Optimisation-based Design of Interval Type-2 Fuzzy Logic Controllers for High Performance Temperature Control

SNEJANA YORDANOVA, DESISLAVA STOITSEVA-DELICHEVA
Faculty of Automation,
Technical University of Sofia,
Sofia, 8 Kliment Ohridski Blvd.,
BULGARIA

Abstract: - The paper focuses on the development of an approach for performance improvement of control systems by designing interval type-2 (IT2) PID fuzzy logic controllers (FLC) with single and two inputs. The approach is demonstrated for the control of the temperature in a laboratory fruit dryer and conforms to the requirements for real-time control via industrial programmable logic controllers. It steps on the integration of IT2 FLC empirical design with optimisation using genetic algorithms. The suggested performance-bound fitness function is computed from simulations of the FLC closed-loop systems using a Takagi-Sugeno-Kang plant model derived from experimental step responses. The optimised FLC parameters are the pre-and post-processing gains and selected output singletons and parameters of the upper membership functions that shape the footprint of the uncertainty of the IT2 PID FLC. The designed IT2 PID FLC control systems outperform in simulations the corresponding type-1 PID FLC systems in an increased dynamic accuracy and smoothness of the control action.

Key-Words: - genetic algorithms optimisation, laboratory fruit dryer, temperature control, type-2 PID fuzzy logic controller, simulations.

Received: November 11, 2024. Revised: February 17, 2025. Accepted: April 12, 2025. Published: May 21, 2025.

1 Introduction and State-of-the-Art

The fuzzy logic controllers (FLC) mark significant progress in broadening their industrial implementations. The reason is the stability, robustness, and energy efficiency of the closed-loop systems for the control of various process variables that ensure coping with the nonlinearity and the model uncertainty of the modern plants. Expert experience compensates for the lack of a plant model. The control algorithm is simple to design and easy to program in programmable logic controllers (PLC) and in real-time operation saving both memory and execution time. The PLC implementation is facilitated using economically described and computed triangular and trapezoidal input membership functions (MF) and singletons for the output MF that determine a weighted average defuzzification of low computational cost. The PID FLC is the most widely distributed for the easily constructed fuzzy rule base from the requirement for closed loop system stability and good performance, [1], [2], [3], [4]. It consists commonly of a single input-single output (SISO) or a two inputs-single output (2ISO) fuzzy unit (FU), a pre-processing for computing of the FU input or inputs from the measured controlled variable y and its reference y_r and an integral, PI or PID post-processing. The input of the SISO FU is the system error $e=y_r-y$. The 2ISO FU has inputs e and its derivative $\dot{e}$.

Type-1 (T1) FLC are developed for robots, drones, cars, heating, ventilation and air-conditioning, dryers, heat exchangers, wastewater treatment, cement industry, tanks, boilers, nuclear generators, refrigeration systems, carbonisation columns for soda ash production, etc., [5], [6], [7], [8], [9], [10], [11], [12], [13], [14]. Most of the FLC systems are implemented as real-time laboratory tests, hardware-in-the-loop simulations, low-cost, and PLC automation using mainly plant models, [4], [14], [15], [16].

The subjectivity of the empirical design as well as the changes of the plant with the operation mode, industrial conditions, and time is tackled by different approaches suitable for PLC implementation in the industrial environment.

Optimisation of the FLC parameters is a widely applied approach based commonly on genetic algorithms (GA), [2], [13], [14], [15]. GA performs a gradient-free random parallel search for a global extremum of a fitness function of many parameters computed from experimental or simulation data. GA parameter optimisation is also used to derive

Takagi-Sugeno-Kang (TSK) nonlinear plant models from experimental data, [17], [18]. The TSK plant models enable the design of the model-based FLC on the principle of parallel distributed compensation (PDC), [18], [19]. They are the basis of the closed-loop control system simulation models, built for the computation of the fitness function from simulations with input data from industrial experiments in the offline GA optimisation of the FLC parameters, [18]. GA optimised FLC are developed for heating, ventilating, and air conditioning systems, fire tube boilers, refrigeration and air conditioning, wind turbines, etc. based mostly on plant models and simulations, [6], [20].

Another approach for compensation of the subjectivity of the empirical design and the change in the plant is the online adaptation via simple FL supervisors or various adaptation mechanisms, [15], [21], [22].

The interval type-2 (IT2) FLC is developed to increase system robustness and reduce the subjectivity in choosing the FLC MF, [3], [23], [24]. The IT2 FU uses interval MF, usually defined by a lower MF (LMF) and an upper MF (UMF). The area between the LMF and the UMF determines the footprint of uncertainty (FoU). The FoU as a measure for uncertainty is accepted as an extra tuning parameter, [24]. Thus, each measured variable pre-processed to yield the FU input is fuzzified in the FU to obtain the degrees of matching to the defined LMF $L\mu$ and UMF $U\mu$. All further FU operations are accomplished according to the rules for processing interval numbers:

$$w_i=(L\mu_{Ai}, U\mu_{Ai}) \text{ AND } (L\mu_{Bi}, U\mu_{Bi}),$$
$$w_i=(Lw_i, Uw_i),\text{with } Lw_i=\min(L\mu_{Ai}, L\mu_{Bi}),$$
$$Uw_i=\min(U\mu_{Ai}, U\mu_{Bi});$$
$$o_i=(Lo_i, Uo_i)=w_i.S_i, \text{ with } Lo_i=Lw_i.S_i,$$
$$Uo_i=Uw_i.S_i;\ o=\Sigma_{i=1}^{N} o_i=(\Sigma Lo_i, \Sigma Uo_i), \quad (1)$$

where for each i-th fuzzy rule (i=1÷N) with output singleton S_i the degree of rule activation w_i is computed via aggregation of premises by AND operator, the qualified conclusion o_i - via fuzzy implication and o – via accumulation of the individual rules qualified conclusions by SUM operator.

Various type-reducers are suggested to process the accumulated qualified rules conclusions before the developed for T1 FU defuzzification, [24,] [25]. Most of them are not suitable for PLC real-time applications for iterations or complex computations. The IT2 FLC is difficult to design and also needs more PLC resources (memory and processing time) and optimisation of the FoU to get the desired improvement of the system performance.

IT2 FLC is developed for inverted pendulums, tanks, DC motors, manipulators, active power filters, carbonisation columns, etc., [3], [14], [23], [24], [26], [27], [28].

The existing approaches for system performance improvement based on IT2 FLC though promising do not explore all their potential. The FoU is not subjected to optimisation. No proper fitness function and important parameters to be optimised are selected. Besides, the restrictions related to the FLC industrial implementation are not accounted for. Some approaches lack dynamic post-processing or are demonstrated for the control of linear plant models. The improvement of the control system performance achieved is not assessed with respect to the closest simpler optimised T1 FLC system. Often it is rather small and cannot justify the complexity of the IT2 algorithm which makes the IT2 FLC improper for real-time PLC implementation. Good system performance and robustness can be reached in a simpler and more effective way by tuning the pre-and post-processing gains. The online adaptation of these gains is equivalent to a dynamic change of the MF parameters, e.g. a bigger value for the input reduced by a smaller input scaling gain can result in the same MF grade to a given term which concerning the real input is equivalent to an increase of the MF support.

