Finite-time boundedness analysis for a class of uncertain discrete-time systems with interval time-varying delay

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Abstract: - In this paper, the problems of H_∞ finite-time boundedness for a class of discrete-time systems with time-varying delay and norm-bounded disturbance is investigated. By constructing a time-varying Lyapunov-Krasovskii functional and utilizing the linear matrix inequality approach, the H_∞ finite-time boundedness criterion is established to ensure that the discrete-time system with time-varying delay and norm-bounded disturbance is H_∞ finite-time bounded. A numerical example is provided to demonstrate the effectiveness of the theoretical results.

Key-Words: - H_∞ finite-time boundedness; Discrete-time systems; Time-varying delay; Lyapunov-Krasovskii functional

1 Introduction

In the past few years, system stability and feedback stabilization are important aspects of system theory research, and many research results have been obtained [1-3]. But these results mainly focus on the asymptotic stability in the sense of Lyapunov. In [1], stability analysis and observer design for discrete-time systems with interval time- varying delay were considered. In [2], Wang at al. investigated the exponential stability in the mean square for stochastic neural networks with mixed time-delays. In [3], the robust stability of neutral systems with mixed time-varying delays and nonlinear perturbations was considered. Note that most of the previous research works of stability are on the basis of the concept of Lyapunov stability, which is defined over an infinite time interval and used to characterize the steady performance. However, from practical considerations, there exist some systems, whose behavior may be only defined over a finite time interval or state variables are required to be within specific bounds. For this case, it is fundamentally meaningful and important to investigate the finite-time stability [4-7]. Amato at al. [4] studied the robust finite-time stability of impulsive dynamical linear systems subject to norm-bounded uncertainties. Moulay at al. [6] dealt with finite time stability of differential inclusions. In [7], Sun at al. considered finite-time stabilization and H_∞ control for a class of nonlinear Hamiltonian descriptor systems. In [5], finite-time stability and stabilization of time-delay systems were considered. In [8], finite-time stability for impulsive switched delay systems with nonlinear disturbances was investigated. In [9], Zhang at al. considered the robust finite-time stability and stabilization of switched positive systems. In [10], Chen at al. studied finite-time stability of switched positive linear systems. In [11], Yang at al. dealt with Finite-time stability and stabilization for a class of nonlinear time-delay systems. In [12], finite-time stability of fractional delayed neural networks was considered.

However, so far there are few results concerning the finite time stability of time-delay systems [5, 11,13]. The reason is that time delay systems have more complicated dynamic behaviors and are more difficult to deal with than system without delays. As is stated in [5], it is difficult to find a Lyapunov functional to satisfy the derivative condition for finite time stability of time delay systems. Also, it is reported in [13] that some key results in [11] are incorrect. So finite time stability of time-delay systems is still an open problem that needs further investigation.

In this paper, we considered the problem of H_∞ finite-time boundedness for a class of uncertain discrete-time delay systems with time-varying delay and norm-bounded disturbance. Sufficient conditions are given to ensure the systems with time-varying delay and norm-bounded disturbance are H_∞ finite-time bounded. A numerical design example is given to illustrate the proposed results in this paper.

The rest of this paper is organized as follows. In Section 2 the problem formulation and some preliminaries are introduced. The main results, H_∞ finite-time boundedness analysis is given in Sections 3. A numerical example is given in Section 4. Conclusions are given in Section 5.

Notations. Let N^+ stands for the set of nonnegative integers, the superscript "T" denotes the transpose, R^n and $R^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n\times m$ real matrices respectively. $X>0 (X\ge 0)$ denotes a real positive definite (semi-definite) matrix, I is the identity matrix with compatible dimension. We use an asterisk '$*$'to represent a term induced by symmetry. $\lambda_{\min}(\cdot)$ ($\lambda_{\max}(\cdot)$)denote the minimum (maximum) eigenvalue of the real symmetric matrix.

2 Problem Formulation

We consider the uncertain discrete-time systems with time-varying delays and norm-bounded disturbance

$$\begin{aligned} &x(k+1)=(A+\Delta A(k))x(k)+f(x(k),k)\\ &(B+\Delta B(k))x(k-\tau(k))+(C+\Delta C(k))\omega(k),\\ &z(k)=G_1x(k-\tau(k))+G_2\omega(k),\\ &x(\theta)=\varphi(\theta),\quad \theta=-\tau_M,-\tau_M+1,...,0, \end{aligned} \tag{1}$$

where $x(k)\in R^n$ is the state vector, $z(k)\in R^q$ is the measurement output, $\varphi(\theta)$ is an initial condition, A,B,C,G_1,G_2 are appropriate dimension constant matrices. $\tau(k)$ is time varying delay satisfying

$$0<\tau_m\le\tau(k)\le\tau_M, \tag{2}$$

where τ_m,τ_M are positive integers. The parameter uncertainties $\Delta A(k),\Delta B(k),\Delta C(k)$ satisfy

$$[\Delta A(k),\Delta B(k),\Delta C(k)]=N\Delta(k)[E_a,E_b,E_c], \tag{3}$$

where N,E_a,E_b,E_c are known real constant matrices. $\Delta(k)$ is unknown time-varying matrix which satisfying

$$\Delta^T(k)\Delta(k)\le I,\forall k\in N^+. \tag{4}$$

$f(x(k),k)$ denotes unknown nonlinear function which satisfy the following conditions

$$f^T(x(k),k)f(x(k),k)\le x^T(k)F^TFx(k),$$

where F is a known real constant matrix.

