\ * & \tilde{\Sigma}_{22}\end{bmatrix}<0, \quad (24)$$

where

$$\Sigma_{11}=\begin{bmatrix}\tilde{\Sigma}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & -P_2 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \tilde{\Sigma}_3 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -Q & 0 & 0 & 0 & 0\\ * & * & * & * & -R & 0 & 0 & 0\\ * & * & * & * & * & -\alpha_1 I & 0 & 0\\ * & * & * & * & * & * & -\alpha_1 I & 0\\ * & * & * & * & * & * & * & -\gamma^2 I\end{bmatrix},$$

$$\Sigma_{12}=\begin{bmatrix}A^T P_1 & C^T Y^T & E^T - C^T D_F^T\\ 0 & X^T & -C_F^T\\ A_\tau^T P_1 & C_\tau^T Y^T & E_\tau^T - C_\tau^T D_F^T\\ 0 & 0 & 0\\ 0 & 0 & 0\\ P_1 & 0 & 0\\ P_1 & 0 & 0\\ B^T P_1 & D^T Y^T & F^T - D^T D_F^T\end{bmatrix},$$

$$\Sigma_{22}=diag\{-P_1,-P_2,-I\},$$

$$\tilde{\Sigma}_1=-P_1+Q+R+\bar{\tau}W+\alpha_1 L_1^T L_1,\ \tilde{\Sigma}_3=-W+\alpha_2 L_2^T L_2.$$

Moreover, the filter gains can be obtain by

$$A_F=P_2^{-1}X,\ B_F=P_2^{-1}Y.$$

4. Numerical example

In this section, we present one example to demonstrate the effectiveness of our results.

Example 1. Consider the system (1) with the following parameters

$$A=\begin{bmatrix}0.38 & 0.22\\ 0.24 & -0.19\end{bmatrix},\quad A_\tau=\begin{bmatrix}0.01 & 0.15\\ -0.1 & -0.15\end{bmatrix},$$

$$B=\begin{bmatrix}0.23\\ 0.8\end{bmatrix},\quad C=\begin{bmatrix}0.31 & 0.21\end{bmatrix},\quad C_\tau=\begin{bmatrix}0.4 & 0.6\end{bmatrix},$$

$$D=1.31,\quad E=\begin{bmatrix}0.21 & 0.22\end{bmatrix},\quad E_\tau=\begin{bmatrix}0.35 & 0.26\end{bmatrix},$$

$$F=-0.5,\quad H(k)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad L_1=\begin{bmatrix}0.01 & 0.01\\ 0.02 & 0.01\end{bmatrix},$$

$$L_2=\begin{bmatrix}0.01 & 0.01\\ 0.02 & 0.01\end{bmatrix},\quad M_1=\begin{bmatrix}0.1\\ 0.1\end{bmatrix},\quad N_1=\begin{bmatrix}0.1 & 0.01\end{bmatrix},$$

$$N_2=\begin{bmatrix}0.1 & 0.1\end{bmatrix},\quad M_2=0.11,\quad N_3=\begin{bmatrix}0.1 & 0.1\end{bmatrix},$$

$$N_4=\begin{bmatrix}0.1 & 0.1\end{bmatrix},\quad d(k)=2+\sin(k\pi),\ \bar{H}(k)=1,$$

$$\omega(k)=\begin{pmatrix}e^{-0.1k}\sin(k)\\ e^{-0.1k}\cos(k)\end{pmatrix},\ f(x(k))=\begin{pmatrix}0.01\sin(x_1(k))\\ 0.01\sin(x_1(k))\end{pmatrix},$$

$$g(x(k-\tau(k)))=\begin{pmatrix}0.02\sin(x_1(k-\tau(k)))\\ 0.02\sin(x_1(k-\tau(k)))\end{pmatrix},\gamma=7.77.$$

With the above parameters and by using the Matlab LMI Toolbox, we solve the LMI (10), and obtain

