Denoising ECG Signals using Weighted Iterative UFIR Filtering

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Abstract: - The electrocardiogram (ECG) holds paramount importance in diagnosing heart disease, and as it persists leading cause of global mortality. Over the past decades, diverse techniques have emerged for processing ECG signals, with denoising taking a prominent role in enhancing feature extraction. Nonetheless, achieving heightened accuracy remains an enduring challenge. In this study, we introduce an innovative approach involving the application of a weighted unbiased finite impulse response (UFIR) filter. Under the same noise conditions and in terms of the root mean square error (RMSE) and signal-to-noise ratio (SNR), our proposed method showcases worthy performance in comparison to the weighted Savitzky-Golay (SG) filter. This research contributes to the progressive evolution of ECG signal processing, offering the potential for more precise and dependable detection of cardiac diseases.

Key-Words: - weighted UFIR, Savitzky-Golay filter, ECG signals, root mean square error (RMSE), Denoising ECG Signals, Signal-Noise to Ratio (SNR).

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1 Introduction

An electrocardiogram (ECG) is a recording that represents the electrical activity of the heart and is a vital diagnostic tool for detecting cardiac pathology. The P-QRS-T waves that make up the ECG signal provide valuable information, with the (QRS) complex playing a particularly important role in identifying cardiac arrhythmias, [1], [2]. Early detection of arrhythmia is crucial for predicting and preventing heart attacks, making the ECG a critical tool in saving lives. However, the accuracy of arrhythmia detection can be compromised by noise and artifacts, which can result in incorrect diagnoses. To address this issue, a preprocessing step is essential. Preprocessing is a widely used and indispensable process for ECG signal analysis, aimed at reducing noise and improving the quality of the ECG signal to ensure accurate feature extraction and diagnosis. Hence, it is necessary to continuously monitor ECG signals over extended periods to enable precise diagnoses using electronic analog and digital devices for data acquisition and processing. Several techniques have been proposed to remove noise and artifacts from ECG signals, with smoothing techniques of special interest.

Recent studies have presented wavelet-based digital filters and conventional filters as effective techniques for reducing noise in biomedical signals that have been digitized using embedded systems, [3], [4], [5], [6], [7], [8], [9]. In one study, a unique methodology was proposed for processing ECG signals by utilizing wavelet-based transform techniques and wireless IoT devices to monitor the behavior of the heart, [10]. Another study proposed the use of the adaptive Fourier-Bessel domain wavelet transform (FBDAWT) for the automatic detection of anxiety stages, utilizing the signal from a single-channel portable electrocardiogram (ECG) sensor, [11]. Additionally, a study evaluated the performance of Butterworth-type low-pass filters configured with fourth and eighth order, compared to other filters like Chebyshev-type, [12]. Other approaches proposed a deep learning-based artificial intelligence technique to classify and reduce the noise associated with ECG signals, [13], [14], [15], [16]. Moreover, the use of generative adversarial neural networks (GANs) for the blind restoration of ECG signals was also suggested, [17]. It is worth noting that GANs have been utilized for generating and classifying ECG signals, [18], [19], [20], [21], [22], [23], [24], [25].

Finally, a study provided a general overview of the stages of ECG signal processing and the application of machine learning techniques, [26], [27]. Overall, these studies highlight the effectiveness of wavelet-based digital filters and conventional filters in reducing noise in biomedical signals, particularly those that have been digitized with embedded systems. The presented techniques, including deep learning-based artificial intelligence and generative adversarial neural networks (GANs), demonstrate the potential for advanced signal processing in the biomedical field. The studies also showcase the importance of monitoring the behavior of the heart, detecting anxiety stages, and utilizing machine learning techniques in ECG signal processing.

However, despite the notable benefits of these works, they have limitations associated with their structure. In the case of the wavelet transform, finding a mother wavelet function and its optimal parameters is a process that takes computational time. In the case of the conventional filters mentioned, the filtered signal tends to produce delays in the time domain, which is an inherent characteristic of these types of filters. Although deep learning-based approaches can help reduce signal noise, their computational efficiency still lags other filtering techniques. Additionally, the ECG signal filtering process cannot be easily understood using deep learning techniques.