Assumption1. [14] The exogenous disturbance $\omega(k)$ is time-varying, and there exists a constant $d>0$ such that

$$\sum_{k=0}^{M+1}\omega^T(k)\omega(k)<d^2, \tag{5}$$

where $M>0$ is a constant to be given.

Lemma 1. [15] For any appropriately dimensioned matrices $O>0, O\in R^{n\times n}, U\in R^{m\times n}$, positive integers $d_1,d_2>d_1$, and a positive scalar β, the following inequality holds

$$\begin{aligned} &-\sum_{j=k-d_2}^{k-d_1-1}\beta^{k-j}y^T(j)Oy(j)\le\xi^T(k)\rho UO^{-1}U^T\xi(k)\\ &+2\xi^T(k)U(x(k-d_1)-x(k-d_2)), \end{aligned} \tag{6}$$

where $y(k)=x(k+1)-x(k),\xi(k)\in R^{m\times 1}$ is suitably selected vector function of the state vector and ρ is positive constant which is defined by

$$\rho=\begin{cases} d_2-d_1, & \beta=1,\\ (\beta^{-d_1}-\beta^{-d_2})/(\beta-1), & \beta\ne 1. \end{cases}$$

Definition 1. (Finite-time boundedness) Given four positive constants α_1,α_2,d,M with $\alpha_1<\alpha_2$ and $M\in Z^+$, a positive-definite matrix L, the discrete-time delay system (1) with exogenous disturbance is said to be finite-time bounded with respect to $(\alpha_1,\alpha_2,d,L,M)$, if

$$\sup_{\theta\in\Gamma}\{x^T(\theta)Lx(\theta)\}\le\alpha_1$$
$$\Rightarrow x^T(k)Lx(k)<\alpha_2,\ \forall k\in\{1,...,M\},$$

for any exogenous disturbance $\omega(k)$ satisfying (5).

Definition 2. (H_∞ finite-time boundedness) Given five positive constants $\alpha_1,\alpha_2,d,\gamma,M$ with

$\alpha_1 < \alpha_2$ and $M \in Z^+$, a positive definite matrix L, discrete-time system (1) is H_∞ finite-time bounded if the following two conditions hold:

(1) System (1) is finite-time bounded with respect to $(\alpha_1, \alpha_2, d, L, M)$;

(2) Under zero-initial condition, for any exogenous disturbance $\omega(k)$ satisfying assumption1, system (1) has a finite-time l_2- gain γ, that is,

$$\sum_{k=0}^{M} z^T(k)z(k) \le \gamma^2 \sum_{k=0}^{M} \omega^T(k)\omega(k). \quad (7)$$

3. Main results

The system (1) can be written as

$$\begin{aligned} x(k+1) &= Ax(k) + Bx(k-\tau(k)) + f(x(k),k) \\ &\quad + NH(k) + C\omega(k), \\ z(k) &= G_1 x(k-\tau(k)) + G_2\omega(k), \\ H(k) &= \Delta(k)(E_a x(k) + E_b x(k-\tau(k)) + E_c\omega(k)), \\ x(\theta) &= \varphi(\theta), \quad \theta = -\tau_M, -\tau_M + 1, \ldots, 0, \end{aligned} \quad (8)$$

Theorem 1. Under Assumption 1, the system (8) is H_∞ finite-time boundedness with respect to $(\alpha_1, \alpha_2, d, \gamma, \mathbb{F}, M)$, if there exist positive defined matrices P, Q, R_1, R_2, R_3, $T = \left[T_1^T T_2^T T_3^T T_4^T T_5^T T_6^T T_7^T\right]^T$, $S = \left[S_1^T S_2^T S_3^T S_4^T S_5^T S_6^T S_7^T\right]^T$, and positive scalars ε, μ, such that the following inequalities hold:

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ * & \Pi_{22} & \Pi_{23} \\ * & * & \Pi_{33} \end{bmatrix} < 0, \quad (9)$$

$$\begin{aligned} &\alpha_1[(\lambda_{\max}(\mathbb{Z}) + \varepsilon_1\lambda_{\max}(R_1) + \varepsilon_2\lambda_{\max}(R_2) \\ &+ \varepsilon_3\lambda_{\max}(R_3)) + d_{12}\varepsilon_4\lambda_{\max}(Q)] + \gamma^2 d^2 < \lambda_{\min}(\mathbb{Z})\alpha_2, \end{aligned} \quad (10)$$

where

$$\Pi_{11} = \begin{bmatrix} \Theta_{11} & d_{12}S_1 & \Theta_{13} & -d_{12}T_1 \\ * & \Theta_{22} & \Theta_{23} & d_{12}(S_4^T - T_2) \\ * & * & \hat{\Theta}_{33} & \Theta_{34} \\ * & * & * & \Theta_{44} \end{bmatrix},$$

