Nonlinear Control of a Single-Link Flexible Joint Manipulator via Predictive Control

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Abstract: - In this paper, a NCGPC (Nonlinear Continuous Time Generalized Predictive Control) is proposed for a single-link flexible joint manipulator. This control is developed by using a property of NCGPC, the demonstration that the selection of particular design parameters, such as control order and predictor order leads to well-known feedback linearization. Simulations are presented in order to illustrate the effectiveness of the approach.

Key-Words: - Nonlinear control, Predictive Control, Feedback linearization, flexible joint manipulator.

1 Introduction

A very important feature in modern robot system is the flexibility, which is required in specific applications of industrial automation and space system. Meanwhile in order to increase the control performance, it is necessary to consider flexible joints instead rigid joints, then the mathematical model becomes more complex if the joint flexibility is taken into account. Many controllers have been designed for this kind of manipulator, as examples [1]-[4]. Model based predictive control has recently received much attention from researchers, as a popular control technique in linear and nonlinear systems. The NCGPC, [5, 6] is an alternative nonlinear predictive controller; this controller was developed in a different way than conventional nonlinear predictive controllers. The NCGPC is based in the prediction of the system output and due to the fact that it was not derived with the objective of canceling nonlinearities, as feedback linearization techniques do, the NCGPC has three advantages: First, it can constrain the predicted control through N_u -additionally the response becomes slow and the control is not very active-, and second, when $N_u < N_y - r$, there is not zero dynamics cancellation and then the internal stability is preserved. Also, the NCGPC [7] provides a nice way of handling systems with unstable zero dynamics. And the last advantage is the control weight λ, it plays a very important role in the cost function. In [8], the non-regular nonlinear system is treated by using the last two advantages of the NCGPC. Another of the main advantages of NCGPC control schemes is that, when N_u= N_y -r they do not require on-line optimization and asymptotic tracking of the smooth reference signal is guaranteed. This last advantage will be used in this paper in order to control a single link manipulator with flexible joint Simulations are presented in this paper in order to show the effectiveness of the control strategy.

2 Review of NCGPC

This paper considers the nonlinear SISO systems with all system states assumed to be accessible, affine in the input with the following state-space representation:

$$\dot{x}(t) = f(x) + g(x)u \qquad (1)$$
$$y(t) = h(x)$$

where *f*, *g* and *h* are differentiable N_y times with respect to each argument. $x \in R^n$ is the vector of the state variables, $u \in R$ is the manipulated input and $y \in R$ is the output to be controlled. It has a well-defined relative degree and its zero dynamics are stable.

The development of the NCGPC [5, 6] was carried out following the receding horizon strategy of its linear counterpart [9].

The output prediction is approximated for a Maclaurin series expansion of the system output as follows.

$$y^*(t,T) = y(t) + \dot{y}(t)T + y^{(2)}(t)\frac{T^2}{2!} + \cdots + y^{(N_y)}(t)\frac{T^{N_y}}{N_y!} \qquad (2)$$

or

$$y^*(t,T) = T_{N_y} Y_{N_y} \qquad (3)$$

where

$$Y_{N_y} = [y \quad \dot{y} \quad y^{(2)} \quad \cdots \quad y^{(N_y)}]^T \qquad (4)$$

and

$$T_{N_y} = [1 \quad T \quad \frac{T^2}{2!} \quad \cdots \quad \frac{T^{N_y}}{N_y!}] \tag{5}$$

The predictor order N_y is chosen less than the number of the times that the output has to be differentiated in order to obtain terms not linear in u.

The NCGPC is based in taking the derivatives of the output, which are obtained as follows

$$\begin{aligned}
&\dot{y}(t) = L_f h(x)\\
&y^{(2)}(t) = L_f^2 h(x)\\
&\vdots\\
&y^{(r)}(t) = L_f^r h(x) + L_g L_f^{r-1} h(x) u(t)\\
&y^{(r+1)}(t) = S_1(x) + J_1(x)u(t) + L_g L_f^{r-1} h(x)\dot{u}(t)\\
&y^{(r+2)}(t) = S_2(x) + J_2(x)u(t) + I_1(x)\dot{u}(t) + L_g L_f^{r-1} h(x) u^{(2)}(t)\\
&\vdots\\
&y^{(N_y)}(t) = S_{(N_y-r)}(x) + J_{(N_y-r)}(x)u(t) + I_{(N_y-r)}(x)\dot{u}(t) + I_{(N_y-r+1)}(x)u^{(2)}(t) +\\
&\qquad I_{(N_y-r-1)}(x)u^{(N_y-r-1)}(t) + L_g L_f^{r-1} h(x) u^{(N_y-r)}(t)
\end{aligned} \tag{6}$$