Various techniques can be used to process ECG signals, including the Kalman filter. The extended Kalman filter (EKF) is the most popular among these methods due to its compatibility with dynamic models. Some studies have utilized adaptive filter banks with EKF to denoise ECG signals, [28], while others have concentrated on analyzing morphological features by segmenting the ECG signal, [29]. Additionally, some researchers have estimated the breathing rate by smoothing ECG and photoplethysmogram (PPG) signals using the Kalman filter, [30], [31], [32], [33]. Other research was conducted to evaluate the effectiveness of the Kalman filter in reducing noise in telehealth systems, [34]. Although the filter showed impressive results, it has limitations when the model is unknown. The success of this filter depends on the formulation of the ECG signal model, which can vary over time. As a result, establishing the appropriate parameters of the model can be difficult. There are several techniques available for smoothing ECG signals and achieving promising results, [35], [36], [37]. In one approach presented in, [38], a smoothing filter was designed based on the delay differential equation (DDE), which requires the regularization parameter and the delay. The regularization parameter is related to the cutoff frequency, while the delay is related to the tuning provided by the user. Another technique proposed in, [39], is the quantum smoothing filter (QSF), which is advantageous in terms of runtime complexity compared to other methods like discrete wavelet transform (DWT) and Empirical Mode Decomposition (EMD). However, the QSF method requires quantum computers to work. In, [40], the Complete Ensemble Empirical Mode Decomposition with Adaptive Noise (CEEMDAN) was used to reduce noise in ECG signals. The sample entropy was then utilized to identify the noisy intrinsic mode functions (IMFs) and subsequently apply the non-local mean smoothing technique. However, this method is only suitable for ECG signals with low Signal Noise to Ratio (SNR). When studying cardiac pathologies, decomposition methods are often more suitable, such as dynamic mode decomposition (DMD), [41].

The Savitzky-Golay (SG) smoothing filter, [42], on the other hand, is a widely used filter in ECG analysis. It is known for yielding significant insights, [43], [44]. However, it has certain limitations concerning its parameters. For instance, the length N of the horizon parameter must be odd, otherwise, fractional values arise at the boundaries of the summation. Additionally, the fixed delay is positioned at the center of the horizon, which may not align with the needs of certain applications where optimal delays may vary. Despite the limitations, the SG filter is still one of the standard methods for denoising ECG signals, [45], [46].

Once the frequency bands in the ECG are removed, a proper smoothing technique can improve the quality of an ECG signal. A p−shift finite–length Unbiased Finite Impulse Response filter (p−shift UFIR) has been widely used for denoising ECG, [47]. The p-shift UFIR filter was used in, [48], to achieve an adaptive averaging horizon: optimal for slow ECG behaviors and minimal for fast excursions. Additionally, in, [49], the p-shift UFIR filter was employed to estimate the QRS interval based on the information provided by its second state. The p-shift UFIR filter has been applied to denoise ECG signals, aiming to extract ECG signal features, [50].

In this paper, we present a robust weighted iterative UFIR filter designed for enhancing ECG signals and compare it to the weighted SG filter. This work is organized into the following sections: Section II outlines different methods utilized in this study. In Section III, we delve into the key findings, as assessed through RMSE and SNR metrics, with an additional focus on the practical implementation of real ECG signals. Finally, the last section describes the conclusions of the work.

2 Preliminaries

2.1 ECG Signal State Space Model

The discrete-time model for representing ECG signals is given in, [51], where the signal is represented on a horizon $[m, n]$ of length N, where $m = n - N + 1$. The degree polynomial used in the representation is determined in space-state, providing a precise representation of the ECG signal within the specified time frame. The ECG signal is time-invariant and deterministic. It is supposed that measurement of the ECG signal is corrupted by zero-mean noise with an unknown, standard deviation and not necessary Gaussian distribution. Under such conditions, an ECG signal can be represented as follows:

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{\mathbf{k+1}} \tag{1}$$

$$y_k = \mathbf{C}\mathbf{x}_{\boldsymbol{k+1}} + v_n \tag{2}$$

where x_k is the process vector of the ECG signal, y_k is the measurement observation of the ECG signal, v_k is the zero mean measurement noise with unknown distribution, $\mathbf{C}$ is the observation matrix $\mathbf{C} = [10\ldots0]$ and, the matrix $\mathbf{A}$ defined as:

$$\mathbf{A} = \begin{bmatrix} 1 & \tau & \frac{(\tau)^2}{2} & \cdots & \frac{(\tau)^{K-1}}{(K-1)!} \\ 0 & 1 & \tau & \cdots & \frac{(\tau)^{K-2}}{(K-2)!} \\ 0 & 0 & 1 & \cdots & \frac{(\tau)^{K-3}}{(K-3)!} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{3}$$

2.2 The p-shift UFIR Filter with Weights

The UFIR approach assumes that a shift to the past can be achieved at point k using data taken from $[m + p, k + p]$ with a positive smoother lag $q = -p$, a shift to the future at point k using data taken from $[m + p, k + p]$ with a positive prediction step p > 0, and that p = 0 means filtering. Thus, the p-shift UFIR filtering estimate can be defined as:

$$\hat{x}_{n|n-p_n} = \sum_{k=p}^{N-1+p} h_n^{(i)}(p) y_{n-k} \tag{4}$$

$$= \sum_{k=p}^{N-1+p} h_n^{(i)}(p) y_{n-k} \tag{5}$$

where the components of the gain $\widehat{H}_N(p)$ are formed by $\widehat{H}_N(p) = [H_0(p)H_1(p)\ldots H_{N-1}(p)]$ each matrix $H_n(p)$ is a diagonal matrix specified by $H_n(p) = \mathrm{diag}(h_n^{K-1}(p)\ h_n^{K-2}(p)\ \ldots h_n^{(0)}(p))$, whose components, in turn, are the values of the function $h_n^{(i)}(p)$, [52]. The function $h_n^{(i)}(p)$. is called the ith degree polynomial impulse response and can be calculated by the following equation:

$$h_n^{(i)}(p) = \sum_{j=0}^{i} a_{ji}(p) n^j, \tag{6}$$

where $n \in [p, N - 1 + p]$ and $i \in [0, K - 1]$ and finally, the term $a_{ji}(p)$ is determined by:

$$a_{ji}(p) = (-1)^j \frac{M_{(j+1)1}^{(i)}}{|\Lambda_i(p)|} \tag{7}$$

where $\Lambda_i(p)$ is a matrix defined as:

$$\Lambda_i(p) = \begin{bmatrix} c_0(p) & c_1(p) & c_2(p) & \cdots & c_i(p) \\ c_1(p) & c_2(p) & c_3(p) & \cdots & c_{i+1}(p) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ c_i(p) & c_{i+1}(p) & c_{i+2}(p) & \cdots & c_{2i}(p) \end{bmatrix} \tag{8}$$

The determinant of matrix $\Lambda_i(p)$ is $|\Lambda_i(p)|$. The mth component c_m , $m \in [0, 2i]$, of $|\Lambda_i(p)|$ is calculated using a power series based on the minor $M_{(j+1)1}^{(i)}(p)$ of $\Lambda_i(p)$,

$$c_m(p) = \sum_{j=p}^{N-1} i^m \,, \tag{9}$$

this expression can also be expressed differently,

$$c_m(p) = \frac{1}{m+1}[\mathcal{B}_{m+1}(N+p) - \mathcal{B}_{m+1}(p)] \tag{10}$$

where the term $\mathcal{B}_{m+1}$ is called Bernoulli polynomial.

The UFIR theory suggests that the process can be estimated on $[m, n]$ in the following batch state-space form,

$$\hat{\mathbf{x}}_n = (\mathbf{W}_{n,m}^{\mathbf{T}}\mathbf{W}_{n,m})^{-1}\mathbf{W}_{n,m}^{\mathbf{T}}\,\mathrm{Y}_{n,m} \tag{11}$$

As has been shown in, [52], [53], the UFIR filtering estimate is $\hat{\mathbf{x}}_n \triangleq \hat{\mathbf{x}}_{n|n}$, $\mathbf{Y}_{n,m}$ is the extended observation vector and $\mathbf{W}_{n,m}$ is the augmented measurement matrix. The p-shift estimate is given by:

$$\hat{x}_{n|n-p} = \mathbf{A}^{-p}\hat{x}_{n|n} \tag{12}$$

where $p = \frac{N-1}{2}$ is known as the digital optimal lag 2 which is also used in SG smoothing. Unlike the SG, the UFIR Smoother can minimize the MSE with an optimum horizon N_{opt} . Specifically, without the reference signal (ground truth), by minimizing the trace of mean square value (MSV) derivative of the residual matrix V (N), the optimum horizon N_{opt} is calculated as:

$$N_{opt} = \arg\min_N \frac{\partial \operatorname{tr} V(N)}{\partial N} + 1 \tag{13}$$