$$\Pi_{12} = \begin{bmatrix} 0 & 0 & \varepsilon E_a^T E_c \\ d_{12}S_5^T & d_{12}S_6^T & d_{12}S_7^T \\ \Theta_{35} & \Theta_{36} & \hat{\Theta}_{37} \\ -d_{12}T_5^T & -d_{12}T_6^T & -d_{12}T_7^T \end{bmatrix},$$

$$\Pi_{13} = \begin{bmatrix} A^T P & (A-I)^T Q & d_{12}T_1^T & d_{12}S_1^T \\ 0 & 0 & d_{12}T_2^T & d_{12}S_2^T \\ B^T P & B^T Q & d_{12}T_3^T & d_{12}S_3^T \\ 0 & 0 & d_{12}T_4^T & d_{12}S_4^T \end{bmatrix},$$

$$\Pi_{22} = \begin{bmatrix} -\mu I & 0 & 0 \\ * & -\varepsilon I & 0 \\ * & * & \hat{\Theta}_{77} \end{bmatrix},$$

$$\Pi_{23} = \begin{bmatrix} P & Q & d_{12}T_5^T & d_{12}S_5^T \\ N^T P & N^T P & d_{12}T_6^T & d_{12}S_6^T \\ C^T P & C^T Q & d_{12}T_7^T & d_{12}S_7^T \end{bmatrix},$$

$$\Pi_{33} = \begin{bmatrix} -P & 0 & 0 & 0 \\ * & -(d_{12}^2)^{-1}Q & 0 & 0 \\ * & * & -d_{12}^{-1}Q & 0 \\ * & * & * & -d_{12}^{-1}Q \end{bmatrix},$$

$$\begin{aligned} \Theta_{11} &= R_2 + R_3 - P + \mu F^T F + \varepsilon E_a^T E_a, \\ \Theta_{13} &= \varepsilon E_a^T E_b + d_{12}(T_1 - S_1), \\ \Theta_{22} &= (R_1 - R_3) + d_{12}(S_2 + S_2^T), \\ \Theta_{23} &= d_{12}(T_2 - S_2 + S_3^T), \\ \hat{\Theta}_{33} &= d_{12}(T_3 + T_3^T - S_3 - S_3^T) \\ &\quad + \varepsilon E_b^T E_b + G_1^T G_1, \\ \Theta_{34} &= d_{12}(T_4^T - T_3 - S_4^T), \\ \Theta_{35} &= d_{12}(T_5^T - S_5^T), \\ \Theta_{36} &= d_{12}(T_6^T - S_6^T), \\ \hat{\Theta}_{37} &= d_{12}(T_7^T - S_7^T) + \varepsilon E_b^T E_c + G_1^T G_2, \\ \Theta_{44} &= (-R_1 - R_2) - d_{12}(T_4 + T_4^T), \\ \hat{\Theta}_{77} &= \varepsilon E_c^T E_c - \gamma^2 I + G_2^T G_2, \quad d_{12} = \tau_M - \tau_m, \\ \mathbb{Z} &= \mathbb{F}^{-1/2} P \mathbb{F}^{-1/2} \end{aligned} \quad (11)$$

$$\varepsilon_1 = d_{12}\lambda_{\max}(\mathbb{F}^{-1}), \varepsilon_2 = \tau_M\lambda_{\max}(\mathbb{F}^{-1}), \varepsilon_3 = \tau_m\lambda_{\max}(\mathbb{F}^{-1}),$$
$$\varepsilon_4 = 2\left(d_{12}+1\right)d_{12}\lambda_{\max}(\mathbb{F}^{-1}).$$

Proof. Construct Lyapunov–Krasovskii functional candidate

$$V(k) = V_1(k) + V_2(k) + V_3(k), \quad (12)$$

where

$$V_1(k)=x^T(k)Px(k),$$
$$V_2(k)=\sum_{j=k-\tau_M}^{k-\tau_m-1}x^T(j)R_1x(j)+\sum_{j=k-\tau_M}^{k-1}x^T(j)R_2x(j)+\sum_{j=k-\tau_m}^{k-1}x^T(j)R_3x(j), \quad (13)$$
$$V_3(k)=d_{12}\sum_{i=-\tau_M}^{-\tau_m-1}\sum_{j=k+i}^{k-1}\eta^T(j)Q\eta(j),$$
$$\eta(k)=x(k+1)-x(k), d_{12}=\tau_M-\tau_m.$$

