A Closed-Form Solution to Observer Design Problem for Ostensible Metzler Takagi-Sugeno Systems

DUŠAN KROKAVEC
Department of Cybernetics and Artificial Intelligence
Faculty of Electrical Engineering and Informatics
Technical University of Košice
Letná 9, 042 00 Košice
SLOVAKIA

Abstract: - This paper addresses the state estimation problems related to the generalized fuzzy observer design for ostensible Metzler Takagi-Sugeno (T-S) systems. Attention is focused on design constraints for the concept of diagonal stabilization and positivity of observer gain matrices. On the basis of some new interpretations, the parameterizations of ostensible Metzler T-S fuzzy systems is presented, which opens the way to the solution of the design problem using only the principle of linear matrix inequalities. The same approach is intended to ensure the stability of the dynamics of the estimation error. The presented method extends and generalizes the results that have been presented in the literature so far.

Key-Words: - Metzler Takagi-Sugeno fuzzy system, ostensible Metzler matrix structures, diagonal stabilization, parametric constraints, state-space methods, linear matrix inequalities.

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1 Introduction

Many of the nonlinear systems can be represented as an aggregation of a set of linear mathematical models using the fuzzy implication of local dynamics by the fuzzy Takagi-Sugeno (T-S) approach, [1]. Based on this idea, specific fuzzy design methods have appeared especially in the field of fuzzy control, [2], [3], [4]. Since TS fuzzy models are differential inclusions, these methods resort to stability analysis using quadratic Lyapunov function when using the representation of models in the state space, although other fuzzy inference systems can be used, [5], [6]. Currently, T-S fuzzy systems are growing in popularity because they have powerful modeling and control in ship control systems, [7], [8], multi-agent systems, [9], data-oriented flexible-joint robot structures, [10], and T-S network control systems, [11], [12]. A similar trend can be seen in the use of T-S fuzzy observers in fault diagnosis, [13], [14], and in fault-tolerant control, [15], of nonlinear systems.

As for positive nonlinear systems, modern methods of synthesis of their control are aimed at less restrictiveness than offered by methods based on linearization techniques. Inspired by linear positive systems, [16], [17], the use of linear state space theory in solving the control synthesis problem has led to T-S structures of positive nonlinear systems based on the matrices of the Metzler structure, [18]. Since the design conditions are in the form of linear matrix inequalities (LMIs), the problem can be numerically solved by the latest LMI techniques. Already the first technological applications have shown that the structure of the ostensible Metzler matrix suits reality, [19]. This class is represented by ostensible Metzler T-S fuzzy systems, which includes models of chaotic systems given by the Lorenz equation with an input term, [20], [21], [22]. New control algorithms based on fuzzy T-S methods have been proposed for water turbine control systems, [23], [24], actuator fault-tolerant control of wind turbines, [25], and extended fuzzy control for aircraft engines, [26], whose T-S models are based on ostensible Metzler matrix structures.

In the mentioned papers, the authors focus only on the synthesis of the control of T-S fuzzy positive systems, and in [27], only design conditions for T-S fuzzy positive observers with strictly Metzler matrices can be found, which generalize the results for linear positive systems, [28]. In general, a positive observer is driven by the output of a positive system such that the estimation error is asymptotically stable and positive. Because ostensible Metzler T-S fuzzy systems are not internally positive, for state observers based on this class of models it is necessary to modify the standard conditions of synthesis due to incomplete positivity. Based on these facts, the paper formulates a new approach to the design of fuzzy observers for ostensible Metzler T-S fuzzy systems.

A convex optimization technique is used to represent design constraints within the concept of diagonal stabilization and positivity of observer gain matrices. To solve the limitations generated by ostensible Metzler matrix structures, a new efficient approach is presented to find feasible solutions and parameterize the gains of the ostensible Metzler T-S fuzzy observer. Under LMI-based design conditions, positive observer gains are determined such that all modes of the observer subsystems are asymptotically stable and ostensible Metzler. Mixed pair of ostensible and strictly Metzler matrices are used as an example to demonstrate the effectiveness of the proposed method.

This correspondence is structured as follows. In Sect. 2 essential limiting features are included with reference to Metzler T-S fuzzy models and their projection into the task of Metzler T-S fuzzy observer design is outlined in Sect. 3. The formulation of the fuzzy observer stability problem using the LMI approach is the subject of Sect. 4. In Sect. 5 a numerical example is presented to illustrate the synthesis procedure proposed in this paper and Sect. 6 draws conclusions regarding the effectiveness of the presented approaches and potential future research directions in this area.

The following notations are used throughout the paper: $\boldsymbol{x}^{\mathrm{T}}$, $\boldsymbol{X}^{\mathrm{T}}$ denotes the transpose of the vector $\boldsymbol{x}$, respectively the matrix $\boldsymbol{X}$, $\operatorname{diag}(\cdot)$ denotes the (block) diagonal matrix, the notation $\prec 0$ for the square symmetric matrix $\boldsymbol{X}$ means that $\boldsymbol{X}$ is negative definite, the symbol $\boldsymbol{I}_n$ denotes the identity matrix of the nth order, $\mathbb{R}_+$, $\mathbb{R}_+^n$ are sets of nonnegative real numbers and n-dimensional real vectors, $\mathbb{R}_+^{n\times n}$ refers to the set of nonnegative real matrices and $\mathbb{R}_{-+}^{n\times n}$, $\mathbb{R}_{-\oplus}^{n\times n}$ covers the set strictly or ostensible Metzler matrices.

