

# Method for Determining the Conditions for Ensuring Stable Rearward Movement of a Semi-trailer Combination with Non-turning Semi-trailer Wheels

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*Abstract:* - The efficiency of organising freight transport and using motor transport in a Smart City depends on a set of its properties, which in the process of operation determine its suitability for use in given operating conditions. Vehicles have different overall dimensions and designs, which determine their maneuverability and directional stability, their ability to make a curve-line movement in the city, as well as to move safely in reverse. Nowadays, tractor-trailer combinations with non-turning semi-trailer wheels are widely used for freight transport. Such a scheme of vehicle construction does not ensure its directional stability when reversing, which can lead to the folding of the tractor and semi-trailer and loss of mobility of the combination. In the absence of additional special systems, the driver controls the rearward movement of the road train using the steering wheel and rear-view mirrors, and the accuracy of delivery to the object depends on the level of the driver's training. The research of driver's reaction time spent on situation assessment, decision making, and realisation has been carried out. It has been revealed that with manual control, there are difficulties connected with training of drivers to estimate the parameters of movement and control of the road train, in particular, with the process of parking and its accurate delivery to the object. With the help of the developed mathematical model of the rearward movement of the road train and the proposed new turn control law, the research of maneuvering of the road train during its delivery to the object has been carried out and the results confirming the possibilities of building an automated system allowing to reduce the influence of the human factor on the parking process, as well as to reduce the time required to perform the maneuver have been obtained. The conducted analysis of system stability with the help of Routh–Hurwitz criterion has determined the conditions of providing stability of the system moving in reverse. The method of controlling the backward movement of the road train and the design of the device realising it has been developed. The results of simulation modeling allowed us to find the necessary values for the control law for the movement of road trains of different lengths, as well as the preferred variants of the initial positions of the road train for the realisation of its precise delivery to the object.

*Key-Words:* - Smart City, Maneuverability, Road Train, Instability, Semi-road train, Laplace operator, Routh–Hurwitz criterion, Method.

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## 1. Introduction

### 1.1 Motivation and Goal of the Paper

The creation of industrial systems of the Internet of Things, Smart City systems and Smart Intersection systems implies the solution of many problems. These include network problems, logistics

problems, optimisation problems, traffic control problems, and others.

When solving the problems of logistics and debugging the transport system operation, it is necessary to solve a number of problems, in particular, the organisation of large-size trucks and road trains movement within the city.

In the conditions of the Smart City system, there is a subsystem of smart transport. It includes a number of subsystems that provide logistic tasks, signal transmission and traffic safety. The vehicle, which is the subject of research in this paper, has different maneuverability and directional stability when moving forward and backward, which complicates logistic tasks in the Smart City system and can lead to a violation of traffic safety. In particular, there is a problem in the delivery of the road train for loading: it is necessary to accurately deliver the side of the road train to the platform for loading or loading the cargo into the mine. For this purpose, a method must be devised to ensure safe maneuvering on the smallest possible footprint.

Road trains with semi-trailers are mostly used for the transport of long goods. The basic regularities of motion of such road trains with steerable and unsteerable wheels of semi-trailers on different types of turns are now practically important, [1], [2], [3], [4], [5], [6]. However, the majority of scientific works are related to the study of their maneuverability and stability when moving forward. Up to now, the question of estimating the maneuvering properties of a road train moving in reverse remains insufficiently developed, [7], [8], [9], [10], [11], [12], [13], [14]. In order to deploy a road train on narrow roads, it has to maneuver using both forward and reverse movements to accurately deliver to the object. It is difficult to steer the combination when reversing, as even a slight turn of the steering wheel causes the combination to fold. When reversing, the position of the semi-trailer must be monitored and if the direction of travel of the semi-trailer changes slightly, it must be immediately leveled in the axis of travel. To steer the semi-trailer, turn the steering wheel in the direction of the change of direction. It can also be leveled by moving it forward. However, this maneuvering of the vehicle does not allow the semi-trailer to turn toward the object accurately and quickly. The task is especially difficult when the base length of the semi-trailer exceeds the base length of the tractor.

The main reason for the development of unmanned means of cargo transport was the need to exclude humans from driving the vehicle, due to the large influence of the human factor on the driving process. Currently, the majority of road trains are manually driven. Manual driving poses difficulties in training drivers to drive the truck and, in particular, in the process of parking. To make the driver's job easier, special markings on the maneuvering area can be used to facilitate a precise approach to the unloading point. There are

tasks of positioning the road train so that the wheels on the left or right side of the vehicle are on the same line. This line is drawn on the road.

This line is called the collision line. The difficulty of accurately reversing of a road train to an object indicates the need to automate the process of driving a road train, which would reduce the influence of human factor on the process of parking, as well as reduce the time required to perform the maneuver, [5]. At the moment there are no systems of automatic control of a road train, which could provide the process of perpendicular parking of a semi-trailer in a given sector. Therefore, it is urgent to create such a system that provides autonomous performance of this operation. Such a system is intended for use on tractors with semi-trailers, as well as on other types of lorry transport having a semi-trailer in its composition. The movement of the semi-trailer behind the tractor has some peculiarities, which must be taken into account when controlling the movement. The main feature is that the wheel axle of the semi-trailer moves at a smaller radius than that of the tractor. This difference depends on three main parameters: the length of the tractor; the length of the semi-trailer; and the angle between the axles of the tractor and the semi-trailer. In automated and non-automated traffic control systems, the driver is involved. Based on the information received by the senses, the driver acts on the tractor controls to perform a certain maneuver. The driver receives most of the information visually. The driver determines the position of the road train relative to the collision line. The time taken by the driver to assess the situation, the position of the road train relative to the object of approach, and to make and implement a decision is called the driver's reaction time, we suggest calculating the time value by the formula, [1]:

$$t_r = t_1 + t_2,$$

where:  $t_1$  – latent time, i.e. the time elapsing from the moment when the driver fixes a certain situation to the beginning of the implementation of the decision;  $t_2$  – motor reaction time (turning the steering wheel);  $t_r$  - reaction time, which depends on the driver's experience, fatigue, psychological state, situation, speed and varies widely.