The aim of the present research is to develop an approach for the improvement of the control system performance by the optimisation-based design of IT2 Sugeno SISO and 2ISO PI FLC suitable for PLC implementation. The approach is demonstrated for the control of the temperature in a laboratory batch convective dryer for fruits which is a nonlinear plant operating in an environment close to the industrial. Its effectiveness is assessed via simulation and comparison with the designed T1 Sugeno PI FLC in previous research. The optimisation of both the IT2 and T1 FLC steps on a small number of selected parameters, GA with specific fitness function which minimisation aims at increasing the system dynamic accuracy by a smooth control action. The fitness function is computed from simulations of the control system with a derived from experimental data TSK plant model. The investigation is carried out with the help of MATLAB™ and its toolboxes for GA and FL, [29], [30].

Drying fruits, or food, is selected for demonstration of the approach since it is a widely spread complex process with temperature as the most commonly controlled variable. The requirements in drying refer to a fast and energy-efficient process that also preserves the quality (moisture, flavour, aroma, shape, colour, etc.) of the product. A variety of FLC - T1, IT2, Mamdani, adaptive and multivariable, are developed for different dryers and food - fish, coffee beans, grain, etc., [11], [21], [31], [32].

The further organization of the paper is the following. Section 2 describes the derived TSK plant model and the designed T1 PI FLC in previous research. In Section 3 the optimisation-based approach for the design of IT2 SISO and 2ISO PI FLC from the requirements for improving the closed-loop system performance is explained. Section 4 presents simulation investigations of the systems with the designed IT2 PI FLC and an assessment of the performance improvement achieved with respect to the T1 PI FLC systems. Section 5 contains the conclusion and a vision for future research.

2 Foundations of the Study from Previous Research

The plant output is the controlled air temperature $y=\theta^{o}$C in a laboratory convective batch dryer for fruits. Its input is the PLC computed control action U sent for pulse width modulation (PWM) to form the necessary duty ratio of switching the electrical heater to the nets, [18]. Thus, the plant includes also the PWM, the electrical heater, and the temperature transducer. It is highly energy-consuming, nonlinear, inertial, and with model uncertainty.

The GA optimisation-based design of the IT2 FLC for the control of the dryer's temperature steps on closed loop system simulations using MATLAB™ Simulink and experimental data. The block diagram of the simulation model of the T1 or the IT2 FLC closed loop system is presented in Fig. 1. It needs a TSK plant model.

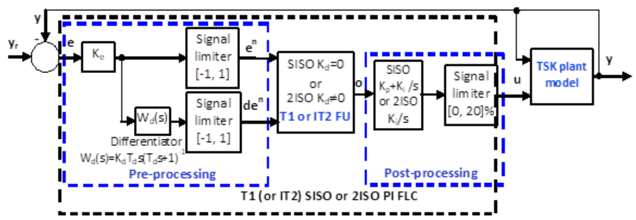

Fig. 1: Closed loop system with T1 PI FLC on SISO or 2ISO FU for control of TSK plant model

2.1 TSK Plant Model

The accepted TSK plant model is shown in Fig. 2. It is derived in [17], [18], from a number of experimental plant responses y_{ex} to step changes of the input U with increasing magnitudes in the range $U\in[0,\ 20]$. The TSK model structure is expert-defined from the three linearisation zones of the static characteristic computed from the step responses. The time lags $P_i(s)=K_i.(T_is+1)^{-1}$ in each i^{th} zone of linearisation, i=1÷3, correspond to the expert-assessed dynamic behavior of the plant in each zone. The time lag $P_4(s)=(T_4s+1)^{-1}$ serves to increase the order, i.e. the inertia, of all linear local for each zone plant models. The Sugeno model has an input y with standard orthogonal MF - a triangle (a, b=c, d) for the term 'Zone 2' and trapezoids (a, b, c, d) on both sides for 'Zone 1' and 'Zone 3' which parameters a, b, c and d comply with the static characteristic. The outputs $o1_i=\mu_i$ are the degrees of belonging μ_i of y to the three linearisation zones which is achieved by a specific rule base of three fuzzy rules of the type $\mathbf{R^1}$: **If** θ **is** Zone 1 **Then** $o_1^1=1$, $o_2^1=0$, $o_3^1=0$, where o_i^j is the i^{th} output in the j^{th} rule and $o_i^j=1$ for $j=i$ while the other outputs are zeros. The TSK plant model output y_{TSK} is computed as the weighted average of the outputs of the local plant models y_i processed by the time lag $P_4(s)$.

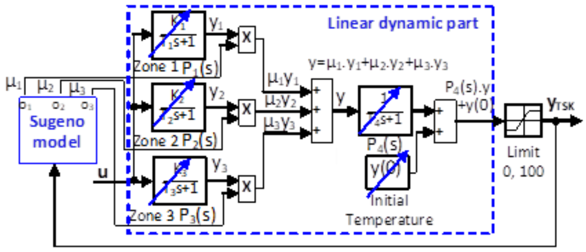

Fig. 2: TSK plant model

The parameters of the four time lags and the initial temperature $y(0)$ are the result of a GA minimisation

of a fitness function of the modelling error $e_m=y_{TSK}-y_{ex}$, $\mathbf{q}_{TSK}^{o}=[K_1=3.8\ K_2=3.5\ K_3=3.6\ T_1=23.5\ T_2=33\ T_3=57\ T_4=82\ y(0)=27]$.

The TSK plant model is validated from experimental data for the plant input and output during its present real-time linear PLC PID-PWM control.

2.2 T1 FLC

The improvement of the system performance by IT2 PI FLC, designed via optimisation, is assessed with respect to the performance of the systems with designed corresponding T1 PI FLC. Here the empirically designed and GA optimised in [17], T1 Sugeno PI FLC on SISO and 2ISO FU are used. Their T1 FU has the smallest possible number of MF with a simple mathematical description - orthogonal standard triangular and trapezoidal for the inputs and singletons for the output. The error normalisation gain K_e is computed for the maximal expected absolute system error $|e|_{max}$, $K_e=1/|e|_{max}$. The control action U is limited in the range [0, 20] % by a signal limiter to comply with the range of the input for which the TSK plant model is derived. The post-processing gains are computed from empirical tuning relationships, [17] [18]: $K_p=A.T/(K_{eq}.\tau)$, $K_i=B/(K_{eq}.\tau)$, where A=0.1÷2, B=0.03÷3 shape the desired closed loop system dynamic behavior, $K_{eq}=K_e.K_{flc}.k$ is the equivalent plant gain, K_{flc} is the linearisation gain of the FU, (k T τ) are the parameters of a linear Ziegler-Nichols plant model $P(s)=k.e^{-\tau s}(T.s+1)^{-1}$, assessed from one of the plant step responses.

