Optimized TRIAD-Based Nanosatellite Attitude Determination Using Horizon Sensor and Magnetometer

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Abstract: - The aim of this article is to investigate the accuracy of vector measurement-based attitude determination methods for a nanosatellite. Measurements from the horizon sensor and magnetometer are therefore modeled on the body frame. The triaxial Attitude Determination (TRIAD) technique is a widely used and effective method to determine the attitude of a nanosatellite. In this study, the TRIAD method is used with three different approaches to obtain the smallest orientation error of a nanosatellite equipped with magnetometers and horizon sensors. Analysis of covariance is conducted to evaluate the validity and reliability of the attitude determination process. Three modifications of the TRIAD algorithm were tested for accuracy and the most accurate was determined. The analysis provides information on the sources of error and uncertainty associated with the measurement and estimation process. This information is used to improve system performance and the accuracy of attitude outputs.

Key-Words: - nanosatellite orientation, orbital reference frame, attitude angles, Nadir direction, triaxial attitude determination, attitude sensors.

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1 Introduction

Attitude determination is the process of determining the orientation of a nanosatellite in relation to a reference coordinate frame. Attitude determination is critical to a nanosatellite's effective operation and mission success. Knowing a nanosatellite's attitude allows its instruments to aim in the appropriate direction, communicate with the ground station, and execute the required activities.

A nanosatellite's attitude can be determined using a variety of technologies, including star trackers, horizon sensors, sun sensors, magnetometers, and gyros, [1], [2], [3], [4], [5], [6], [7]. These sensors measure the direction and strength of the magnetic field, the position of the stars, the direction to the Earth, the Sun's direction, and the nanosatellite's angular velocity. The data from these sensors is then processed by algorithms to estimate the nanosatellite's attitude in real-time.

Satellite attitude can be determined by using at least two vectors measured by the attitude sensors (Sun sensor, Earth horizon sensor, magnetometer, etc.) in the satellite body frame and the models describing the corresponding directions (to the Sun, nadir, and magnetic field) in the reference frame, [2].

The first published approach to determining the orientation of satellites is an algebraic method proposed in 1964, [8]. [9], also presents a method called TRIAD, which stands for Three-Axis Attitude Determination. Finding the transformation matrix between the satellite body coordinate system and the reference frame is the goal of the TRIAD algorithm.

Satellite attitude determination using the TRIAD algorithm has been considered in many researches, and various algorithms have been proposed to improve the estimation accuracy, [10], [11], [12], [13].

Optimized TRIAD which is studied in works [10], [11], and [12], combines the two transformation matrices held in two different TRIAD systems which two different direction vectors act as anchors one by one.

In this study, TRIAD optimization is performed using three different approaches to obtain the smallest attitude error of a nanosatellite that has magnetometers and horizon sensors as onboard attitude sensors. The accuracy of these three modifications of the TRIAD algorithm was compared and the optimized algorithm with the highest accuracy was determined.

2 Attitude Motion Model of the Nanosatellite

The nanosatellite's rotational motion is mathematically described in terms of Euler angles and angular velocities, and the problem is solved iteratively using the initial values of these parameters. Below is a mathematical representation of the rotational motion of a nanosatellite around its center of mass.

Expressions for Euler angles

$$\psi_{(i+1)} = \psi_{(i)} + \Delta t\left(-w_{x_{(i)}} tan(\theta_{(i)}) * \cos(\psi_{(i)}) + w_{y_{(i)}} \sin(\psi_{(i)}) * \tan(\theta_{(i)}) + w_{z_{(i)}}\right) \quad (1)$$

$$\theta_{(i+1)} = \theta_{(i)} + \Delta t\left(w_{x_{(i)}} \sin(\psi_{(i)}) + w_{y_{(i)}} \cos(\psi_{(i)})\right) \quad (2)$$

$$\phi_{(i+1)} = \phi_{(i)} + \Delta t\left(w_{x_{(i)}} \cos(\psi_{(i)}) - \frac{w_{y_{(i)}} \sin(\psi_{(i)})}{\cos(\theta_{(i)})}\right) \quad (3)$$

