Robust Digital LQ Control of Time-delay Systems

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Abstract: - Time-delays are mainly caused by the time required to transport mass, energy or information, but they can also be caused by processing time or accumulation. Typical examples of such processes are e.g. pumps, liquid storing tanks, distillation columns or some types of chemical reactors. This paper deals with a design of universal and robust digital control algorithms for control of great deal of processes with time-delay. These algorithms are realized using the digital Smith Predictor (SP) based on polynomial approach – by minimization of the Linear Quadratic (LQ) criterion. For minimization of the LQ criterion is used spectral factorization principle with application of the MATLAB Polynomial Toolbox. The designed polynomial digital SPs were verified in simulation conditions (and also using an experimental model of a laboratory heat exchanger). The main contribution of this paper is an experimental simulation examination of the robustness of the designed control algorithms. The program system MATLAB/SIMULINK was used for this purpose.

Key-Words: - Digital control, LQ control, Polynomial approach, Simulation of control loops, Smith Predictor, Time-delay, Robustness.

1 Introduction

Time-delay may be defined as the time interval between the start of an event at one point in a system and its resulting action at another point in the system. Delays are also known as transport lags or dead times; they arise in physical, chemical, biological and economic systems, as well as in the process of measurement and computation.

The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability. One of available approaches of optimal control such processes is a design of the robust controllers.

Historically, first modifications of time-delay algorithms were proposed for continuous-time (analog) controllers (see e.g. [1] - [6]. When a high performance of the control process is desired or the relative time-delay is very large, a predictive control strategy is one of possible approaches for a control of time-delay processes. The predictive control strategy includes a model of the process in the structure of the controller. The first time-delay compensation algorithm was proposed by Smith [7] in 1957. This time-delay compensator (TDC) known as the SP contained a dynamic model of the process and it can be considered as the first model predictive algorithm.

In industrial practice the implementation of the time-delay compensators based on continuous-time technique was difficult. Therefore the SPs and its modified versions can be implemented since 1980s together with the use of microprocessors in the industrial controllers. The first digital time-delay compensators are presented (see e.g. in [8]).

One of possible approaches to control of processes with time-delay is digital SP based on polynomial theory. Polynomial methods are design techniques for complex systems (including multivariable), signals and processes encountered in Control, Communications and Computing that are based on manipulations and equations with polynomials, polynomial matrices and similar objects. Systems are described by input-output relations in fractional form and processed using algebraic methodology and tools. The design procedure is thus reduced to algebraic polynomial equations [9]. Controller design consists in solving polynomial (Diophantine) equations. The Diophantine equations can be solved using the uncertain coefficient method – which is based on comparing coefficients of the same power. This is transformed into a system of linear algebraic equations [10]. Because the classical continuous-time SP is not suitable for control of unstable and integrating time-delay processes, the polynomial digital LQ SP for control of unstable and integrating time-delay processes has been designed in [11].

Much attention is currently paid to Model Predictive Control (MPC) of time-delay systems [8], [12], [13]. Disadvantage of MPC methods are quite complicated optimization calculations. And moreover, in case of the adaptive MPC it is necessary to apply recursive algorithms for estimation of process model parameters. The proposed digital LQ SPs eliminate these computational disadvantages.

It is obvious that the majority processes met in industrial practice are influenced by uncertainties. The uncertainties suppression can be solved either by implementation adaptive control or robust control. Some adaptive (self-tuning) modifications of the digital SPs are designed in [14] – [16]. Two versions of these controllers were implemented into MATLAB/SIMULINK Toolbox [17], [18].

Until recently, robust control and adaptive control have been viewed as two control techniques which are used for controller design in the presence of process model uncertainty (process model variations) [19].

From a robust control point of view, adaptive control is a method used for reducing the uncertainty level of the process model in closed control loops. Furthermore, the design of a robust controller deals in general with designing the controller in the presence of process uncertainties. This can simultaneously be: parameter variations (affecting low- and medium-frequency ranges) and unstructured model uncertainties (often located in high-frequency range). While in adaptive control the adaptation suppresses the parametric variations, the problem of suppressing unstructured model uncertainties remains.

The aim of this paper is the experimental examination of the robustness of control of time-delay processes. Robustness is the property when the dynamic response of closed control loop (including stability of course) is satisfactory not only for the nominal process transfer function used for design but also for the entire (perturbed) class of transfer functions that express uncertainty of the designer about dynamic environment in which real controller is expected to operate. The design of robust digital controllers for systems with time delay is investigated in [20]. A particular class of digital controller is considered, namely based on the pole assignment approach.

A more comprehensive discussion of robustness is referred in literature when design using frequency methods is considered. For root locus design, the natural measure of robustness is in effect gain margin. One can readily compare the system gain at the desired operating point and the point(s) of onset of instability to determine how much gain change is acceptable. Just this method will be used for investigation of the robustness control time-delay processes.