2 Metzler T-S Fuzzy Models

Considering the T-S fuzzy approach to positive nonlinear MIMO continuous-time dynamic systems then the system state and output, $\boldsymbol{q}(t), \boldsymbol{y}(t)$, are given by

$$\dot{\boldsymbol{q}}(t) = \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\vartheta}(t))(\boldsymbol{A}_i\boldsymbol{q}(t) + \boldsymbol{B}_i\boldsymbol{u}(t)) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \tag{2}$$

where $\boldsymbol{q}(t) \in \mathbb{R}_+^n$, $\boldsymbol{u}(t) \in \mathbb{R}^r$, $\boldsymbol{y}(t) \in \mathbb{R}_+^m$, $\boldsymbol{A}_i \in \mathbb{M}_{-+}^{n\times n}$, $\boldsymbol{B}_i \in \mathbb{R}_+^{n\times r}$, $\boldsymbol{C} \in \mathbb{R}_+^{m\times n}$ and $\mathrm{h}_i(\boldsymbol{\theta}(t))$ is averaging weight for the i-th rule, where

$$0 \le \mathrm{h}_i(\boldsymbol{\vartheta}(t)) \le 1,\ \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\vartheta}(t)) = 1\ \forall\, i \in \langle 1, s\rangle \tag{3}$$

while s is the number of linear sub-models) and

$$\boldsymbol{\vartheta}(t) = \begin{bmatrix} \theta_1(t) & \theta_2(t) & \cdots & \theta_q(t) \end{bmatrix} \tag{4}$$

is q-dimensional vector of measured premise variables. Further details can be found in [29].

To reflect the system positiveness, nonnegative matrices $\boldsymbol{B}_i \in \mathbb{R}_+^{n\times r}$ and $\boldsymbol{C} \in \mathbb{R}_+^n$ have to be considered when matrices $\boldsymbol{A}_i \in \mathbb{R}_{-+}^{n\times n}$ are Metzler.

Definition 1 [30] *A square matrix $\boldsymbol{A} \in \mathbb{M}_{-+}^{n\times n}$ is strictly Metzler if its diagonal elements are negative and its off-diagonal elements are positive and the following constraints result*

$$a_{lh} < 0,\ l = h,\ a_{lh} > 0, l \neq h,\ \forall l, h \in \langle 1, n\rangle \tag{5}$$

Since positive systems are only diagonally stabilizable, [31], using the circulant permutation matrix $\boldsymbol{L} \in \mathbb{R}_+^{n\times n}$ they matrix parameters have to be diagonally parameterized.

Definition 2 [32] *The square matrix $\boldsymbol{L} \in \mathbb{R}_+^{n\times n}$ is circulant permutation matrix if*

$$\boldsymbol{L} = \begin{bmatrix} \boldsymbol{0}^{\mathrm{T}} & 1 \\ \boldsymbol{I}_{n-1} & \boldsymbol{0} \end{bmatrix}. \tag{6}$$

Definition 3 [33] *If Metzler $\boldsymbol{A} = \{a_{lj}\} \in \mathbb{M}_{-+}^{n\times n}$ is represented in the equivalent rhombic structure*

$$\boldsymbol{A}_{\Theta} = \begin{bmatrix} a_{11} & & & & \\ a_{21} & a_{22} & & & \\ a_{31} & a_{32} & a_{33} & & \\ \vdots & \vdots & \vdots & \ddots & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \\ & a_{12} & a_{13} & \cdots & a_{1n} \\ & & a_{23} & \cdots & a_{2n} \\ & & & \ddots & \vdots \\ & & & & a_{n-1,n} \end{bmatrix} \tag{7}$$

then the set of diagonal matrix inequalities

$$\begin{cases} \boldsymbol{A}_{\Theta}(p+h, p) \prec 0, & h = 0 \\ \boldsymbol{A}_{\Theta}(p+h, p) \succ 0, & h = 1, \ldots, n-1 \\ \boldsymbol{A}_{\Theta}(p+h, p) = & \\ \operatorname{diag}\left[a_{1+h,1} \cdots a_{n,n-h}\, a_{1,n-h+1} \cdots a_{h,n}\right] & \end{cases} \tag{8}$$

implying from diagonals of (7) is equivalent to (6).