Moreover, an iterative UFIR smoothing algorithm akin to the Kalman filter is presented in, [54], recursively in two distinct phases: prediction and update. This algorithm re-calibrates the generalized noise power gain (GNPG), with its adjustment reliant on a gain derived from batch processing. In contrast to the Kalman filter, the UFIR algorithm outlined in algorithm 1 presents a significant advantage in that it does not necessitate any prior knowledge regarding measurement noise. This feature endows the UFIR algorithm with a superior level of robustness and reliability. In, [55], [56], it was shown an improvement of the robustness of the UFIR filter with the weight γ, defined by:

$$\gamma = \frac{1}{\lfloor N/2 \rfloor}\sum_{i=k_0}^{k} \sqrt[K]{\eta_i/}\,\eta_{i-1} \tag{14}$$

where $k_0 \;= k - \lfloor N/2 \rfloor + 1, \lfloor N/2 \rfloor$ and K are the iintegerpart of $N/2$ and the numbers of states. The root mean square (RMS) deviation $\eta_k$of the estimate is calculated using the innovation residual as:

$$\eta_{k=}\sqrt{\frac{1}{K}(y_k - \boldsymbol{H}_k\hat{x}_k)^T(y_k - \boldsymbol{H}_k\hat{x}_k)} \tag{15}$$

where k is the dimension of the target motion.

Algorithm 1: Iterative UFIR Filtering Algorithm

1: Data: $\boldsymbol{y\_k}$
2: Begin
3: for $k = N-1, N$ **… do**
4: $\quad m = k - N + 1, s = k - N + K$**;**
5: $\quad G_s = (\boldsymbol{C}_{\boldsymbol{m},\boldsymbol{s}}^{\boldsymbol{T}}\boldsymbol{C}_{\boldsymbol{m},\boldsymbol{s}})(\boldsymbol{Y}_{m,s})$;
6: $\quad \hat{\boldsymbol{x}}_{\boldsymbol{s}} = G_s\boldsymbol{C}_{\boldsymbol{m},\boldsymbol{s}}^{\boldsymbol{T}})(\boldsymbol{Y}_{m,s})$;
7: $\quad$ **for** $i = s+1 : k$ **do**
8: $\quad\quad \tilde{\boldsymbol{x}}_i^- = \boldsymbol{A}_i\tilde{\boldsymbol{x}}_{i-1}$;
9: $\quad\quad \boldsymbol{G}_i = [\boldsymbol{H}_i^T\boldsymbol{H}_i + \left(\boldsymbol{A}_i\boldsymbol{G}_{i-1}\boldsymbol{F}_i^T\right)^{-1}]^{-1}$;
10: $\quad\quad K_i = \boldsymbol{G}_i\boldsymbol{H}_i^T$
11: $\quad\quad \tilde{\boldsymbol{x}}_i = \tilde{\boldsymbol{x}}_i^- + K_i(y_i - H_i\tilde{\boldsymbol{x}}_i^-)$
12: end for
13: end for
14: Result: $\hat{x}_k$

2.3 Savitzky-Golay Filter for ECG Signals

The SG filter can be considered as a special case of the UFIR smoothing filter as shown in, [57]. The convolution-based smoothed estimate with a lag $p = \frac{N-1}{2}$ in the middle of the averaging horizon is given by

$$\hat{s}_{k|k-\frac{N-1}{2}} = \sum_{n=-N-1)/2}^{(N-1)/2} \varphi_n\, y_{k-n} \tag{16}$$

where φ_n represents the convolution coefficients determined by the linear least square (LS) method to configure with commonly low-degree polynomials systems. The coefficients φ_n can be extracted from the FIR function $h_n^i\,(-p)$. The SG filter has the following restrictions to ensure accurate calculations, the horizon length N must always be an odd number. If an even number is used, the sum limits would include fractional values, which is not desirable.