The accepted five input MF for the normalized error e^n of the SISO FU input and respectively the five output singletons are:

MFe=[MFe$_j$]=[NGe; Ne; Ze; Pe; PGe]=[-1 -1 -0.6 -0.2; -0.6 -0.2 0; -0.2 0 0.2; 0 0.2.0.6; 0.2 0.6 1 1] (2)

S=[S$_j$]= [NGo No Zo Po PGo]=[-1 -0.8 0 0 8 1], j=1÷5. (3)

The five fuzzy rules are of the type of **R$_j$**: **If** e^n **is** MFe$_j$ **Then** o **is** S$_j$. The FU linearisation gain K_{flc} is the gain of the upper line of the sector that bounds the FU control curve, [14]. So, the computed PI post-processing parameters are K_p=4.84, K_i= 0.027.

The 2ISO FU has the same MF for the normalized error e^n, MFe from (2), and 3 MF for the normalized derivative of error de^n:

MFde= [Nde; Zde; Pde] =[-1 -1 -0.3 0; -0.3 0 0.3; 0 0.3 1 1] (4)

The derivative of error is computed by a first-order noise-filtering differentiator $W_d(s)=K_d.T_d.s(T_d.s+1)^{-1}$ with empirically tuned parameters $T_d=(2\div10)\Delta t$=3min and K_d=20% for a sample period Δt=1min. The five output singletons differ from (3) in No=-0.2 and Po=0.2:

S=[S$_j$]= [-1 -0.2 0 0.2 1] (5)

Standard soft fuzzy rules are used. The FU linearisation gain K_{flc} is the gain of the sector upper line that bounds the o-e^n projection of the control surface. The integrator gain is K_i=0.13.

Four selected parameters of the SISO and the 2ISO PI FLC **q**= [No Po (K_p or K_d) K_i] are GA optimised to minimise a fitness function **F** of two components:

F=F1+w. F2 (6)

where **F1** is the system mean squared error (MSE) for a sample size N, **F1**= $\frac{1}{N}\sum_{k=1}^{N} e_k^2$, **F2** is the maximal span of the control action from all system reference step response $\mathbf{F2} = \max_j|u_{\max j} - u_{\min j}|$, j=1÷3, and **w** is a weighting factor.

The optimal parameters for the SISO PI FLC and **w**=1 are $\mathbf{q}_{SISO}^{opt}$= [No^{opt}=-0.1, Po^{opt}=0.64, K_p^{opt}=4.4 K_i^{opt}=0.038] and for the 2ISO PI FLC and **w**=3.7 - $\mathbf{q}_{2ISO}^{opt}$=[No^{opt}=-0.28, Po^{opt}=0.1, K_d^{opt}=23 K_i^{opt}=0.21].

3 Optimisation-based Design of IT2 Sugeno PI FLC

In order to further improve the performance of the T1 FLC closed loop systems IT2 FLC are designed using GA optimisation of selected tuning parameters of the MF, FoU, pre- and post-processing. The GA generates random chromosomes or individuals, i.e. combinations of values for the parameters (genes) from their given ranges, to make the population of the first generation. Then the system in Fig. 1 with IT2 FU is simulated for each chromosome and an accepted fitness function F is computed. The chromosomes are rated according to the value of the fitness function. Then offspring of new chromosomes is produced through mating, crossover of genes, and mutation using different approaches. The offspring is rated in the same manner and if better than the parents enter the next generation. The process of mating, crossover, and mutation is repeated till the new generation reaches the same number of populations. Then another cycle starts till an end condition is met. The end condition is either a reached number of generations or a reached minimal value of F. After the end of the GA optimisation the system with the optimal parameters is simulated and tested for satisfaction of auxiliary criteria for system performance improvement. In this research, the step responses of the simulated IT2 FLC system serve for assessment of the performance indicators for dynamic accuracy via F1 in (6), overshoot and settling time, and for smooth and economic control action via F2 in (6) and enable the comparison with the same indicators of the T1 FLC systems. The optimisation ends in case of satisfactory IT2 FLC system performance improvement. Otherwise, it is repeated with random generation of the initial population or changed GA input data – parameters to be optimised, their ranges, fitness function, etc.

Here it is assumed that the FoU is enclosed by the orthogonal MF of the T1 FLC which is accepted for LMF, and UMF computed from GA optimisation of the MF parameters with $\mu(e^n)=0$ and $\mu(de^n)=0$. In case of three normalized in the range [-1, 1] orthogonal LMF – trapezoidal LMF1=[-1 -1 c1 d1], triangular LMF2=[a2 b2 c2] and trapezoidal LMF3=[a3 b3 1 1], the parameters of the UMF to be optimized are selected as corresponding to the parameters of the LMF for which $\mu=0$, $\mathbf{q}_{\mathrm{UMF}}$=[d11; a22 c22; a33], i.e. the parameter d11 of the UMF corresponds to d1 of the LMF, etc. Besides, it should be observed that d11>d1, a22<a2, c22>c2, and a33<a3 in order to form the UMF with respect to the accepted LMF. As a result, UMF1= [-1 -1 c1 d11], UMF2= [a22 b2 c22] and UMF3= [a33 b3 1 1]. Thus, by preserving the UMF parameters for $\mu=1$ the same as in the LMF and proper selection of the ranges of the parameters in $\mathbf{q}_{\mathrm{UMF}}$ the linguistic sense of the MF is preserved. The other tuning parameters are the singletons $\mathbf{q}_{\mathrm{S}}$ which are different from 0, 1, and (-1) in the normalized universe of discourse [-1, 1], e.g. for five output singletons $\mathbf{q}_{\mathrm{S}}$=[No Po], and all parameters of the pre- and post-processing $\mathbf{q}_{\mathbf{pr}}=[K_d\ K_p\ K_i]$. So, the IT2 PI FLC parameters to be optimised are $\mathbf{q}_{\mathbf{FLC}}=[\mathbf{q}_{\mathrm{UMF}}\ \mathbf{q}_{\mathrm{S}}\ \mathbf{q}_{\mathbf{pr}}]$. The fitness function **F** to be minimised is (6) and is computed from simulations using the system model in Fig. 1 with IT2 FU. The optimal parameters are searched in expert-specified ranges around their empirically defined values from the T1 FLC design and accounting for the restriction that the optimal MF parameters have to build UMF with respect to the accepted LMF. The optimisation is successful when the minimal value of **F** of the IT2 FLC system becomes *n*-times smaller than the minimal value of the corresponding T1 FLC system, i.e. a desired performance improvement is reached.