Lemma 1 [33] *Using (6)-(8) the parameterization of a Metzler matrix $\boldsymbol{A} \in \mathbb{M}_{-+}^{n\times n}$ means the relation*

$$\boldsymbol{A} = \sum_{h=0}^{n-1} \boldsymbol{L}^h \boldsymbol{A}(p+h, p) \tag{9}$$

Applying for a Metzler $\boldsymbol{A}_e = \boldsymbol{A} - \boldsymbol{J}\boldsymbol{C} \in \mathbb{M}_{-+}^{n\times n}$, $\boldsymbol{J} \in \mathbb{R}_+^{n\times m}$, $\boldsymbol{C} \in \mathbb{R}_+^{m\times n}$ address the following parametrization

$$\boldsymbol{A}_e = \sum_{h=0}^{n-1} \boldsymbol{L}^h \big(\boldsymbol{A}(p+h, p) - \sum_{k=0}^{m} \boldsymbol{J}_{kh}\boldsymbol{C}_k\big) \tag{10}$$

where diagonal $\boldsymbol{J}_{kh}, \boldsymbol{C}_k \in \mathbb{R}_+^{n\times n}$ are given as

$$\boldsymbol{C}^{\mathrm{T}} = [\boldsymbol{c}_1 \cdots \boldsymbol{c}_m]\,, \quad \boldsymbol{C}_{dk} = \operatorname{diag}\left[\boldsymbol{c}_k^{\mathrm{T}}\right] \tag{11}$$

$$\boldsymbol{J} = [\boldsymbol{j}_1 \cdots \boldsymbol{j}_m]\,,\ \boldsymbol{J}_k = \operatorname{diag}\left[\boldsymbol{j}_k\right] \tag{12}$$

where $\boldsymbol{J}_{kh} = \boldsymbol{L}^{h\mathrm{T}}\boldsymbol{J}_k\boldsymbol{L}^h$.

The results of Lemma 1 show that all terms related to the parametrization of the matrix $\boldsymbol{A}_e$ are diagonal matrices.

3 Metzler T-S Fuzzy Observer

The system state estimation assumes the observer to Metzler T-S fuzzy system (1), (2) in the form

$$\begin{aligned}\dot{\boldsymbol{q}}_e(t) = \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\vartheta}(t))(\boldsymbol{A}_i\boldsymbol{q}_e(t) + \boldsymbol{B}_i\boldsymbol{u}(t)) + \\ + \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\vartheta}(t))\boldsymbol{J}_i\boldsymbol{C}(\boldsymbol{q}(t) - \boldsymbol{q}_e(t))\end{aligned} \tag{13}$$

$$\boldsymbol{y}_e(t) = \boldsymbol{C}\boldsymbol{q}_e(t) \tag{14}$$

where $\boldsymbol{q}_e(t) \in \mathbb{R}_+^n$ is the estimation of the system state vector, $\boldsymbol{J}_i \in \mathbb{R}_+^{n\times m}$, $i = 1, \dots, s$ is the set of the observer gain matrices. Since (13), (14) implies

$$\begin{aligned}\dot{\boldsymbol{q}}_e(t) = \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\vartheta}(t))(\boldsymbol{A}_i - \boldsymbol{J}_i\boldsymbol{C})\boldsymbol{q}_e(t) + \\ + \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\vartheta}(t))(\boldsymbol{B}_i\boldsymbol{u}(t) + \boldsymbol{J}_i\boldsymbol{C}\boldsymbol{q}(t))\end{aligned} \tag{15}$$

and for every Metzler matrix $\boldsymbol{A}_i \in \mathbb{M}_{-+}^{n\times n}$ diagonal constraints can be obtained analogously to (8)

$$\begin{cases}\boldsymbol{A}_{\Theta i}(p+h,p) \prec 0, & h = 0 \\ \boldsymbol{A}_{\Theta i}(p+h,p) \succ 0, & h = 1, \dots, n-1 \\ \boldsymbol{A}_{\Theta i}(p+h,p) = \\ \mathrm{diag}\left[a_{i,1+h,1} \cdots a_{i,n,n-h}\, a_{i,1,n-h+1} \cdots a_{i,h,n}\right] \end{cases} \tag{16}$$

the parametrization of $\boldsymbol{A}_{ei} = \boldsymbol{A}_i - \boldsymbol{J}_i\boldsymbol{C} \in \mathbb{M}_{-+}^{n\times n}$ is

$$\boldsymbol{A}_{ei} = \sum_{h=0}^{n-1} \boldsymbol{L}^h \big(\boldsymbol{A}_{\Theta\, i}(p+h,p) - \sum_{k=0}^{m} \boldsymbol{J}_{ikh}\boldsymbol{C}_k\big) \tag{17}$$

and the diagonal matrices $\boldsymbol{J}_{ikh}, \boldsymbol{C}_k \in \mathbb{R}_+^{n\times n}$ are defined as

$$\boldsymbol{C}^{\mathrm{T}} = [\boldsymbol{c}_1 \cdots \boldsymbol{c}_m], \quad \boldsymbol{C}_k = \mathrm{diag}\left[\boldsymbol{c}_k^{\mathrm{T}}\right] \tag{18}$$

$$\boldsymbol{J}_i = [\boldsymbol{j}_{i1} \cdots \boldsymbol{j}_{im}], \; \boldsymbol{J}_{ik} = \mathrm{diag}\left[\boldsymbol{j}_{ik}\right] \tag{19}$$

where $\boldsymbol{J}_{ikh} = \boldsymbol{L}^{h\mathrm{T}}\boldsymbol{J}_{ik}\boldsymbol{L}^h$.