Expressions for angular velocities

$$\omega_{x_{(i+1)}} = \omega_{x_{(i)}} + \frac{\Delta t}{J_x}\left(\omega_{z_{(i)}}\omega_{y_{(i)}} + N_T\right)(J_y - J_z) \quad (4)$$

$$\omega_{y_{(i+1)}} = \omega_{y_{(i)}} + \frac{\Delta t}{J_y}\left(\omega_{x_{(i)}}\omega_{z_{(i)}} + N_T\right)(J_z - J_x) \quad (5)$$

$$\omega_{z_{(i+1)}} = \omega_{z_{(i)}} + \frac{\Delta t}{J_z}\left(\omega_{x_{(i)}}\omega_{y_{(i)}} + N_T\right)(J_x - J_y) \quad (6)$$

In the equations (1)-(6) ϕ is the roll angle, θ is the pitch angle, ψ is the yaw angle, w_x, w_y, and w_z are the angular velocities, J_x,J_y, and J_z are the moments of inertias of the nanosatellite, w_{orbit} is the angular orbit velocity of the nanosatellite, N_T is the disturbance torque acting on the nanosatellite, Δt is the sample time and the N is the iteration number.

3 Mathematical Modeling of Reference Direction Sensors

The focus of this section is characterizing the horizon and magnetic field sensors which are used in the TRIAD algorithm as reference direction vectors for the simulation environment.

3.1 Mathematical Model of Earth Magnetic Field

This section is to investigate how the Earth's magnetic field vector, one of the reference vectors used in the TRIAD approach, behaves when the orbital position of the nanosatellite varies. The magnetic field vector varies significantly with the orbital parameters as the nanosatellite travels along its path as described in [14]. The magnetic field tensor vector that affects satellites can be proven analytically as a function of time if those parameters are known as follows,

$$H_1(t) = \frac{M_e}{r_0^3}\left[\cos(\omega_0 t)\left(\cos(\varepsilon)\sin(i) - \sin(\varepsilon)\cos(i)\cos(\omega_e t)\right) - \sin(\omega_0 t)\sin(\varepsilon)\sin(\omega_e t)\right] \quad (7)$$

$$H_2(t) = -\frac{M_e}{r_0^3}\left[\cos(\varepsilon)\cos(i) + \sin(\varepsilon)\sin(i)\cos(\omega_e t)\right] \quad (8)$$

$$H_3(t) = \frac{2M_e}{r_0^3}\left[\sin(\omega_0 t)\left(\cos(\varepsilon)\sin(i) - \sin(\varepsilon)\cos(i)\cos(\omega_e t)\right) - 2\sin(\omega_0 t)\sin(\varepsilon)\sin(\omega_e t)\right] \quad (9)$$

In the equations (7)-(9) M_e is the magnetic dipole moment of the Earth, μ is the Earth Gravitational constant, i is the orbit inclination, ω_e is the spin rate of the Earth, ε is the magnetic dipole tilt, r_0 is the distance between the center of mass of the nanosatellite and the Earth, ω_0 is the angular velocity of the orbit with respect to the inertial frame, found as $\omega_0 = (\mu/r_0^3)^{1/2}$. To find the direction of the magnetic field vector, we can track its direction cosines, which are computed as

$$H_0 = \begin{bmatrix} H_{x0} \\ H_{y0} \\ H_{z0} \end{bmatrix} = \frac{1}{\sqrt{H_x^2 + H_y^2 + H_z^2}} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} \quad (10)$$

where H_0 is the direction cosine vector of the magnetic field in the orbital coordinate system. The direction cosine vector measurement can be obtained in the body frame as follows

$$H_B(k) = A(k)H_o(k) + \eta_H \quad (11)$$

Here η_H is the zero mean Gaussian white noise of the magnetometer and $A(k)$ is the direction cosine matrix in terms of Euler angles which is given below in equation (12),

$$A = \begin{bmatrix} c(\theta)c(\psi) & c(\theta)s(\psi) & -s(\theta) \\ -c(\varphi)s(\psi) + s(\varphi)s(\theta)c(\psi) & c(\varphi)c(\psi) + s(\varphi)s(\theta)s(\psi) & s(\varphi)c(\theta) \\ s(\varphi)s(\psi) + c(\varphi)s(\theta)c(\psi) & -s(\varphi)c(\psi) + c(\varphi)s(\theta)s(\psi) & c(\varphi)c(\theta) \end{bmatrix} \quad (12)$$

where c and s define trigonometric functions as the cosine and the sine of the angle respectively.