3.1 Design of IT2 SISO PI FLC

The IT2 SISO FU accepts the same fuzzy rules of the T1 SISO FU, the same five singletons from (3) in the rules' conclusions, and the MFe from (2) as LMF:

LMF= [LNG; LN; LZ; LP; LPG] = [-1 -1 c1 d1; a2 b2 c2; a3 0 c3; a4 b4 c4; a5 b5 1 1] (7)

where c1=-0.6, d1=-0.2; a2=-0.6, b2=-0.6, c2=0; a3=0.2, c3=0.2; a4=0, b4=0.2, c4=0.6; a5=0.2, b5=0.6.

The UMF are searched in the form:

UMF= [UNG; UN; UZ; UP; UPG] = [-1 -1 c1 d11; a22 b2 c22; a33 0 c33; a44 b4 c44; a55 b5 1 1]. (8)

The selected parameters to be optimised $\mathbf{q}_{\mathbf{SISO}}$= [$\mathbf{q}_{\mathbf{UMF\_SISO}}$ **q**], where **q**= [$\mathbf{q}_{\mathbf{S}}$ $\mathbf{q}_{\mathbf{pr}}$] are the same parameters of T1 SISO PI FLC that are optimised, $\mathbf{q}_{\mathbf{pr}}=[K_p\ K_i]$ are the gains of the PI post-processing and $\mathbf{q}_{\mathbf{S}}$= [No Po] are the singletons different from (-1), 0 and 1 in (3). The UMF parameters to be optimised are those with membership grade $\mu=0$, $\mathbf{q}_{\mathbf{UMF\_SISO}}$= [d11 a22 c22 a33 c33 a44 c44 a55]. The rest of the UMF parameters are the same as the LMF.

A simulation model of the IT2 SISO FU in Fig. 1 is developed and shown in Fig. 3. The normalized by the help of K_e system error e in the range [-1 1], $e^n=K_e.e$ is fed to the SISO FU. There are fuzzification results in the membership grades which are interval numbers $\mu(e^n)$= (Lμ, Uμ), defined by lower Lμ and upper Uμ bounds. The qualified conclusion $o_i=\mu_i.S_i$ in each rule $\mathbf{R_i}$ is also an interval number o_i= (Lo$_i$, Uo$_i$). The weighted average defuzzification is performed using the operations with interval numbers (1):

$o=\Sigma o_i/\Sigma\mu_i=\Sigma(\mathrm{Lo}_i,\ \mathrm{Uo}_i)/\Sigma(\mathrm{L}\mu_i,\ \mathrm{U}\mu_i)$, o= ($\Sigma\mathrm{Lo}_i$, $\Sigma\mathrm{Uo}_i$)/ ($\Sigma\mathrm{L}\mu_i$, $\Sigma\mathrm{U}\mu_i$) =[L(o/μ), U(o/μ)] (9)

where L(o/μ) =min[$\Sigma\mathrm{Lo}_i.(1/\Sigma\mathrm{L}\mu_i)$, $\Sigma\mathrm{Uo}_i.(1/\Sigma\mathrm{L}\mu_i)$, $\Sigma\mathrm{Uo}_i.(1/\Sigma\mathrm{U}\mu_i)$, [$\Sigma\mathrm{Lo}_i.(1/\Sigma\mathrm{U}\mu_i)$ and

U(o/μ) =max[$\Sigma\mathrm{Lo}_i.(1/\Sigma\mathrm{L}\mu_i)$, $\Sigma\mathrm{Uo}_i.(1/\Sigma\mathrm{L}\mu_i)$, $\Sigma\mathrm{Uo}_i.(1/\Sigma\mathrm{U}\mu_i)$, [$\Sigma\mathrm{Lo}_i.(1/\Sigma\mathrm{U}\mu_i)$.

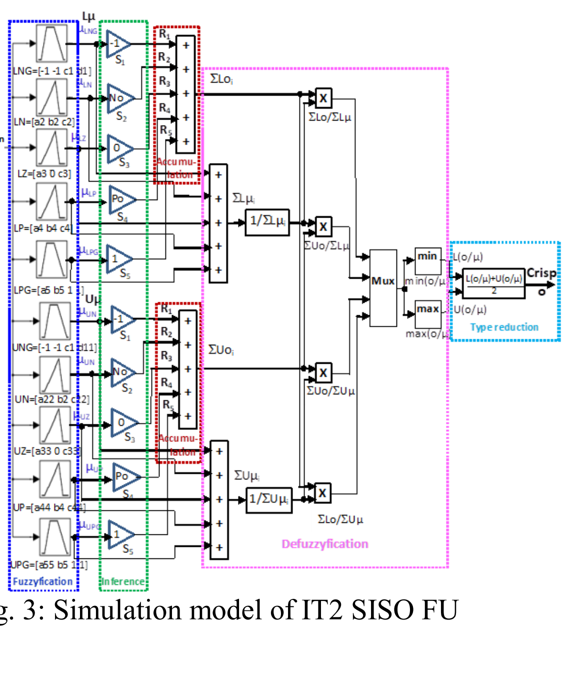

Fig. 3: Simulation model of IT2 SISO FU

The crisp FU output o_{crisp}=[L(o/μ) + L(o/μ)]/2 is computed as a mean value of L(o/μ) and U(o/μ) and is bounded in the range [-1 1]. The IT2 SISO FU algorithm, described in Fig. 3, with the optimised parameters, can easily be programmed in PLC for real-time IT2 FLC control.

The minimisation of **F** from (6) for **w**=1 results in the following optimal values of the parameters: $\mathbf{q}_{SISO}^{opt}$=[d11=-0.1; a22=-0.7; c22=0.04; a33=-0.27; c33=0.29; a44=-0.15; c44=0.8; a55=-0.13; No=-0.1; Po=0.14; K_p=6.33; K_i=0.037].

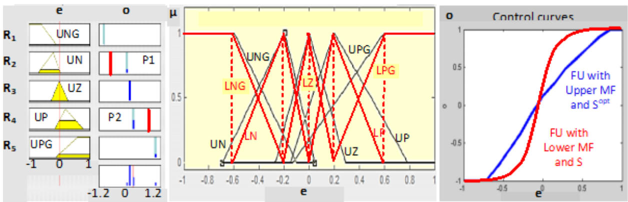

Fig. 4: Fuzzy rules, MF, singletons, and control curves of empirically designed T1 FU (MFe=LMF, S) and GA optimised T1 FU^{opt} (UMF, S^{opt})

The tuning parameters of the T1 and IT2 SISO PI FLC from the empirical design and after GA optimisation are summarized in Table 1, where the computed parameters in GA optimisation are in red. In Fig. 4 are depicted the fuzzy rules, the MF, the singletons, and the control curves of T1 FU with empirically designed MFe=LMF and **S** (in red) and T1 FU^{opt} with UMF and S^{opt} from GA optimisation. Both FUs have the same fuzzy rules. The T1 FU^{opt} has a control curve close to a line. The FLC based on UMF often causes static errors, [14].