The paper is organized in the following way. The general problem of a control of the time-delay systems with regard to robustness is described in Section 1. The fundamental principle of digital SP is described in Section 2. Two versions of the primary polynomial LQ controller, which are components of the digital SP, are proposed in Section 3. The simulation verification of individual control-loops in term of their robustness are presented in Section 4. Results of simulation experiments for the control of the laboratory heat exchanger are introduced in Section 5. The discussion and compare of both control algorithms (advantages and availability for application in real-time conditions) are given in Section 6.

2 Principle of Digital SP

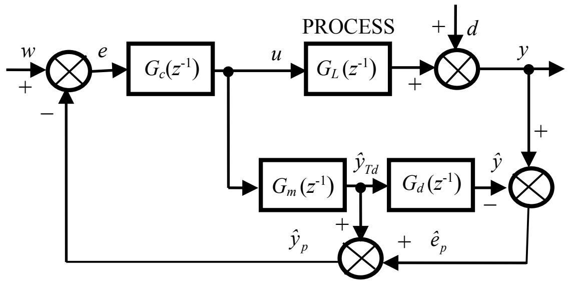

Fig. 1. Block Diagram of a Digital SP

The discrete versions of the SP and its modifications are suitable for time-delay compensation in industrial practice. The block diagram of a digital SP (see [14], [15]) is shown in Fig. 1. The function of the digital version is similar to the classical continuous-time version.

Number of higher order industrial processes can be approximated by a reduced order model with a pure time-delay. In this paper the following second-order linear model with a time-delay is considered

$$G_L\left(z^{-1}\right)=\frac{B\left(z^{-1}\right)}{A\left(z^{-1}\right)}z^{-d}=\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}z^{-d} \qquad (1)$$

The term z^{-d} represents the pure discrete time-delay. The time-delay is equal to dT_0 where T_0 is the sampling period.

The block $G_m(z^{-1})$ represents process dynamics without the time-delay and is used to compute an open-loop prediction. The numerator in transfer function (1) is replaced by its static gain $B(1)$, i.e. for

$z = 1$. This is to avoid problem of controlling a model with a $B(z^{-1})$, which has non-minimum phase zeros caused by a high sampling period or fractional delay. Since $B(z^{-1})$ is not controllable as in the case of a time-delay, it is moved out of the prediction model $G_m(z^{-1})$ and is treated together with the time-delay block, as shown in Fig. 1. The difference between the output of the process y and the model including time-delay $\hat{y}$ is the predicted error $\hat{e}_p$ as shown in Fig. 1, whereas e and d are the error and the measured disturbance, w is the reference signal. The primary (main) controller $G_c(z^{-1})$ can be designed by different approaches (for example digital PID control or methods based on polynomial approach). The outward feedback-loop through the block in Fig. 1 is used to compensate load disturbances and modelling errors.

3 Design of Primary Polynomial 2DOF Controller

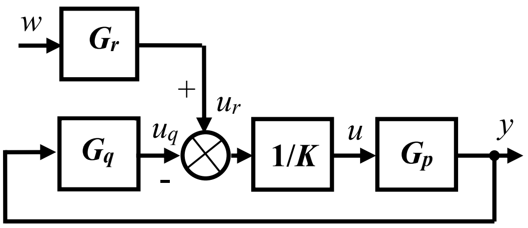

Fig. 2 Block diagram of a closed loop 2DOF control system

Polynomial control theory is based on the apparatus and methods of linear algebra. The design of the controller algorithm is based on the general block scheme of a closed-loop with two degrees of freedom (2DOF) according to Fig. 2.

The controlled process is given by the transfer function in the form

$$G_p(z^{-1}) = \frac{Y(z)}{U(z)} = \frac{B(z^{-1})}{A(z^{-1})} \tag{2}$$

where A and B are the second order polynomials. The controller contains the feedback part G_q and the feedforward part G_r. Then the digital controllers can be expressed in the form of a discrete transfer functions

$$G_r\left(z^{-1}\right) = \frac{R\left(z^{-1}\right)}{P\left(z^{-1}\right)K\left(z^{-1}\right)} \tag{3}$$

$$G_q\left(z^{-1}\right) = \frac{Q\left(z^{-1}\right)}{P\left(z^{-1}\right)} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{\left(1 + p_1 z^{-1}\right)\left(1 - z^{-1}\right)} \tag{4}$$

where $K\left(z^{-1}\right) = 1 - z^{-1}$.

According to the scheme presented in Fig. 2 and equations (2) – (4) it is possible to derive a polynomial Diophantine equation for computation of feedback controller parameters as coefficients of the polynomials Q and P

$$A(z^{-1})K(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1}) = D(z^{-1}) \tag{5}$$

where $D\left(z^{-1}\right)$ is the characteristic polynomial.