3.2 Mathematical Model of Nadir Direction

The nadir direction, denotes the vector pointing directly towards the center of the celestial body being orbited, typically Earth for satellites in low Earth orbit.

Model is a vector looking to nadir direction in orbit reference system with equation (13) as,

$$N_o(k) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{13}$$

Vector in the orbital frame can be converted to body frame as follows,

$$N_B(k) = A(k)N_o(k) + \eta_N \tag{14}$$

where $A(k)$ is the direction cosine matrix, η_N is the zero mean Gaussian white noise of the horizon sensor.

4 TRIAD Algorithm

TRIAD is an attitude determination algorithm that uses a vector-based approach to estimate the satellite's attitude based on measurements from two sets of known vectors. The name "TRIAD" can be considered either as the word "triad" or as an acronym for Triaxial Attitude Determination, [2]. TRIAD, the earliest published algorithm for satellite attitude determination from two vector measurements, has been widely used in both ground-based and onboard attitude determination.

4.1 Classic TRIAD

TRIAD uses two unparallel unit vectors to construct a new coordinate. The attitude matrix was calculated by the algebraic method as stated in [8]. The equation is given as,

$$\begin{gathered} r_1 = v_1 \\ r_2 = (v_1 \times v_2)/|v_1 \times v_2| \\ r_3 = (v_1 \times r_2)/|v_1 \times r_2| \end{gathered} \tag{15}$$

The equation includes two vectors, v_1 and v_2, which are magnetic field and nadir direction vectors in the orbital frame.

$$\begin{gathered} s_1 = w_1 \\ s_2 = (w_1 \times w_2)/|w_1 \times w_2| \\ s_3 = (w_1 \times s_2)/|w_1 \times s_2| \end{gathered} \tag{16}$$

and w_1 and w_2 are the magnetic field and horizon sensor measurements in the body frame.

$$M_o = [r_1\ r_2\ r_3] \tag{17}$$

After orthogonalizing the reference vectors, the base vectors of the orbital frame matrix M_O are represented by r_1, r_2, and r_3

$$M_b = [s_1\ s_2\ s_3] \tag{18}$$

The base vectors of body frame matrix, M_b, are represented by s_1, s_2, and s_3. Then the attitude matrix is obtained in the equation (19)

$$A = M_b * M_o' \tag{19}$$

For the application of TRIAD algorithm, two-unit vectors in two different reference frames is needed. H_0 is the Earth magnetic field vector in the orbital frame, H_B is the magnetometer measurement vector in the body frame, N_o is the nadir direction vector in the orbital frame and N_B is the nadir sensor measurement vector in the body frame.

After the application of the classic TRIAD, in equation (12) a rotation matrix can be obtained in the 3-2-1 Euler-angle sequence. It is observed that attitude matrix is weighted twice by the first vector, indicating that the first vector is the major vector and plays a critical role in determining the attitude accurately. If the noise associated with the major vector is higher than that of the secondary vector, the classic TRIAD algorithm may not provide the optimized solution. This issue is addressed by the proposed algorithm in this study. The error covariance for TRIAD was presented in [9] as;

$$P_{\text{TRIAD}} = \sigma_1^2 I + \frac{1}{|\hat{w}_1 + \hat{w}_2|^2}[\sigma_1^2(\hat{w}_1 \cdot \hat{w}_2)(\hat{w}_1\hat{w}_2^T + \hat{w}_2\hat{w}_1^T) + (\sigma_2^2 - \sigma_1^2)\hat{w}_1\hat{w}_1^T] \tag{20}$$

The attitude covariance matrix is denoted by P_{TRIAD}, and I represents the identity matrix. The variances of two vector sensors are denoted by σ_1^2 and σ_2^2. Equation (20) shows that if σ_1^2 is greater than σ_2^2, the error covariance matrix is larger than the case when σ_1^2 is less than σ_2^2.