Having in mind constraints (19) it is not hard to establish the following:

Lemma 2 [34] *The Metzler T-S fuzzy observer (13), (14) is stable if there exist positive definite diagonal matrices $\boldsymbol{P}, \boldsymbol{R}_{ik} \in \mathbb{R}_+^{n\times n}$ such that with $\boldsymbol{l}^{\mathrm{T}} = [1 \cdots 1]$ for $i = 1, 2, \dots, s$, $h = 1, 2, \dots, n-1$, $k = 1, 2, \dots, m$,*

$$\boldsymbol{P} \succ 0, \quad \boldsymbol{R}_{ik} \succ 0 \tag{20}$$

$$\boldsymbol{P}\boldsymbol{A}_{\Theta i}(p,p) - \sum_{k=1}^{m} \boldsymbol{R}_{ik}\boldsymbol{C}_k \prec 0 \tag{21}$$

$$\boldsymbol{P}\boldsymbol{L}^h\boldsymbol{A}_{\Theta i}(p+h,p)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=1}^{m} \boldsymbol{R}_{ik}\boldsymbol{L}^h\boldsymbol{C}_k\boldsymbol{L}^{h\mathrm{T}} \succ 0 \tag{22}$$

$$\boldsymbol{P}\boldsymbol{A}_i + \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P} - \sum_{k=1}^{m} (\boldsymbol{R}_{ik}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{C}_k + \boldsymbol{C}_k\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{R}_{ik}) \prec 0 \tag{23}$$

When the above conditions hold, the set of strictly positive $\boldsymbol{J}_i$ is given by (19), where

$$\boldsymbol{J}_{ik} = \boldsymbol{P}^{-1}\boldsymbol{R}_{ik}, \quad \boldsymbol{j}_{ik} = \boldsymbol{J}_{ik}\boldsymbol{l} \tag{24}$$

and $\boldsymbol{A}_{ei} \in \mathbb{M}_{-+}^{n\times n}$ are strictly Metzler and Hurwitz.

Remark 1 *The conditions (21) guarantee that elements on the main diagonals of $\boldsymbol{A}_{ei}$ are strictly negative and the set of LMIs (22) guarantee that the off-diagonal elements of $\boldsymbol{A}_{ei}$ are strictly positive ($\boldsymbol{A}_{ei}$ are strictly Metzler) if $\boldsymbol{A}_i$ are strictly Metzler, $\boldsymbol{C}$ is non-negative and $\boldsymbol{J}_i$ are strictly positive. The Lyapunov matrix inequalities (23) force that all $\boldsymbol{A}_{ei}$ will be Hurwitz. The condition and solution for the existence of observer gains $\boldsymbol{J}_i$ are clearly obtained from Lemma 2.*

4 Ostensible Metzler T-S Systems

In the considered case, the state space equation of a system is also (1) but minimally one system matrix from the set of $\boldsymbol{A}_i \in \mathbb{M}_{-\odot}^{n\times n}$, $i \in \langle 1, s\rangle$, is ostensible Metzler.

Definition 4 *The matrix $\boldsymbol{A} \in \mathbb{M}_{-\ominus}^{n\times n}$ is ostensible Metzler if in the structure $\boldsymbol{A}$ there is at least one negative off-diagonal element, while the number of positive off-diagonal elements of $\boldsymbol{A}$ is predominant and all diagonal elements of $\boldsymbol{A}$ are negative.*

Parametric structure design limits of an ostensible Metzler matrix can be eliminated by using the matrix eigenvalue principle. The essentials of the approach are most easily understood in case of dealing with the following lemma:

Lemma 3 [35] *If for $\boldsymbol{U}, \boldsymbol{V} \in \mathbb{R}^{n\times n}$ is*

$$\boldsymbol{V} = c\boldsymbol{U} + d\boldsymbol{I}_n \tag{25}$$

with $c, d \in \mathbb{R}$, $c \neq 0$ and $\boldsymbol{I}_n \in \mathbb{R}^{n\times n}$, then the eigenvectors of $\boldsymbol{U}$ and $\boldsymbol{V}$ are identical and

$$\eta_k = c\lambda_k + d \tag{26}$$

where η_k, $k = 1, \dots, n$ are eigenvalues of $\boldsymbol{V}$ and λ_k runs over the eigenvalues of $\boldsymbol{U}$,

In such a way, the proposed approach is based on separation of an ostensible Metzler matrix $\boldsymbol{A} \in \mathbb{M}_{-\ominus}^{n\times n}$ to ensure the following is met:

$$\boldsymbol{A} = \boldsymbol{A}_p + \boldsymbol{A}_m \tag{27}$$

where $\boldsymbol{A}_p \in \mathbb{M}_{-+}^{n\times n}$ is strictly Metzler and $\boldsymbol{A}_m \in \mathbb{R}_{-}^{n\times n}$ is constructed as element-wise negative and Hurwitz using Lemma 3. Therefore, this concept transforms the task of synthesis for ostensible Metzler matrices into a task for strictly Metzler ones, for which standard methods are available, parameterizing only the matrix $\boldsymbol{A}_p \in \mathbb{M}_{-+}^{n\times n}$ with its rhombic diagonals.