Where $L_f h(x)$ represents the Lie derivative S_i, J_i and I_i, are some functions of x (and not u). These output derivatives are obtained from the system of equation (1) and N_y is chosen less than the number of the times that the output has to be differentiated in order to obtain terms not linear in u, r is the relative degree. Output and its derivatives can be rewritten by

$$Y_{N_y}(t) = O(x(t)) + H(x(t))u_{N_y} \tag{7}$$

where

$Y_{N_y} = [y \quad \dot{y} \quad y^{(2)} \quad \cdots \quad y^{(N_y)}]^T$ and

$u_{N_y} = [u \quad \dot{u} \quad u^{(2)} \quad \cdots \quad u^{(N_y-r)}]^T$

$$O = \begin{bmatrix} y \\ L_f h(x) \\ L_f^2 h(x) \\ \vdots \\ L_f^r h(x) \\ S_1(x) \\ S_2(x) \\ \vdots \\ S_{(N_y-r)}(x) \end{bmatrix} \quad H = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ L_g L_f^{r-1} h(x) & 0 & \cdots & 0 \\ J_1(x) & L_g L_f^{r-1} h(x) & \cdots & 0 \\ J_2(x) & I_1(x) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ J_{N_y-r}(x) & I_{N_y-r}(x) & \cdots & L_g L_f^{r-1} h(x) \end{bmatrix} \tag{8}$$

In order to drive the predicted output along a desired smooth path (reference trajectory) to a set point. Two different reference trajectories are chosen in order to demonstrate the properties of NCGPC. The first reference trajectory is the output of the following reference model [9]

$$W_r(t,s) = \frac{R_n(s)}{R_d(s)} \frac{w(t) - y(t)}{s} \tag{9}$$

Considering the following approximation

$$\frac{R_n(s)}{R_d(s)} \approx \sum_{i=0}^{N_y} r_i s^{-i} \tag{10}$$

The reference trajectory is given by

$$w_r^*(t,T) = [r_0 + r_1 T + r_2 \frac{T^2}{2!} + \cdots + r_r \frac{T^{N_y}}{N_y!}][w - y(t)] + y(t)$$

where w is the set point, or rewriting this equation

$$w_r^*(t,T) = T_{N_y} w_r + y(t) \tag{11}$$

where

$w_r = [r_0 \quad r_1 \quad \cdots \quad r_r]^T (w - y(t))$

and T_{N_y} is given by (5)

The second reference trajectory $y_r(t)$ is the output of the reference model represented by

$$\begin{aligned} \dot{x}(t) &= A_r x_r(t) + B_r w(t) \\ y(t) &= C_r x_r(t) \end{aligned} \tag{12}$$

$x_r \in R^{n_r}, A_r \in R^{n_r \times n_r}, B_r \in R^{n_r \times 1}, C_r \in R^{1 \times n_r}, w \in R$

In order to define the predicted output of the reference trajectory $y_r(t,T)$ a truncated Taylor series is used, obtaining:

$$y_r(t,T) = y_r + \dot{y}_r T + \ddot{y}_r \frac{T^2}{2!} + \cdots + y_r^{N_y} \frac{T^{N_y}}{N_y!},$$

where the derivatives are easy to obtain from the reference model simulation. Rewriting this equation

$$w_r^*(t,T) = T_{N_y} w_r(t) \tag{13}$$

where

$w_r(t) = [y_r, \dot{y}_r, y_r^{(2)}, \ldots, y_r^{(N_y)}]$

and T_{N_y} is given by (5)