4.2 Optimized TRIAD

Different types of optimized TRIAD is proposed in this section and there is a comparison study between these algorithms in the simulation section. There are several variations of the optimized TRIAD algorithm, which aim to improve the accuracy and computational efficiency of the original algorithm.

The optimized TRIAD algorithm is a mixture of two TRIAD algorithms, namely TRIAD-I and TRIAD-II. The former generates matrix A_1, while the latter generates matrix A_2. The approach involves creating two sets of vector triads using four vector measurements, two in the orbital reference frame (v_1 and v_2) and two in the body reference frame (w_1 and w_2). To implement the optimized TRIAD algorithm, one must compute the matrices r_i in the body coordinate and the corresponding column matrices s_i in the orbital frame. For the TRIAD-I algorithm, a specific definition is assigned

$$r_1 = \frac{v_1}{|v_1|} \quad r_2 = \frac{(r_1 \times v_2)}{|r_1 \times v_2|} \quad r_3 = r_1 \times r_2 \tag{21}$$

$$s_1 = \frac{w_1}{|w_1|} \quad s_2 = \frac{(s_1 \times w_2)}{|s_1 \times w_2|} \quad s_3 = s_1 \times s_2 \tag{22}$$

The expression for matrix A_1 is given by;

$$A_1 = \mathrm{r}_1 \cdot \mathrm{s}_1^{\mathrm{T}} + \mathrm{r}_2 \cdot \mathrm{s}_2^{\mathrm{T}} + \mathrm{r}_3 \cdot \mathrm{s}_3^{\mathrm{T}} \quad (23)$$

The definition for the TRIAD-II algorithm is as follows;

$$\mathrm{r}_5 = \frac{\mathrm{v}_2}{|\mathrm{v}_2|} \;\; \mathrm{r}_2 = \frac{(\mathrm{r}_1 \times \mathrm{v}_2)}{|\mathrm{r}_1 \times \mathrm{v}_2|} \;\; \mathrm{r}_4 = \mathrm{r}_5 \times \mathrm{r}_2 \quad (24)$$

$$s_5 = \frac{w_2}{|w_2|} \;\; s_2 = \frac{(s_1 \times w_2)}{|s_1 \times w_2|} \;\; s_4 = s_5 \times s_2 \quad (25)$$

The expression for matrix A_2 is given by,

$$A_2 = \mathrm{r}_5 \cdot \mathrm{s}_5^{\mathrm{T}} + \mathrm{r}_2 \cdot \mathrm{s}_2^{\mathrm{T}} + \mathrm{r}_4 \cdot \mathrm{s}_4^{\mathrm{T}} \quad (26)$$

4.2.1 Optimized TRIAD - 1

The computation of the optimized attitude matrix $\widehat{\mathrm{A}}$ follows as;

$$\widehat{\mathrm{A}}' = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\mathrm{A}_1 + \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\mathrm{A}_2 \quad (27)$$

Finally, the attitude matrix of the optimized TRIAD can be written according to studies in [10]

$$\mathrm{A}_{opt1} = 0.5\left[\widehat{\mathrm{A}}' + \left((\widehat{\mathrm{A}}')^{-1}\right)^{\mathrm{T}}\right] \quad (28)$$

Matrix, A_{opt1}, performs an optimized solution for the Classic TRIAD algorithm. The error covariance of the optimized TRIAD is given in the equation (29) as stated in [12],

$$P_{\text{Opt -TRIAD}} = \sigma_{opt}^2 I + \frac{1}{|\hat{w}_1 + \hat{w}_2|^2}\left[\sigma_{opt}^2(\hat{w}_1 \cdot \hat{w}_2)(\hat{w}_1 \hat{w}_2{}^T + \hat{w}_2 \hat{w}_1{}^T)\right] \quad (29)$$

where

$$\sigma_{opt}^2 = \left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)^{-1} \quad (30)$$