While the fixed lag is typically set as $p = (N-1)/2$, it is important to note that different applications may require alternative lag values. The optimal lag might not necessarily be equal to this default value. It is also worth mentioning that the

UFIR smoothing filter explained by, [53], extends the functionality of the SG filter for arbitrary $N > 1$ and lags $0 < p < N - 1$. However, in the specific case of an odd N and $p = (N - 1)/2$, the UFIR filter corresponds to the SG filter. Also note that the q -lag can be optimized for even-degree polynomials.

3 Main Findings

3.1 ECG Signals Denoising by Different Filters

Displayed in Figure 1 are the effects of various estimators on filtering results. To conduct our experiment, we utilized a simulated ECG signal with Gaussian white noise that possessed a standard deviatiequal .0316. This ECG signal was generated using the model introduced by, [58], which can be readily executed on platforms like MATLAB or Octave. The SG filters, labeled as SGK-1 and SGK-2, use the Kaiser window-like weights vector with values of $\beta = 38$ and $\beta = 18$, respectively. The WUFIR q -lag 2 is the UFIR filter with GPNG weight and $p = p_{opt}$ lag determined by the following expression:

$$p = \frac{N_{opt}-1}{2} - \frac{1}{2}\sqrt{\frac{N_{opt}^2+1}{5}} \quad (17)$$

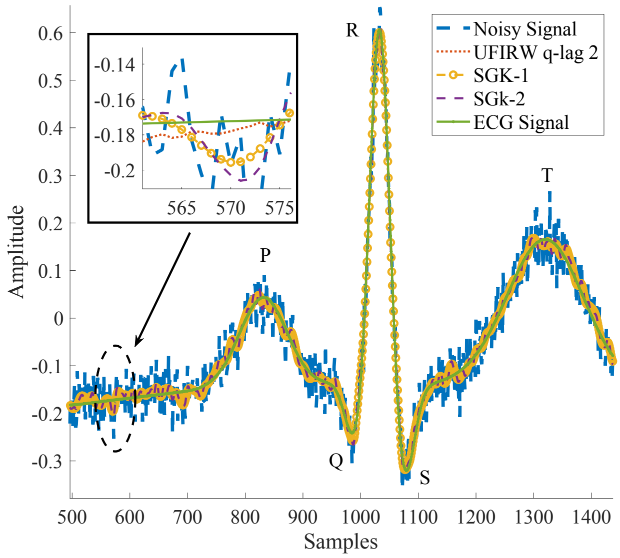

Fig. 1: Synthetic ECG estimations from filters based on weighted SG and UFIR filters.

Upon examining Figure 2, it is evident that the proposed UFIRW method provides greater variability compared to the SG estimator. Notably, the estimation provided by UFIRW is closer to the reference ECG signal.

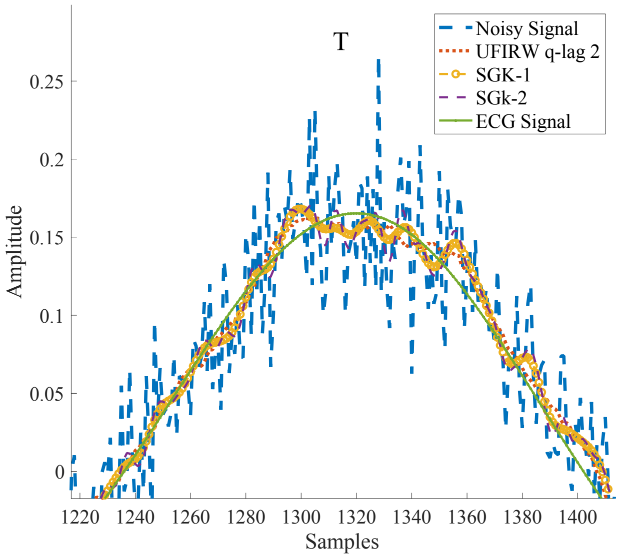

Fig. 2: A detailed visualization of the filtering process of the T wave in synthetic ECG signals