NCGPC calculates the future controls from a predicted output over a time frame. The first element *u(t)* of the predicted controls is then applied to the system and the same procedure is repeated at the next time instant. Thus predicted output depend on the input *u(t)* and its derivatives, and the future controls being function of *u(t)* and its N_u-derivatives. The cost function is:

$$J(u_{N_y}) = \int_{T_1}^{T_2} [y^*(t,T) - w_r^*(T,t)]^2 dT \tag{14}$$

with the substitution of equations (7) and (12) or (13) the cost function the minimization results in

$$\boldsymbol{u_{N_y} = K(w_r - O)} \tag{15}$$

where

$$T_y = \int_{T_1}^{T_2} T_{N_y}^T T_{N_y} dT \tag{16}$$

The *ijth* element of T_y is:

$$T_{y_{ij}} = \frac{T_2^{i+j-1} - T_1^{i+j-1}}{(i-1)!(j-1)!(i+j-1)!}$$

and $K=[H^T T_y H]^{-1}[H^T T_y]$ (17)

As explained above, just the first element of u_{Nu} is applied. The control law is given by

$$u(t)=k[w_r - O] \quad (18)$$

3 Geometric Interpretation

In this section the input-output feedback linearization [10] and [11] is shown to be equivalent to NCGPC.

The control law given by equation (18) is analyzed; the matrix H equation (8) is decomposed as

$$H=\begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \quad (19)$$

H_1 is a zero matrix with dimension $r\times(N_y - r+1)$, and H_2 is a lower triangular matrix with dimension $(N_y - r+1)\times(N_y - r+1)$ given by

$$H_2=\begin{bmatrix} L_g L_f^{r-1}h(x) & 0 & \cdots & 0 \\ J_1(x) & L_g L_f^{r-1}h(x) & \cdots & 0 \\ J_2(x) & I_1(x) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ J_{N_y-r}(x) & I_{N_y-r}(x) & \cdots & L_g L_f^{r-1}h(x) \end{bmatrix} \quad (20)$$

The matrix T_y equation (5) is decomposed as

$$T_y=\begin{bmatrix} T_{y_{11}} & T_{y_{12}} \\ T_{y_{21}} & T_{y_{22}} \end{bmatrix} \quad (21)$$

Where

$T_{y_{11}}$ is $r\times r$

$T_{y_{12}}$ is $r\times(N_y - r+1)$

$T_{y_{21}}$ is $(N_y - r+1)\times r$

$T_{y_{22}}$ is $(N_y - r+1)\times(N_y - r+1)$

The equation (17) can be written as

$$K=H_2^{-1}[T_{y_{22}}^{-1}T_{y_{21}} I] \quad (22)$$

The unitary matrix I has dimension $(N_y - r+1)\times(N_y - r+1)$. The first row of the inverse of H_2 is given by

$$h_2^{-1}=\left[1/L_g L_f^{r-1}h(x) \quad 0 \quad \ldots \quad 0\right] \quad (23)$$

Then, the first row of K, which will be called k

$$k=\frac{1}{L_g L_f^{r-1}h(x)}\left[t_1 \quad t_2 \quad \ldots \quad t_r \quad 1 \quad 0 \quad \ldots \quad 0\right] \quad (24)$$

where $t_1, t_2, t_3, \ldots, t_r$ are elements of the first row of $T_{y_{22}}^{-1}T_{y_{21}}$. If the reference trajectory is chosen as the equation (11), the control law is given by

$$u(t)=\frac{(t_1 r_0+\ldots+t_e r_{r-1})(w-y(t))-\sum_{i=1}^{r-1} t_{i+1}L_f^i h(x)-L_f^r h(x)}{L_g L_f^{r-1}h(x)} \quad (25)$$

$$u(t)=\frac{(w-y(t))-\sum_{i=1}^{r}\beta_i L_f^i h(x)}{\beta_r L_g L_f^{r-1}h(x)} \quad (26)$$

where

$$\beta_r = 1/(t_1 r_0 + t_2 r_1+\cdots+t_r r_{r-1})$$
$$\beta_i = t_{i+1}/(t_1 r_0 + t_2 r_1+\cdots+t_r r_{r-1}) \qquad i=1,2\ldots,r-1 \quad (27)$$

We can notice, that large N_y does not require a bigger computational effort, because as we can see from equation (26), the control depends just on the r-first derivatives, thus the rest of the derivatives only have influence in obtaining the parameters of t_i , which just depends on T. Moreover, N_y can be chosen as the smallest predictor order, which is such that the predicted output depends on $u(t)$. Because of this, the relative degree r will be the smallest predictor order N_y. We can conclude if $N_u= N_y -r$, the control law is independent of the last N_y $-r$ derivatives. Then it is possible to calculate the parameters β_i considering the largest N_y without the use of the remaining derivatives. Substituting equation (26) into the rth derivative given by equation (6) leads to:

$$y^r(t)=\frac{1}{\beta_r}(w-y)-\sum_{i=1}^{r-1}\frac{\beta_i}{\beta_r}y^i \quad (28)$$

