High-order sliding mode design based on geometric homogeneity and fast terminal sliding mode

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Abstract: - A high-order sliding mode control algorithm is proposed for nonlinear uncertain systems based on geometric homogeneity and fast terminal sliding mode. Homogeneity approach used to stabilize the nominal system, and fast terminal sliding mode used to reject the uncertainty, are applied to constructing the controller. High-order sliding mode is established by the proposed algorithm, the boundary of the uncertainty is not required, and the global robustness is achieved. Additionally, the resulting control law is continuous, which is beneficial to reduce the chattering effect. Simulation results of the proposed algorithm confirm the effectiveness.

Key-Words: - Nonlinear Systems, High-order Sliding Mode, Geometric Homogeneity, Fast Terminal Sliding Mode, Robustness

1 Introduction

Due to the strong robustness against the matching uncertainties including the parametric uncertainty, unmodelled dynamics and external disturbances, sliding mode control (SMC) has been widely utilized to deal with systems operating in the uncertain context [1–3]. However, the chattering effect caused by high-frequency control switching, is a main problem to many practical applications. In order to overcome this drawback, many papers are available to solve it, such as dynamic sliding mode [4], high-gain control [5], and sliding-sector method [6].

In addition, high-order sliding mode control (HOSMC), which inherits advantages of the standard SMC, was proposed to reduce and (or) remove the chattering effect [7–9]. Moreover, better accuracy can be achieved by using HOSMC [10]. There exist many second order sliding mode algorithms [11–13]. Arbitrary order sliding mode controllers have been proposed as well. In [14], new families of high-order sliding mode controllers (HOSMCs) were developed based on homogeneity approach. A approach, called quasi-continuous HOSMCs, was presented in [15], allowing the control to be practically continuous function with respect to time in the case of relative degree $r > 1$. Homogeneity reasoning and integral sliding mode were used to design HOSMCs in [16, 17]. Although the mentioned techniques above can admit the

tracking of desired signals through adjusting only one sufficiently large gain parameter, they did not provide constructive conditions on the parameter adjustment. Solutions can be obtained in [18, 19], where the HOSMC is equivalent to finite time stabilization of higher order integrator chain system with a bounded uncertainty. And the controller was designed based upon the concept of integral sliding mode control. However, the boundary of the uncertainty must be determined in advance, which may be difficult or impossible in some applications.

Compared to the standard SMC, Terminal sliding mode (TSM) originating from the terminal attractor [20], adopts a nonlinear switching manifold with finite time mechanism, which can guarantee the time to reach the origin for system trajectories will be finite [21]. Fast terminal sliding mode (FTSM) is a revision form of the TSM, achieving better performance in the case of the initial state far away from the origin. In [22], the controller was designed via the fast terminal sliding mode in both reaching phase and sliding phase. As a result, the system shows superior robustness in system uncertainties and external disturbances.

The objective of this paper is to develop a new controller based on homogeneity and fast terminal sliding mode, characteristics of the controller are as follows:

1) High-order sliding mode can be established in any given accuracy.
2) The control law is continuous such that chattering effect can be removed or decreased.
3) The prior knowledge of boundaries of uncertain terms is not needed.
4) Global robustness is guaranteed.

The rest of this paper is organized as follows. Section 2 states the problem and some hypotheses. In section 3, the high-order sliding mode control algorithm is derived. A numerical simulation of a permanent magnet stepper motor is carried out to verify effectiveness of the proposed algorithm in section 4. Section 5 concludes the paper.

2 Problem Formulation

Consider a dynamic system of the form:

$$\begin{cases} \dot{x} = f(x,t) + g(x,t)u \\ y = s(x,t) \end{cases} \tag{1}$$

where $x \in R^n$, $u \in R$ are the state variable and the control input, respectively. $f(x,t)$ and $g(x,t) \neq 0$ are uncertain smooth functions with proper

dimensions, and $s(x,t) \in R$ is a only smooth measured output. To make the output $s(x,t)$ vanish in finite time is the objective of control.

Assumption 1. The relative degree r of system (1) with respect to $s(x,t)$ is constant and known, and the associated zero dynamics are stable.