Asymptotic tracking of the reference signal w is provided by the feedforward part of the controller which is given by solution of the following polynomial Diophantine equation

$$S(z^{-1})D_w\left(z^{-1}\right) + B(z^{-1})R(z^{-1}) = D(z^{-1}) \tag{6}$$

For a step-changing reference signal value w, polynomial $D_w(z^{-1}) = 1 - z^{-1}$ and S is an auxiliary polynomial which does not enter into the controller design. Then it is possible to derive the polynomial R from equation (6) by substituting $z = 1$

$$R = r_0 = \frac{D(1)}{B(1)} \tag{7}$$

The 2DOF controller output is given by

$$u(k) = \frac{r_0}{K\left(z^{-1}\right)P\left(z^{-1}\right)} w(k) - \frac{Q\left(z^{-1}\right)}{K\left(z^{-1}\right)P\left(z^{-1}\right)} y(k) \tag{8}$$

Two primary polynomial LQ controllers are derived in this paper using minimization of the LQ criterion [21]. For the minimization procedure is used spectral factorization by means of the MATLAB Polynomial Toolbox 3.0 [22].

3.1 Minimization of LQ Criterion Using u(k)

In the first case the linear quadratic control methods try to minimize the quadratic criterion by penalization the value of the square controller output $u(k)$

$$J = \sum_{k=0}^{\infty}\left\{\left[w(k) - y(k)\right]^2 + q_u\left[u(k)\right]^2\right\} \tag{9}$$

where q_u is the so-called penalization constant (really positive number), which gives the rate of the controller output on the value of the criterion (where the constant at the first element of the criterion is considered equal to one). The standard procedure for minimization of criterion (9) is based on the state description of the system and leads to solution of the Riccati equation. In this paper, criterion minimization will be realized through the spectral factorization for an input-output description of the system

$$A(z)q_u A\left(z^{-1}\right)+B(z)B\left(z^{-1}\right)=D(z)\delta D\left(z^{-1}\right) \quad (10)$$

where δ is a constant chosen so that $d_0 = 1$.

Spectral factorization of polynomials of the first and the second degree can be computed simply by an analytical way [11], [23]; the procedure for higher degrees must be performed iteratively. Although $A(z^{-1})$ and $B(z^{-1})$ are the second degree polynomials (spectral factorization (10) can be computed by an analytical way), the MATLAB Polynomial Toolbox is used for this computation. The factorized polynomial must by also of second degree

$$D_2\left(z^{-1}\right)=1+d_{21}z^{-1}+d_{22}z^{-2} \quad (11)$$

The file *spf.m* by command

$$\mathrm{d}=\mathrm{spf}\left(\mathrm{a}*\mathrm{qu}*\mathrm{a}'+\mathrm{b}*\mathrm{b}'\right) \quad (12)$$

was used in this paper for computation of spectral factorization (10).

It is obvious that by using of the spectral factorization, only two parameters d_{21} and d_{22} of the second degree polynomial $D_2(z^{-1})$ (11) can be computed. This approach is applicable only for control of processes without time-delay (out of SP). The design of primary controller in the digital SP structure requires usage of the fourth degree polynomial

$$D_4\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2}+d_3z^{-3}+d_4z^{-4} \quad (13)$$

in equations (5) and (6). The polynomial $D_2(z^{-1})$ (11) has two different real poles α, β or one complex conjugated pole $z_{1,2}=\alpha\pm j\beta$ (in the case of oscillatory systems). These poles must be included into polynomial $D_4(z^{-1})$ (15) and other two poles γ, δ are user-defined real poles. A suitable pole assignment was designed for both types of the processes:

1st possibility:
Polynomial (13) has two different real poles α, β (computed from (12)) and user-defined real poles γ, δ. Then it is possible to write polynomial (13) as a product root of factor

$$D_4(z)=(z-\alpha)(z-\beta)(z-\gamma)(z-\delta) \quad (14)$$

and its individual parameters can be expressed as

$$\begin{aligned} d_1 &= -(\alpha+\beta+\gamma+\delta) \\ d_2 &= \alpha\beta+\gamma\delta+(\alpha+\beta)(\gamma+\delta) \\ d_3 &= -\left[(\alpha+\beta)\gamma\delta+(\gamma+\delta)\alpha\beta\right] \\ d_4 &= \alpha\beta\gamma\delta \end{aligned} \quad (15)$$

2nd possibility:
Polynomial (13) has the complex conjugate pole $z_{1,2}=\alpha\pm j\beta$ (computed from (12)) and user-defined real poles γ, δ. Then polynomial (13) has the form

$$D_4(z)=(z-\alpha-j\beta)(z-\alpha+j\beta)(z-\gamma)(z-\delta) \quad (16)$$

and its individual parameters can be expressed as

$$\begin{aligned} d_1 &= -(2\alpha+\gamma+\delta) \\ d_2 &= 2\alpha(\gamma+\delta)+\alpha^2+\beta^2+\gamma\delta \\ d_3 &= -\left[2\alpha\gamma\delta+\left(\alpha^2+\beta^2\right)(\gamma+\delta)\right] \\ d_4 &= \left(\alpha^2+\beta^2\right)\gamma\delta \end{aligned} \quad (17)$$

The control algorithm based on the LQ control method contains the following steps:

The parameters of the polynomial $D_2(z^{-1})$ are computed using command (12).

If the polynomial (13) has the real poles α, β, its parameters are computed according to equations (15), otherwise, they are computed according to equations (17).