Rearranging and taking Laplace transforms, the resulting closed-loop transfer function is given by:

$$Y(s)=G(s)W(s)$$

$$G(s)=\frac{1}{\beta_r s^r+\beta_{r-1}s^{r-1}+\ldots+\beta_1 s+1} \quad (29)$$

Note that, by using the Routh-Hurwitz criterion, we can show that the systems are stable only for systems with $r\leq 4$.

If the reference trajectory is chosen as the equation (12), following the same procedure the control law is given by

$$u(t)=\frac{\sum_{i=0}^{r} t_{i+1}\left[y_r^i L_f^i h(x)\right]-L_f^i h(x)+y_r^r}{L_g L_f^{r-1}h(x)} \quad (30)$$

We can see that the control law is identical to the control law presented by Isidori [10], which solves the problem known as asymptotic model matching.

4 Control law of a Single-Link Flexible Joint Manipulator

In this section the predictive control (30) of the single-link flexible joint manipulator is obtained, the derivatives are required for this controller, which are obtained of the single-link flexible joint manipulator model. The single-link flexible joint manipulator is shown in Fig. 1, which has a difference between the angular position of the motor and that of the driven link, i.e. joint flexibility exists. The mathematical model is given by [4].

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{MgL}{I}\sin(x_1) - \frac{K}{I}(x_1 - x_3) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{K}{J}(x_1 - x_3) + \frac{1}{J}u \\
y &= x_1
\end{aligned} \tag{31}$$

where

I Inertia of flexible manipulator *0.03 kgm²*

J Inertia of rotational platform *0.004 kgm²*

g Gravitational acceleration *9.81 N/m*

L Distance to center of gravity of rotational platform *0.135m*

M Mass of the flexible joint *0.6 Kg*

k Flexibility coefficient joint *31.0 Nm/rad*

The values are considered from [4], the output is the link angular displacement x_1 and the control u is the torque given by the motor.

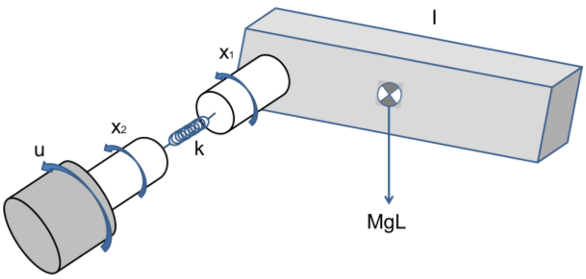

Figure 1. Single-link flexible joint manipulator

The reference trajectory $y_r(t)$ is the output of the reference model represented by

$$\begin{aligned}
\dot{y}_r &= y_{r1} \\
\dot{y}_{r1} &= y_{r2} \\
\dot{y}_{r2} &= y_{r3} \\
\dot{y}_{r3} &= -16y_{r3} - 96y_{r2} - 256y_{r1} - 256y_r + 256w
\end{aligned} \tag{32}$$

The derivatives are obtained as follows

$$\begin{aligned}
h(x) &= x_1 \\
L_f h(x) &= x_2 \\
L_f^2 h(x) &= -\frac{MgL}{I}\sin(x_1) - \frac{K}{I}(x_1 - x_3) \\
L_f^3 h(x) &= -\frac{MgL}{I}x_2\cos(x_1) - \frac{K}{I}(x_2 - x_4) \\
L_f^4 h(x) &= \frac{MgL}{I}\sin(x_1)\left[x_2^{\,2} + \frac{MgL}{I}x_2\cos(x_1) + \frac{K}{I}\right] + \\
&\quad \frac{K}{I}(x_1 - x_3)\left[\frac{K}{J} + \frac{K}{I} + \frac{MgL}{I}x_2\cos(x_1)\right] \\
L_g L_f^3 h(x) &= \frac{K}{JI}
\end{aligned} \tag{33}$$

5 Simulations

In order to show the effectiveness of the proposed controller simulation will be presented. In this simulation design parameters are chosen as: setpoint equal $\pi/2$, $r=4$, $N_y=4$, $N_u=N_y-r=0$, $T_2=1$ and $T_1=0$. The output system and the reference are shown Fig. 2, Fig.3 show the error tracking and Fig. 4 the systems states.