Definition 1 ([23]). Consider a smooth dynamic system with a smooth output function $s(x,t)$, and let the system be closed by some possibly dynamical discontinuous feedback. Then, provided the successive total time derivatives $s(x,t)$, $\dot{s}(x,t)$,..., $s^{(r-1)}(x,t)$ are continuous functions of the closed-system state-space variables, and the set $s(x,t) = 0$, $\dot{s}(x,t) = 0$, ..., $s^{(r-1)}(x,t) = 0$ is non-empty and consists locally of Filippov trajectories [24], the motion on the set $s(x,t) = 0$, $\dot{s}(x,t) = 0$, ..., $s^{(r-1)}(x,t) = 0$ is said to exist in rth-sliding mode (r-order sliding mode). The rth derivative $s^{(r-1)}(x,t)$ is mostly supposed to be discontinuous or nonexistent.

By defining a suitable discontinuous control, the r-order SMC allows the finite time stabilization to zero both the sliding variable $s(x,t)$ and its $(r-1)$ first time derivatives. Computing the rth total time derivative of $s(x,t)$ along the trajectories of the system (1) yields the following equation:

$$s^{(r)}(x,t) = \varphi(x,t) + \gamma(x,t)u \tag{2}$$

where $\varphi(x,t) = L_{f(x,t)}^r s(x,t)$, $\gamma(x,t) = L_{g(x,t)} \cdot L_{f(x,t)}^{r-1} s(x,t)$ are uncertain functions, $L_{f(x,t)} s(x,t) = \nabla s(x,t) f(x,t)$, while $\nabla s(x,t) = \dfrac{\partial s(x,t)}{\partial x}$ represents the gradient of $s(x,t)$.

Assumption 2. The solutions are understood in the Filippov sense [24], and system trajectories are supposed to be infinite extendible in time for any bounded Lebesgue measurable input. In practice, it means that the system is weakly minimum phase.

Assumption 3. There exist $K_m \in R^+$, $K_M \in R^+$, $C \in R^+$ such that the following inequalities hold at least locally.

$$K_m \leq \gamma(x,t) \leq K_M, |\varphi(x,t)| \leq C \tag{3}$$

The control objective is to drive the sliding variable $s(x,t)$ to zero in finite time, and to keep it

exactly by defining a suitable control. The problem of r-order SMC of system (1) with respect to the sliding variable $s(x,t)$ is equivalent to finite time stabilization of

$$\begin{cases} \dot{z}_i = z_{i+1} & i=1,2,...,r-1 \\ \dot{z}_r = \varphi(x,t) + \gamma(x,t)u \end{cases} \quad (4)$$

where

$$z = [z_1, z_2,...,z_r]^\mathrm{T} = [s(x,t), \dot{s}(x,t),..., s^{(r-1)}(x,t)]^\mathrm{T}$$
.

3 High-order Sliding Mode Control Based on Geometric Homogeneity and Fast Terminal Sliding Mode

Based on the concept of integral sliding mode, the high-order sliding mode controller is designed in two steps: the design of a finite time controller stabilizing the nominal system at the origin and the design of another controller suppressing the uncertainty.

To facilitate the description of the design process, consider the following uncertain integrator system.

$$\begin{cases} \dot{x}_i = x_{i+1} & i=1,2,...,n-1 \\ \dot{x}_n = u + d \end{cases} \quad (5)$$

where d is the uncertain term and its absolute value is less than a known positive number D, i.e. $|d| \le D$.

3.1 Finite Time Controller for Integrator Systems

For a chain of integrators without uncertainties, the existence of a continuous finite time stabilizing feedback controller has been proved, which provides an explicit construction involving a small parameter in [25]. The controller renders the closed-loop system asymptotically stable and homogeneous of negative degree with respect to a suitable dilation such that finite-time stability is achieved.

Proposition 1([25]). Let $k_1,...,k_n > 0$ be such that the polynomial $s^n + k_n s^{n-1} + \cdots + k_2 s + k_1$ is Hurwitz, and consider the system:

$$\begin{cases} \dot{x}_i = x_{i+1} & i=1,2,...,n-1 \\ \dot{x}_n = w_0 \end{cases} \quad (6)$$

There exists $\varepsilon \in (0,1)$ such that, for every $\alpha \in (1-\varepsilon,1)$, the origin is a globally finite-time-stable equilibrium for the system (6) under the feedback:

$$w_0 = -k_1 \text{sign}(x_1)|x_1|^{\alpha_1} - \cdots - k_n \text{sign}(x_n)|x_n|^{\alpha_n} \quad (7)$$

where α_1, ..., α_n satisfy

$$\alpha_{i-1} = \frac{\alpha_i \alpha_{i+1}}{2\alpha_{i+1} - \alpha_i}, \quad i=2,...,n \quad (8)$$

with $\alpha_{i+1} = 1$ and $\alpha_n = \alpha$.