Then the digital 2DOF controller (8) can be expressed in the form

$$\begin{aligned} &\left[1+(p_1-1)z^{-1}-p_1z^{-2}\right]u(k) \\ &\quad = r_0w(k)-\left(q_0+q_1z^{-1}+q_2z^{-2}\right)y(k) \end{aligned} \quad (18)$$

where

$$r_0=\frac{1+d_1+d_2+d_3+d_4}{b_1+b_2} \quad (19)$$

and parameters q_0, q_1, q_2 are computed from (5). The primary 2DOF controller output is given by

$$\begin{aligned} u(k) &= r_0w(k)-q_0y(k)-q_1y(k-1)-q_2y(k-2) \\ &\quad +(1-p_1)u(k-1)+p_1u(k-2) \end{aligned} \quad (20)$$

3.2 Minimization of LQ Criterion Using Increment Δu(k)

In the second case the linear quadratic control methods try to minimize the quadratic criterion by penalization of the incremental value of the controller output $\Delta u(k)$

$$J=\sum_{k=0}^{\infty}\left\{\left[w(k)-y(k)\right]^2+q_u\left[\Delta u(k)\right]^2\right\} \qquad (21)$$

Equation (10) for computation of the spectral factorization changes into

$$\begin{aligned}&(1-z)A(z)q_u\left(1-z^{-1}\right)A\left(z^{-1}\right)\\&\quad+B(z)q_uB\left(z^{-1}\right)=D(z)\delta D\left(z^{-1}\right)\end{aligned} \qquad (22)$$

It is obvious that after arrangement and substitution the first term of the left side (22) has this form

$$\left(1+a_{s1}z+a_{s2}z^2+a_{s3}z^3\right)q_u\left(1+a_{s1}z^{-1}+a_{s2}z^{-2}+a_{s3}z^{-3}\right) \qquad (23)$$

where

$$A_s\left(z^{-1}\right)=1+a_{s1}z^{-1}+a_{s2}z^{-2}+a_{s3}z^{-3} \qquad (24)$$

and $a_{s1}=a_1-1;\quad a_{s2}=a_2-a_1;\quad a_{s3}=-a_2.$

Because (24) is the third degree polynomial whose parameters and poles α, β and γ it is impossible to compute by an analytical way, the MATLAB Polynomial Toolbox 3 was used for their computation using command (12).

The characteristic polynomial is the sixth degree polynomial in this case

$$\begin{aligned}D_6\left(z^{-1}\right)&=1+d_1z^{-1}+d_2z^{-2}+d_3z^{-3}\\&\quad+d_4z^{-4}+d_5z^{-5}+d_6z^{-6}\end{aligned} \qquad (25)$$

Spectral factorization (22) gives three optimal parameters of polynomial (24) and then it is possible to write characteristic polynomial (25) as a combination of polynomial (24) $A_s(z)$ and product root of factors in positive power of variable z

$$D_6(z)=\left(z^3+a_{s1}z^2+a_{s2}z+a_{s3}\right)(z-\lambda)(z-\mu)(z-\nu) \qquad (26)$$

where λ, μ, ν are user-defined real poles. After modification (25) the characteristic polynomial is in the following form

$$D_6(z)=z^6+d_1z^5+d_2z^4+d_3z^3+d_4z^2+d_5z+d_6 \qquad (27)$$

After comparison of (26) and (27) it is possible to obtain expressions for computation of individual parameters of polynomial (27)

$$\begin{aligned}d_1&=a_{s1}-\lambda-\mu-\nu\\d_2&=a_{s2}+\nu(\lambda+\mu)-a_{s1}(\lambda+\mu+\nu)+\lambda\mu\\d_3&=a_{s1}\nu(\lambda+\mu)-a_{s2}(\lambda+\mu+\nu)-\lambda\mu\nu+a_{s3}\\d_4&=a_{s2}\mu(\lambda+\mu)-a_{s1}\lambda\mu\nu+a_{s2}\lambda\mu-a_{s3}(\lambda+\mu+\nu)\\d_5&=a_{s3}\lambda\mu-a_{s2}\lambda\mu\nu+a_{s3}\nu(\lambda+\mu)\\d_6&=-a_{s3}\lambda\mu\nu\end{aligned} \qquad (28)$$

Then the 2DOF controller design consists of determination of parameters (28) of polynomial (27) using command (12) from the Polynomial Toolbox and solution of the Diophantine equation for computation of feedback controller parameters

$$A_s(z^{-1})K(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})=D_6(z^{-1}) \qquad (29)$$

where

$$\begin{aligned}&K\left(z^{-1}\right)=1-z^{-1};\quad P\left(z^{-1}\right)=1+p_1z^{-1}+p_2z^{-2};\\&Q\left(z^{-1}\right)=q_0+q_1z^{-1}+q_2z^{-2}+q_3z^{-3}\end{aligned} \qquad (30)$$

and from expression (14)

$$\begin{aligned}u(k)&=r_0w(k)-q_0y(k)-q_1y(k-1)-q_2y(k-2)\\&\quad+(p_1-p_2)u(k-2)-p_2u(k-3)\end{aligned} \qquad (31)$$

The primary 2DOF controller output is given by

$$\begin{aligned}u(k)&=r_0w(k)-q_0y(k)-q_1y(k-1)-q_2y(k-2)\\&\quad+(p_1-p_2)u(k-2)-p_2u(k-3)\end{aligned} \qquad (32)$$

4 Simulation Verification

Numerical study, modelling and simulation are useful tools for the design of control systems [24] - [26].