The tasks performed by the driver at this time can be divided into logical and reflexive tasks. Logical solutions arise in the process of thinking. In reflexive actions, the role of conditioned reflex - automaticity of behavior developed on the basis of experience - is great.

The driver's reaction in non-automated traffic control systems is complex. He has to evaluate the situation, choose a certain solution from several possible solutions, and perform a motor action. Therefore, the driver's reaction time can vary widely. Numerous statistical studies have shown that with probability  $p = 0.9$  this time is between 0.1 and 1.5 seconds. Significant reaction time leads to untimely or erroneous actions, and thus to an incorrect approach to the object. It is particularly difficult to control the movement of a road train if the semi-trailer is long-wheelbase and its width significantly exceeds the width of the tractor. Some vehicle control systems depend on supporting infrastructure (e.g. the use of traffic management systems, sensors embedded in the road, Smart Traffic Lights). Through the use of various measurement tools, video cameras, satellite navigation systems, and radar, advanced technologies make it possible to simulate human presence at the level of decision-making about vehicle orientation and speed. There are two main directions for creating such systems: complex automation of a car and automation of individual modes of vehicle movement (parking, traffic jams, driving on highways, passing through 'smart' traffic lights).

Modern cars contain electronic driver assistance systems with varying degrees of automation, such as stability control, collision avoidance, cruise control, parking distance control, and others. Electronic systems provide part of the vehicle control functions, such as automatic control of speed, acceleration, turning, and parking maneuvering. When solving logistic tasks, it is necessary to take into account the movement parameters of large vehicles. They can make maneuvers moving backward, for example, in a warehouse for loading, or they can move at an intersection and create an obstacle on the road for other types of transport. It is especially important to take into account the maneuvers of such transport vehicles in the case of the automatic type of control - with the help of auto-pilot because the wrong trajectory of both the vehicle and its semi-trailer can lead to catastrophic consequences, [1].

The results obtained in this work can be used in the development of road trains, including unmanned ones, which have non-turning wheels of semi-trailers.

## 1.2 Work-related Analysis

Vehicle maneuverability is one of its main operational properties, which are described in a number of works.

The paper [1] presents the developed mathematical models of road train movement in a given direction in reverse and the results of their study.

In [2], decision making about drivers' behavior when receiving speed recommendations related to energy consumption and safety-free speed is modeled through vehicle-infrastructure communication.

In [3], the results of modeling the rearward movement of a trailed road train are presented and technical solutions for automatic folding prevention are proposed, which allows for automated parallel parking in combination with a path-planning algorithm.

In [4], the reverse motion of an articulated vehicle, namely a tractor-trailer with a single trailer on the axle, is analyzed and a fully autonomous driving system is developed that allows reverse parking in the presence of static obstacles.

In [5], it is shown how a vehicle with passive trailers can be easily driven using the proposed driver assistance system and traffic control scheme. Since the keypad is an optional device for the driver assistance system, the proposed scheme can be implemented using conventional trucks without many hardware modifications. A manual push-pull control strategy is established. A kinematic scheme of a vehicle with trailers for push-pull control is proposed.

In [6] it is shown that the complexity of controlling a road train is due to pronounced nonlinearities, as well as instability of the control object when reversing, often leading to the phenomenon known as a folding knife.

In [7] based on kinematic and dynamic models three control approaches for dynamic stabilization in a road train configuration are proposed, as well as a methodology for tuning the control gains using three possible actuators.

Paper [8] presents the results of developing a path tracking and cascade controller for controlling an unmanned tractor-trailer vehicle during reversing.

In [9], a deep learning method based on an auto-vehicle model is proposed to determine the parameters of front and rear stiffness of an auto-vehicle when cornering.

In [10], taking into account drifting methods applicable to front-wheel drive (FWD) drivetrain configurations, the cornering balance was calculated using a car model with front-wheel drive and rear wheels 'locked' at zero angular velocity with a handbrake applied using a controller.

In [11], a V2I architecture is proposed that can operate in both autonomous and manually controlled vehicles and coordinate their movement based on V2I communication.

In [12], the reverse motion of a vehicle and trailer combination is investigated. A single-vehicle path model is used with a quasi-static tyre model to develop a simple feedback linear controller that can provide a stable reversing motion along a straight path.

In [13], technical solutions are proposed to control the reversing motion of a trailed road train. For this purpose, two feedback controllers are developed that support the driver with auto-automatic steering inputs in different situations.

In [14], the problem of path tracking control of a mobile robot with two trailers is solved, for which a path tracking control algorithm (using LOS (Line of sight) method and PID (Proportion, differential and integral) control algorithm) is proposed.

The analysis of the literature has shown that no one deals with the solution of such a problem by the method proposed in this paper, namely, bringing an unstable system to a stable controlled motion along the guideway.

The goal of our research is to improve the efficiency and safety of traffic control in the system Smart City by reducing the maneuvering time and providing accurate accident-free rearward delivery of the road train to the object or to the object.

## 2 Theoretical Justification of Road Train Maneuverability

Let us consider the initial position of the semi-trailer train with the wheels of the semi-trailer not rotated relative to the frame before the beginning of its movement back to the object. Let us replace the real wheels of the road train links with conditional average reduced wheels located in the longitudinal planes of the links, which allows us to consider kinematic links in the form of a 'bicycle' scheme, [1], [6], [8].