3.2 Fast Terminal Sliding Mode

The fast terminal sliding mode, which can accelerate the moving rate of the sliding mode compared to the standard terminal sliding mode when the initial position is far away from the origin, is provided in [21], given by

$$s = \dot{x}_1 + \alpha x_1 + \beta x_1^{q/p} \quad (9)$$

where $x_1 \in R$, α, $\beta > 0$. The time to reach the origin can be calculated after the sliding mode occurs, expressed as

$$t_s = \frac{p}{\alpha(p-q)} \ln \frac{\alpha x_1(0)^{(p-q)/p} + \beta}{\beta} \quad (10)$$

At the same time, it can be proved that the x_1 is a terminal attractor using Lyapunov theory.

3.3 Disturbances Attenuation Controller Design

In the presence of disturbances, the effect caused by them can not be completely compensated by the finite time controller. Therefore, an additional controller should be designed to do it. As mentioned in Introduction, the sliding mode control can be capable of doing this. However, in the conventional SMC design principle $s\dot{s} < -k|s|$, which results in a discontinuous controller that produces the chattering effect. As seen in subsection 3.2, the fast terminal sliding mode bears the mechanism of finite time convergence, hence it can be used to design the controller instead of the conventional design principle.

For system (5), define the sliding variable as

$$\sigma = z_n + \zeta \quad (11)$$

where ζ is a auxiliary variable, determined by

$$\dot{\zeta} = -w_0 \qquad (12)$$

with w_0 being the finite time controller. Therefore the attenuation controller can be designed as

$$w_1 = -\phi\sigma - \gamma\sigma^{q/p} \qquad (13)$$

Theorem 1. Consider the system (5), and define γ and γ' as

$$\gamma = \frac{D}{\left|\sigma^{q/p}\right|} + \eta \qquad (14)$$

$$\gamma' = \gamma - \frac{d}{\sigma^{q/p}} \qquad (15)$$

with $\eta > 0$. The control law

$$u = w_0 + w_1 \qquad (16)$$

ensures that x_i $(i = 1, 2, 3)$ convergences on zero in any given accuracy, and w_0, w_1 are given by (7) and (13), respectively.

Proof: Taking the first order derivative of σ with respect to the time obtains

$$\dot{\sigma} = -\phi\sigma - \gamma\sigma^{q/p} + d \qquad (17)$$

Based on the equations (14) and (15), the equation (17) can be rewritten as

$$\dot{\sigma} = -\phi\sigma - \gamma'\sigma^{q/p} \qquad (18)$$

In order to have the fast terminal convergence, the condition $\gamma' > 0$ should be hold. According to the equation (15), it can be seen

$$\begin{aligned}\gamma' &= \frac{D}{\left|\sigma^{q/p}\right|} - \frac{d}{\sigma^{q/p}} + \eta \\ &\geq \frac{D}{\left|\sigma^{q/p}\right|} - \frac{|d|}{\left|\sigma^{q/p}\right|} + \eta \geq \eta > 0 \end{aligned} \qquad (19)$$

Based on the equation (14), it is concluded that

$$|\sigma| < \left(\frac{D}{\gamma}\right)^{p/q} \qquad (20)$$

Remark 1. Choose large enough γ and p/q, $|\sigma|$ can be smaller than any given positive number.

Remark 2. To realize the global robustness, the initial condition of ζ should be equal to the initial value of z_n, that is $\zeta(0) = z_n(0)$.