A simulation verification of the designed control algorithms was performed in MATLAB/SIMULINK environment. The robustness of individual control loops was experimentally investigated by a change of the static gain of the nominal process model. From the point of view of the robust theory it is possible to consider these experiments on behalf of the gain margin determination by the parametric uncertainty influence.

The experimental process model was described by the second order continuous-time transfer function

$$G(s)=\frac{K}{T^2s^2+2\xi Ts+1}e^{-Ls} \qquad (33)$$

and for the nominal model the following parameters were choice: $K=2;\quad T=2;\quad \xi=1.25;\quad L=8$. Then the continuous–time transfer function (nominal continuous-time model) is in the form

$$G(s) = \frac{2}{(4s+1)(s+1)} e^{-8s} \tag{34}$$

The individual simulation experiments are realized subsequently: the static gain K was increased as far as the control closed-loop was in the stability boundary (no damping oscillation was achieved).

4.1 Control Using Primary Controller (20)

Let us now discretize (34) using a sampling period $T_0 = 2$ s. The discrete form of these transfer function is the nominal discrete model and it is expressed by

$$G_L(z^{-1}) = \frac{0.4728z^{-1} + 0.2076z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-4} \tag{35}$$

For all experiments the penalization factor was chosen as $q_u = 2$.

The characteristic polynomial:

$$D_4(z) = z^4 - 1.1461z^3 + 0.4409z^2 - 0.00652z + 0.0032$$

The individual poles:

$\alpha = 0.3796; \quad \beta = -0.7419; \quad \gamma = 0.1; \quad \delta = 0.5.$

The primary controller (16):

$$u(k) = 0.342w(k) - 0.9455y(k) + 0.6775y(k-1) - 0.0740y(k-2) + 0.8513u(k-1) + 0.487u(k-2) \tag{36}$$

The control courses of the process output and controller output for the nominal model are shown in Fig. 3.

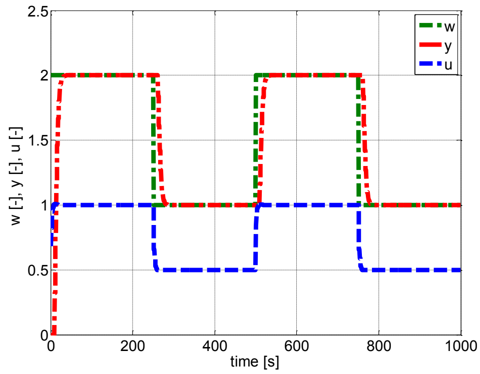

Fig. 3 Control of nominal model $G_L(z^{-1})$ - $K = 2$

Perturbed models (with different static gain K):

$$K = 3: \quad G_{P1}(z^{-1}) = \frac{0.7092z^{-1} + 0.3114z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-4} \tag{37}$$

$$K = 4: \quad G_{P2}(z^{-1}) = \frac{0.9456z^{-1} + 0.4153z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-4} \tag{38}$$

$$K = 4.4: \quad G_{P3}(z^{-1}) = \frac{1.0402z^{-1} + 0.4568z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-4} \tag{39}$$