Then, we have

$$\Delta V_1(k)=[Ax(k)+Bx(k-\tau(k))+f(x(k),k)+NH(k)+C\omega(k)]^TP[Ax(k)+Bx(k-\tau(k))+f(x(k),k)+NH(k)+C\omega(k)]-x^T(k)Px(k), \quad (14)$$

$$\begin{aligned}\Delta V_2(k)&=\sum_{j=k-\tau_M+1}^{k-\tau_m}x^T(j)R_1x(j)-\sum_{j=k-\tau_M}^{k-\tau_m-1}x^T(j)R_1x(j)\\&+\sum_{j=k-\tau_M+1}^{k}x^T(j)R_2x(j)-\sum_{j=k-\tau_M}^{k-1}x^T(j)R_2x(j)\\&+\sum_{j=k-\tau_m+1}^{k}x^T(j)R_3x(j)-\sum_{j=k-\tau_m}^{k-1}x^T(j)R_3x(j)\\&=x^T(k)(R_2+R_3)x(k)+x^T(k-\tau_m)R_1x(k-\tau_m)\\&-x^T(k-\tau_M)R_1x(k-\tau_M)-x^T(k-\tau_M)R_2x(k-\tau_M)\\&-x^T(k-\tau_m)R_3x(k-\tau_m),\end{aligned} \quad (15)$$

$$\begin{aligned}\Delta V_3(k)&=d_{12}(\sum_{i=-\tau_M}^{-\tau_m-1}\sum_{j=k+1+i}^{k}\eta^T(j)Q\eta(j)-\sum_{i=-\tau_M}^{-\tau_m-1}\sum_{j=k+i}^{k-1}\eta^T(j)Q\eta(j))\\&=d_{12}\sum_{i=-\tau_M}^{-\tau_m-1}(\eta^T(k)Q\eta(k)-\eta^T(k+i)Q\eta(k+i))\\&=\eta^T(k)(d_{12}^2)Q\eta(k)-d_{12}\sum_{j=k-\tau_M}^{k-\tau_m-1}\eta^T(j)Q\eta(j).\end{aligned} \quad (16)$$

According to Lemma 1, we obtain

$$\begin{aligned}-\sum_{j=k-\tau_M}^{k-\tau_m-1}\eta^T(j)Q\eta(j)&=-\sum_{j=k-\tau_M}^{k-\tau(k)-1}\eta^T(j)Q\eta(j)-\sum_{j=k-\tau(k)}^{k-\tau_m-1}\eta^T(j)Q\eta(j)\\&\le\varsigma^T(k)d_{12}TQ^{-1}T^T\varsigma(k)\\&+2\varsigma^T(k)T(x(k-\tau(k))-x(k-\tau_M))\\&+\varsigma^T(k)d_{12}SQ^{-1}S^T\varsigma(k)\\&+2\varsigma^T(k)S(x(k-\tau_m)-x(k-\tau(k)))\\&\le\varsigma^T(k)d_{12}TQ^{-1}T^T\varsigma(k)+\varsigma^T(k)d_{12}SQ^{-1}S^T\varsigma(k)\\&+2\varsigma^T(k)([0\quad 0\quad T\quad -T\quad 0\quad 0\quad 0]\\&+[0\quad S\quad -S\quad 0\quad 0\quad 0\quad 0]\varsigma(k),\end{aligned} \quad (17)$$

where

$$\varsigma^T(k)=[x^T(k),x^T(k-\tau_m),x^T(k-\tau(k)),x^T(k-\tau_M),f^T(x(k)),H^T(k),\omega^T(k)].$$

So,

$$\begin{aligned}\Delta V_3(k)&\le\eta^T(k)(d_{12}^2)Q\eta(k)\\&+d_{12}\varsigma^T(k)\delta TQ^{-1}T^T\varsigma(k)+d_{12}\varsigma^T(k)\delta SQ^{-1}S^T\varsigma(k)\\&+2d_{12}\varsigma^T(k)[0\quad 0\quad T\quad -T\quad 0\quad 0\quad 0]\varsigma(k)\\&+2d_{12}\varsigma^T(k)[0\quad S\quad -S\quad 0\quad 0\quad 0\quad 0]\varsigma(k).\end{aligned} \quad (18)$$

In addition, for positive scalars $\varepsilon,\mu,$ we have

$$\mu x^T(k)F^TFx(k)-\mu f^T(x(k),k)f(x(k),k)\ge 0 \quad (19)$$

and

$$\begin{aligned}&\varepsilon x^T(k)(E_a^TE_a)x(k)+2\varepsilon x^T(k)(E_a^TE_b)x(k-\tau(k))\\&+2\varepsilon x^T(k)(E_a^TE_c)\omega(x(k))\\&+\varepsilon x^T(k-\tau(k))(E_b^TE_b)x(k-\tau(k))\\&+2\varepsilon x^T(k-\tau(k))(E_b^TE_c)\omega(x(k))\\&+\varepsilon\omega^T(x(k))(E_c^TE_c)\omega(x(k))-\varepsilon H^T(k)H(k)\ge 0.\end{aligned} \quad (20)$$