3.4 High-order Sliding Mode Control Design
In fact, the system (4) can be divided into the nominal part and the uncertain part, therefore it can be rewritten as

$$\begin{cases} \dot{z}_i = z_{i+1} & i = 1, 2, ..., r-1 \\ \dot{z}_r = \varphi_0(x,t) + \gamma_0(x,t)u + f(x,t) \end{cases} \qquad (21)$$

where $\varphi_0(x,t)$, $\gamma_0(x,t) \neq 0$ are determinate terms, $\Delta\varphi(x,t)$, $\Delta\gamma(x,t)$ are the disturbance ones, $f(x,t) = \Delta\varphi(x,t) + \Delta\gamma(x,t)u$ indicates the whole disturbance.

Design the following law for the system (21), like that

$$u = \frac{1}{\gamma_0(x,t)}(-\varphi_0(x,t) + w) \qquad (22)$$

with w being the auxiliary control input. Hence the system (21) is transformed into the following form.

$$\begin{aligned} \dot{z}_i &= z_{i+1} \quad i = 1, 2, ..., r-1 \\ \dot{z}_r &= w + f(x,t) \end{aligned} \qquad (23)$$

Obviously, the system (23) is identical to the system (5) in the form, it follows that
Theorem 2. Consider the nonlinear uncertain system (1) with a relative degree r with respect to the sliding variable $s(x,t)$. Assume that the hypotheses *1-3* are satisfied, the control law (22) can allow the establishment of an r-order sliding mode control with respect to $s(x,t)$ in any given accuracy with large enough parameters.
Remark 3. To execute the design of the controller, the sliding variable and its $(r-1)$ order time derivative must be obtained, whilst the sliding variable $s(x,t)$ is commonly an only measurable output. This problem can be solved by the $(r-1)$ order differentiator provided in [14, 15].

3.5 Example
Consider an uncertain second order integrator system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u + d \end{cases} \quad (24)$$

Choose the simulation parameters as $\phi = 10$, $\gamma = 10$, $q = 1$, $p = 3$, $k_1 = 1$, $k_2 = 1.5$, $v_1 = 0.60$, $v_2 = 0.75$, and $d = 5\sin x_1$.
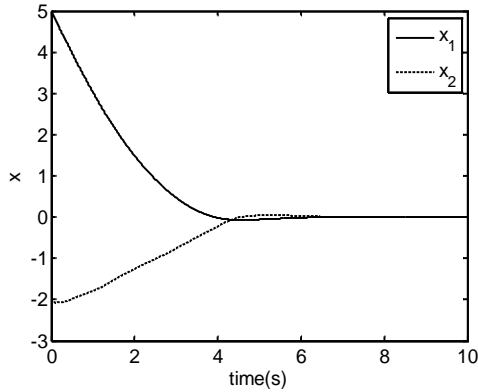

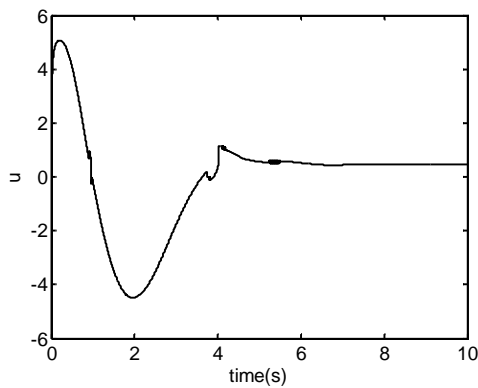Fig. 1 State trajectories versus time
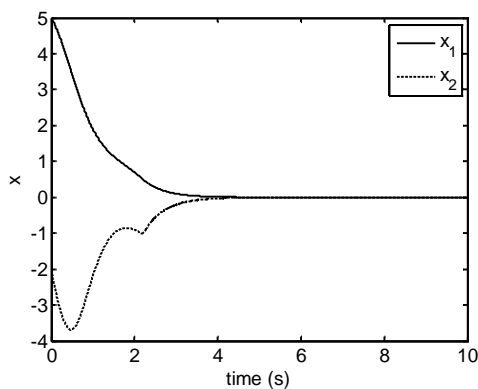