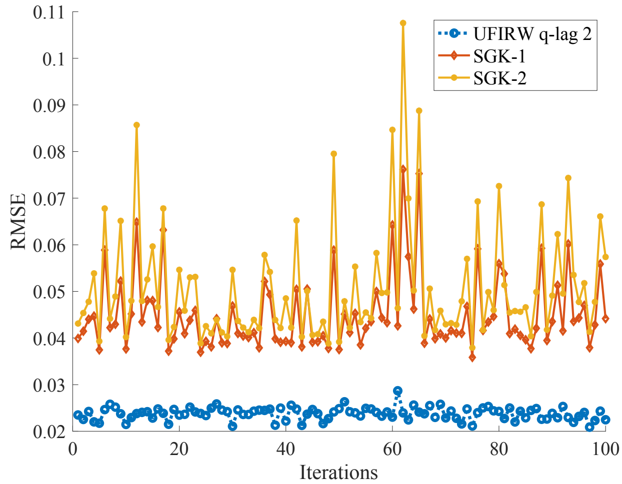

Fig. 3: Performance of mean square error calculated from estimations.

3.2 RMSE Analysis

Given the filtering estimate, to find the better estimator, we calculate the root mean square error (RMSE) determined by

$$RMSE = \frac{1}{L}\sqrt{\sum_{i=0}^{L}(x_i - \hat{x}_i)^2} \quad (18)$$

where L is the sample length of x_i and $\hat{x}_i$. Individually, x_i and $\hat{x}_i$ are the samples associated with the ECG synthetic signal and estimation of the filter. Under the same noise conditions, we conducted an experiment, where we tested the performance of several filters by iterating the process 100 times. The results are presented in

Figure 3, where we can see the performance of each filter. The filters based on Savitzky-Golay (SGK-1 and SGK-2) showed an RMSE between 0.04 and 0.1, which was lower than the RMSE provided by the proposed UFIRW method, which showed an RMSE value between 0.02 and 0.03. It is well-known that the UFIR method provides good stability, while methods based on SG exhibit high variability, which indicates susceptibility to noise.

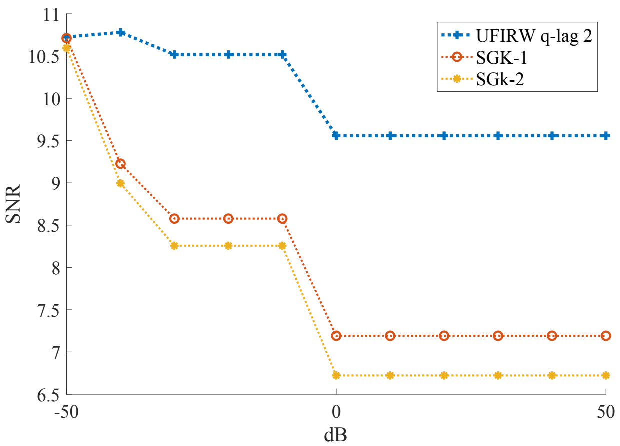

Fig. 4: Signal-to-Noise Ratio (SNR) obtained from varying levels of noise in decibels (dB).

3.3 Signal-to-Noise Ratio (SNR) Analysis

A recent study was conducted to examine how different levels of noise affect ECG signals. The study used Gaussian noise ranging from -50 to 50 dB and analyzed the signal-to-noise ratio provided by two filters shown in Figure 4 - the UFIR weighted estimator and the SG filter. The results showed that the UFIR weighted estimator outperformed the SG filter in producing a clearer ECG signal with less random noise. This study helps in understanding the response of each filter to noise and highlights the superiority of the proposed UFIR method over SG estimators in terms of SNR output.

3.4 Applications to Real ECG Signals

Following a rigorous analysis of the Root Mean Square Error (RMSE) and Signal-to-Noise Ratio (SNR), this study has identified the Unbiased Finite Impulse Response (UFIR) filter with weights as the most appropriate option for accurately estimating real Electrocardiogram (ECG) signals. The analysis has confirmed the filter's superior performance over other filters under consideration. The study focuses on two types of pathologies, namely normal sinus rhythm and premature ventricular complex (PVC), and the ECG signal estimates are showcased visually in Figures 5 and Figure 6. The UFIRW-qlag2 filter provides a precise fit to the ECG real signal, as attested by our analysis.