Let the steering wheels of the tractor are rotated relative to its frame by angle  $\varphi_0$ , the frame of the tractor is rotated by angle  $\beta_0$ , and the frame of the semi-trailer is rotated relative to the longitudinal line of the collision by angle  $\alpha_0$ . The wheels of the semi-trailer are displaced relative to the longitudinal line of the collision by the value  $Z_0$  (Figure 1), [1]. Let us make the following assumptions: the motion of the road train is flat, parallel to a fixed support surface, with constant

speed; there is no lateral drift of elastic tyres; the influence of the suspension on the trajectory is not taken into account. Let us use the known system of equations describing the motion of the road train. The velocity of the road train links is given by the vector of the tractor's driving wheels velocity  $\vec{V}$ .

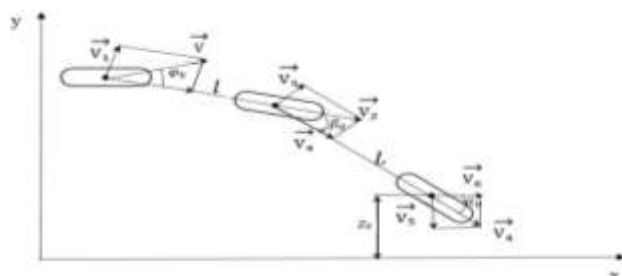


Fig. 1: Kinematic diagram characterizing the position of the vehicle before reversing (X axis coincides with the collision line), [1]

Determine the values of projections of velocity vectors  $V_1...V_6$  (Figure 1), [1]:

$$\begin{aligned} V_1 &= V \cdot \sin\varphi; \\ V_2 &= V \cdot \cos\varphi; \\ V_3 &= V \cdot \cos\varphi \cdot \sin\beta; \\ V_4 &= V \cdot \cos\varphi \cdot \cos\beta; \\ V_5 &= V \cdot \cos\varphi \cdot \cos\beta \cdot \sin\alpha; \\ V_6 &= V \cdot \cos\varphi \cdot \cos\beta \cdot \cos\alpha. \end{aligned}$$

Let's write down the equations of linkages of the road train:

$$\left\{ \begin{aligned} \frac{d\beta}{dt} &= -\frac{V_1}{l} + \frac{V_3}{L}; \\ \frac{d\alpha}{dt} &= -\frac{V_3}{L}; \\ \frac{dZ}{dt} &= V_5; \\ \frac{dx}{dt} &= V_6, \end{aligned} \right.$$

where:  $l$  – tractor base length;

$L$  – semi-trailer base length;

$\frac{d\beta}{dt}$  – angular velocity of folding of the tractor frame with the semi-trailer frame;

$\frac{d\alpha}{dt}$  – angular velocity of the semi-trailer axle to the longitudinal collision line;

$\frac{dZ}{dt}$  – rate of change of displacement of semi-trailer wheels relative to the longitudinal collision line;

$\frac{dx}{dt}$  – wheel speed of the semi-trailer to the object.

Taking into account the velocity values, the equations of motion of the links can be rewritten in the following form [1], [2]:

$$\begin{cases} \dot{\beta} = -\frac{V \cdot \sin \varphi}{l} + \frac{V \cdot \cos \varphi \cdot \sin \beta}{L}; \\ \dot{\alpha} = \frac{V \cdot \cos \varphi \cdot \sin \beta}{L}; \\ Z = V \cdot \cos \varphi \cdot \cos \beta \cdot \sin \alpha; \\ X = V \cdot \cos \varphi \cdot \cos \beta \cdot \cos \alpha. \end{cases} \quad (1)$$

In order to provide control over the reversing movement of the road train to the object in reverse, the system of equations (1) is supplemented with a feedback equation, assuming that the control over  $\varphi(t)$  should depend on the values  $\beta(t)$ ,  $\alpha(t)$ ,  $Z(t)$  measured both in the initial position of the road train relative to the object before the start of movement and during its movement. Taking into account the assumptions made, let us choose a linear law of controlling the rotation of the steering wheels of the tractor, different from the one used in [1].

$$\varphi(t) = K_1 \cdot \beta(t) + K_2 \cdot \alpha(t) + K_3 \cdot Z(t), \quad (2)$$

where  $K_1$ - gain by  $\beta(t)$ ;  $K_2$ - gain by  $\alpha(t)$ ;  $K_3$ - gain by  $Z(t)$ .

Then the complete system of equations of motion of the road train taking into account the control will take the following form:

$$\begin{cases} \dot{\beta} = -\frac{V \cdot \sin \varphi}{l} + \frac{V \cdot \cos \varphi \cdot \sin \beta}{L}; \\ \dot{\alpha} = \frac{V \cdot \cos \varphi \cdot \sin \beta}{L}; \\ Z = V \cdot \cos \varphi \cdot \cos \beta \cdot \sin \alpha; \\ X = V \cdot \cos \varphi \cdot \cos \beta \cdot \cos \alpha; \\ \varphi = K_1 \cdot \beta + K_2 \cdot \alpha + K_3 \cdot Z. \end{cases} \quad (3)$$

Thus, we have obtained a mathematical model of the movement of a road train with non-turning wheels relative to the frame of a semi-trailer in reverse to the object. Let us determine the necessary and sufficient conditions for finding the coefficients  $K_1 \dots K_3$ . For this purpose we linearise the system of equations (3), taking into account that at small values of angles  $\cos \varphi=1$ ,  $\sin \varphi=1$ ,  $\cos \beta=1$ ,  $\sin \beta=\beta$ ,  $\cos \alpha=1$ ,  $\sin \alpha=\alpha$ . Let us study the system of equations describing the motion of the road train along the Y axis only, as well as its angular displacements.

Then the system of equations (3) can be represented in the following form:

$$\begin{cases} \dot{\beta} = -\frac{V \cdot \varphi}{l} + \frac{V \cdot \beta}{L}; \\ \dot{\alpha} = -\frac{V \cdot \beta}{L}; \\ Z = V \cdot \alpha; \\ \varphi = K_1 \cdot \beta + K_2 \cdot \alpha + K_3 \cdot z. \end{cases} \quad (4)$$

To simplify the calculation of automatic control systems, the operator method of description is used. The equations of dynamics are written in the form of images of functions obtained by means of the direct Laplace transform (operator form of equation writing).