By combining (14)-(20), we can get

$$\begin{aligned}\Delta V(k)&\le[Ax(k)+Bx(k-\tau(k))+f(x(k),k)\\&+NH(k)+C\omega(k)]^TP[Ax(k)+Bx(k-\tau(k))\\&+f(x(k),k)+NH(k)+C\omega(k)]\\&+x^T(k)(R_2+R_3-P+\mu F^TF+\varepsilon E_a^TE_a)x(k)\\&+2\varepsilon x^T(k)(E_a^TE_b)x(k-\tau(k))\\&+2\varepsilon x^T(k)(E_a^TE_c)\omega(x(k))\\&+\varepsilon x^T(k-\tau(k))(E_b^TE_b)x(k-\tau(k))\\&-x^T(k-\tau_M)(R_1+R_2)x(k-d_M)\\&+2\varepsilon x^T(k-\tau(k))(E_b^TE_c)\omega(x(k))\\&-x^T(k-\tau_m)(R_3-R_1)x(k-\tau_m)\\&-\mu f^T(x(k),k)f(x(k),k)\\&+\varepsilon\omega^T(x(k))(E_c^TE_c)\omega(x(k))-\varepsilon H^T(k)H(k)\\&+\eta^T(k)d_{12}^2Q\eta(k)+d_{12}{}^2\varsigma^T(k)TQ^{-1}T^T\varsigma(k)\\&+d_{12}{}^2\varsigma^T(k)SQ^{-1}S^T\varsigma(k)+\\&2d_{12}\varsigma^T(k)[0\quad 0\quad T\quad -T\quad 0\quad 0\quad 0]\varsigma(k)\\&+2d_{12}\varsigma^T(k)[0\quad S\quad -S\quad 0\quad 0\quad 0\quad 0]\varsigma(k)\end{aligned}$$

$$\begin{aligned}&\le \varsigma^T(k)\Xi\varsigma(k)+d_{12}{}^2\varsigma^T(k)TQ^{-1}T^T\varsigma(k)\\&+d_{12}{}^2\varsigma^T(k)SQ^{-1}S^T\varsigma(k)+\eta^T(k)d_{12}^2Q\eta(k)\\&+[Ax(k)+Bx(k-\tau(k))+f(x(k),k)\\&+NH(k)+C\omega(k)]^TP[Ax(k)+Bx(k-\tau(k))\\&+f(x(k),k)+NH(k)+C\omega(k)],\end{aligned}\tag{21}$$

where

$$\Xi=\begin{bmatrix}\bar{\Pi}_{11} & \bar{\Pi}_{12}\\ * & \bar{\Pi}_{22}\end{bmatrix},\tag{22}$$

$$\bar{\Pi}_{11}=\begin{bmatrix}\Theta_{11} & d_{12}S_1 & \Theta_{13}\\ * & \Theta_{22} & \Theta_{23}\\ * & * & \Theta_{33}\end{bmatrix},$$

$$\bar{\Pi}_{12}=\begin{bmatrix}-d_{12}T_1 & 0 & 0 & \varepsilon E_a^TE_c\\ d_{12}S_4^T-d_{12}T_2 & d_{12}S_5^T & d_{12}S_6^T & d_{12}S_7^T\\ \Theta_{34} & \Theta_{35} & \Theta_{36} & \Theta_{37}\end{bmatrix}$$

$$\bar{\Pi}_{22}=\begin{bmatrix}\Theta_{44} & -d_{12}T_5^T & -d_{12}T_6^T & -d_{12}T_7^T\\ * & -\mu I & 0 & 0\\ * & * & -\varepsilon I & 0\\ * & * & * & \varepsilon E_c^TE_c\end{bmatrix}$$

$$\begin{aligned}&\Theta_{11}=R_2+R_3-P+\mu F^TF+\varepsilon E_a^TE_a,\\&\Theta_{13}=\varepsilon E_a^TE_b+d_{12}(T_1-S_1),\\&\Theta_{22}=(R_1-R_3)+d_{12}(S_2+S_2^T),\Theta_{23}=d_{12}(T_2-S_2+S_3^T),\\&\Theta_{33}=d_{12}(T_3+T_3^T-S_3-S_3^T)+\varepsilon E_b^TE_b,\\&\Theta_{34}=d_{12}(T_4^T-T_3-S_4^T),\\&\Theta_{35}=d_{12}(T_5^T-S_5^T),\quad \Theta_{36}=d_{12}(T_6^T-S_6^T),\\&\Theta_{37}=d_{12}(T_7^T-S_7^T)+\varepsilon E_b^TE_c,\\&\Theta_{44}=\alpha^{\tau_M}(-R_1-R_2)-d_{12}(T_4+T_4^T).\end{aligned}$$

Applying Schur complement to (21)，we have that

$$\Delta V(k)\le 0,\tag{23}$$

if

$$\begin{bmatrix}\hat{\Pi}_{11} & \hat{\Pi}_{12} & \Pi_{13}\\ * & \hat{\Pi}_{22} & \Pi_{23}\\ * & * & \Pi_{33}\end{bmatrix}<0\tag{24}$$

where

$$\hat{\Pi}_{11}=\begin{bmatrix}\Theta_{11} & d_{12}S_1 & \Theta_{13} & -d_{12}T_1\\ * & \Theta_{22} & \Theta_{23} & d_{12}(S_4^T-T_2)\\ * & * & \Theta_{33} & \Theta_{34}\\ * & * & * & \Theta_{44}\end{bmatrix},$$

$$\hat{\Pi}_{12}=\begin{bmatrix}0 & 0 & \varepsilon E_a^TE_c\\ d_{12}S_5^T & d_{12}S_6^T & d_{12}S_7^T\\ \Theta_{35} & \Theta_{36} & \Theta_{37}\\ -d_{12}T_5^T & -d_{12}T_6^T & -d_{12}T_7^T\end{bmatrix},$$

$$\hat{\Pi}_{22}=\begin{bmatrix}-\mu I & 0 & 0\\ * & -\varepsilon I & 0\\ * & * & \varepsilon E_c^TE_c\end{bmatrix},$$