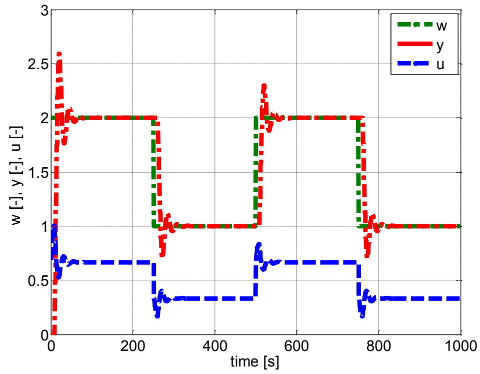

Fig. 4 Control of perturbed model (37) - $K = 3$

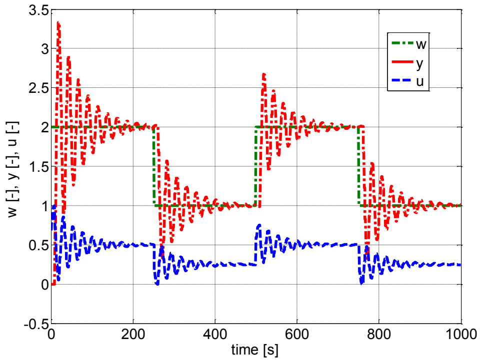

Fig. 5 Control of perturbed model (38) - $K = 4$

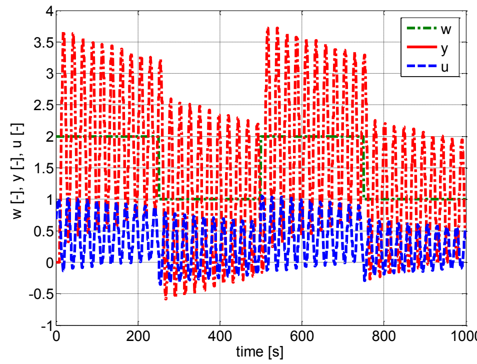

Fig. 6 Control of perturbed model (39) - $K = 4.4$

The control courses of the process output and controller output for the individual perturbed models (37) – (39) are shown in Figs. 4 - 6. It is obvious from Fig. 6 that for the static gain $K = 4.4$ is the closed-loop control on the stability boundary.

4.2 Control Using Primary Controller (32)

Nominal discrete model (35) and penalization factor $q_u = 2$ was used for all simulation experiments.

The characteristic polynomial:

$$D_6(z) = z^6 - 1.6483z^5 + 1.0427z^4 - 0.3720z^3$$
$$+ 0.0743z^2 - 0.0067z + 2.2920e-04$$

The individual poles:

$$\alpha, \beta = 0.4567 \pm 0.2867i; \;\; \gamma = 0.1346;$$
$$\lambda = 0.1; \;\; \mu = 0.2; \;\; \nu = 0.3.$$

The primary controller (32):

$$u(k) = 0.1386w(k) - 0.6963y(k) + 0.8863y(k-1)$$
$$-0.3581y(k-2) + 0.0648u(k-2) - 0.006u(k-3) \quad (40)$$

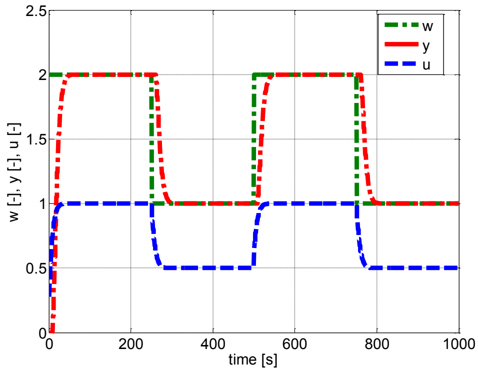

Fig. 7 Control of nominal model $G_{\Delta L}(z^{-1})$ - $K = 2$

The control courses of the process output and controller output for the nominal model are shown in Fig. 7.

Perturbed models (with different static gain *K*):

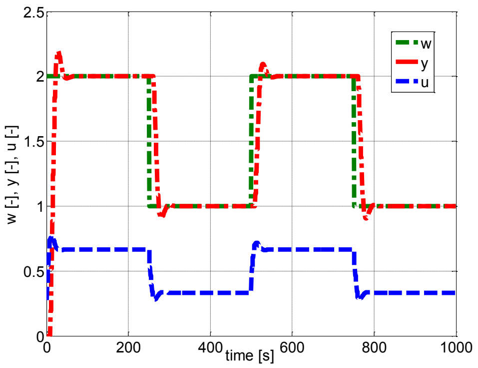

Fig. 8 Control of perturbed model with $K = 3$

The control courses of the process output and controller output for the individual perturbed models for K = 3; 4 and 6.6 are shown in Figs. 8 - 10. It is obvious from Fig. 10 that for model (41) with the K = 6.6 is the closed-loop control on the stability boundary.

$$K = 6.6: \quad G_{P4}(z^{-1}) = \frac{0.4728z^{-1} + 0.2076z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-4} \quad (41)$$

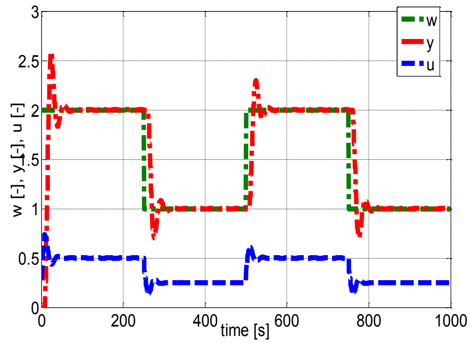

Fig. 9 Control of perturbed model with $K = 4$

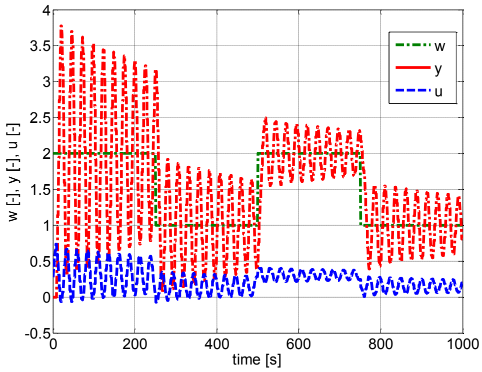

Fig. 10 Control of perturbed model (41) - $K = 6.6$

5 Simulation Control of Heat Exchanger

Heat exchangers are used for the purpose of transferring heat from a hot fluid to a cold fluid. They are requisite in a range of industrial technologies, particularly in the energetic, metallurgical, chemical and processing of polymer and rubber materials. A new universal SP was successfully verified by control of a laboratory heat exchanger in simulation conditions.