For such application the approach is reformulated to the following:

Lemma 4 [36] *For ostensible Metzler* $\boldsymbol{A} \in \mathbb{M}_{-\ominus}^{n\times n}$ *there exists a strictly Metzler* $\boldsymbol{A}_p \in \mathbb{M}_{-+}^{n\times n}$ *and an element-wise negative and Hurwitz* $\boldsymbol{A}_m \in \mathbb{R}_{-}^{n\times n}$ *satisfying (27) with relation to positive scalars* $\eta, \delta \in \mathbb{R}_+$ *only if*

$$\boldsymbol{A}_p = \boldsymbol{A}_d + \boldsymbol{A}^+ + \eta\boldsymbol{\Sigma} + p\boldsymbol{I}_n = \boldsymbol{A}_p^\circ + p\boldsymbol{I}_n \quad (28)$$

$$\boldsymbol{A}_m = \boldsymbol{A}^- - \eta\boldsymbol{\Sigma} - p\boldsymbol{I}_n = \boldsymbol{A}_m^\circ - p\boldsymbol{I}_n \quad (29)$$

$$\boldsymbol{A}_d = diag\,[-a_{11} \cdots - a_{nn}]\,,\ \boldsymbol{A}_d + p\boldsymbol{I}_n \prec 0 \quad (30)$$

$$\boldsymbol{A}^+ = \begin{bmatrix} 0 & a_{12}^+ & \cdots & a_{1,n-1}^+ & a_{1n}^+ \\ a_{21}^+ & 0 & \cdots & a_{2,n-1}^+ & a_{2n}^+ \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{n-1,1}^+ & a_{n-1,2}^+ & \cdots & 0 & a_{n-1,n}^+ \\ a_{n1}^+ & a_{n2}^+ & \cdots & a_{n-1,n-1}^+ & 0 \end{bmatrix} \quad (31)$$

$$\boldsymbol{A}^- = \begin{bmatrix} 0 & a_{12}^- & \cdots & a_{1,n-1}^- & a_{1n}^- \\ a_{21}^- & 0 & \cdots & a_{2,n-1}^- & a_{2n}^- \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{n-1,1}^- & a_{n-1,2}^- & \cdots & 0 & a_{n-1,n}^- \\ a_{n1}^- & a_{n2}^- & \cdots & a_{n-1,n-1}^- & 0 \end{bmatrix} \quad (32)$$

$$\begin{array}{ll} a_{ij}^+ = \begin{cases} a_{ij} \text{ if } a_{ij} > 0, \\ 0 \ \text{ if } a_{ij} < 0, \end{cases} & i,j \in \langle 1,n\rangle \\ a_{ij}^- = \begin{cases} a_{ij} \text{ if } a_{ij} < 0, \\ 0 \ \text{ if } a_{ij} > 0, \end{cases} & i \neq j \end{array} \quad (33)$$

$$\begin{array}{l} \lambda_k \in \rho(\boldsymbol{A}_m^\circ) \\ \lambda_o = \max\limits_k(\lambda_k^+ \,|\, \lambda_k^+ = \mathrm{real}(\lambda_k) > 0) \end{array} \quad (34)$$

$$p = \lambda_o + \delta, \qquad \boldsymbol{\Sigma} = \begin{bmatrix} 0 & 1 & \cdots & 1 & 1 \\ 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \quad (35)$$

Note, since any $\boldsymbol{A}_i$ is not strictly Metzler, the system (1) in the related mode is not internally positive.

Remark 2 *If* $\boldsymbol{A}_p^\circ$*,* $\boldsymbol{A}_m^\circ$ *are constructed for any* $\eta \in \mathbb{R}_+$ *as (28), (29), it yields*

$$\boldsymbol{A} = (\boldsymbol{A}_d + \boldsymbol{A}^+ + \eta\boldsymbol{\Sigma}) + (\boldsymbol{A}^- - \eta\boldsymbol{\Sigma}) = \boldsymbol{A}_p^\circ + \boldsymbol{A}_m^\circ \quad (36)$$

Then, evidently, $\boldsymbol{A}_p^\circ$ *takes strictly Metzler structure and* $\boldsymbol{A}_m^\circ$ *is a matrix which diagonal elements are zeros and all its off-diagonal elements are negative.*

Assuming (without loss of generality) that all eigenvalues of $\boldsymbol{A}_m^\circ$ *are distinct then*

$$\boldsymbol{A}_m^\circ - \lambda_o\boldsymbol{I}_n \preceq 0 \quad (37)$$

where λ_o *is defined in (34). Since* $\lambda_o > 0$*, to obtain a stable D-stability region it can be set*

$$\boldsymbol{A}_m = \boldsymbol{A}_m^\circ - (\lambda_o + \delta)\boldsymbol{I}_n = \boldsymbol{A}_m^\circ - p\boldsymbol{I}_n \prec 0 \quad (38)$$

where $\delta \in \mathbb{R}_+$ *is a tuning parameter.*

Introducing the error in the state observations as

$$\boldsymbol{e}(t) = \boldsymbol{q}(t) - \boldsymbol{q}_e(t) \quad (39)$$

then, exploiting (1) and (13), it is obtained

$$\begin{aligned} \dot{\boldsymbol{e}}(t) = &\sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{A}_i(\boldsymbol{q}(t) - \boldsymbol{q}_e(t)) - \\ &- \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{J}_i(\boldsymbol{y}(t) - \boldsymbol{y}_e(t)) \end{aligned} \quad (40)$$

which can be written using (2), (17) as follows

$$\dot{\boldsymbol{e}}(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{A}_{ei}\boldsymbol{e}(t) \quad (41)$$

where, with the above modifications,

$$\begin{array}{ll} \boldsymbol{A}_{ei} = \boldsymbol{A}_i - \boldsymbol{J}_i\boldsymbol{C} & strictly\ Metzler\ mode \\ \boldsymbol{A}_{ei} = \boldsymbol{A}_{pi} - \boldsymbol{J}_i\boldsymbol{C} & ostensible\ Metzler\ mode \end{array} \quad (42)$$

and the above presented approach can be modified to derive the stable ostensible Metzler T-S fuzzy observer.