Since $\Omega^T\Omega\ge 0$, where

$$\Omega=\begin{bmatrix}0 & 0 & G_1 & 0 & 0 & 0 & \mathrm{G}_2 & 0 & 0 & 0 & 0\end{bmatrix},$$

we get that condition (9) implies that

$$\begin{bmatrix}\hat{\Pi}_{11} & \hat{\Pi}_{12} & \Pi_{13}\\ * & \tilde{\Pi}_{22} & \Pi_{23}\\ * & * & \Pi_{33}\end{bmatrix}<0\tag{25}$$

with

$$\tilde{\Pi}_{22}=\begin{bmatrix}-\mu I & 0 & 0\\ * & -\varepsilon I & 0\\ * & * & \varepsilon E_c^TE_c-\gamma^2I\end{bmatrix},$$

Then, we get

$$V(k+1)-V(k)-\gamma^2\omega^T(k)\omega(k)<0,\tag{26}$$

i.e.

$$\begin{aligned}V(k)&\le V(0)+\gamma^2\Big(\sum_{l=0}^{k-1}\omega^T(l)\omega(l)\Big)\\&<V(0)+\gamma^2d^2.\end{aligned}\tag{27}$$

A lower bound of the LKLF can be written as

$$V(k)\ge x^T(k)\mathbb{F}^{1/2}\mathbb{F}^{-1/2}P\mathbb{F}^{-1/2}\mathbb{F}^{1/2}x(k).\tag{28}$$

Define $\mathbb{Z}=\mathbb{F}^{-1/2}P\mathbb{F}^{-1/2}$. $\mathbb{Z}$ is a positive defined matrix and we have

$$V(k)\ge \lambda_{\max}(\mathbb{Z})x^T(k)\mathbb{F}x(k).\tag{29}$$

On the other hand，

$$
\begin{aligned}
V(0) &\le \lambda_{\max}(\mathbb{Z})x^T(0)\mathbb{F}x(0)+\lambda_{\max}(R_1)\sum_{j=-\tau_M}^{-\tau_m-1}x^T(j)x(j)\\
&+\lambda_{\max}(R_2)\sum_{j=-\tau_M}^{-1}x^T(j)x(j)+\lambda_{\max}(R_3)\sum_{j=-\tau_m}^{-1}x^T(j)x(j)\\
&+d_{12}\lambda_{\max}(Q)\sum_{i=-\tau_M}^{-\tau_m-1}\sum_{j=i}^{-1}\eta^T(j)\eta(j)\\
&\le \alpha_1(\lambda_{\max}(\mathbb{Z})+\varepsilon_1\lambda_{\max}(R_1)+\varepsilon_2\lambda_{\max}(R_2)\\
&+\varepsilon_3\lambda_{\max}(R_3))+\varepsilon_4 d_{12}(\lambda_{\max}(Q)).
\end{aligned}
\tag{30}
$$

So, from inequality (24), (25) and (26), we have

$$
\begin{aligned}
&\lambda_{\min}(\mathbb{Z})x^T(k)\mathbb{F}x(k)<\alpha_1[(\lambda_{\max}(\mathbb{Z})+\varepsilon_1\lambda_{\max}(R_1)\\
&+\varepsilon_2\lambda_{\max}(R_2)+\varepsilon_3\lambda_{\max}(R_3))+d_{12}\varepsilon_4\lambda_{\max}(Q)]+\gamma^2 d^2.
\end{aligned}
$$

From (10), we have

$$x^T(k)\mathbb{F}x(k)<\alpha_2.$$

Thus, it can be see that the system (8) is finite-time bounded with respect to $(\alpha_1,\alpha_2,d,\mathbb{F},M)$.

Now, we will prove the system (8) is H_∞ finite-time bounded. From (9), we get

$$V(k+1)-V(k)+z^T(k)z(k)-\gamma^2\omega^T(k)\omega(k)<0 \tag{31}$$

i.e.

$$V(k+1)<V(k)-z^T(k)z(k)+\gamma^2\omega^T(k)\omega(k).$$

We have

$$V(k)<V(0)+\sum_{l=0}^{k-1}(\gamma^2\omega^T(l)\omega(l)-z^T(l)z(l)).$$

Under zero-initial condition, we have

$$\sum_{l=0}^{M}z^T(l)z(l)\le\sum_{l=0}^{M}\gamma^2\omega^T(l)\omega(l).$$

Thus, according to Definition 2, the system (8) is H_∞ finite-time bounded with respect to $(\alpha_1,\alpha_2,d,\gamma,\mathbb{F},M)$. The proof is completed.