Such transformations allow us to pass from differential equations to algebraic equations.

Let us represent the system (4) by means of Laplace transforms with zero conditions, [1]:

$$\sigma(s) = \int_0^t \sigma(t) \cdot e^{-st} dt.$$

The image  $\sigma(s)$  of the original  $\sigma(t)$  is a function of the complex variable  $s$  given by the integral:

Using the substitution rule, let us exclude the variables  $\varphi$ ,  $\beta$ ,  $\alpha$  from the system (5). To do this, we first substitute expression (2) into the first equation of system (5), and then substitute expressions for  $\beta$ ,  $\alpha$  into the second and third equations of system (5), respectively. Let us exclude from (5) the variables  $\varphi$ ,  $\beta$ ,  $\alpha$ .

Such transformations allow us to pass from differential equations to algebraic equations.

In this case, the differentiation operation (original)  $\frac{d^n \sigma}{dt^n}$  is replaced by an image  $s^n \sigma(s)$ , integration operation  $\int_0^t \sigma(t) dt$  – depiction  $\frac{\sigma(s)}{s}$  etc.

Let us represent the system (4) by means of Laplace transformations with zero conditions:

$$\begin{cases} s\beta = -\frac{V \cdot \varphi}{l} + \frac{V \cdot \beta}{L} + \beta_0; \\ s\alpha = -\frac{V \cdot \beta}{L} + \alpha_0; \\ sZ = V \cdot \alpha + Z_0; \\ \varphi = K_1 \cdot \beta + K_2 \cdot \alpha + K_3 \cdot Z. \end{cases} \quad (5)$$

Using the rule of substitution, let us exclude the variables  $\varphi$ ,  $\beta$ ,  $\alpha$  from the system (5). To do this, we first substitute the expression (2) into the first equation of the system (5), and then substitute the expressions for  $\beta$ ,  $\alpha$  into the second and third equations of the system (5), respectively. Excluding from (5) the variables  $\varphi$ ,  $\beta$ ,  $\alpha$ .

$$Z = \frac{\frac{V^2}{L} \beta_0}{\left[ s^3 + \left( \frac{V}{L} K_1 - \frac{V}{L} \right) s^2 + \frac{V^2}{L \cdot l} K_2 s + \frac{V^3}{L \cdot l} K_3 \right]} + \frac{V \cdot \left( s + \frac{V}{L} K_1 - \frac{V}{L} \right) \alpha_0}{\left[ s^3 + \left( \frac{V}{L} K_1 - \frac{V}{L} \right) s^2 + \frac{V^2}{L \cdot l} K_2 s + \frac{V^3}{L \cdot l} K_3 \right]} + \frac{\left[ s^2 + \left( \frac{V}{L} K_1 - \frac{V}{L} \right) s + \frac{V^2}{L \cdot l} K_2 \right] \cdot Z_0}{\left[ s^3 + \left( \frac{V}{L} K_1 - \frac{V}{L} \right) s^2 + \frac{V^2}{L \cdot l} K_2 s + \frac{V^3}{L \cdot l} K_3 \right]} \quad (6)$$

Since a control system with a linear control law of the front wheels of the tractor is chosen to control the movement of the tractor, we use the Routh–Hurwitz criterion to assess the conditions of its stability, which refers to algebraic stability criteria that impose restrictions on the coefficients of the characteristic equation. The characteristic equation of the considered closed system is the denominator in equation (6) and has the form:

$$s^2 + \left( \frac{V}{L} \cdot K_1 - \frac{V}{L} \right) \cdot s^2 + \frac{V^2}{L \cdot l} \cdot K_2 + \frac{V^3}{L \cdot l} \cdot K_3 = 0. \quad (7)$$

It can be visualized in a different way:

$$a_0 s^3 + a_1 s^2 + a_2 s^2 + a_0 = 0, \quad (8)$$

where:  $a_0 = 1$ ;

$$a_1 = \frac{V}{L} \cdot K_1 - \frac{V}{L}; a_2 = \frac{V^2}{L \cdot l} \cdot K_2; a_3 = \frac{V^3}{L \cdot l} \cdot K_3.$$

Since the characteristic equation (7) is determined by (8),  $\alpha_0 > 0$ , it is necessary and sufficient for the stability of the linear control system that the three Routh–Hurwitz determinants are positive. For the considered characteristic equation of the third order, the determinant will have the following form:

$$\Delta_2 = \begin{vmatrix} a_1 & a_2 & 0 \\ a_0 & a_2 & 0 \\ 0 & a_1 & a_2 \end{vmatrix}.$$

All diagonal minors of the Routh–Hurwitz determinant must be greater than zero, i.e.:

$$\begin{aligned} \Delta_1 &= a_0 > 0; \\ \Delta_2 &= \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} = a_1 \cdot a_2 - a_0 \cdot a_2 > 0; \\ \Delta_3 &= \begin{vmatrix} a_1 & a_3 & 0 \\ a_0 & a_2 & 0 \\ 0 & a_1 & a_3 \end{vmatrix} = a_3 \cdot \Delta_2 > 0, \text{ hence, } a_3 > 0. \end{aligned}$$

Hence, in this case not only the positivity of all coefficients of the characteristic equation is required, but also the observance of the condition:

$$\Delta_2 > 0; a_0, a_1 > 0, a_2 > 0, a_3 > 0; \\ a_1 \cdot a_2 > a_0 \cdot a_3.$$

Whence it follows that  $K_1 > 0, K_2 > 0, K_3 > 0$ .