4.2.2 Optimized TRIAD - 2

The second method is based on the sensor fusion technique presented in [4]. The second method uses covariance matrix of the TRIAD-1 and TRIAD-2 rather than using covariance of the reference direction vectors

$$\hat{x} = \frac{\sigma_{cov1}^2 x_{A_2} + \sigma_{cov2}^2 x_{A_1}}{\sigma_{cov1}^2 + \sigma_{cov2}^2} \quad (31)$$

where $\hat{x}$ is the angle that is optimized, σ_{cov1}^2 is the angle error variance when TRIAD-1 algorithm is used, σ_{cov2}^2 is the angle error variance when TRIAD-2 algorithm is used, x_{A_1} is the angle that is found by the TRIAD-1 algorithm and x_{A_2} is the angle that is found by the TRIAD-2 algorithm.

The error variance of the optimized angle $\hat{x}$ can be written as in equation (32),

$$D_x = \frac{\sigma_{cov1}^2 \sigma_{cov2}^2}{\sigma_{cov1}^2 + \sigma_{cov2}^2} \quad (32)$$

4.2.3 Method 3

The third method is based on the sensor fusion technique when three type TRIAD method outputs are processing as mentioned in [4]. The third method uses TRIAD-1 and TRIAD-2 and also optimized TRIAD which is calculated in the method 1. Each angle is calculated by the equation (33) as follows;

$$\hat{x} = \frac{\sigma_{cov1}^2 \sigma_{cov2}^2 x_{A_{opt}} + \sigma_{cov1}^2 \sigma_{cov3}^2 x_{A_2} + \sigma_{cov2}^2 \sigma_{cov3}^2 x_{A_1}}{\sigma_{cov1}^2 \sigma_{cov2}^2 + \sigma_{cov1}^2 \sigma_{cov3}^2 + \sigma_{cov2}^2 \sigma_{cov3}^2} \quad (33)$$

where $\hat{x}$ is the angle that is optimized, σ_{cov3}^2 is the angle error variance when the optimized TRIAD-1 method is used.

The error variance of the optimized angle $\hat{x}$ can be determined as in equation (34),

$$D_x = \frac{\sigma_{cov1}^2 \sigma_{cov2}^2 \sigma_{cov3}^2}{\sigma_{cov1}^2 \sigma_{cov2}^2 + \sigma_{cov1}^2 \sigma_{cov3}^2 + \sigma_{cov2}^2 \sigma_{cov3}^2} \quad (34)$$

where D_x is the angle error variance.

5 Simulation Results of TRIAD Algorithm

To visualize the data, MATLAB is utilized. The required programming algorithm is created using a mathematical model of the nanosatellite's rotating motion (1)-(6).

Simulations are performed in order to estimate attitude of a nanosatellite using the classic TRIAD algorithm and optimization of TRIAD with different modifications.

Algorithm is run for 54000 iterations with time step of 0.1 seconds. Standard deviations of error for the magnetometer and horizon sensor are taken as 0.08 and 0.06 respectively.

In Figure 1, when the angle between the reference directions is close to parallel, which means that the angle is close to 0 or 180 degrees, the TRIAD errors increase (Figure 2). Also, as the pitch angle is close to ±90 degree TRIAD error increases as in simulation results of classic TRIAD in Figure 2.