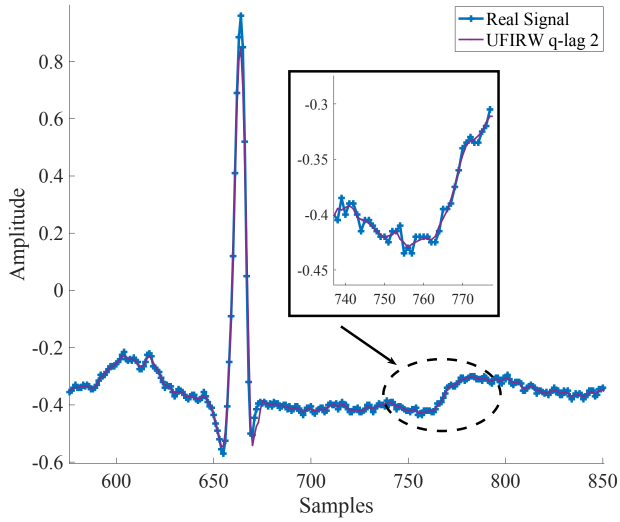

Fig. 5: Estimation of real ECG signal with normal sinus rhythm, [59].

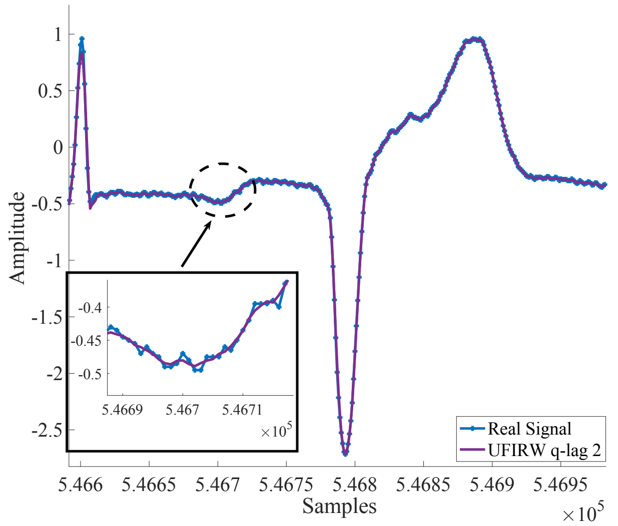

Fig. 6: Estimation of real ECG signal with premature ventricular complex (PVC), [60].

Our study has demonstrated that the UFIR filter with weights is a reliable method for estimating ECG signals, and its application can significantly improve the accuracy of ECG signal estimation, particularly in the context of the two pathologies analyzed. As such, the UFIR filter with weights is highly recommended for accurately estimating real ECG signals.

4 Conclusions

We evaluated the effectiveness of our UFIR filter in comparison to the SG filter using Root Mean Square Error (RMSE) and Signal-to-Noise Ratio (SNR)

metrics. Our results showed that the UFIR filter outperformed the SG filter, demonstrating its adaptability and effectiveness in various ECG signal pathologies. The UFIRW method displayed superior error stability compared to the SG methods. However, the weighted SG method showed high variance and was easily affected by random noise.
The UFIR filter performance remained consistent despite cardiac variability, and we could adjust the horizon parameter, *N*, to obtain optimal results for different noise levels.

In environments with high levels of noise, it is imperative to adopt pre-processing steps to achieve optimal results. However, implementing the recommended approach on low-power devices in such settings may present some challenges. This is due to the method requiring a significant number of points, represented by horizon N, which could potentially impact the memory capacity of the device. This limitation creates a promising avenue for future research that could lead to the development of more efficient solutions for applying the method in low-power and high-noise scenarios.

We will continue to explore various pathologies to identify patterns associated with significant diseases. Ultimately, this project has the potential to greatly enhance ECG signal denoising and advance medical diagnostics.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- The research project was led by Carlos Lastre-Dominguez, who supervised the simulation, algorithm implementation, paper preparation and review, and paper editing.
- Victor Jiménez-Ramos was responsible for project administration and methodology of work.
- Hector Azcaray-Rivera and Eduardo Pérez-Campos carried out the writing, reviewing, and editing of the paper.
- Jorge Munoz-Minjares was responsible for the simulation and paper preparation, review, and editing.
- Yuriy S. Shmaliy carried out writing, reviewing, and editing as well.

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