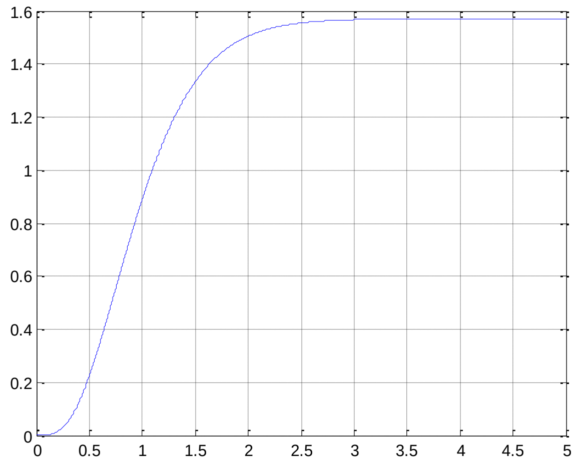

Figure 2. System Output and reference trajectory

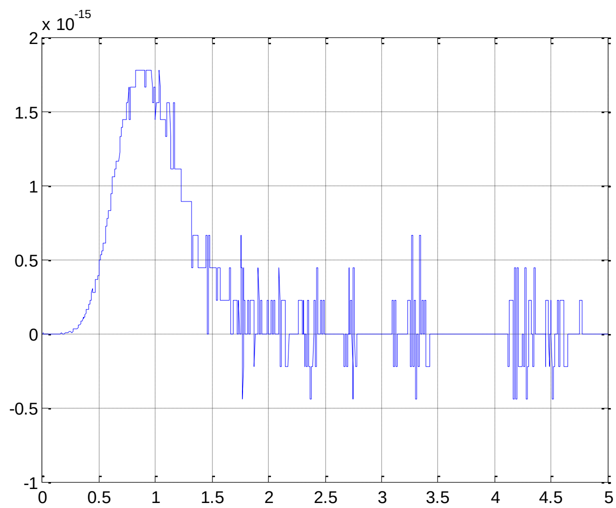

Figure 3. System Tracking error

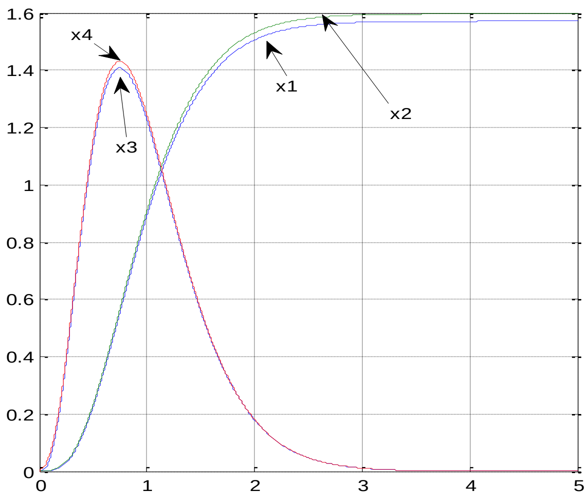

Figure 4. Systems states

4 Conclusion

The selection of particular design parameters NCGPC (Nonlinear Continuous Time Generalized Predictive Control), such as control order and predictor order leads to well-known feedback linearization. The response of closed loop is influenced by the prediction horizon and the reference model. Simulations are presented in order to demonstrate the effectiveness of NCGPC.

Another of the main advantages of NCGPC control schemes is that, when $N_u= N_y$ $-r$ they do not require on-line optimization and asymptotic tracking of the smooth reference signal is guaranteed. We can show, that incredible as it may seem, large N_y does not require a bigger computational effort, because the control depends just on the r-first derivatives, thus the rest of the derivatives only have influence in obtaining the parameters of t_i , which just depends on T. Then it is possible to calculate the parameters β_i considering the largest $N_{y.}$, without the use of the remaining derivatives. Additionally, it is shown that the control law is another feedback linearization, thus closed-loop stability is ensured.

A closed-loop transfer function was found, it is possible to infer that, by using the Routh-Horwitz criterion, systems are stable only for systems with $r\leq 4$.

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