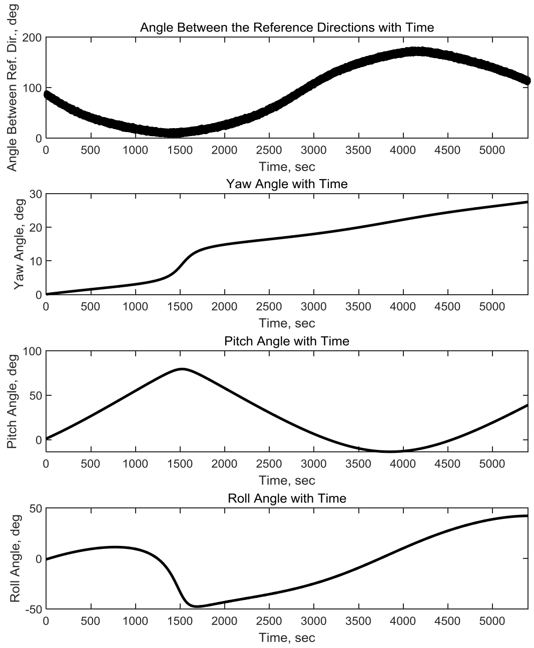

Fig. 1: Angle between the reference directions; and Euler Angles

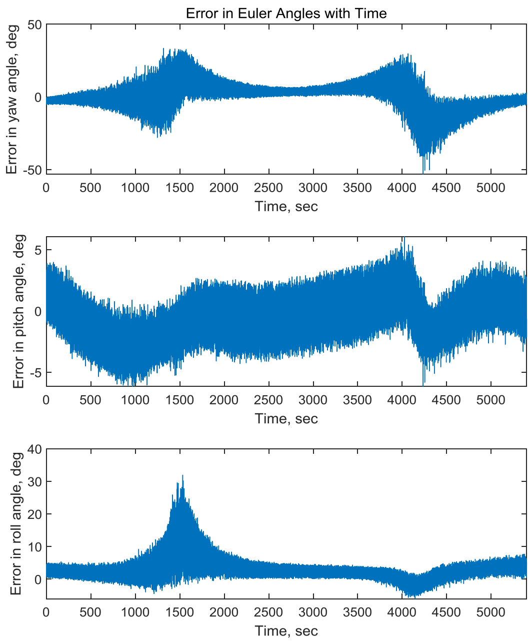

Fig. 2: Error in Euler angles when using Classic TRIAD

In the Classic TRIAD (TRIAD-1) algorithm magnetic field vector is used as an anchor. As shown in Figure 2, estimation errors are generally low but pitch angle affects the results badly when the angles is close to ±90 degrees at 1500th second. Also, at 4200th second, the angle between the reference directions gets close to parallel which is another reason to get higher estimation errors.

All the TRIAD algorithms are simulated and their error in Euler angles and the error variances are shown in Figure 3 and Figure 4 respectively. TRIAD-1 which is also named Classic TRIAD, uses a magnetic field vector as its dominant part, TRIAD-2 uses horizon vector as its main vector and all three optimization methods use both TRIAD-1 and TRIAD-2 to obtain the optimized result. Based on the simulation results, Optimized-3, which is an optimized TRIAD algorithm, generally gives the best result in both Euler angle errors and error variances of the TRIAD algorithms.