Fig. 2 Control input versus time

Fig. 3 State trajectories versus time using SMC

Fig. 1 and Fig. 2 are simulation results under the control of the proposed strategy, Fig. 3 and Fig. 4 are the ones under the control of SMC. It is concluded that the system trajectories can converge on zero under the two control strategies. However, the manipulated variable is continuous using the proposed

algorithm, which removes the chattering effect. In contrast, the manipulated variable is serious chattering using SMC, which is harmful to many applications. Comparative study testifies the advantage of the proposed strategy.
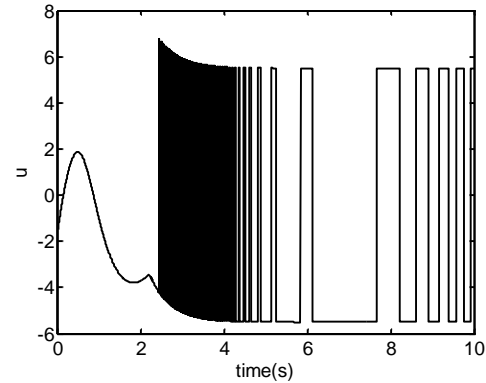

Fig. 4 Control input versus time using SMC

4 Tracking Control of a Permanent Magnet Stepper Motor

To illustrate the aforementioned procedure, consider the high-order sliding mode control of a permanent magnet stepper motor (PMSM).

4.1 Mathematical Model

Consider the following uncertain PMSM model, derived in [26].

$$\begin{cases} \dfrac{d\theta}{dt} = \omega \\ \dfrac{d\omega}{dt} = \dfrac{k_M}{J}(-i_a \sin p\theta + i_b \cos p\theta) - \dfrac{F}{J}\omega - \dfrac{T_L}{J} \\ \dfrac{di_a}{dt} = -\dfrac{R}{L}i_a + \dfrac{k_M}{L}\omega \sin p\theta + \dfrac{u_a}{L} \\ \dfrac{di_b}{dt} = -\dfrac{R}{L}i_b - \dfrac{k_M}{L}\omega \cos p\theta + \dfrac{u_b}{L} \end{cases} \quad (25)$$

where θ is the rotor angle, ω is the rotor speed, T_L is the load torque, $i = \begin{bmatrix} i_a & i_b \end{bmatrix}$ and $u = \begin{bmatrix} u_a & u_b \end{bmatrix}$ are the stator current and voltage, respectively(see [26] for more details on the other parameters).

Based on Park's transformation [27], i.e. the vectors u and i expressed in the fixed stator frame (a,b) are transformed into the vectors expressed in a frame (d,q).

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} \cos p\theta & \sin p\theta \\ -\sin p\theta & \cos p\theta \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} \qquad (26)$$

The dynamics (25) expressed in terms of currents and voltages in the frame (d,q) become the equation (27).

$$\begin{cases} \dfrac{d\theta}{dt} = \omega \\[2mm] \dfrac{d\omega}{dt} = \dfrac{k_M}{J}i_q - \dfrac{F}{J}\omega - \dfrac{T_L}{J} \\[2mm] \dfrac{di_d}{dt} = -\dfrac{R}{L}i_d + p\omega i_q + \dfrac{u_d}{L} \\[2mm] \dfrac{di_q}{dt} = -\dfrac{R}{L}i_q - p\omega i_d - \dfrac{k_M}{L}\omega + \dfrac{u_q}{L} \end{cases} \qquad (27)$$

4.2 Tracking Control Using the Proposed Algorithm

The control objective is to design a robust control law such that the θ tracks the following reference trajectory $\theta_r(t)$. In addition, in order to minimize the Joule losses and maximize the motor torque, it is sensible to choose a direct current trajectory $i_d = 0$.

$$\theta_r(t) = \frac{6}{1 + e^{-9.63(t-0.993)}} \qquad (28)$$

Define the following two sliding variables as

$$\begin{cases} \sigma_1 = \theta - \theta_r \\ \sigma_2 = i_d \end{cases} \qquad (29)$$

then

$$\begin{cases} \dddot{\sigma}_1 = -(\dfrac{k_M R}{JL} + \dfrac{F k_M}{J^2})i_q - \dfrac{k_M}{J}p\omega i_d - \\[2mm] \qquad (\dfrac{k_M^2}{JL} - \dfrac{F^2}{J^2})\omega + \dfrac{k_M}{JL}u_q + \dfrac{F T_L}{J^2} - \dfrac{T_L}{J} - \dddot{\theta}_r \\[2mm] \dot{\sigma}_2 = -\dfrac{R}{L}i_d + p\omega i_q + \dfrac{1}{L}u_d \end{cases} \quad (30)$$