Table 1. Parameters of T1 and IT2 SISO PI FLC

SISO FLC	T1, w=1 in (6)	IT2, w=1 in (6)
MF	MFe=[NGe Ne Ze Pe PGe] empirically designed NGe=[-1 -1 -0.6 -0.2], Ne=[-0.6 -0.2 0], Ze=[-0.2 0 0.2], Pe=[0 0.2.0.6], PGe=[0.2 0.6 1 1]	LMF=[LNG LN LZ LP LPG]=MFe UMF=[UNG UN UZ UP UPG] UNG=[-1 -1 -0.6 -0.1], UN=[-0.7 -0.2 0.04], UZ=[-0.27 0 0.29], UP=[-0.15 0.2.0.8], UPG=[-0.13 0.6 1 1]
Singletons	**S**= [NGo=-1,No=-0.8,Zo=0,Po=0.8, PGo=1] **S**^{opt}=[NGo=-1,No=-0.1, Zo=0, Po=0.64, PGo=1]	**S**=[NGo=-1,No=-0.1,Zo=0,Po=0.14, PGo=1]
Post-processing	K_p=4.84, K_i=0.027 - empirically designed K_p^{opt}=4.4 K_i^{opt}=0.038	K_p=6.33, K_i=0.037

3.2 Design of IT2 2ISO PI FLC

Here a reduced number from 5 to 3 orthogonal MF with respect to the main input *e* with broader support can be accepted for IT2 2ISO FU since the effect on the system performance is compensated by the FoU, [14]. Thus the empirically designed FU is 3x3 (3 MF for *e* and 3 – for *de*) with orthogonal trapezoidal on both ends and triangular in the middle MF for e^n which constitute $LMFe_3$=[LNe LZe LPe] with LNe=[-1 -1 c1 d1], LZe=[a2 0 c2] and LPe=[a3 b3 1 1], where c1=-0.5, d1=0, a2=-0.5, b2=0.5, a3=0 and b3=0.5. The $UMFe_3$ is searched in the form $UMFe_3$= [UNe UZe UPe] with UNe= [-1 -1 c1 d11], UZe= [a22 0 c22] and UPe= [a33 b3 1 1]. The output singletons **S** from (3) and the MFde from (4) for the second input de^n as LMFde are accepted from the T1 2ISO FU, where LMFde=MFde=[LNde LZde LPde] with LNde=[-1 -1 c4 d4], LZde=[a5 0 c5] and LPde=[a6 b6 1 1], where c4=-0.3, d4=0, a5=-0.3, b5=0.3, a6=0 and b6=0.3. The UMFde is searched in the form UMFde= [UNde UZde UPde] with UNde= [-1 -1 c4 d44], UZe= [a55 0 c55] and UPe= [a66 b6 1 1]. The fuzzy rule base in Table 2 is used.

Table 2. Fuzzy rule base

e/*de*	**Nde**	**Zde**	**Pde**
Ne	NGo	No	No
Ze	No	Zo	Po
Pe	Po	Po	PGo

The selected parameters to be optimised are $\mathbf{q}_{2ISO}$=[$\mathbf{q}_{UMF\_2ISO}$ **q**], where $\mathbf{q}_{UMF\_2ISO}$=[d11 a22 c22 a33; d44 a55 c55 a66] are the parameters of $UMFe_3$ and UMFde with μ=0, **q**=[$\mathbf{q}_S$ $\mathbf{q}_{pr}$] are the same parameters of T1 2ISO PI FLC with $\mathbf{q}_S$=[No Po] - the singletons different from (-1), 0 and 1 in (3) and $\mathbf{q}_{pr}$=[K_d K_i] - the gains of the differentiator and the I post-processing respectively.

The simulation model of the 3x3 IT2 2ISO FU, used in the system model in Fig. 1 and for PLC implementation, is shown in Fig. 5. It is developed based on the operations with interval numbers (1), the defuzzification and the type-reduction with interval numbers (9).

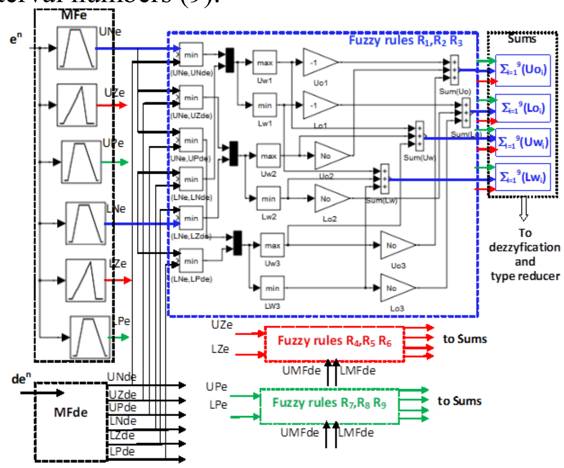

Fig. 5: Simulation model of IT2 2ISO FU

The optimal values of the parameters computed in the GA optimisation for **w**=2 is:
$\mathbf{q_{2ISO}^{opt}}$= [d11=0.007, a22=-0.68, c22=0.7, a33=-0.18; d44=0.007, a55=-0.32, c55=0.65, a66=-0.1; No=-0.5; Po=0.13; K_d=12.2; K_i=0.27].

The tuning parameters of the T1 and IT2 2ISO PI FLC from the empirical design and after GA optimisation are summarized in Table 3, where the computed parameters in GA optimisation are in red. Fig. 6 depicts the LMF (in red),the UMF, and the control surfaces of T1 FU with empirically designed MFe=LMF and **S** and T1 FU^{opt} with UMF^{opt} and S^{opt} from GA optimisation. The control surface of T1 FU^{opt} with (UMF^{opt}, S^{opt}) is smoother.

Table 3. Parameters of T1 and IT2 2ISO PI FLC

3x3 2ISO FLC	T1, w=3.7 in (6)	IT2, w=2 in (6)
MF	MFe=[NGe Ne Ze Pe PGe] empirically designed NGe=[-1 -1 -0.6 -0.2], Ne=[-0.6 -0.2 0] Ze=[-0.2 0 0.2], Pe=[0 0.2.0.6] PGe=[0.2 0.6 1 1] MFde=[Nde Zde Pde] empirically designed Nde=[-1 -1 -0.3 0], Zde=[-0.3 0 0.3], Pde=[0 0.3 1 1]	LMFe$_3$=[LNe LZe LPe] empirically designed Ne=[-1 -1 -0.5 0], Ze=[-0.5 0 0.5], Pe=[0 0.5.1 1]; UMFe$_3$=[UNe UZe UPe] UNe=[-1 -1 -0.5 0.007], UZe=[-0.68 0 0.7], UPe=[-0.18 0.5 1 1]; LMFde=[LNde LZde LPde]=MFde UMFde=[UNde UZde UPde] UNde=[-1 -1 -0.3 0.007], UZde=[-0.32 0 0.65], UPde=[-0.1 0.3 1 1]
Singletons	**S**=[NGo=-1, No=-0.2, Zo=0, Po=0.2, PGo=1] **S**^{opt}=[NGo=-1, No=-0.28,Zo=0,Po=0.1,PGo=1]	**S**=[NGo=-1,No=-0.5,Zo=0,Po=0.13, PGo=1]
Post-processing	K_d=20, K_i=0.13 empirically designed K_d^{opt}=23, K_i^{opt}=0.21	K_d=12.2, K_i=0.27