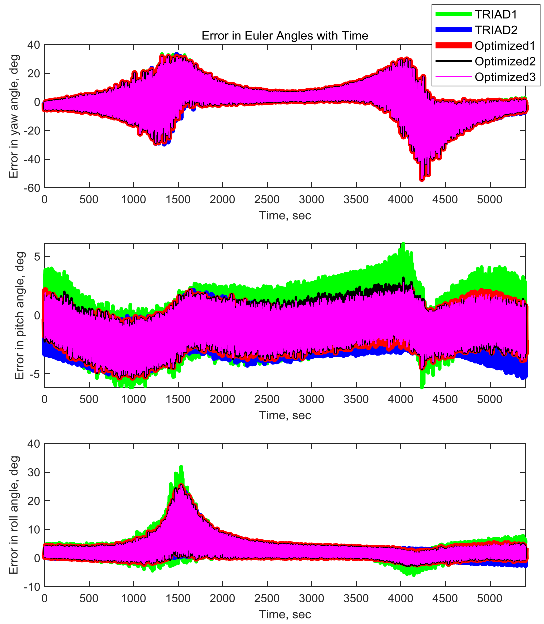

Fig. 3: Comparison of TRIAD algorithms' errors in Euler angles

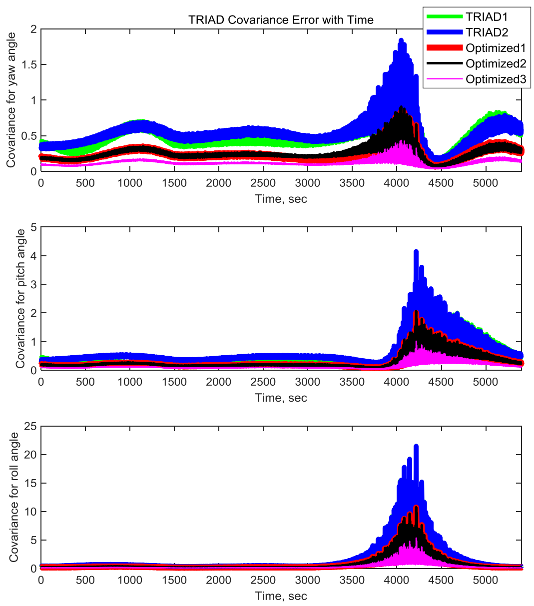

Fig. 4: Comparison of error variances of TRIAD algorithms

A closer look at 1 to 60 seconds is shown in Figure 5 and Figure 6. From time to time as illustrated in Figure 5, the lowest error of Euler angles cannot be obtained by just one algorithm, this is because of the changing performance or accuracy of the direction sensors which is also simulated. Since optimization has lots of dependences such as pitch angle's degree, the parallelism of the two sensors, and changing performance of the sensor; it is not suitable to define a definite optimized algorithm. TRIAD error variances with time for the first 60 seconds are given in Figure 6.

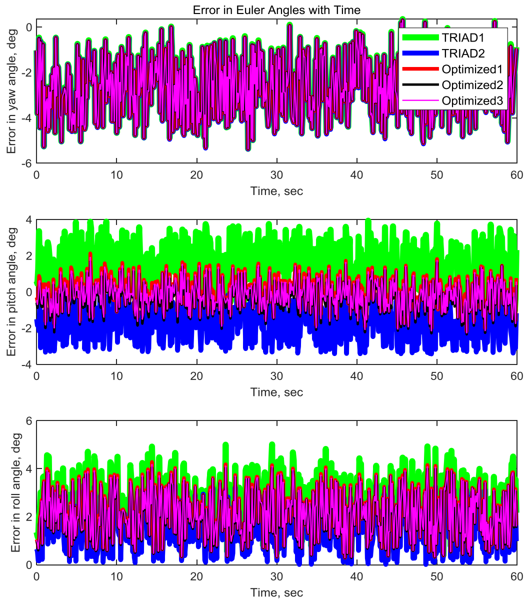

Fig. 5: Error in Euler angles for the first 60 seconds

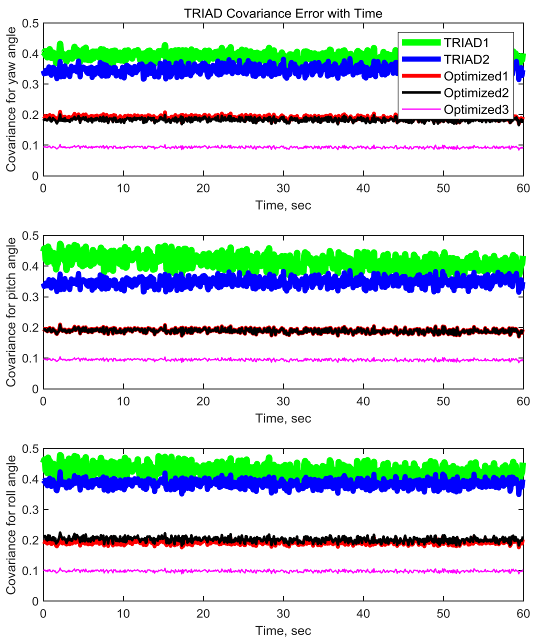

Fig. 6: TRIAD error variances with time for the first 60 seconds

6 Conclusion

In this study, the TRIAD algorithm is optimized using three distinct ways to get the lowest error for the attitude of a nanosatellite equipped with magnetometers and horizon sensors. The accuracy of these three TRIAD algorithm modifications is compared, and a high-accuracy optimized algorithm is identified.