From the stability conditions  $\Delta_1$  should be greater than zero, i.e.:

$$\frac{V}{l} \cdot K_1 - \frac{V}{L} > 0.$$

Hence,

$$K_1 > \frac{l}{L}.$$

The determinant  $\Delta_2$  is found as follows:

$$\left( \frac{V}{l} \cdot K_1 - \frac{V}{L} \right) \cdot \left( \frac{V^2}{L \cdot l} \cdot K_2 \right) - \frac{V^3 \cdot K_3}{l \cdot L} > 0.$$

Hence

$$\left( K_1 - \frac{l}{L} \right) \cdot \frac{K_2}{L} > K_3.$$

Thus, in order to ensure a stable movement of a semi-trailer combination with wheels not rotated relative to the frame of the semi-trailer in reverse to the object at the selected law of controlling the rotation of the front steerable wheels of the tractor (2), the following conditions must be met:

$$\begin{cases} \left( K_1 - \frac{l}{L} \right) \cdot \frac{K_2}{L} > K_3 > 0; \\ K_1 > \frac{l}{L}; \\ K_2 > 0. \end{cases} \quad (9)$$

Fulfillment of the obtained conditions of stability of operation on the movement will provide the road train with stable movement along the longitudinal collision line from different initial positions of its location relative to the collision line. In this case, the time of arrival to the stable motion will be the less, the more successfully the proportionality coefficients  $K_1 \dots K_3$  will be selected and depends, as it can be seen from (9), on the geometrical dimensions of the tractor and semi-trailer. For different tractors and semi-trailers, the coefficients will be different.

Let us again turn to the system of equations (3). There are five unknowns in the system (3) of five equations. However, it is not possible to solve the system of equations by direct integration. To find

solutions, we can use a method of numerical integration of the system of differential equations with high accuracy, for example, the Runge-Kutta method. The theoretical studies carried out earlier can serve for the development of algorithms for solving the set problems. In addition, they have shown that the road train can move to the object in different ways.

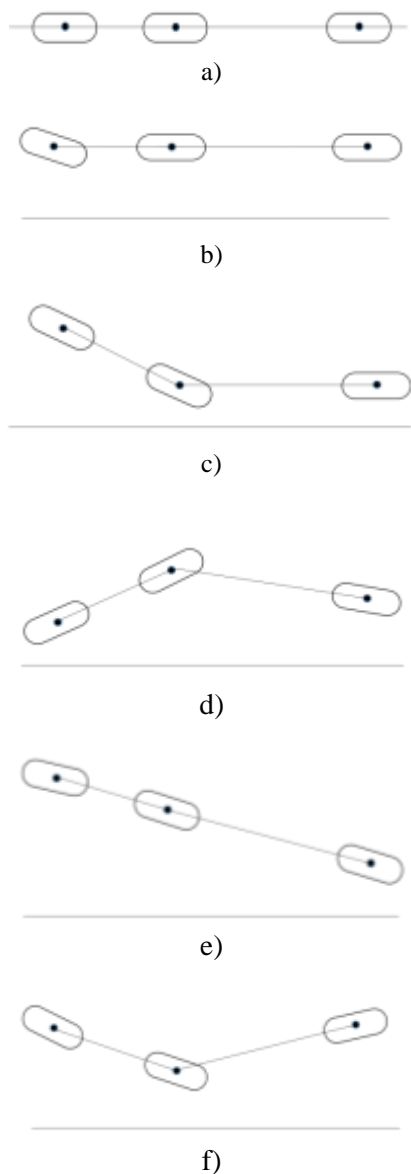


Fig. 2: Calculated cases of road train positioning relative to the collision line: a)  $\beta_0=0, \alpha_0=0, Z_0=0$ ; b)  $\beta_0=0, \alpha_0=0, 0 < Z_0 \leq 2$  m; c)  $0 < \beta_0 \leq \pi/3, \alpha_0=0, Z_0=0$ ; d)  $-\pi/3 \leq \beta_0 \leq -\pi/6, \pi/6 \leq \alpha_0 \leq \pi/3, -0,8 \text{ m} \leq Z_0 \leq 0,8 \text{ m}$ ; e)  $\beta_0=0, \pi/6 \leq \alpha_0 \leq \pi/3, 0 \leq Z_0 \leq 1,5 \text{ m}$ ; f)  $0 \leq \beta_0 \leq \pi/3, -\pi/3 \leq \alpha_0 \leq -\pi/6, 0 < Z_0 \leq 1,5 \text{ m}$

At one ratio of coefficients, the road train is able to come to a stable movement along the collision

line for a short time, at another - the road train will make oscillatory movements relative to the collision line for a long time before it comes to a stable movement along the collision line. In this paper, we consider the scheme of a specific road train, for which the proportionality coefficients are chosen based on the geometric dimensions of the tractor with the base from the front wheels to the support point of the semi-trailer ( $l = 5.2$  m) on the tractor and semi-trailer with the base taking into account the conditions (9). The study of the character of maneuvering for the case of its approach to the object from one possible initial position relative to the collision line does not allow us to fully evaluate the work of the control system with the adopted control law (2). Therefore, it is necessary to analyse the rearward movement of the road train from other possible initial positions relative to the collision line. By investigating the system (3) under different initial conditions, taking into account the calculated limitation on the steering angle of the driven wheels of the road train and the assumptions made, the most unfavorable variants of the position of the road train have been determined, at which the control system cannot bring the road train to a stable movement along the collision line in a relatively short time. Six variants of possible initial positions of the road train relative to the collision line are shown in Figure 2, [1].

The nature of change of parameters  $\beta(t), \alpha(t), Z(t)$  for all considered variants of the initial placement of the road train is shown in Figure 3, Figure 4 and Figure 5.

In the first variant (Figure 2(a)) the wheels are on the longitudinal line of the collision, i.e.:  $\beta = 0, \alpha = 0, Z = 0$ . At this initial position, the road train will move steadily along the collision line.

In the second variant (Figure 2(b)) the road train is stretched in a line, having  $\beta = 0, \alpha = 0$  and be at a distance of up to 2 m from it. Calculations have shown that at initial values of displacements  $0 < Z \leq 2 \text{ m}$  the road train will always move steadily along the line of approach to the object. As can be seen from the figures, the road train comes to a stable motion along the collision line in 40 seconds at a given driving speed  $V = 0.3 \text{ m/s}$ . In this case, the tractor performs such a maneuver, at which the displacement of the semi-trailer wheels relative to the collision line gradually decreases.