Consider the following system

$$
\begin{aligned}
x(k+1)&=Ax(k)+Bx(k-\tau(k))+f(x(k),k)+C\omega(k),\\
z(k)&=G_1x(k-\tau(k))+G_2\omega(k),\\
x(\theta)&=\varphi(\theta),\quad \theta=-\tau_M,-\tau_M+1,\dots,0,
\end{aligned}
\tag{32}
$$

The following Corollary can be obtained.

Corollary 1. Under Assumption 1, the system (32) is H_∞ finite-time boundedness with respect to $(\alpha_1,\alpha_2,d,\gamma,\mathbb{F},M)$, if there exist positive defined matrices P,Q,R_i $(i=1,2,3)$, $T=\left[T_1^T\ T_2^T\ T_3^T\ T_4^T\ T_5^T\ T_6^T\right]^T$, $S=\left[S_1^T\ S_2^T\ S_3^T\ S_4^T\ S_5^T\ S_6^T\right]^T$, a positive scalars μ, such that the following inequalities hold:

$$\begin{bmatrix}\Phi_{11} & \Phi_{12}\\ * & \Phi_{22}\end{bmatrix}<0, \tag{33}$$

$$
\begin{aligned}
&\alpha_1[(\lambda_{\max}(\mathbb{Z})+\varepsilon_1\lambda_{\max}(R_1)+\varepsilon_2\lambda_{\max}(R_2)+\varepsilon_3\lambda_{\max}(R_3))\\
&+d_{12}\varepsilon_4\lambda_{\max}(Q)]+\gamma^2d^2<\lambda_{\min}(\mathbb{Z})\alpha_2,
\end{aligned}
\tag{34}
$$

where

$$
\Phi_{11}=\begin{bmatrix}
\Theta_{11} & d_{12}S_1 & \Theta_{13} & -d_{12}T_1 & 0\\
* & \Theta_{22} & \Theta_{23} & d_{12}(S_4^T-T_2) & d_{12}S_5^T\\
* & * & \hat{\Theta}_{33} & \Theta_{34} & \Theta_{35}\\
* & * & * & \Theta_{44} & -d_{12}T_5^T\\
* & * & * & * & -\mu I
\end{bmatrix},
$$

$$
\Phi_{12}=\begin{bmatrix}
0 & A^TP & (A-I)^TQ & d_{12}T_1^T & d_{12}S_1^T\\
d_{12}S_6^T & 0 & 0 & d_{12}T_2^T & d_{12}S_2^T\\
\hat{\Theta}_{36} & B^TP & B^TQ & d_{12}T_3^T & d_{12}S_3^T\\
-d_{12}T_6^T & 0 & 0 & d_{12}T_4^T & d_{12}S_4^T\\
0 & P & Q & d_{12}T_5^T & d_{12}S_5^T
\end{bmatrix},
$$

$$
\Phi_{22}=\begin{bmatrix}
\hat{\Theta}_{66} & C^TP & C^TQ & d_{12}T_6^T & d_{12}S_6^T\\
* & -P & 0 & 0 & 0\\
* & * & -(d_{12}^2)^{-1}Q & 0 & 0\\
* & * & * & -d_{12}^{-1}Q & 0\\
* & * & * & * & -d_{12}^{-1}Q
\end{bmatrix},
$$

$$
\begin{aligned}
\Theta_{11}&=R_2+R_3-P+\mu F^TF,\\
\Theta_{13}&=d_{12}(T_1-S_1),\\
\Theta_{22}&=(R_1-R_3)+d_{12}(S_2+S_2^T)\\
\Theta_{23}&=d_{12}(T_2-S_2+S_3^T),\\
\hat{\Theta}_{33}&=d_{12}(T_3+T_3^T-S_3-S_3^T)+G_1^TG_1,\\
\Theta_{34}&=d_{12}(T_4^T-T_3-S_4^T),\\
\Theta_{35}&=d_{12}(T_5^T-S_5^T),\\
\hat{\Theta}_{36}&=d_{12}(T_6^T-S_6^T)+G_1^TG_2,\\
\Theta_{44}&=(-R_1-R_2)-d_{12}(T_4+T_4^T),\\
\hat{\Theta}_{66}&=-\gamma^2I+G_2^TG_2,\\
d_{12}&=\tau_M-\tau_m,
\end{aligned}
\tag{35}
$$