Theorem 1 *The Metzler T-S fuzzy observer (13), (14) with ostensible Metzler modes is stable if there exist positive definite diagonal matrices* $\boldsymbol{P}, \boldsymbol{R}_{ik} \in \mathbb{R}_+^{n\times n}$ *such that with* $\boldsymbol{l}^{\mathrm{T}} = [1 \cdots 1]$ *for* $i = 1, 2, \ldots, s$*,* $h = 1, 2, \ldots, n-1$*,* $k = 1, 2, \ldots, m$*,*

$$\boldsymbol{P} \succ 0\,, \quad \boldsymbol{R}_{ik} \succ 0 \quad (43)$$

$$\boldsymbol{P}\boldsymbol{A}_{\ominus i}(p,p) - \sum_{k=1}^{m} \boldsymbol{R}_{ik}\boldsymbol{C}_k \prec 0 \quad (44)$$

$$\boldsymbol{P}\boldsymbol{L}^h\boldsymbol{A}_{\ominus i}(p{+}h,p)\boldsymbol{L}^{h\mathrm{T}} - \sum_{k=1}^{m} \boldsymbol{R}_{ik}\boldsymbol{L}^h\boldsymbol{C}_k\boldsymbol{L}^{h\mathrm{T}} \succ 0 \quad (45)$$

$$\boldsymbol{P}\boldsymbol{A}_i + \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P} - \sum_{k=1}^{m} (\boldsymbol{R}_{ik}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{C}_k + \boldsymbol{C}_k\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{R}_{ik}) \prec 0 \quad (46)$$

When the above conditions hold, the set of strictly positive $\boldsymbol{J}_i$ *is given by (19), where*

$$\boldsymbol{J}_{ik} = \boldsymbol{P}^{-1}\boldsymbol{R}_{ik}, \quad \boldsymbol{j}_{ik} = \boldsymbol{J}_{ik}\boldsymbol{l} \quad (47)$$

Proof: Defining the Lyapunov function

$$v(\boldsymbol{e}(t)) = \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}(t) > 0 \quad (48)$$

where $\boldsymbol{P} \in \mathbb{R}_+^{n\times n}$ is positive definite diagonal matrix, then (48) implies for strictly negative of time derivative of Lyapunov function

$$\dot{v}(\boldsymbol{e}(t)) = \dot{\boldsymbol{e}}(t)\boldsymbol{P}\boldsymbol{e}(t) + \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\boldsymbol{e}}(t) \quad (49)$$

Substituting (41) into (49) gives

$$\begin{aligned}&\dot{v}(\boldsymbol{e}(t))\\ &= \boldsymbol{e}^{\mathrm{T}}(t)\sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))(\boldsymbol{P}\boldsymbol{A}_{ei}+\boldsymbol{A}_{ei}^{\mathrm{T}}\boldsymbol{P})\boldsymbol{e}(t)\\ &< 0\end{aligned} \tag{50}$$

$$\boldsymbol{P}\boldsymbol{A}_{ei}+\boldsymbol{A}_{ei}^{\mathrm{T}}\boldsymbol{P} \prec 0 \;\forall i \tag{51}$$

respectively. Since the diagonal matrix representation is necessary, the expression (50) implies the following result:

$$\begin{aligned}&\boldsymbol{P}(\boldsymbol{A}_i-\sum_{k=1}^{m}\boldsymbol{J}_{ik}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{C}_k)+\\ &+(\boldsymbol{A}_i-\sum_{k=1}^{m}\boldsymbol{J}_{ik}\boldsymbol{l}\boldsymbol{l}^{\mathrm{T}}\boldsymbol{C}_k)^{\mathrm{T}}\boldsymbol{P}\prec 0\end{aligned} \tag{52}$$

and with the notation

$$\boldsymbol{R}_{ik} = \boldsymbol{P}\boldsymbol{J}_{ik} \tag{53}$$

(52) implies (46).

Applying the circular shifted structures to cover sets of algebraic constraints then (17) for $h = 0$ implies

$$\boldsymbol{A}_{\Theta\,i}(p,p)-\sum_{k=1}^{m}\boldsymbol{J}_{ik}\boldsymbol{C}_{dk}\prec 0 \tag{54}$$

and multiplying the left side by a positive definite diagonal matrix $\boldsymbol{P}\in\mathbb{R}_{+}^{n\times n}$ then (54) implies

$$\boldsymbol{P}\boldsymbol{A}_i(p,p)_{\Delta}-\sum_{k=1}^{m}\boldsymbol{P}\boldsymbol{J}_{ik}\boldsymbol{C}_k\prec 0 \tag{55}$$

and with the notation (53) then (55) implies (44).