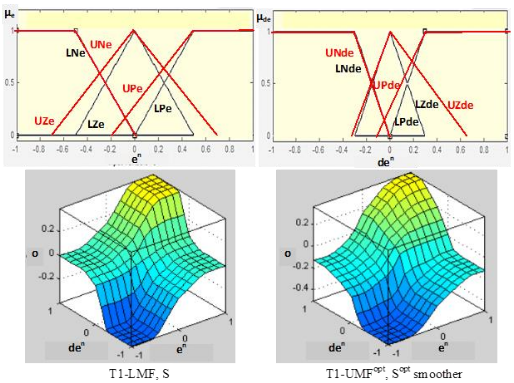

Fig. 6: LMF and UMF and control surfaces for T1 FU with (LMF, S) and T1 FU^{opt} with (UMF^{opt}, S^{opt})

4 Simulation Investigations and Assessment of Performance Improvement

The simulation investigations aim to compare the performance indicators MSE (F1), F2, overshoot $\sigma=(y\text{-}y_r)/\Delta y_r$, %, settling time t_s, min, and control action span U_{span} of the systems with the following controllers:

- T1 SISO FLC with 5 MF, empirical tuning, and four GA optimised parameters $\mathbf{q^{opt}}$;
- IT2 SISO FLC with 5 MF with twelve GA optimised parameters $\mathbf{q_{SISO}^{opt}}$ for **w**=1 and **w**=1.5 to stress on the requirement for smooth control action $\mathbf{q_{SISO}^{opt1}}$= [-1 -1 -0.6 0.054; -0.83 -0.2 0.02; -0.22 0 0.28; -0.15 0 0.69; -0.08 0.6 1 1; No=-0.15, Po=0.52; K_p=3.86; K_i=0.028];
- T1 2ISO FLC with 5x3 FU with empirical tuning and four GA optimised parameters $\mathbf{q_{2ISO}^{opt}}$;
- IT2 2ISO FLC with 3x3 FU with twelve GA optimised parameters $\mathbf{q_{2ISO}^{opt}}$.

The performance indicators are assessed from the systems step responses of temperature and control action to three successive reference changes Δy_r=10ºC in order to study the impact of the plant nonlinearity in the different operation points. The step responses of the SISO FLC systems are presented in Fig. 7 and of the 2ISO FLC system – in Fig. 8.

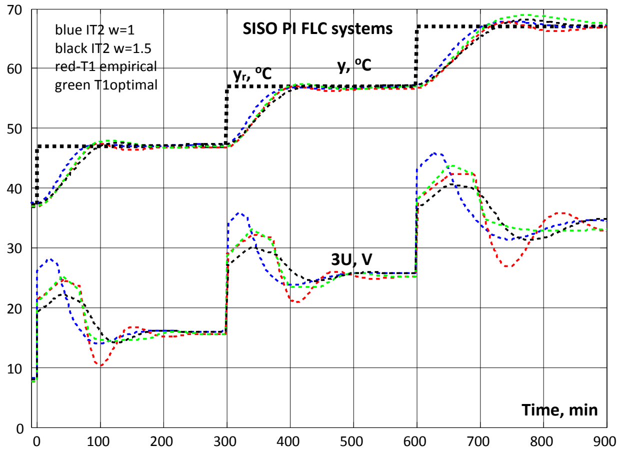

Fig. 7: Step responses with respect to temperature and control action of T1 and IT2 SISO PI FLC systems

The performance indicators of the investigated systems are summarized in Table 3. In bold are their best (smallest) values and the grey highlighted are the worst (with maximal value). Their relative (normalized) dimensionless values in brackets and blue are computed with respect to the maximal value for the indicator for all systems. The sum of the relative values for all performance indicators for a given system in the last column of Table 3 enables the rating of the systems. The highest rated with the smallest sum of 2.06 is the IT2 SISO FLC system followed by the IT2 2ISO FLC system with a sum of 2.33.

Table 3. Performance indicators for investigated systems

	PI FLC Systems	Performance indicators from step responses (1, 2, 3) F1 (MSE)	F2	σ, % (σ_1, σ_2, σ_3) meanσ $\Delta\sigma$	t_s, min (t_{s1}, t_{s2}, t_{s3}) means Δt_s	Sum of relative (improved)
SISO 5 MFe	T1 empirical **q**=[No=-0.8, Po=0.8; K_p=4.84, K_i=0.027]	30.2 (0.83)	6.2 (0.93)	(6, 0, 7) 4.3 (0.34) $\Delta\sigma$=7	(200,230,300) 243 (0.88) Δt_s=100	2.98 Best T1
	T1 optimal (**w**=1) **q**^{opt}=[No^{opt}=-0.1,Po^{opt}=0.64;K_p^{opt}=4.4,K_i^{opt}=0.038]	19.9 (0.55)	6.6 (0.99)	(12, 6, 20) 12.7 (1) $\Delta\sigma$=14	(170,220,300) 230 (0.83) Δt_s=130	3.37
	IT2 (**w**=1) **q**$_{SISO}^{opt}$=[8x1 **q**$_{UMF\_SISO}^{opt}$; No^{opt}=-0.1, Po^{opt}=0.14; K_p^{opt}=6.3, K_i^{opt}=0.037]	**9.98** (0.28)	6.7 (1)	(5, 0, 7) **4** (0.32) $\Delta\sigma$=7	(80,100,200) **127** (0.46) Δt_s=120	**2.06** **(1.45)**
	IT2 (**w**=1.5) **q**$_{SISO}^{opt1}$=[8x1**q**$_{UMF\_SISO}^{opt1}$; No^{opt1}=-0.15,Po^{opt1}=0.52;K_p^{opt1}=3.86,K_i^{opt1}=0.028]	13.3 (0.37)	**4.96** (0.74)	(9, 5, 10) 8 (0.63) **$\Delta\sigma_{min}$=5**	(150,160,230) 180 (0.65) **Δt_{smin}=80**	2.39
2ISO	T1 empirical, 5x3 MF **q**=[No=-0.2, Po=0.2; K_d=20, K_i=0.13]	36.2 (1)	5.36 (0.8)	(12, 6, 20) 12.7 (1) $\Delta\sigma_{max}$=14	(300,170,350) 273 (0.99) Δt_s=180	3.79
	T1 optimal (**w**=3.7), 5x3 MF **q**^{opt}=[No^{opt}=-0.28,Po^{opt}=0.1;K_d^{opt}=23,K_i^{opt}=0.21]	31.4 (0.87)	5.26 (0.79)	(10, 5, 15) 10 (0.79) $\Delta\sigma$=10	(320,160,350) 277 (1) Δt_{smax}=190	3.45
	IT2 (**w**=2), **3**x3 MF **q**$_{2ISO}^{opt}$=[8x1**q**$_{UMF\_2ISO}^{opt}$; No^{opt}=-0.5, Po^{opt}=0.13; K_p^{opt}=12.2, K_i^{opt}=0.027]	**14.63** (0.41)	**5.1** (0.76)	(10, 5, 15) **5** (0.39) $\Delta\sigma$=10	(220,120,300) **213** (0.77) Δt_s=180	**2.33** **(1.28)**
	Maximal indicator's value	36.2	6.7	12.7	277	1.45
	Most robust $\Delta\sigma_{min}$,Δt_{smin} (relative to $\Delta\sigma_{max}$,Δt_{smax})			5 (0.36)	80 (0.42)	