5.1 Laboratory Heat Equipment

A scheme of the laboratory heat equipment [28] is depicted in Fig. 11. The heat transferring fluid (e. g. water) is transported using a continuously controllable DC pump (6) into a flow heater (1) with max. power of 750 W. The temperature of a fluid at the heater output T_1 is measured by a platinum thermometer. Warmed liquid then goes through a 15 meters long insulated coiled pipeline (2) which causes the significant delay (20 – 200 s) in the system. The air-water heat exchanger (3) with two cooling fans (4, 5) represents a heat-consuming appliance. The speed of the first fan can be continuously adjusted, whereas the second one is of on/off type. Input and output temperatures of the cooler are measured again by platinum thermometers as T_2, respective T_3. The platinum thermometer T_4 is dedicated for measurement of the outdoor-air temperature. The laboratory heat equipment is connected to a standard PC via a technological multifunction I/O card MF 624. This card is designed for the need of connecting PC compatible computers to real world signals. The card is designed for standard data acquisition, control applications and optimized for use with Real Time Toolbox for SIMULINK.

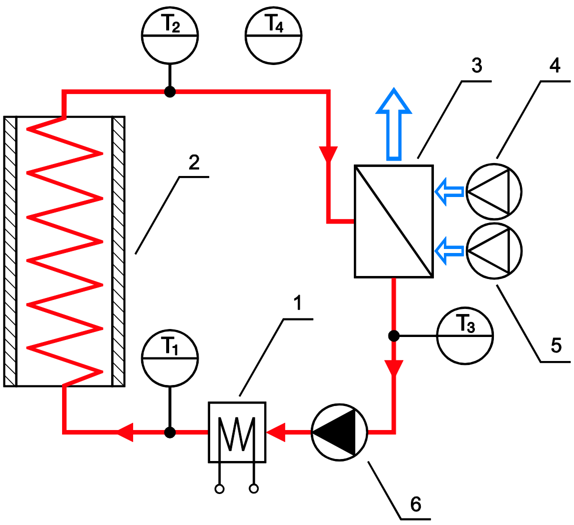

Fig. 11 Scheme of Laboratory Heat Equipment

On the basis of several identification experiments [12], one of the discrete model in the following form

$$G\left(z^{-1}\right)=\frac{0.1494z^{-1}+0.028z^{-2}}{1-0.6376z^{-1}-0.1407z^{-2}}z^{-2} \qquad (42)$$

with a sampling period T_0 = 50 s was used for a simulation verification of the designed control algorithms. The simulation experiments have been realized using minimization of both criterions (9) and (17). The process which is described by transfer function (42) was used in the Simulink control scheme for the verification of the dynamical behaviour for different penalization factors q_u .

The following control conditions have been chosen for individual simulation experiments:

5.2 Control Using Primary Controller (20)

1. Experiment, $q_u = 0.01$
   The poles:
   $\alpha = 0.2089; \quad \beta = -0.1720; \quad \gamma = 0.01; \quad \delta = 0.1$
   The characteristic polynomial:
   $$D_4(z) = z^4 - 0.1478z^3 - 0.0309z^2 + 0.0039z - 3.6100e-05$$
2. Experiment, $q_u = 1$
   The poles:
   $\alpha = 0.7647; \quad \beta = -0.1725; \quad \gamma = 0.01; \quad \delta = 0.1$
   The characteristic polynomial:
   $$D_4(z) = z^4 - 0.7012z^3 - 0.0667z^2 + 0.0140z - 1.3270e-04$$

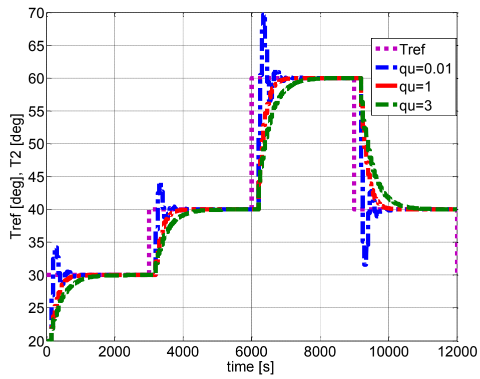

Fig. 12 Courses of process outputs, controller (16)

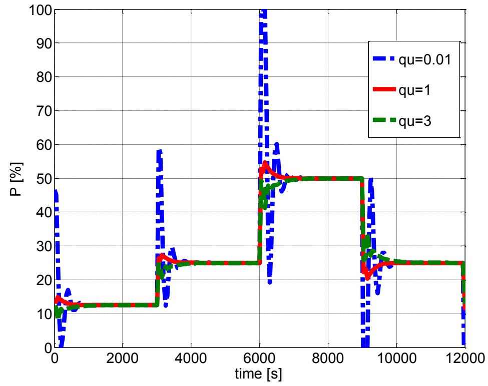

Fig. 13 Courses of controller outputs, controller (20)

3. Experiment, $q_u = 3$
   The poles:
   $\alpha = 0.8597; \quad \beta = -0.7219; \quad \gamma = 0.01; \quad \delta = 0.1$
   The characteristic polynomial:
   $$D_4(z) = z^4 - 0.2478z^3 - 0.6044z^2 + 0.0681z - 6.2060e-04$$

The courses of the process outputs and controller outputs for individual penalization factors q_u are shown in Figs. 12 and 13. From these Figs. follows that for low value of q_u the control courses oscillate. By increasing of q_u the courses of the control variables are without overshoots.

