Tuning Kalman Filter in Linear Systems

Jan Thore Lassen and Paolo Mercorelli
Institute of Product and Process Innovation
Leuphana University Lueneburg
Universitätsalle 1, 21335 Lueneburg, Germany
mercorelli@uni.leuphana.de

Abstract: Kalman filters are used in many different areas that require a solution to discrete-data linear filtering problems. Especially in the field of electric controls, Kalman filters represent a used approach and they are an integral part of many states of the art of electric controls. However, the practical implementation of the Kalman Filter often presents difficulties due to the challenging task of getting a good estimate of the covariance matrix of the process noise and covariance matrix of the measurement noise. A fitting and simultaneous choice of these two matrices based on a feedback loop within the Kalman filter realized by the filter itself can directly lead to an asymptotically stable operating Kalman filter after a reasonable amount of iterations. In this paper an approach to apply a feedback loop enabling dynamic values of the covariance matrix process noise and covariance matrix of the measurement noise is presented. This approach will be applied in simulations using Matlab/Simulink.

Key-words: Kalman Filter, Linear Systems, DC-Drives, Sensors

1 Introduction

Since its publication more than five decades ago the Kalman filters found usage in many different areas that require a solution to discrete-data linear filtering problems. Especially in the field of electric controls Kalman filters presented a revolutionary opportunity and disruptively changed the design of certain electric controls. Kalman Filters (KF) are used for state estimation and provide accurate estimates in the presence of deterministic and stochastic disturbances and produce results superior to most other previously used approaches [1]. This paper aims to present an approach for a Kalman filter tuning within the Kalman filters of a DC drive and a virtual sensor. Both devices are intended for industrial use primarily: the motor in several classic industrial applications and the sensor to observe a 1 Farad capacitor intended for use in the automotive industry. The actual tuning within the Kalman filter aims to set Q_k and R_k as dynamic variables that are calculated again in a feedback loop with every iteration of the simulated time. If executed carefully and correctly, this approach should enable the user of the observed system to obtain far more precise estimations for Q_k and R_k than a static estimation of Q_k and R_k gathered by "trial-and-error" methods [2]. The approach intends to rather find a mathematical solution to the problem than a heuristically found solution. This solution will be based on the principal formulas for the covariance matrix P_k and the Kalman gain k_k [3]. The validity and the effectiveness of the proposed Kalman filter tuning method will finally be tested in simulations of both in a DC-Motor and in a sensor. The approach should then be applicable to nearly every similar problem. The paper is organized in the following way. Section 2 is devoted to the modelling of the DC-Motor and the sensor. Section 3 considers the design of matrix Q_k and R_k. Section 4 shows the two applications using simulations performed by Matlab.

2 Modelling

2.1 Modelling of the electric drive

DC Drives are very commonly used in numerous industrial applications among a huge variety of manufacturing processes. This design of an electric drive is still the most common type of motor velocity control for applications that are requiring very fine precise control over wide ranges of velocity with a simultaneously high torque. For the first part of the assignment to implement dynamic Q_k and R_k the following model of a DC-Drive will be considered:

$$U_A = I_A R_A + L_A \frac{dI_A}{dt} + U_q \tag{1}$$

$$k_\varphi I_A - k_f\omega - J\frac{d\omega}{dt} = 0 \quad (2)$$

$$U_q = k_\varphi \omega \quad (3)$$

where U_A is the input voltage, I_A is the current, L_A and R_A represent the inductance and resistance respectively of the DC drive. Parameters k_φ, k_f and J are the motor coefficient, friction coefficient and inertia of the motor respectively. Finally, ω is the velocity and U_q is the induced voltage.

2.2 Modelling of the Sensor

By being present in any form of feedback control virtual sensors represent one of the most important topics in the fields of measurement and control. The second simulation in this paper will therefore emphasize the possibility of the use of a Kalman filter as an observer to obtain measurements which can often require sophisticated and expensive devices. Especially the measurement of velocity in mechanical systems and the measurement of current in electrical systems is usually either very expensive or not precise. The sensor viewed in this paper is based on the following low-pass-filter with the input variable u_1 representing the external voltage, the output variable u_2 representing the capacitor voltage.