It can be see that the relative degrees of the system with respect to the sliding variables equal 3 and 1, respectively. So a 3rd and a 1st order sliding controllers are designed. According to the Section 3, high-order sliding mode control is equivalent to finite time stabilization of the following uncertain integral systems.

$$\begin{cases} \dot{z}_{1,1} = z_{1,2} \\ \dot{z}_{1,2} = z_{1,3} \\ \dot{z}_{1,3} = A_1 + B_1 u_q + f_1 \end{cases} \qquad (31)$$

$$\dot{z}_{2,1} = A_2 + B_2 u_d \qquad (32)$$

where

$$z_1 = \begin{bmatrix} z_{1,1} & z_{1,2} & z_{1,3} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \sigma_1 & \dot{\sigma}_1 & \ddot{\sigma}_1 \end{bmatrix}^{\mathrm{T}}, z_{2,1} = \sigma_2$$

, $B_1 = \dfrac{k_M}{JL}$, $f_1 = (\dfrac{F}{J^2} - \dfrac{1}{J})T_L$, $A_2 = -\dfrac{R}{L}i_d + p\omega i_q$,

$$A_1 = -(\frac{k_M R}{JL} + \frac{F k_M}{J^2})i_q - \frac{k_M}{J}p\omega i_d - (\frac{k_M^2}{JL} - \frac{F^2}{J^2})\omega - \dddot{\theta}_r$$

, $B_2 = \dfrac{1}{L}$. Herein, the term f_1 is viewed as a disturbance caused by the load torque T_L.

For the system (31), the control law can be defined as

$$u_q = B_1^{-1}(-A_1 + w_1) \qquad (33)$$

where w_1 is a auxiliary control input, expressed as

$$w_1 = w_{1,0} + w_{1,1} \qquad (34)$$

where $w_{1,0}$ is the nominal controller, given by

$$w_{1,0} = -k_{1,1}\mathrm{sign}(z_{1,1})\left|z_{1,1}\right|^{\alpha_{1,1}} - k_{1,2}\mathrm{sign}(z_{1,2})$$
$$\left|z_{1,2}\right|^{\alpha_{1,2}} - k_{1,3}\mathrm{sign}(z_{1,3})\left|z_{1,3}\right|^{\alpha_{1,3}} \qquad (35)$$

Define the following sliding variable:

$$s_1 = z_{1,3} + \xi_1 \qquad (36)$$

where

$$\dot{\xi}_1 = -w_{1,0} \qquad (37)$$

So the attenuation controller $w_{1,1}$ can be designed as

$$w_{1,1} = -\phi_1 s_1 - \eta_1 s_1^{q_1/p_1} \qquad (38)$$

Similarly, the following results can be obtained for the system (32).

$$u_d = B_2^{-1}(-A_2 + w_2) \qquad (39)$$

$$w_2 = w_{2,0} + w_{2,1} \qquad (40)$$

$$w_{2,0} = -k_{2,1}\mathrm{sign}(z_{2,1})\left|z_{2,1}\right|^{\alpha_{2,1}} \qquad (41)$$

$$s_2 = z_{2,1} + \xi_2 \tag{42}$$

$$\dot{\xi}_2 = -w_{2,0} \tag{43}$$

$$w_{2,1} = -\phi_2 s_2 - \eta_2 s_2^{q_2/p_2} \tag{44}$$

4.3 Simulation Results and Analysis

The motor nominal data are referred to [26], and the simulation parameters are chosen as $\phi_1 = \eta_1 = \phi_2 = \eta_2 = 10$, $q_1/p_1 = 3/5$, $q_2/p_2 = 1/5$.
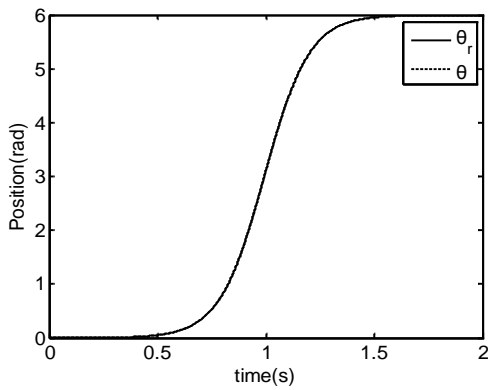