The third variant (Figure 2(c)) assumes the placement of the semi-trailer of the road train relative to the collision line similar to the second variant, i.e.  $\alpha = 0$  and  $0 < Z \leq 2 \text{ m}$ , except that

the tractor is rotated relative to the semi-trailer by the folding angle, which varies within the most possible limits during operation of the combination:  $0 < \beta \leq \pi/3$ . Under these conditions, it takes approximately 40 seconds for the road train to reach a stable movement relative to the collision line, maneuvering with a slight increase in the area required. As the initial offset increases, the steady-state time increases to 45 to 50 seconds.

In the fourth variant (Figure 2(d)), the position of the road train is set within the following limits:  $-\pi/3 \leq \beta \leq -\pi/6$ ,  $\pi/6 \leq \alpha \leq \pi/3$ ,  $-0.8m \leq Z \leq 0.8m$ . If  $\pi/4 \leq \alpha \leq \pi/3$ , the road train will come to a stable movement in 40 - 50 seconds, while the time of coming to a stable movement increases to 70 ÷ 80 seconds. Increasing the maneuvering time to a stable movement along the guide leads to an increase in the distance from the initial position to the object required for an accurate approach. Therefore, it is desirable to set the road train in the initial position at angles not exceeding 45 degrees.

The fifth variant (Figure 2(e)) is a special case of the variant and is characterized by the same regularities. Sixth option (Figure 2(f)) at  $0 \leq \beta \leq \pi/3$ ;  $-\pi/3 \leq \alpha \leq -\pi/6$ ;  $0 \leq Z \leq 1.5m$  is the most unfavorable for the control system. For this position of the combination vehicle relative to the collision line, the maneuvering time will be long and the combination vehicle may not have time to come to a stable movement along the guide rail, having approached the object inaccurately.

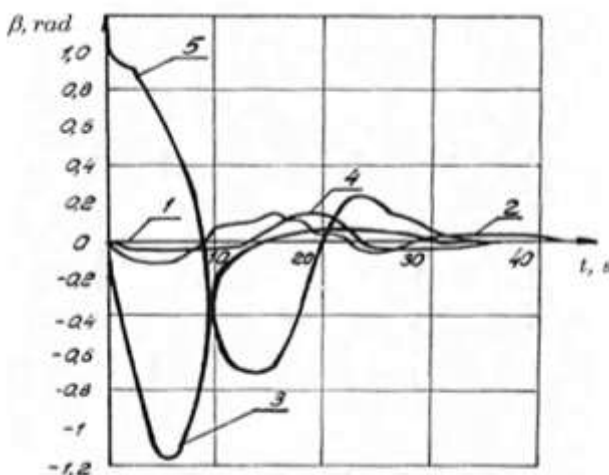


Fig. 3: Dependence of the folding angle  $\beta = f(t)$  when moving to the approach object from the initial positions (Figure 2)

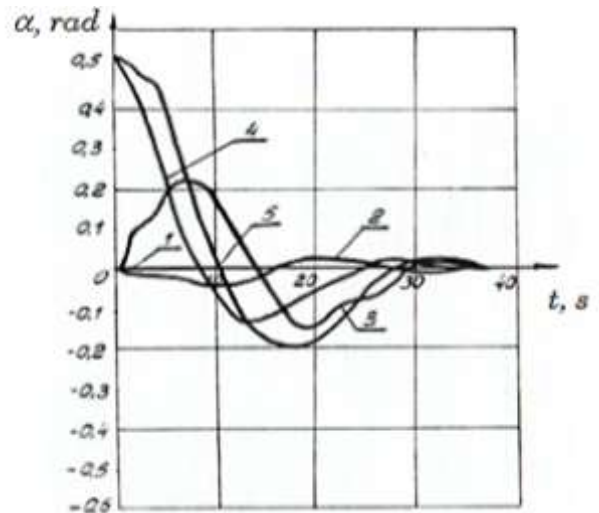


Fig. 4: Dependences of angles of rotation of the semi-trailer frame relative to the longitudinal collision line  $\alpha=f(t)$  when moving to the object of approach from the initial positions (Figure 2)

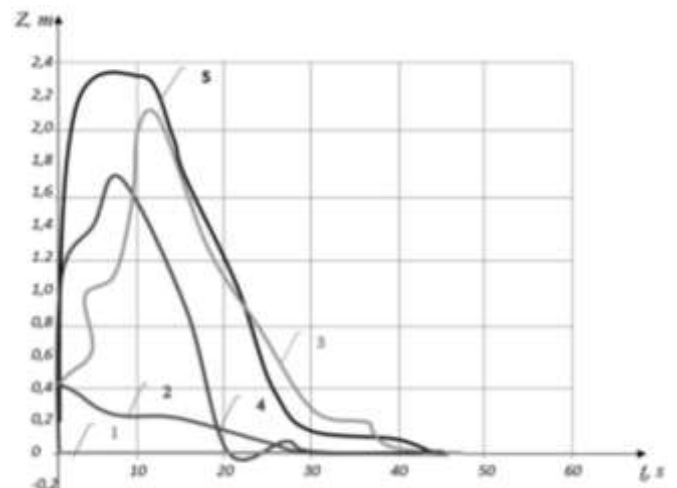


Fig. 5: Dependence of displacements of semi-trailer wheels relative to the collision line  $z = f(t)$  when moving to the object of approach from the initial positions (Figure 2)

### 3 Discussion of Research Results of Modeling Processes of Road Train Movements

Investigations of the processes of modelling the motions of the road train, analysis of dependencies (Figure 3, Figure 4 and Figure 5) were carried out  $\beta=f(t)$ ,  $\alpha=f(t)$ ,  $Z=f(t)$ , obtained under different initial conditions, taking into account the restrictions on the rotation of the steerable wheels of the tractor, which reduce the functional capabilities of the control system to bring the road train to a stable reversible movement along the line of approach to the object, it is necessary to



determine the maneuvering properties of the road train in the design, taking into account the design factors of its systems.