After considering the basic dynamics of a capacitor we can obtain that:

$$\begin{pmatrix} \frac{du_c(t)}{dt} \\ \frac{di_c(t)}{dt} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{C} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_c(t) \\ i_c(t) \end{pmatrix} \quad (4)$$

It can be concluded that only the constant a_0 must be used to estimate the current i_c because C is assumed to be known. Following the Forward Euler Approximation, the following discrete dynamic system can then be derived:

$$\begin{pmatrix} u_c(k) \\ i_c(k) \end{pmatrix} = \begin{pmatrix} 1 & \frac{T_S}{C} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_c(k-1) \\ i_c(k-1) \end{pmatrix} \quad (5)$$

together with the required matrix Q:

$$Q = \begin{pmatrix} q_{11} & 0 \\ 0 & q_{22} \end{pmatrix} \quad (6)$$

with q_{22} >> q_{11} the Kalman filter can be assembled according to the general laws that apply for all Kalman filters and are described in the next chapter [6].

3 Design and Tuning of Q_k and R_k

One of the most critical parts while designing a Kalman filter is the choice of the covariance matrices Q_k and R_k. A suitable choice for these matrices is key to a smooth operation and performance of the Kalman filter and the whole control system. In the context of the Kalman filter, the Matrix Q_k represents the statistical description of the drive model. An increased Q_k therefore indicates either a heavy system noise of our increased uncertainty concerning the given parameters. Likewise, increasing elements of the Q_k matrix will result in a higher Kalman gain and therefore in a faster dynamic of the filter. The confidence with regards to the measurement noise is represented by the covariance matrix R_k. Therefore, the higher the values of the elements of R_k are increased, the higher the measurements are affected by noise. As a result, the confidence of the measurement results is lower the higher R_k is.
For the lack of statistical data to conclude their off-diagonal terms the covariance matrices are commonly assumed and represented by a static value. In order to predict the behavior of the control, we need to find a recursion for Q_k based on P_k^-:

$$P_k^- = AP_{k-1}A^T + Q_k \quad (7)$$

and P_{k-1}^-:

$$P_{k-1}^- = AP_{k-2}A^T + Q_{k-1}. \quad (8)$$

Therefore, P_k^- minus P_{k-1}^- equals:

$$P_k^- - P_{k-1}^- = AP_{k-1}A^T - AP_{k-2}A^T + Q_k - Q_{k-1}. \quad (9)$$

As a result, Q_k results as

$$Q_k = Q_{k-1} + \Delta P_k^- - AP_{k-1}A^T + AP_{k-2}A^T, \quad (10)$$

and finally

$$Q_k = Q_{k-1} + \Delta P_k^- - A\Delta P_{k-1}A^T, \quad (11)$$

where $\Delta P_k^- = P_k^- - P_{k-1}^-$, and

$\Delta P_{k-1} = P_{k-1} - P_{k-2}$.

This equation allows Q_k to be dynamically calculated with every iteration within the simulation. Similarly, a solution for R_k is needed. For that solution we have to consider the formula of the Kalman gain k_k in the form of:

$$k_k = P_k^- H^T (HP_k^- H^T + R_k)^{-1} \tag{12}$$

which can be written as

$$P_k^- H^T = k_k HP_k^- H^T + k_k R_k. \tag{13}$$

Multiplied with k_k^T the following relation is obtained:

$$k_k^T P_k^- H^T = k_k^T k_k HP_k^- H^T + k_k^T k_k R_k. \tag{14}$$

Therefore, it is possible to calculate R_k as

$$R_k = (k_k^T k_k)^{-1}(k_k^T P_k^- H^T - k_k^T k_k HP_k^- H^T) \tag{15}$$

and finally:

$$R_k = -HP_k^- H^T + (k_k^T k_k)^{-1} k_k^T P_k^- H^T. \tag{16}$$

In the same way the following relation is obtained:

$$R_{k-1} = -HP_{k-1}^- H^T + (k_{k-1}^T k_{k-1})^{-1} k_{k-1}^T P_{k-1}^- H^T. \tag{17}$$

To conclude, subtracting (17) from (16), then

$$R_k = R_{k-1} - HP_k^- H^T + (k_k^T k_k)^{-1} k_k^T P_k^- H^T$$

$$+ HP_{k-1}^- H^T - (k_{k-1}^T k_{k-1})^{-1} k_{k-1}^T P_{k-1}^- H^T, \tag{18}$$

which can be written in the following way:

$$R_k = R_{k-1} - H\Delta P_k^- H^T + (k_k^T k_k)^{-1} k_k^T P_k^- H^T$$

$$-(k_{k-1}^T k_{k-1})^{-1} k_{k-1}^T P_{k-1}^- H^T,$$

where $\Delta P_k^- = P_k^- - P_{k-1}^-$.

4 Application

4.1 Application within a DC drive

The Kalman filter described in the previous chapter is now used within the DC-drive in a simulation using Matlab/Simulink. Within the DC-drive, the Kalman filter is used to control the velocity of the DC-drive.