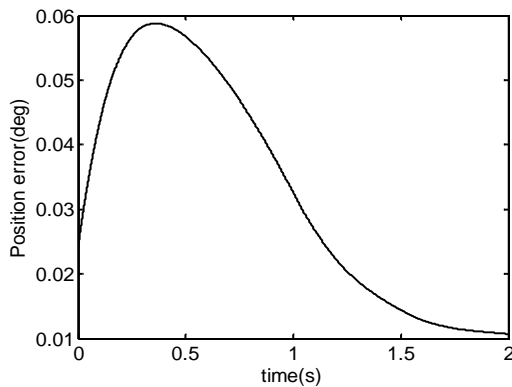Fig. 5 Actual and reference position versus time
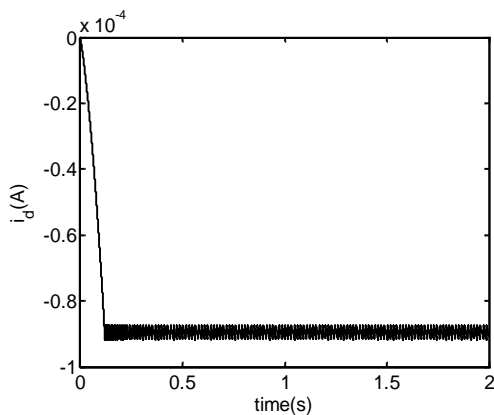

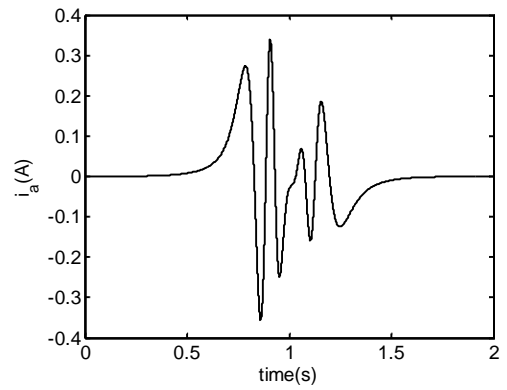Fig. 6 Position error versus time

Fig. 7 Direct current i_d versus time

Fig. 8 Stator current i_a versus time
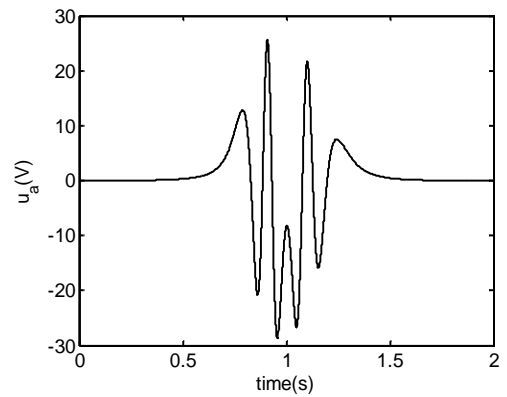
Fig. 9 Stator voltage u_a versus time
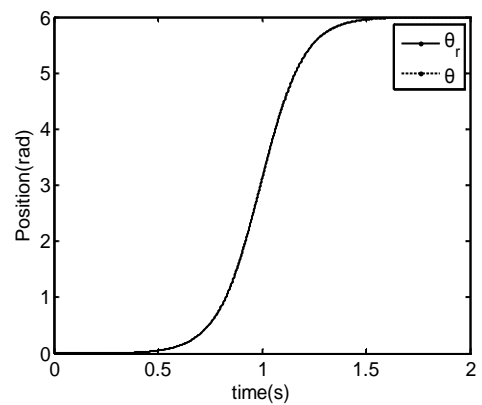