Since $\boldsymbol{J}_{ikh} = \boldsymbol{L}^{h\mathrm{T}}\boldsymbol{J}_{ik}\boldsymbol{L}^{h}$, $\boldsymbol{L}^{h}\boldsymbol{L}^{h\mathrm{T}} = \boldsymbol{I}_n$, pre-multiplying the left side by $\boldsymbol{P}$ and post-multiplying the right side by $\boldsymbol{T}^{h\mathrm{T}}$ then (17) results

$$\begin{aligned}&\boldsymbol{P}\boldsymbol{L}^{h}\boldsymbol{A}_{\Theta\,i}(p+h,p)\boldsymbol{L}^{h\mathrm{T}}-\\ &-\sum_{k=1}^{m}\boldsymbol{P}\boldsymbol{L}^{h}\boldsymbol{L}^{h\mathrm{T}}\boldsymbol{J}_{ik}\boldsymbol{L}^{h}\boldsymbol{C}_k\boldsymbol{L}^{h\mathrm{T}}\succ 0\end{aligned} \tag{56}$$

and with the notation (53) then (56) implies (45). This concludes the proof.

Note, the proposed design conditions have no tuning parameters, tuning parameters are occurred only in separation of a ostensible Metzler matrices. The invertibility of $\boldsymbol{P}$ is guaranteed by the fact that $\boldsymbol{P}\in\mathbb{R}_{+}^{n\times n}$ is positive definite diagonal matrix.

5 Illustrative Example

The example reflects the ostensible Metzler T-S equations (1), (2), where for $s = 2$, $n = 3$, $m = 2$

$$\boldsymbol{A}_1=\begin{bmatrix}-0.272 & 1.94 & 1.45\\ 0.058 & -3.96 & 0.10\\ 0.100 & 0.08 & -2.91\end{bmatrix},\;\rho(\boldsymbol{A}_1)=\begin{Bmatrix}-0.1863\\ -2.9634\\ -3.9923\end{Bmatrix}$$

$$\boldsymbol{A}_2=\begin{bmatrix}-0.272 & 1.94 & -0.45\\ 0.058 & -3.14 & 0.10\\ 0.100 & -0.04 & -2.91\end{bmatrix},\;\rho(\boldsymbol{A}_2)=\begin{Bmatrix}-0.2474\\ -2.9454\\ -3.1293\end{Bmatrix}$$

$$\boldsymbol{B}_1=\boldsymbol{B}_2=\begin{bmatrix}0.50 & 1.00\\ 1.00 & 0.90\\ 0.70 & 1.10\end{bmatrix},\;\boldsymbol{C}=\begin{bmatrix}1.1 & 0 & 0\\ 0 & 1.5 & 0\end{bmatrix}$$

Since $\boldsymbol{A}_2$ is ostensible Metzler

$$\boldsymbol{A}_{2d} = \operatorname{diag}\left[-0.272\;\; -3.14\;\; -2.91\right]$$

$$\boldsymbol{A}_2^{+}=\begin{bmatrix}0 & 1.94 & 0\\ 0.058 & 0 & 0.10\\ 0.100 & 0 & 0\end{bmatrix},\;\boldsymbol{L}=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$$

$$\boldsymbol{A}_2^{-}=\begin{bmatrix}0 & 0 & -0.45\\ 0 & 0 & 0\\ 0 & -0.04 & 0\end{bmatrix},\;\boldsymbol{\Sigma}=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix},\;\boldsymbol{l}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

Setting $\mu = 0.005$ then

$$\boldsymbol{A}_{2m}^{\circ}=\begin{bmatrix}0 & -0.005 & -0.455\\ -0.005 & 0 & -0.005\\ -0.005 & -0.045 & 0\end{bmatrix}$$

$$\rho(\boldsymbol{A}_{2m}^{\circ})=\{-0.0642\quad 0.0321\pm 0.0238\,\mathrm{i}\}$$

which implies $\lambda_0 = 0.0321$ and with $\delta = 0.003$ gives

$$\boldsymbol{A}_{2m}=\begin{bmatrix}-0.0351 & -0.0050 & -0.4550\\ -0.0050 & -0.0351 & -0.0050\\ -0.0050 & -0.0450 & -0.0351\end{bmatrix}$$

$$\rho(\boldsymbol{A}_{2m})=\{-0.0993\quad -0.0030\pm 0.0238\,\mathrm{i}\}$$

$$\boldsymbol{A}_{2p}=\begin{bmatrix}-0.2369 & 1.9450 & 0.0050\\ 0.0630 & -3.1049 & 0.1050\\ 0.1050 & 0.0050 & -2.8749\end{bmatrix}$$

$$\rho(\boldsymbol{A}_{2p})=\{-0.1919\quad -2.9057\quad -3.1191\}$$

To reflect diagonal LMIs structures, the representations of $\boldsymbol{C}$ are given as

$$\boldsymbol{C}_1 = \operatorname{diag}\left[1.1\quad 0\quad 0\right],\;\boldsymbol{C}_2 = \operatorname{diag}\left[0\quad 0.5\quad 0\right]$$