The improvement of the performance of the IT2 FLC systems is assessed with respect to the best-rated T1 FLC system, which is the T1 SISO FLC with empirical tuning and has a sum of the indicators' relative values of 2.98 (yellow highlighted). The first rated is the IT2 SISO FLC system with performance improvement (2.98/2.06) =1.45 and the second rated is IT2 2ISO FLC with improvement of 1.28.

The IT2 FLC systems show better performances than their T1 counterparts. The SISO systems have the advantage of a smaller number of MF and fuzzy rules and hence are simpler for PLC implementation. The higher weighting factor **w** increases the impact of the second component **F2** on the results of GA optimisation thus the optimal parameters computed ensure a smoother and more economical control action in a trade-off with the reduced system dynamic accuracy. Other factors that influence the performance improvement achieved via IT2 FLC and optimisation of the FoU are the selected fitness function, parameters, and their initial ranges. So, the solution obtained is not unique.

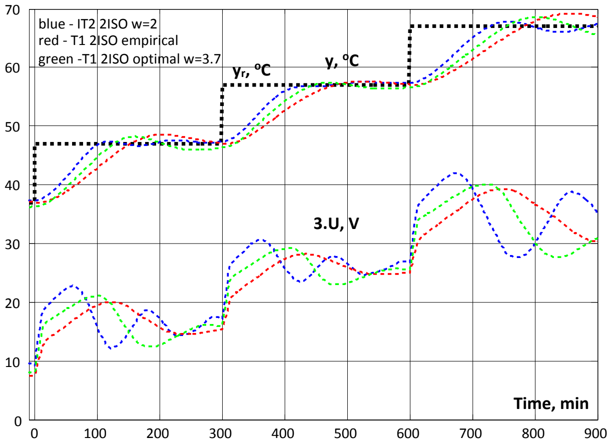

Fig. 8: Step responses with respect to temperature and control action of T1 and IT2 2ISO PI FLC systems

The system robustness can be assessed as the greatest change of σ or t_s for all three step responses which move the operation point along the system nonlinear characteristic where the plant parameters are different: $\Delta\sigma=|\max(\sigma_i)-\min(\sigma_i)|$ and $\Delta t_s=|\max(t_{si})-\min(t_{si})|$.

The system IT2 SISO (**w**=1.5) with the smallest $\Delta\sigma_{min}$=5% and Δt_{smin}=80min is the most robust with respect to the changes in the plant parameters.

Their dynamic accuracy measured by the smaller values for the **MSE**, σ and t_s of the SISO systems is generally better than that of their 2ISO counterparts. The IT2 approach causes a greater improvement in the dynamic accuracy of the 2ISO FLC systems than the SISO FLC systems which shows that IT2 FLC is recommended for a greater performance improvement of 2ISO FLC systems. Besides, the IT2 increases the 2ISO FLC system robustness and reduces the control action span to ensure smooth and economical control and a prolonged lifetime of the final control elements.

5 Conclusion and Future Research

The novel results achieved in the present research conclude with the following.

An approach for improvement of the control system performance by the optimisation-based design of interval type-2 Sugeno SISO and 2ISO PI FLC suitable for PLC implementation is developed. It is demonstrated for the control of the temperature in a laboratory convective dryer for fruits. The plant is nonlinear with model uncertainty and derived in previous research TSK plant model which is necessary for the optimisation.

The adopted fitness function integrates requirements to the mean squared error and the maximal span of the control action. It is minimised by optimisation of the pre-and post-processing gains, selected parameters of the upper membership functions, i.e. of the FoU, and of the output singletons. The fitness function is computed from FLC closed loop systems simulations based on developed IT2 FU models which are also suitable for programming in a PLC for later use in real-time control.

The designed IT2 SISO and 2ISO PI FLC systems are studied by simulation. Their performances are assessed from successive reference step responses in different operation points. The comparison with the performance of the empirically designed and GA-optimised T1 SISO and 2ISO PI FLC systems shows the superiority of the IT2 FLC with the optimised FoU, singletons, and pre-and post-processing. The IT2 SISO PI FLC (**w**=1) system demonstrates the greatest performance improvement with respect to the best T1 FLC system according to an introduced measure based on the sum of relative indicators for the MSE, control action span, overshoot, and settling time. The next rated is the IT2 2ISO PI FLC which is characterized by smoother and more economical control action and greater performance improvement with respect to the T1 2ISO systems. The IT2 SISO system optimised with emphasis on the control action span (**w**=1.5) has the most economical control action and is the most robust of all systems, i.e. the overshoot and the settling time change the least with the operation point, determined by the temperature reference.

Future research will focus on the development and investigation of FLC supervisor-based control systems and the comparison of the system performance with the IT2 FLC systems.

References:

[1] Ataşlar-Ayyıldız, B., Karahan, O. (2020). Design of a MAGLEV system with PID-based fuzzy control using CS algorithm. *Cybern. and Technol.* 20(5), 5-19

[2] Chao, C.-T., Sutarna, N., Chiou, J.-S., Wang, C.-J. (2019). An optimal fuzzy PID controller design based on conventional PID control and nonlinear factors. *Appl. Sci*. 9(6) 1224, 1-18.

[3] Nursoy, H., Başçic, A. (2024). Comparison of the performance of type-1 and interval type-2 fuzzy PI controllers for liquid level control in a coupled tank system. *Journal of Sci and Technol.* 17(2), 523-542.

[4] Saad, M., Alshara, M., Mustafa, K. (2023). Fuzzy PID controller design for a coupled tank liquid level control system. *WSEAS Transactions on Systems and Control,* 18, 401-408. https://doi.org/10.37394/23203.2023.18.42.