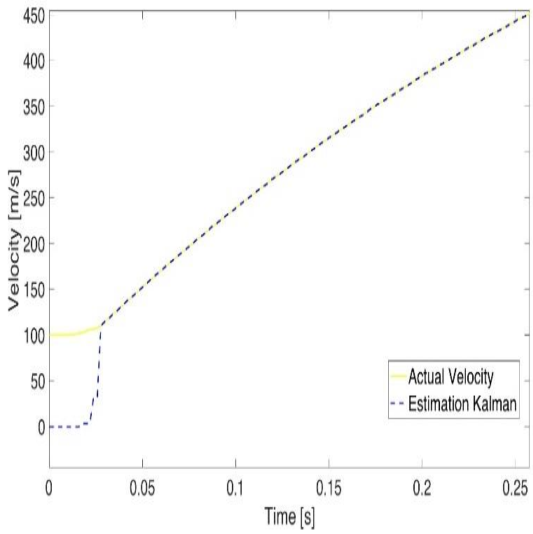

Fig. 3. Actual velocity compared to Kalman estimation

The results of the simulation show a very precise behavior of the Kalman filter: shortly after initializing the filter matches the actual velocity of the DC-drive, see Fig. 3.

4.2 Application within a Kalman filter based sensor

The Kalman based sensor is as well simulated using MatLab/Simulink. In the simulated system the Kalman filter is used to measure the voltage of the system derived from the electric capacity as described in a previous paragraph. The simulated capacitor is based on an existing capacitor model. In the first simulation, the voltage that should have the shape of an ideal cosine wave is estimated by a mathematical derivative, see Fig. 4.

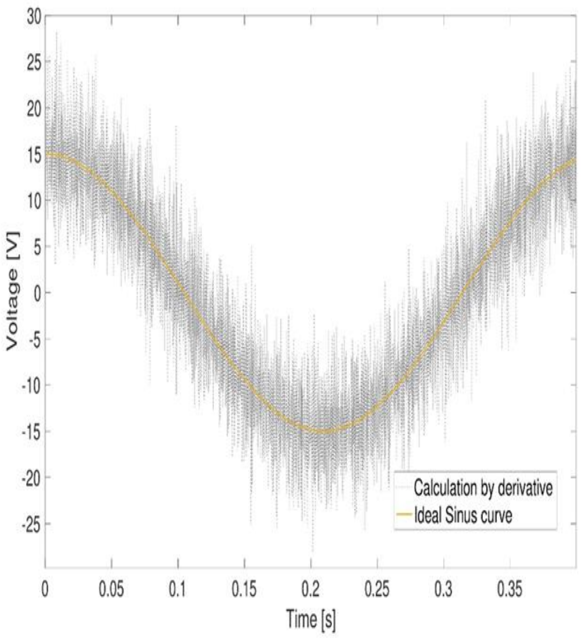

Fig. 4. Ideal Sinus compared to calculation by derivative

Although the estimation generally portrays the shape of a sinus wave, the high mean variation of the estimation is clearly visible. The next step is the use of the actual Kalman filter with dynamic tuning of Q_k and R_k.

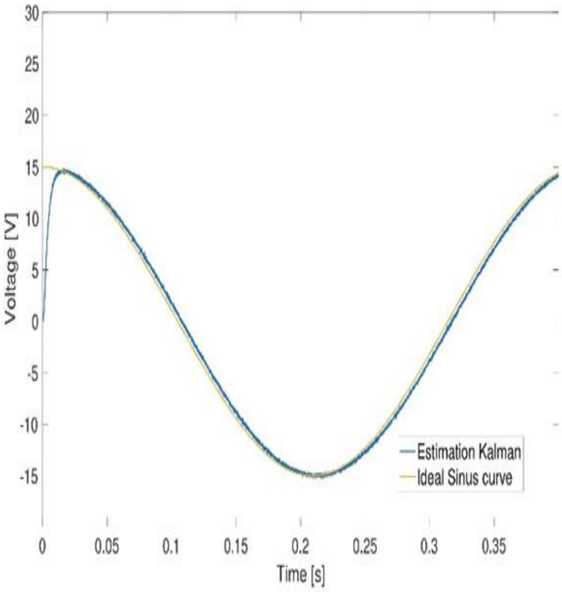

Fig. 5 Ideal Sinus compared to Kalman estimation (Phase shift)

As it is visible, the Kalman filter produces a much more accurate estimation with minimal mean variation from the sinus curve. However, due to the internal processing time of the Kalman filter a slight phase shift becomes visible, see Fig. 5.

5 Conclusion

An approach for tuning Kalman filters in linear cases has been presented. An application concerning a DC-Drive and a sensor has been proposed. The use of this approach is applicable in all linear cases to achieve an acceleration of the convergence of the estimation.

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