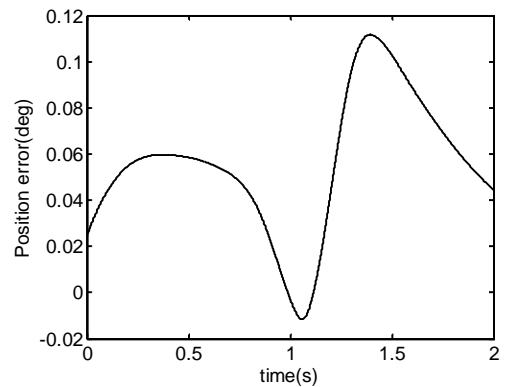Fig. 10 Actual and reference position with uncertainties
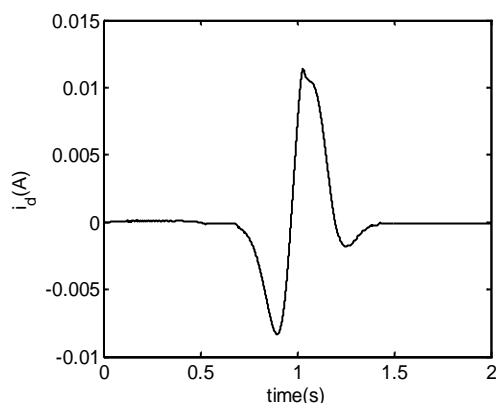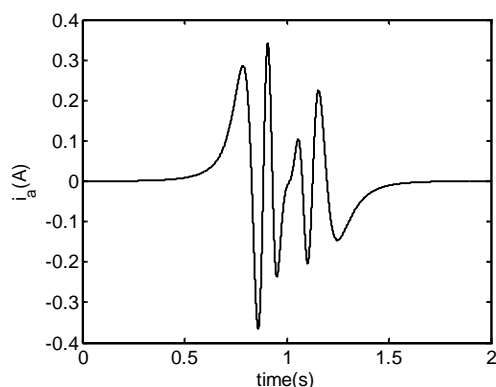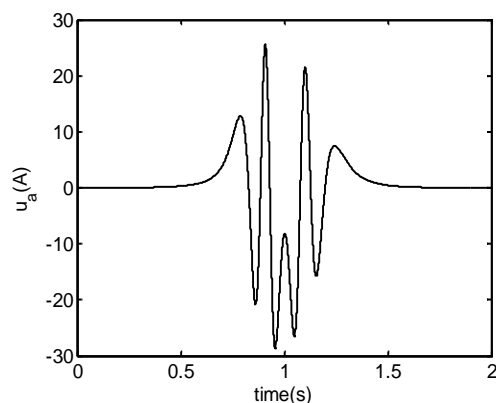

Fig. 11 Position error versus time with uncertainties

Fig. 12 i_d versus time with uncertainties

Fig. 13 i_a versus time with uncertainties

Fig. 14 u_a versus time with uncertainties

The performance of the proposed scheme is shown in Fig. 5~Fig. 9 with nominal values. It can be seen that tracking is performed with high accuracy, as shown in Fig. 5 and Fig. 6. Indeed, the permanent position error is close to 0.015 degree. It is shown in Fig. 7 that the variations of the direct current are slight. Fig. 8 and Fig. 9 show the applied current and voltage in a phase of the motor. In the case of uncertainties including parameter perturbations: $R = 0.8R_N$, $L = 1.3L_N$, $J = 1.4J_N$, $k_M = 0.8k_{MN}$, $F = 0.6F_N$ and external

disturbance, i.e., the load torque $T_L = 2$ Nm, the simulation results are shown in Fig. 10~Fig. 14. The proposed controller can efficiently correct the undesirable deviations due to the uncertainties and manages to accomplish the trajectory tracking with high accuracy, as shown in Fig. 10. It is observed in Fig. 11 that the performance is slightly degraded. The direct current is slight in spite of uncertainties shown in Fig. 12. The demanded current and voltage are described in Fig. 13 and Fig. 14. Other simulation results also show the robustness of the proposed controller with respect to uncertainties.

5 Conclusions

Based on geometric homogeneity and fast terminal sliding mode, a high-order sliding mode control design algorithm is proposed for uncertain nonlinear systems. According to the above simulations and analysis, several conclusions can be drawn: (1) designing a controller for the nominal system is simple using geometric homogeneity; in fact, the controller defines a trajectory to be tracked; and (2) the fast terminal sliding mode concept is used to design another controller for rejecting uncertainties instead of the conventional sliding mode control design principle. As a result, a continuous law is obtained, so chattering effect can be reduced without increasing the relative degree of the system. While in the conventional context, increasing the degree or dynamic sliding mode may be a solution to obtain continuous law; and (3) the system state stays on the sliding manifold from initial instant; therefore the global robustness is guaranteed; and (4) the knowledge of the boundaries of uncertainties is not required, so it is more suitable to practical applications.

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