4 Numerical Example

Consider the system (1) with the following parameters:

$$A=\begin{bmatrix}0.01 & 0\\ 0 & -0.02\end{bmatrix},\quad B=\begin{bmatrix}0.01 & 0.02\\ 0.01 & -0.01\end{bmatrix},$$

$$C=\begin{bmatrix}0.01 & 0\\ 0 & 0.04\end{bmatrix},\quad G_1=\begin{bmatrix}-0.04 & 0\\ 0 & -0.5\end{bmatrix},$$

$$G_2=\begin{bmatrix}-0.2 & 0\\ 0 & 0.1\end{bmatrix},\quad N=\begin{bmatrix}-0.05 & 0\\ 0.1 & -0.02\end{bmatrix},$$

$$E_a=\begin{bmatrix}-0.05 & 0\\ 0.1 & -0.02\end{bmatrix},\quad E_b=\begin{bmatrix}0.1 & 0.1\\ 0 & 0.03\end{bmatrix},$$

$$E_c=\begin{bmatrix}-0.23 & 0.3\\ 0 & 0.01\end{bmatrix},\quad I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, F=\begin{bmatrix}0.2 & 0\\ 0 & 0.05\end{bmatrix},$$

$\tau_m=3,\quad \tau_M=4,\ \gamma=0.4,\quad \alpha_1=4.$

By using Matlab LMI control Toolbox to solve inequalities (9) and (10), we have

$$R_1=\begin{bmatrix}0.6570 & 0.0056\\ 0.0056 & 0.5436\end{bmatrix}, R_2=\begin{bmatrix}0.7689 & 0.0095\\ 0.0095 & 0.6034\end{bmatrix},$$

$$R_3=\begin{bmatrix}3.9838 & 0.0672\\ 0.0672 & 3.4141\end{bmatrix},\ Q=\begin{bmatrix}2.0705 & 0.0389\\ 0.0389 & 1.7139\end{bmatrix},$$

$$P=\begin{bmatrix}9.4682 & 0.1315\\ 0.1315 & 7.3386\end{bmatrix},\quad T_1=\begin{bmatrix}0.3992 & 0.0297\\ 0.0297 & 0.3713\end{bmatrix},$$

$$T_2=\begin{bmatrix}0.2255 & 0.0014\\ 0.0014 & 0.1664\end{bmatrix},\quad T_3=\begin{bmatrix}0.1519 & -0.0010\\ -0.0010 & 0.1005\end{bmatrix},$$

$$T_4=\begin{bmatrix}0.6239 & 0.0005\\ 0.0005 & 0.4905\end{bmatrix}, T_5=\begin{bmatrix}0.7036 & -0.0541\\ -0.0541 & 0.4688\end{bmatrix},$$

$$T_6=\begin{bmatrix}0.1927 & 0.0082\\ 0.0082 & 0.1537\end{bmatrix},\quad T_7=\begin{bmatrix}0.0828 & 0.0121\\ 0.0121 & 0.0819\end{bmatrix},$$

$$S_1=\begin{bmatrix}0.4894 & -0.0053\\ -0.0053 & 0.5808\end{bmatrix},\quad S_2=\begin{bmatrix}0.3835 & 0.0125\\ 0.0125 & 0.3080\end{bmatrix},$$

$$S_3=\begin{bmatrix}0.5176 & 0.0013\\ 0.0013 & 0.5022\end{bmatrix},\quad S_4=\begin{bmatrix}0.7122 & 0.0022\\ 0.0022 & 0.6048\end{bmatrix},$$

$$S_5=\begin{bmatrix}0.9814 & 0.0623\\ 0.0623 & 0.5515\end{bmatrix},\quad S_6=\begin{bmatrix}0.2357 & 0.0029\\ 0.0029 & 0.2077\end{bmatrix},$$

$$S_7=\begin{bmatrix}0.1048 & 0.0196\\ 0.0196 & 0.0902\end{bmatrix},\quad \mu=17.9586,\quad \varepsilon=0.4156,$$

$\alpha_2=18.4679.$

According to Theorem 1, the system (1) is H_∞ finite-time bounded. Fig. 1 shows the state trajectory of the system (1). From Fig. 1, it is easy to see that the system is finite-time bounded.

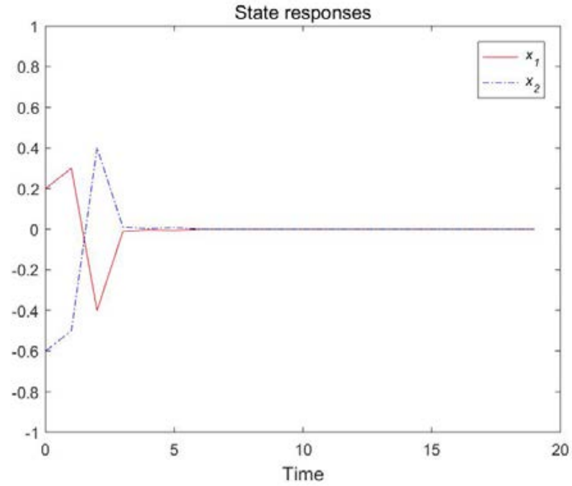

Fig1. The state trajectory of system (1)

5. Conclusion

This paper investigates the H_∞ finite-time boundedness for a class of discrete-time systems with time-varying delay and norm-bounded disturbance. By constructing an appropriate Lyapunov-Krasovskii functional, we have obtained sufficient conditions which ensure that the nonlinear discrete-time systems with norm bounded disturbance is H_∞ finite-time bounded. Finally, we given a numerical example to illustrate the efficiency of proposed methods.

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