For the example under consideration, it is reasonable to exclude setting the semi-trailer to angles within the range when setting the road train in the initial position  $\pi/6 \leq \alpha \leq -\pi/6$ . The first option is the most favorable for the operation of the control system. The use of the road train control system allows you to significantly reduce the time and area of maneuvering with a high accuracy of approach to the object in reverse. Auxiliary automated control systems or an autopilot can be used to control the reversing motion of the road train. Based on the conducted research and taking into account [1] a new design of the electrical scheme for the construction of the motion control system has been developed, shown in Figure 6.

Let us consider an example of a control system for a road train with non-rotating wheels of a semi-trailer traveling in reverse (Figure 6). The device contains the control object 1, sensor 2 of the angle of folding of the tractor with semi-trailer  $\beta$ , sensor 3 of the angle of inclination of the semi-trailer axis to the longitudinal collision line  $\alpha$ , sensor 4 of the displacement of the left wheels of the semi-trailer relative to the longitudinal collision line  $Z$ , amplifiers 5 - 7, adders 8 and 9, zero-indicator 10, steering drive 11, the tracking system 12 of wheel rotation and sensor 13 of the angle of rotation of the steered front wheels of the tractor. The steering actuator 11, the tracking system 12, and the sensor 13 are located on the tractor. Sensors 2, 3, 4 fulfil the function of feedback, i.e. having measured the values of  $\beta$ ,  $\alpha$ ,  $Z$ , they transmit them in the form of signals to the adder 8. The inputs of sensors 2...4 are connected to the control object, and the input of sensor 13 is connected to the output of tracking system 12 for wheel rotation. The outputs of sensors 2 - 4 are connected respectively to the inputs of amplifiers 5...7, and the output 13 is connected to the second input of the adder 9.

Amplifiers 5 - 7 with gain coefficients  $K_1...K_3$ , their outputs are connected respectively to the first, second and third inputs of the adder 8. The output of the adder 8 is connected to the first input of the adder 9. The output of the adder 9 is connected to the input of the zero indicator 10. The output of the zero indicator 10 is coupled to the steering actuator 11, and the steering actuator 11 is coupled to an input of the wheel tracking system 12. The output of the wheel tracking system 12 is coupled to the control object 1.

When reversing the road train, the device works as follows. Before starting the approach of the road train to the object in reverse, it is set near the longitudinal line of the collision, with the axis of the semi-trailer inclined to it at an angle  $\alpha_0$ , the left wheels of the semi-trailer are displaced relative to it by the value  $Z_0$ , and the tractor and semi-trailer have a folding angle  $\beta_0$ . The wheels of the tractor are rotated relative to the frame by the value  $\varphi_0$ . At the moment when the transport vehicle starts reversing to the object, the device is switched on. In this case, sensors 2 - 4 measure the values of  $\alpha$ ,  $\beta$  and  $Z$  and output them in the form of electrical signals to amplifiers 5 - 7 respectively. Amplifiers 5 - 7 amplify the signals  $\alpha$ ,  $\beta$  and  $Z$  to the values  $K_1\beta$ ,  $K_2\alpha$  and  $K_3Z$  and feed them to the first, second and third adders 8 respectively. In the adder 8, the values  $K_2\alpha$  and  $K_3Z$  are algebraically subtracted from the value  $K_1\beta$ . The total signal  $\varphi_{set}$  is given by the adder 9. The sensor 13 measures the actual angle of rotation of the steerable front wheels of the tractor  $\varphi_r$  relative to the frame and outputs an electrical signal to the second input of the adder 9. From the output of the adder 9 the resulting signal is fed to the input of the zero-indicator 10, causing a deviation of its arrow from zero value by an amount equal to the difference between the specified angle of rotation of the steered front wheels of the tractor  $\varphi_{set}$  and the actual real  $\varphi_r$ . By means of the steering drive the driver of the road train influences the tractor wheel steering system so that the arrow of the zero indicators is constantly at zero. With the help of the tracking system of wheel turning the tractor makes a set maneuver along the longitudinal line of the collision, thus, acting on the control object, it will carry out the approach of the road train to the object so that the left wheels of the tractor and semi-trailer were on the longitudinal line of the collision. This will reduce the time for maneuvering and convergence.

The considered system of controlling the movement of the road train in reverse along the longitudinal line of the collision with the proposed parameter gauges allows the mobile system, in compliance with the stability conditions (9), to reach the collision line for the minimum maneuvering time on the minimum area. In addition, the maneuvering time and, accordingly, the maneuvering area can be smaller if the design restrictions on the rotation of the front steerable wheels of the tractor are reduced.

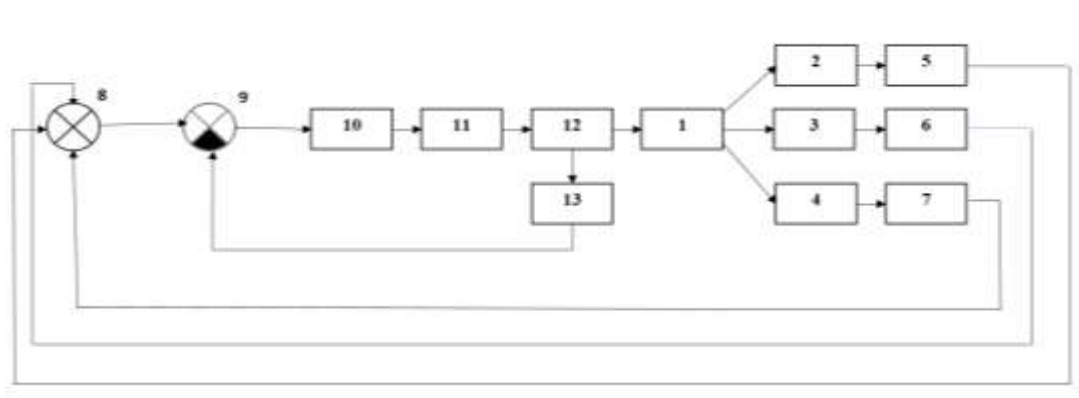


Fig. 6: Schematic diagram of the system of controlling the rearward movement of a road train

The use of such train control systems will significantly reduce the maneuvering time and maneuvering area with high accuracy of approaching the object, especially when the length of the semi-trailer is long.