In the optimization section, it is desired to obtain a better result that has lower error compared to both TRIAD-1 and TRIAD-2 algorithms. The results of the simulation indicated that a single algorithm is unable to yield the lowest error of Euler angles due to the changing accuracy or performance of the sensor measurements. It is not appropriate to define a specific optimized algorithm because optimization involves many dependencies, such as the degree of pitch angle, the parallelism of the two reference directions, and changing sensor performance. However, as this study has demonstrated, it is possible to obtain an optimized algorithm, which is identified as method 3, to have fewer attitude errors and error variances. The simulation results show that the pitch angle of ±90 degrees and the parallelism of the two reference direction vectors result in higher errors, as expected.

References:

[1] J. R. Wertz, "*Spacecraft Attitude Determination and Control*," Springer Dordrecht, 2012.

[2] F.L. Markley and J.L. Crassidis, "*Fundamentals of Spacecraft Attitude Determination and Control*," Springer, 2014.

[3] V. V. Unhelkar and H. B. Hablani, "Spacecraft Attitude Determination with Sun Sensors, Horizon Sensors and Gyros: Comparison of Steady-State Kalman Filter and Extended Kalman Filter," in *Advances in Estimation, Navigation, and Spacecraft Control*, D. Choukroun, Y. Oshman, J. Thienel and M. Idan (Eds.) Berlin, Heidelberg: Springer Berlin Heidelberg, 2015, pp.413–437.

[4] C. Hajiyev, H.E. Soken, "*Fault Tolerant Attitude Estimation for Small Satellites*," Boca Raton: CRC Press, 2021, https://doi.org/10.1201/9781351248839.

[5] J.J.R. Critchley-Marrows, X. Wu, and I.H. Cairns, "Treatment of Extended Kalman Filter Implementations for the Gyroless Star Tracker," *Sensors*, Vol. 22, 9002, https://doi.org/10.3390/s22229002.

[6] T. Mekky and A. Habib, "Magnetometer-Only Kalman Filter Based Algorithms for High Accuracy Spacecraft Attitude Estimation (A Comparative Analysis)," *International Journal of Robotics and Control Systems,* Vol. 3, No. 3, 2023, pp. 433-448.

[7] T. Mekky and A. Habib, "Three-Axis High-Accuracy Spacecraft Attitude Estimation Via Sequential Extended Kalman Filtering of Single-Axis Magnetometer Measurements," *Aerospace Systems*, Published online: April, 2023, https://doi.org/10.1007/s42401-023-00221-w.

[8] H.D. Black, "A Passive System for Determining the Attitude of a Satellite," *AIAA Journal*, Vol. 2, No. 7, 1964, pp. 1350-1351.

[9] M.D. Shuster, and S.D. Oh, "Three-Axis Attitude Determination from Vector Observations," *Journal of Guidance and Control*, Vol. 4, No. 1, 1981, pp. 70-77.

[10] I.Y. Bar-Itzhack, and R.R. Harman, "Optimized TRIAD Algorithm for Attitude Determination," *Journal of Guidance and Control*, Vol. 20, No. 1, 1997, pp. 208-211.

[11] M.D. Shuster, "The Optimization of TRIAD," *The Journal of the Astronautical Sciences*, Vol. 55, No 2, 2007, pp. 245–257.

[12] Zhu, X., Ma, M., & Zhou, Z. (2017). An Optimized TRIAD Algorithm for Attitude Determination, Artificial Satellites, Vol. 52, No. 3, pp.41-47, DOI: 10.1515/arsa-2017-0005.

[13] A. Finance, C. Dufour, T. Boutéraon, A. Sarkissian, A. Mangin, P. Keckhut and M. Meftah, "In-Orbit Attitude Determination of the UVSQ-SAT CubeSat Using TRIAD and MEKF Methods," Sensors, Vol. 21, 2021, 7361, https://doi.org/10.3390/s21217361.

[14] P. Sekhavat, Q. G. Q. Gong, and I. M. Ross, "NPSAT1 parameter estimation using unscented Kalman filtering," *2007 American Control Conference.*, New York, USA, pp. 4445-4451, 2007.

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Orhan Kirci has organized and executed the investigation, validation, simulation, data curation, formal analysis, and writing.
- Chingiz Hajiyev carried out the conceptualization, methodology, formal analysis, review & editing.

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Conflict of Interest

The authors have no conflicts of interest to declare.