and the representations of $\boldsymbol{A}_1$, $\boldsymbol{A}_{2p}$ are

$$\boldsymbol{A}_{\Theta\,1}(p,p) = \operatorname{diag}\left[-0.272\; -3.96\; -2.91\right]$$

$$\boldsymbol{A}_{\Theta\,1}(p+1,p) = \operatorname{diag}\left[0.058\; 0.08\; 1.45\right]$$

$$\boldsymbol{A}_{\Theta\,1}(p+2,p) = \operatorname{diag}\left[0.10\; 1.94\; 0.10\right]$$

$$\boldsymbol{A}_{\Theta\,2p}(p,p) = \operatorname{diag}\left[-0.2369\; -3.1049\; -2.8749\right]$$

$$\boldsymbol{A}_{\Theta\,2p}(p+1,p) = \operatorname{diag}\left[0.063\; 0.005\; 0.005\right]$$

$$\boldsymbol{A}_{\Theta\,2p}(p+2,p) = \operatorname{diag}\left[0.105\; 1.945\; 0.105\right]$$

The LMIs taken into account are those of Theorem 1 nad SeDuMi tolbox, [37], determines the feasible solution

$$\boldsymbol{P} = \operatorname{diag}\left[0.8101\; 0.4452\; 0.6971\right]\succ 0$$

$$\boldsymbol{R}_{11} = \operatorname{diag}\left[0.6951\; 0.0133\; 0.0342\right]\succ 0$$

$$\boldsymbol{R}_{12} = \operatorname{diag}\left[0.7080\; 0.3038\; 0.0224\right]\succ 0$$

$$\boldsymbol{R}_{21} = \mathrm{diag}\,[0.6308\,0.0147\,0.0410] \succ 0$$

$$\boldsymbol{R}_{22} = \mathrm{diag}\,[0.7060\,0.3200\,0.0014] \succ 0$$

$$\boldsymbol{J}_1 = \begin{bmatrix} 0.8580 & 0.8740 \\ 0.0299 & 0.6823 \\ 0.0490 & 0.0322 \end{bmatrix}, \ \boldsymbol{J}_2 = \begin{bmatrix} 0.7787 & 0.8715 \\ 0.0329 & 0.7187 \\ 0.0587 & 0.0021 \end{bmatrix}$$

which, with the positive gains, guarantees strictly Metzler and Hurwitz matrices

$$\boldsymbol{A}_{e1} = \begin{bmatrix} -1.0442 & 0.8912 & 1.45 \\ 0.0311 & -4.7788 & 0.10 \\ 0.0559 & 0.0414 & -2.91 \end{bmatrix}$$

$$\rho(\boldsymbol{A}_{e1}) = \{-0.9936 \ -2.9518\, c - 4.7875\}$$

$$\boldsymbol{A}_{e2p} = \begin{bmatrix} -0.9377 & 0.8992 & 0.0050 \\ 0.0334 & -3.9673 & 0.1050 \\ 0.0521 & 0.0025 & -2.8749 \end{bmatrix}$$

$$\rho(\boldsymbol{A}_{e2p}) = \{-0.9269 \ -2.8771 \ -3.9760\}$$

and ostensible Metzler and Hurwitz matrix

$$\boldsymbol{A}_{e2} = \begin{bmatrix} -0.9728 & 0.8942 & -0.4500 \\ 0.0284 & -4.0024 & 0.1000 \\ 0.0471 & -0.0425 & -2.9100 \end{bmatrix}$$

$$\rho(\boldsymbol{A}_{e2}) = \{-0.9746 \ -2.9052 \ -4.0055\}$$

The ostnesible Metzler T-S fuzzy observer design task is transformed into an equivalent form and leads to a standard design task. Based on the diagonalized matrix representations it is verified that the design conditions allow existence of positive $\boldsymbol{J}_i$ for $i \in \langle 1, 2 \rangle$ such that Lyapunov principle establishes asymptotic stability of the observer equilibrium.

Due to the diagonal principle of stabilization, these results are relevant when the fuzzy observer uses the measured premise variables.

6 Concluding Remarks

Observing the properties of the state space model suggests that it is possible to obtain a finite number of LMIs in the design for a strictly Metzler a Hurwitz fuzzy observer dynamics matrices with positive gains. To obtain a solution for the ostensible structures of Metzler matrices, the design method is modified to provide the feasibility of a set of LMIs, where the conditions take into account the separation of the ostensible structures of Metzler matrices and the resulting modified parametric constraints. The approach favors standard LMI representation to handle the stability of the fuzzy observer, where the proposed conditions guarantee asymptotic stability. The separation takes into account those performance requirements that are associated with the positivity of the system's trajectories. Since ostensible Metzler TS fuzzy systems are not internally positive, the initial states of the observer should be chosen appropriately to allow for non-negative observer error.

The proposed approach provides a platform for the development of ostensible Metzler T-S fuzzy observers that are resilient to parameter uncertainties and perturbations. These topics are currently under investigation and will be addressed in the future. It is possible to envisage in the future control structures of T-S fuzzy networks and cooperative control of T-S fuzzy multi-agent systems for agents whose system matrices are ostensible Metzler.

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