5.3 Control Using Primary Controller (32)

1. Experiment, $q_u = 0.01$

   The poles:

   $\alpha = -0.1739; \quad \beta, \gamma = 0.2531 \pm 0.2824i;$

   $\lambda = 0.2; \quad \mu = 0.3; \quad \nu = 0.4$

   The characteristic polynomial:

$$D_6(z) = z^6 - 1.2323z^5 + 0.6149z^4 - 0.1356z^3 - 1.6800e-05z^2 + 0.0052z - 6.0000e-04$$

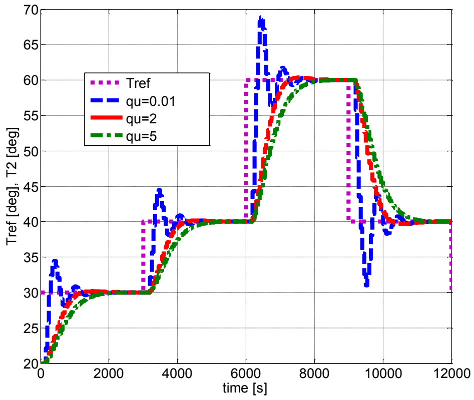

Fig. 14 Courses of process outputs, controller (32)

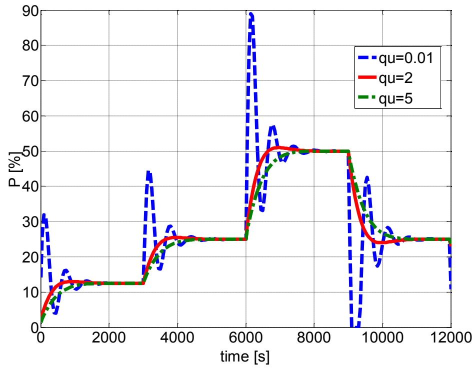

Fig. 15 Courses of controller outputs – controller (32)

2. Experiment, $q_u = 2$

   The poles:

   $\alpha = -0.1735; \quad \beta, \gamma = 0.7482 \pm 0.1661i;$

   $\lambda = 0.2; \quad \mu = 0.3; \quad \nu = 0.4$

   The characteristic polynomial:

$$D_6(z) = z^6 - 2.2228z^5 + 1.7782z^4 - 0.5610z^3 + 0.0252z^2 + 0.0186z - 0.0024$$

3. Experiment, $q_u = 5$

   The poles:

   $\alpha = -0.1735; \quad \beta, \gamma = 0.7915 \pm 0.1296i;$

   $\lambda = 0.2; \quad \mu = 0.3; \quad \nu = 0.4$

   The characteristic polynomial:

$$D_6(z) = z^6 - 2.3066z^5 + 1.8973z^4 - 0.6107z^3 + 0.0293z^2 + 0.0202z - 0.0027$$

The courses of the process outputs and controller outputs for individual penalization factors q_u are shown in Figs. 14 and 15. From these Figs. it is evident that the dynamical behaviour of the control variables is similar as in case of controller (20). However, the transient responses are slower and controller (32) is more conservative and robust than controller (20).

6 Conclusion

The paper presents an experimental simulation investigation of robust algorithms for control of time delay systems. The MATLAB Polynomial Toolbox 3.0 is used for design of the polynomial digital SP. The primary controllers of the digital SP are based on minimization of the LQ criterion using spectral factorization. Two types of minimization of LQ criterions have been designed. In criterion (9) it is minimized a square of the controller output $u(k)$ – controller (20). In criterion (21) it is minimized a square of the increment value of the controller output $\Delta u(k)$ – controller (32). Simulation experiments demonstrated the influence of static gain K (parametric uncertainty) on the course of control variables (robustness of the control-loop). From comparison of both methods it is evident that minimization criterion (9) leads to faster courses of control variables. However the control-loop is in the stability boundary for a lower value of K as in the case of minimization criterion (21). However minimization criterion (21) leads to quieter courses of control variables with their smaller oscillations for grater values of K. The controller (32) is more conservative and robust than controller (20). Both control algorithms were verified by the simulation control of the laboratory heat exchanger. By compare with MPC approach [11], [12], in the designed digital SPs there's no need to use relatively complicated optimization on-line algorithms. From simulation experiments it is evident that both control algorithms are relatively simple and they are suitable for application in real-time conditions [29].

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