Another option for equipping (equipping) a tractor with a semi-trailer with non-turning semi-trailer wheels can be the construction of a traffic control system with autopilot, which allows maneuvering in a limited area without the driver's participation.

#### 4 Features of Unmanned Cars Maneuverability

The process of creation and development of ground-based drones shows that their creation and development follows the following main directions [9], [10], [11], [12], [13], [14]: introduction and expansion of functionality of driver assistance systems; creation of methods and systems of drone traffic control, which are both at the stage of development and testing of prototypes and in operation.

Many global car manufacturers, especially in the USA, Germany, Japan, Italy, China, Great Britain, France, and Korea (General Motors, Ford, Mercedes Benz, Volkswagen, Audi, BMW, Volvo, Cadillac) are working on the development of unmanned cars.

Realisation of the advantages of unmanned vehicles is impossible without the efficient operation of motion control systems, which may be limited by the speed of measuring, computing, and actuating devices. An integrated approach to the creation of an unmanned car is currently realised only by some companies, for example, Google. It should be noted that single-car parking systems have existed for a long time. Toyota, Volkswagen, Valeo, and Ford have achieved the greatest success in creating such systems. Currently, various

automatic parking systems are being developed and implemented, which provide parallel or perpendicular parking of a car in automatic mode. Toyota, BMW, Ford, Mercedes-Benz, Nissan, Opel, and Volkswagen have parking autopilot.

The process of further improvement of the adaptive cruise control system is ongoing, which in the future will allow to realize an automatic mode of car driving in traffic jams. Audi, Ford are conducting research in this direction. Developments of BMW, and Cadillac on automation of movement of cars and road trains on motorways are based on existing active safety systems. A modern car contains electronic driver assistance systems with various degrees of automation of the vehicle control process, such as directional stability, warning, cruise control, parking distance control, and others. Electronic systems provide part of the vehicle control functions, such as automatic speed control, turning, and parking maneuvering functions.

To ensure safe maneuvering, e.g. when parking, in addition to the autopilot, distance sensors (e.g. ultrasonic or laser sensors, which have already been developed and are commercially available) should be installed at points on the road train that define its overall lane when driving in a curve, including in reverse. Similar sensors should also be installed on the front part of the tractor and the rear part of the semi-trailer.

The analysis of control systems for unmanned vehicles has revealed a large number of problems arising before the developers in the process of their creation and in determining the requirements to the motion control system, which is caused by the following objective factors: design features of vehicles; sufficiently high error of measuring the motion parameters; impossibility of most systems to take into account the external conditions, continuously changing in the process of motion; functional limitations of control systems.

Perpendicular parking of a semi-trailer is one of the most complicated maneuvers of a road train and is divided into several stages.

It is obvious that the quality of control system operation directly determines traffic safety, and the developers at the stage of engineering design are obliged to determine the operational capabilities of vehicles, at which the probability of an emergency situation is reduced to a minimum. Therefore, the task of preliminary prediction and evaluation of curvilinear motion characteristics of vehicles (including unmanned vehicles) at the design stages is relevant and important.

## 5 Conclusions

1. The conducted research allowed us to achieve the set research goal and show the ways to achieve it. The obtained modeling results can be used in the development of Smart City systems, Smart Traffic Lights, in solving logistic problems of industrial systems of the Internet of Things.
2. The mathematical model of reversing movement of a truck with a semi-trailer with non-rotating wheels of the semi-trailer has been developed, which allowed to provide and describe the accurate accident-free delivery of cargo to the object by the road train.
3. On the basis of the analysis of the character of movement the main parameters influencing the stable movement of the road train in reverse have been determined, and the law of control of the tractor wheels providing its stable movement by means of introduction of feedbacks has been justified. As well as the construction and application of the design of the electrical scheme of the vehicle control system.
4. Analysing the stability of the system by means of the Routh–Hurwitz criterion has allowed us to obtain the necessary and sufficient conditions for ensuring the stability of the moving system.
5. Modelling of the road train movement with the help of the simulation model allowed to determine the nature and parameters of its movement relative to the collision line from different initial positions relative to the object, as well as to choose the preferred options of the initial location of the vehicle before starting the movement.
6. On the basis of the research results the new method of controlling the directional

movement of the road train and the device realising it are developed.

7. The obtained graphical dependences based on the results of modelling from different initial positions of the road train relative to the collision line allowed to determine the necessary ratios of the parameters of the law of controlling the rotation of the tractor wheels taking into account the design restrictions on the rotation of the wheels.
8. The results of simulation have shown that when designing the schemes of road trains it is necessary to take into account the overall dimensions of the tractor and semi-trailer to ensure the stability of movement, the required maneuverability on a limited area of maneuvering in a short time.
9. According to the results of the work done, a method of determining the conditions for ensuring the stable movement of a semi-trailer train in reverse with non-turning wheels of the semi-trailer has been developed.

The practical significance of the developed method - the results obtained in this work can be used in the stage of development of road trains. It could be used also for unmanned vehicles, with non-turning wheels of semi-trailers.

The future research is planned to continue work on improvement of mathematical apparatus for specification of procedure of definition of values of coefficients of the control law by means of methods of optimisation, and also to work out possibility of full automation of process of control of backward movement of a road train of the considered type.

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