[5] Belman-Flores, J.M.;Rodríguez-Valderrama, D.A., Ledesma, S.; García-Pabón, J.J., Hernández, D., Pardo-Cely, D.M. (2022). A review on applications of fuzzy logic control for refrigeration systems. *Appl. Sci*. 12, 1302.

[6] Berouine, A., Akssas, E., Naitmalek, Y., Lachhab, F., Bakhouya, M., Ouladsine, R., Essaaidi, M. (2019). A fuzzy logic-based approach for HVAC systems control. 2019 6th Int. Conf. on Contr., *Decision and Inform. Technol. (CoDIT*), Paris, France, 1510-1515.

[7] Chojecki, A.; Ambroziak, A.; Borkowski, P. (2023). Fuzzy controllers instead of classical PIDs in HVAC equipment: dusting off a well-known technology and today's implementation for better energy efficiency and user comfort. *Energies* 16, 2967.

[8] Sellitto, M.A., Balugani, E., Gamberini, R., Rimini, B. (2018). A Fuzzy logic control application to the cement industry. *IFAC-Papers OnLine* 51, 1542-1547.

[9] Vasičkaninová, A., Bakošová, M., Mészáros, A. (2021). Fuzzy control design for energy efficient heat exchanger network, *Chem. Eng. Trans*. 88, 529-534.

[10] Vivekanandan, N., Fulambarkar, Dr. A.M., Waghmare, S. (2020). Experimental validation of fuzzy logic based anti-lock braking system used in quarter car model. *Int. J. of Contr. and Autom.* 13(2), 332–348.

[11] Yanti, N., Nur, T., and Randis, R. (2022). Implementation of fuzzy logic in fish dryer design. *ILKOM Journal Ilmiah* 14(1), 39-51.

[12] Yazid, E., Garratt, M., Santoso, F. (2019). Position control of a quadcopter drone using

evolutionary algorithms-based self-tuning for first-order Takagi–Sugeno–Kang fuzzy logic autopilots. *Appl. Soft Comp*., 78, 373-392.

[13] Ye, L., Liang, C., Li, X. and Li, D. (2020). Energy efficiency improvement of eddy-current braking and heating system for electric bus based on fuzzy control. *IET Electr. Syst. Transp*. 10(10), 385-390.

[14] Yordanova, S. Design of Fuzzy Supervisor-based Adaptive Process Control Systems. In: New Approaches in Intelligent Control: Techniques, Methodologies and Applications, *Book series "Intelligent Systems Reference Library",* Eds: K. Nakamatsu, R. Kountchev, Springer Int. Publishing, Switzerland, Chapter 1, 1-42, 2016, ISBN: 978-3-319-32166-0.

[15] Ahmad, S., Ali, S., Tabasha, R. (2020). The design and implementation of a fuzzy gain-scheduled PID controller for the Festo MPS PA compact workstation liquid level control. *Eng. Sci. and Technol.-an International Journal* 23, 307–315.

[16] Aydogmus, Z. (2015). A real-time robust fuzzy-based level control using programmable logic controller. *Elektronika ir Elektrotechnika* 21 (1), 13-17.

[17] Yordanova S., Stoitseva-Delicheva D. (2024). Prospects of intelligent techniques for energy efficient control of drying process, *Proc. IEEE Int. conf. Automatics and Informatics'2024* (ICAI'24), Oct.10-12, Varna, Bulgaria, pp. 205-211.

[18] Yordanova, S., Stoitseva-Delicheva, D. (2025). TSK model-based energy efficient control with application to temperature in laboratory fruit dryer. *Journal of Electrical Systems (JES),* Vol. 20 No. 3 (2024), https://doi.org/10.52783/jes.8397.

[19] Feng, G. *Analysis and Synthesis of Fuzzy Control Systems: A Model Based Approach*. Taylor & Francis, Bosa Roca, US., 2017.

[20] Civelek, Z. (2020). Optimization of fuzzy logic (Takagi-Sugeno) blade pitch angle controller in wind turbines by genetic algorithm. *Eng. Sci. and Technol.-an International Journal* 23(1), 1-9, ISSN: 2215-0986.

[21] Hosseinpour, S., Martynenko, A. (2022). An adaptive fuzzy logic controller for intelligent drying, *Dry. Technol*. 41(7).

[22] Venkataraman, A. (2021). Design and implementation of adaptive PID and adaptive fuzzy controllers for a level process station. *Adv. in Technol. Innov*. 6(2), 90-105.

[23] Raj, R., Mohan, B.M. (2020). General structure of interval type-2 fuzzy PI/PD controller of Takagi–Sugeno type. *Eng. Appl. of Artificial Intell.* 87, 103273.

[24] Wu, D., Mendel, J. M. (2019). Recommendations on designing practical interval type-2 fuzzy systems. *Eng. Appl. of Artificial Intell*. 85, 182–193.

[25] Nie, M., Tan, W.W. (2008). Towards an efficient type-reduction method for interval type-2 fuzzy logic systems. *Proc. IEEE Int. Conf. on Fuzzy Syst*., Hong Kong, China, 1–6 June 2008, 1425–1432.

[26] Al-Mahturi, A., Santoso, F., Garratt, M. A., Anavatti S. G. (2019). An intelligent control of an inverted pendulum based on an adaptive interval type-2 fuzzy inference system, *Proc. 2019 IEEE Int. Conf. on Fuzzy Syst. (FUZZ-IEEE)*, New Orleans, LA, USA, 1-6.

[27] El-Nagar, A. M., El-Bardini, M. (2014). Practical implementation for the interval type-2 fuzzy PID controller using a low cost microcontroller, *Ain Shams Eng. Journal* 5, 475–487.

[28] El-Sotouhy, M. M., Mansour, A. A.,. Marei, M. I., Zaki, A. M., El-Sattar, A. A. (2020). A comparative study between type-1 and type-2 fuzzy logic controllers for 4-leg active power filter. *Int. J. of Contr. and Autom*., 13(02), 1657-1670.

[29] Fuzzy Logic Toolbox: *User's Guide for Use with MATLAB*, The Math Works, Inc. Natick, MA, 1998.

[30] MATLAB – *Genetic Algorithm and Direct Search Toolbox: User's Guide*. The MathWorks, Inc., Natick, MA, 2004.

[31] Júnior, M.P., da Silva, M.T., Guimarães, F.G., Euzébio, T.A.M. (2022). Energy savings in a rotary dryer due to a fuzzy multivariable control application. *Dry. Technol*. 40(6).

[32] Nafisah N., Syamsiana I. N., Putri R. I., Kusuma W., Sumari A. D. W. (2024), Implementation of fuzzy logic control algorithm for temperature control in robusta rotary dryer coffee bean dryer. *MethodsX* 12, ISSN: 2215-0161.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed to the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

No funding was received for conducting this study.

Conflict of Interest

The authors have no conflicts of interest to declare.