$$P_1=\begin{bmatrix}51.6223 & 5.0047\\ 5.0047 & 32.3001\end{bmatrix},$$

$$P_2=\begin{bmatrix}440.3250 & 133.0919\\ 133.0919 & 516.4034\end{bmatrix},$$

$$Q=\begin{bmatrix}2.2723 & -0.5089\\ -0.5089 & 2.1586\end{bmatrix},$$

$$R=\begin{bmatrix}2.2723 & -0.5089\\ -0.5089 & 2.1586\end{bmatrix},$$

$$W=\begin{bmatrix}5.7031 & 0.8571\\ 0.8571 & 5.6032\end{bmatrix},$$

$$X=\begin{bmatrix}-0.7391 & -0.9697\\ -0.4164 & -0.5443\end{bmatrix},$$

$$Y=\begin{bmatrix}-11.9108 & -15.8877\end{bmatrix},$$

$$\alpha_1=733.3014,\quad \alpha_1=565.1790,$$

$$\varepsilon_1=32.5077,\quad \varepsilon_2=12.8881,$$

$$C_F=[2.2434\quad 1.2873],\ D_F=0.3633.$$

Therefore, the filter gains are

$$A_F=\begin{bmatrix}-0.0012 & -0.0007\\ -0.0016 & -0.0009\end{bmatrix},\ B_F=\begin{bmatrix}-0.0193\\ -0.0258\end{bmatrix}.$$

According to Theorem 1, the system (7) is robust asymptotically stable for $\omega(k)=0$ and (8) is satisfied under zero-initial conditions for all nonzero $\omega(k)\in l_2[0,+\infty)$. The state vector of the system and of filter are shown in Fig. 1 and Fig. 2.

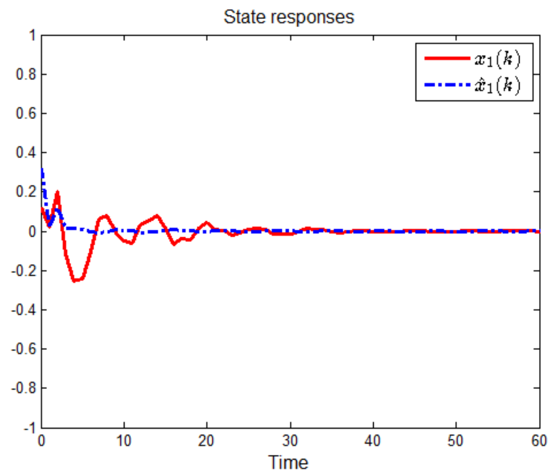

Fig. 1 The state trajectory of $x_1(k),\hat{x}_1(k)$.

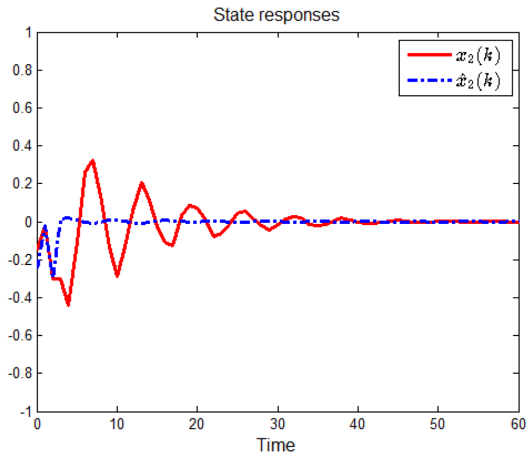

Fig. 2 The state trajectory of $x_2(k), \hat{x}_2(k)$.

The simulations for output of the system $z(k)$ and output of the filter $\hat{z}(k)$ are shown in Fig. 3.

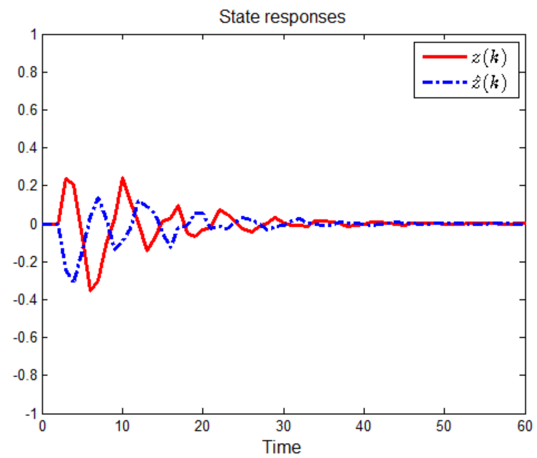

Fig. 3 The trajectory of $z(k), \hat{z}(k)$.

5 Conclusion

The paper studies the robust H_∞ filter for a class of discrete-time nonlinear systems with time-varying delays. The filter is proposed. By building an appropriate Lyapunov-Krasovskii functional, some sufficient conditions are obtained to guarantee the filter error augment system is the robust asymptotically stable with H_∞ performance γ. Furthermore, the calculation methods of filter are given. At the end of this paper, an example is given to verify the validity of the stability criterion. The problem of finite-time H_∞ filter for the discrete-time stochastic switched singular systems with time-varying delays is very meaningful topic that deserves further exploration.

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